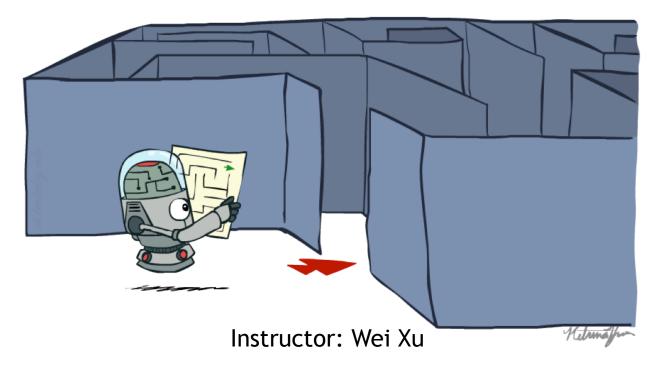
# CS 5522: Artificial Intelligence II

# Uninformed Search Algorithms and Their Computational Complexity



**Ohio State University** 

[These slides were adapted from CS188 Intro to AI at UC Berkeley.]

#### Pac-Man and Al

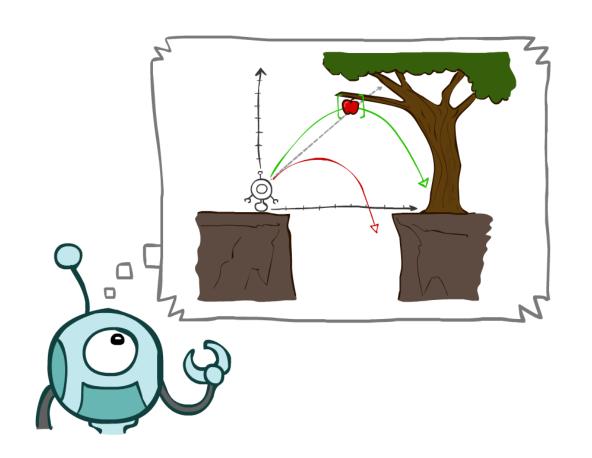
https://www.youtube.com/watch?v=w5kFmdkrluY

