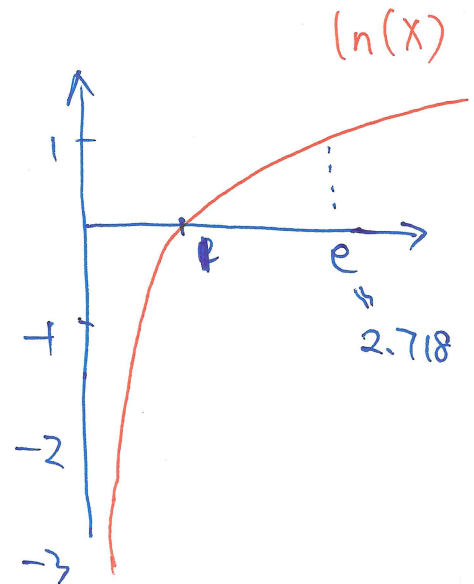# Log Sum Exp Trick 2

$$\log(xy) = \log(x) + \log(y)$$

$$\ln(0.66666) = -0.4055$$
$$\ln(0.00021) = -8.4684$$

$\ln(x)$

2.718

$$P(\text{spam}|w) = \frac{e^{-76.0}}{e^{-76.0} + e^{-80.5}} = \frac{e^{-76.0}}{e^{-76.0 + \ln(1+e^{-4.5})}} = \frac{e^{-76.0}}{e^{-76.0} \cdot e^{\ln(1+e^{-4.5})}}$$

$$= \frac{e^{-76.0}}{e^{-76.0} \cdot (1 + e^{-4.5})}$$

$$= \frac{1}{1 + e^{-4.5}}$$

$$\ln(e^{-76.0} + e^{-80.5})$$
$$= \ln(e^{-76.0} \cdot (1 + e^{-4.5}))$$
$$= \ln(e^{-76.0}) + \ln(1 + e^{-4.5})$$
$$= -76.0 + \ln(1 + e^{-4.5})$$

$$= 0.989$$

Log-Sum-Exp Trick 1

$a_1 = 3.96 \times 10^{-107}$ $k_1 = \log(a_1) = -245$

$a_2 = 1.80 \times 10^{-111}$ $k_2 = \log(a_2) = -255$

compute

$a_1 + a_2 = ?$

$M = \max(k_1, k_2) = -245$

$$\log(a_1 + a_2)$$

$$= \log(e^{k_1} + e^{k_2})$$

$$= \log(e^{M} \cdot (e^{k_1 - M} + e^{k_2 - M}))$$

$$= \log e^{M} + \log(e^{k_1 - M} + e^{k_2 - M})$$

$$= M + \log(e^{0} + e^{-10})$$

$$= -245 + \log(e^{0} + e^{-10})$$