https://www.youtube.com/watch?v=zQyWMHFjewU

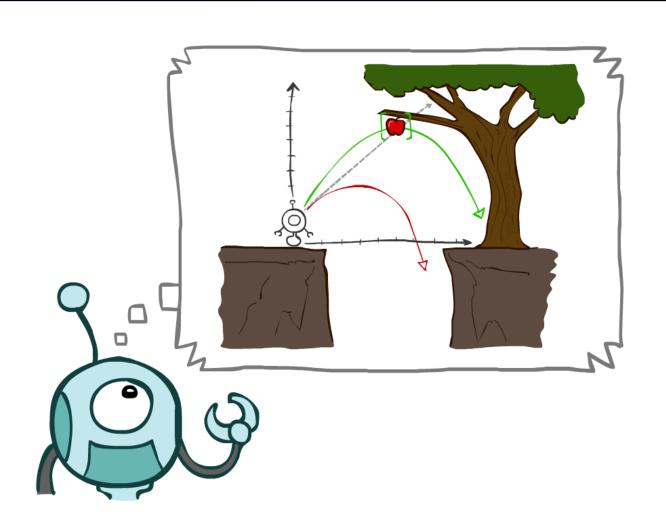
# Today

- Agents that Plan Ahead
- Search Problems

- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

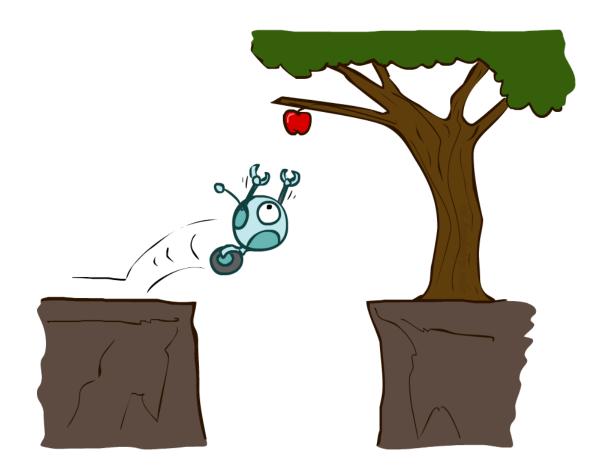


# Agents that Plan



#### Reflex Agents

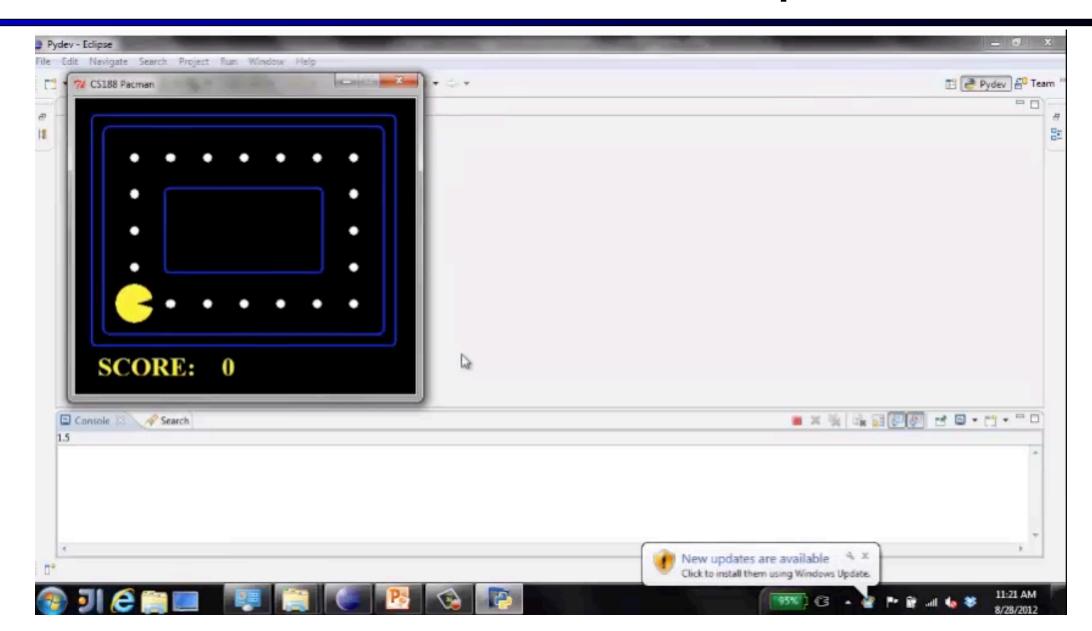
- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS
- Can a reflex agent be rational?



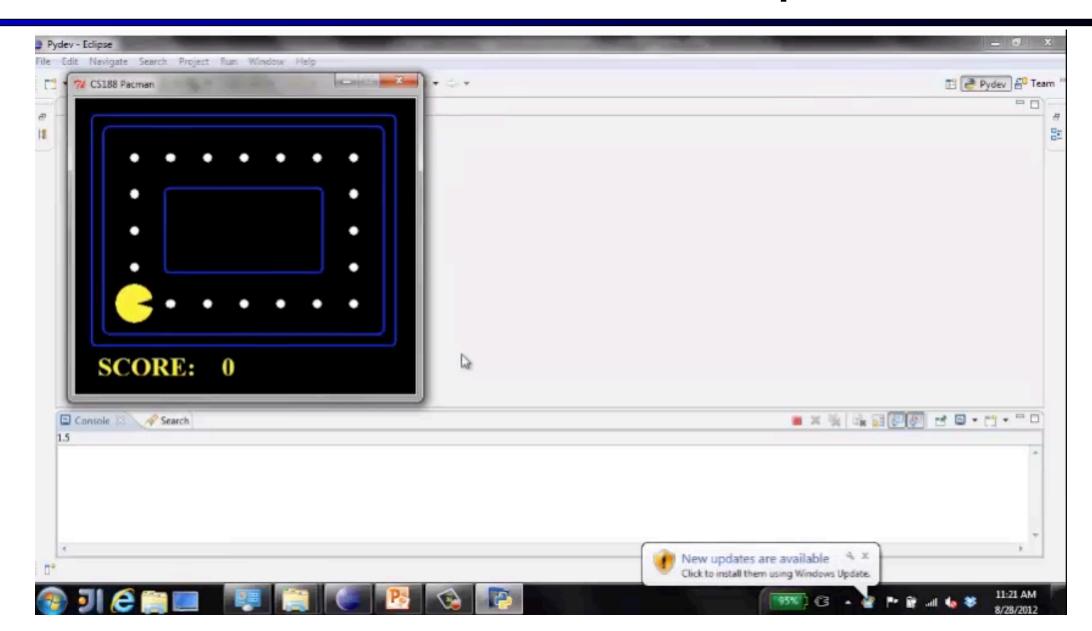
[Demo: reflex optimal (L2D1)]

[Demo: reflex optimal (L2D2)]

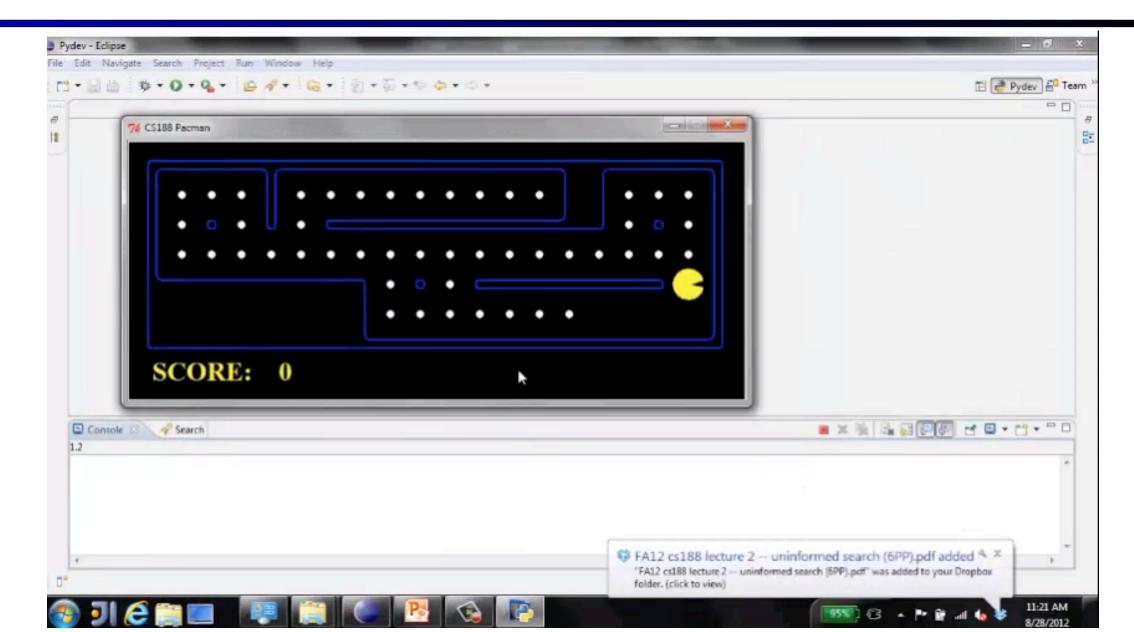
# Video of Demo Reflex Optimal



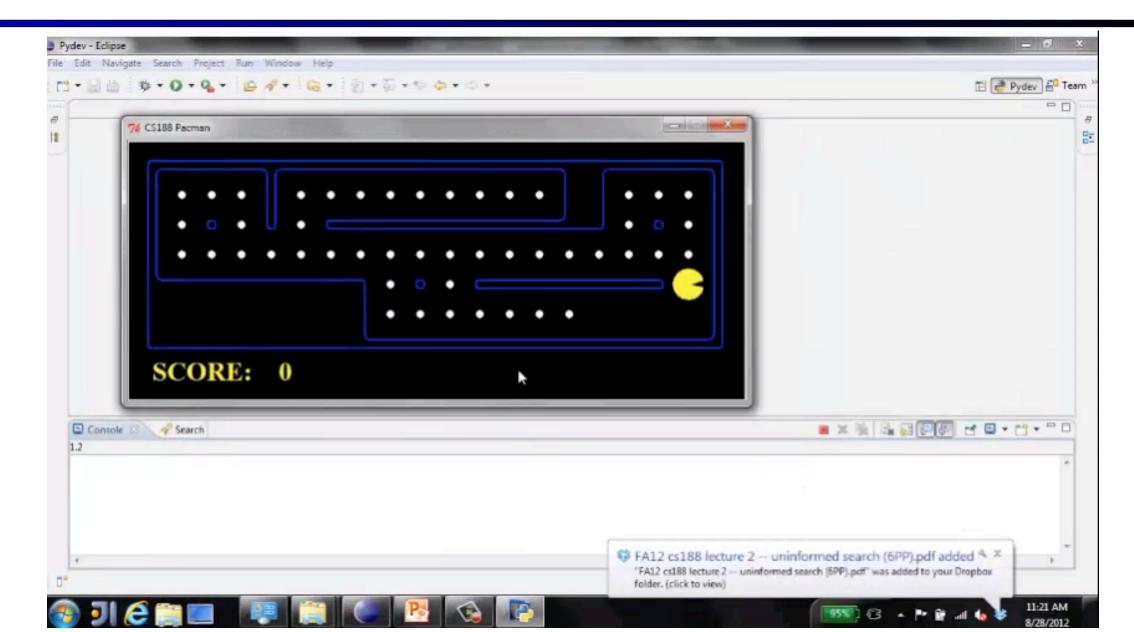
# Video of Demo Reflex Optimal



#### Video of Demo Reflex Odd



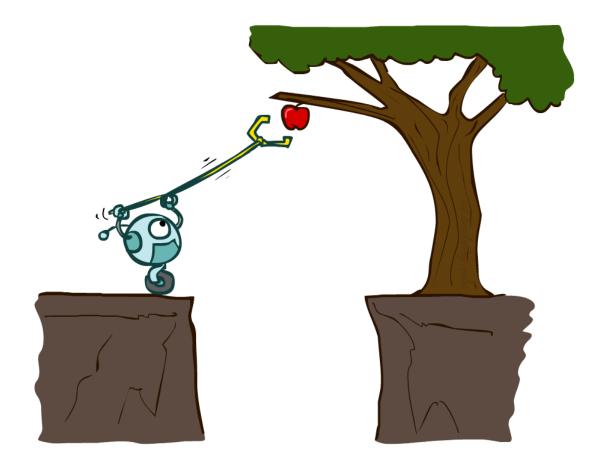
#### Video of Demo Reflex Odd



#### Planning Agents

#### Planning agents:

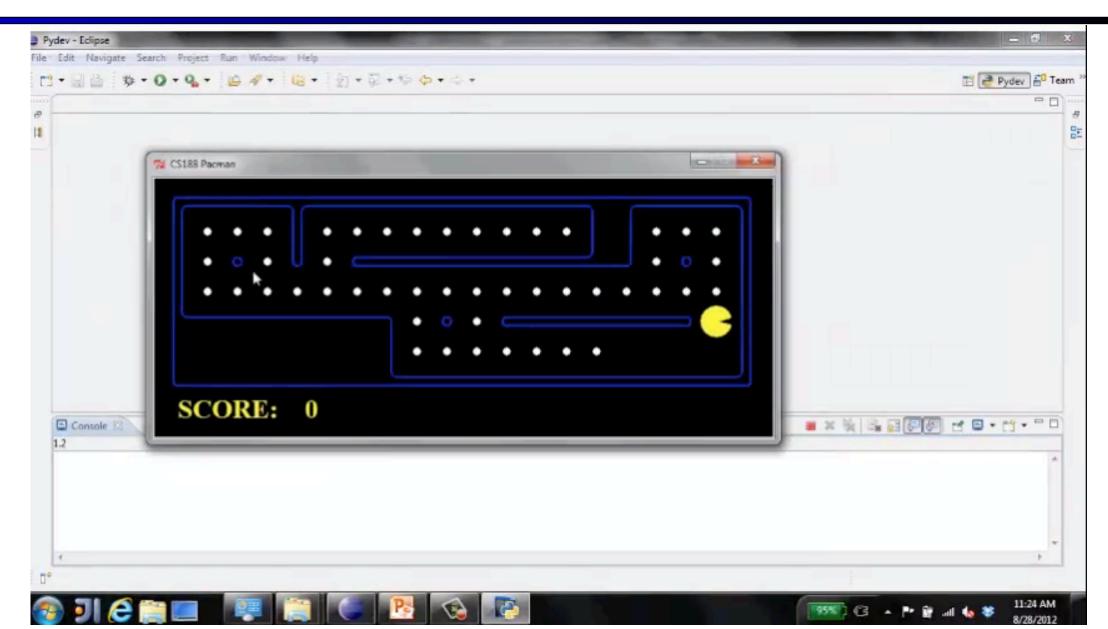
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Must formulate a goal (test)
- Consider how the world WOULD BE
- Optimal vs. complete planning
- Planning vs. replanning



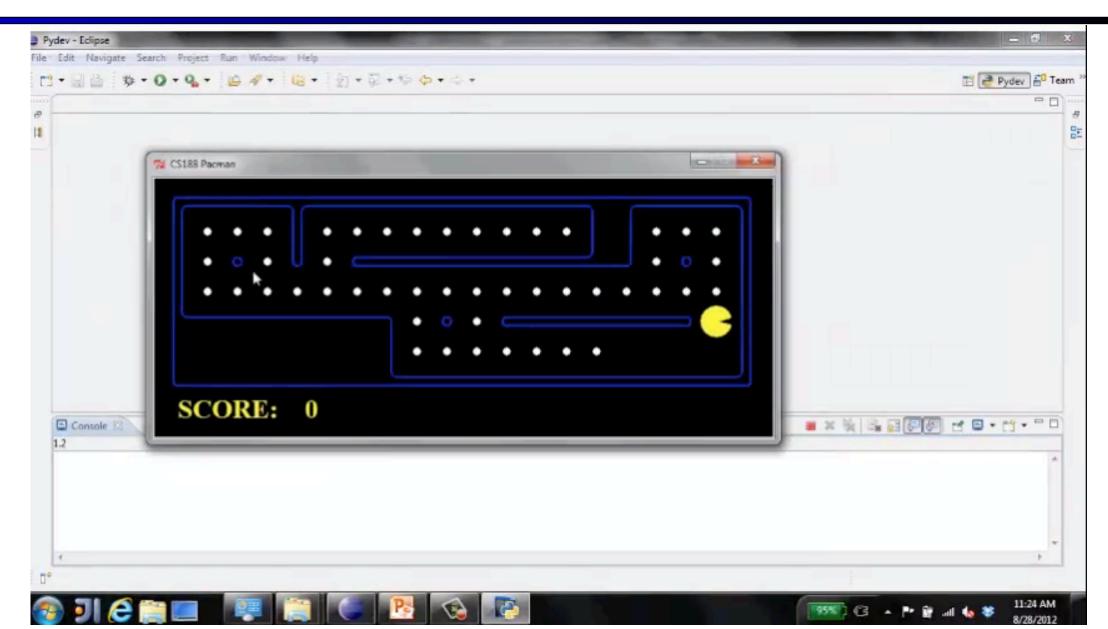
[Demo: replanning (L2D3)]

[Demo: mastermind (L2D4)]

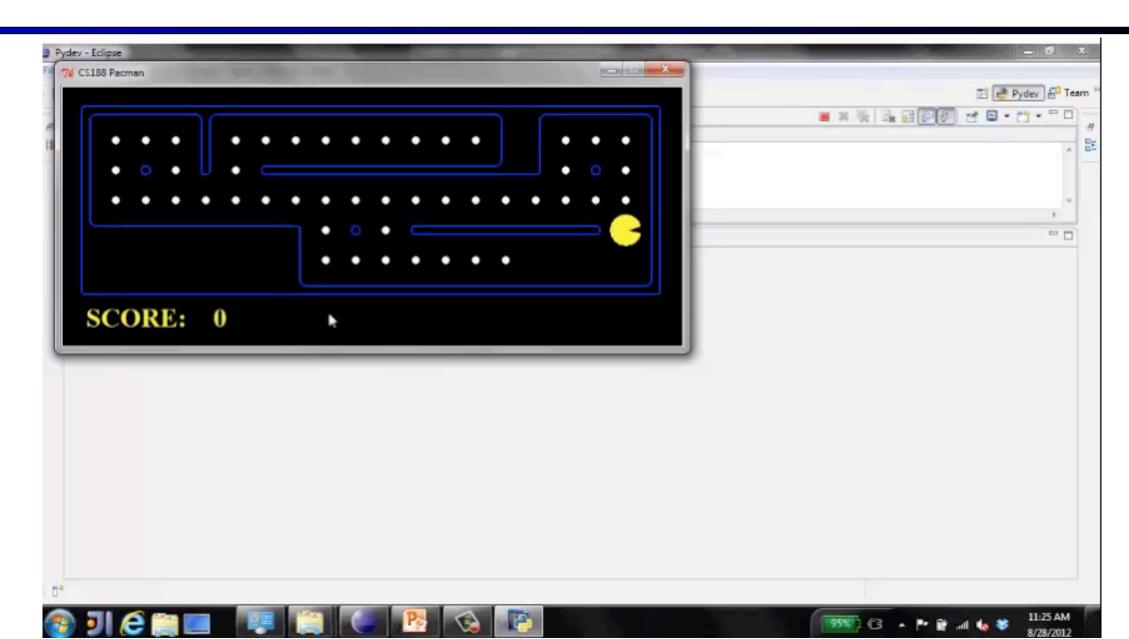
# Video of Demo Replanning



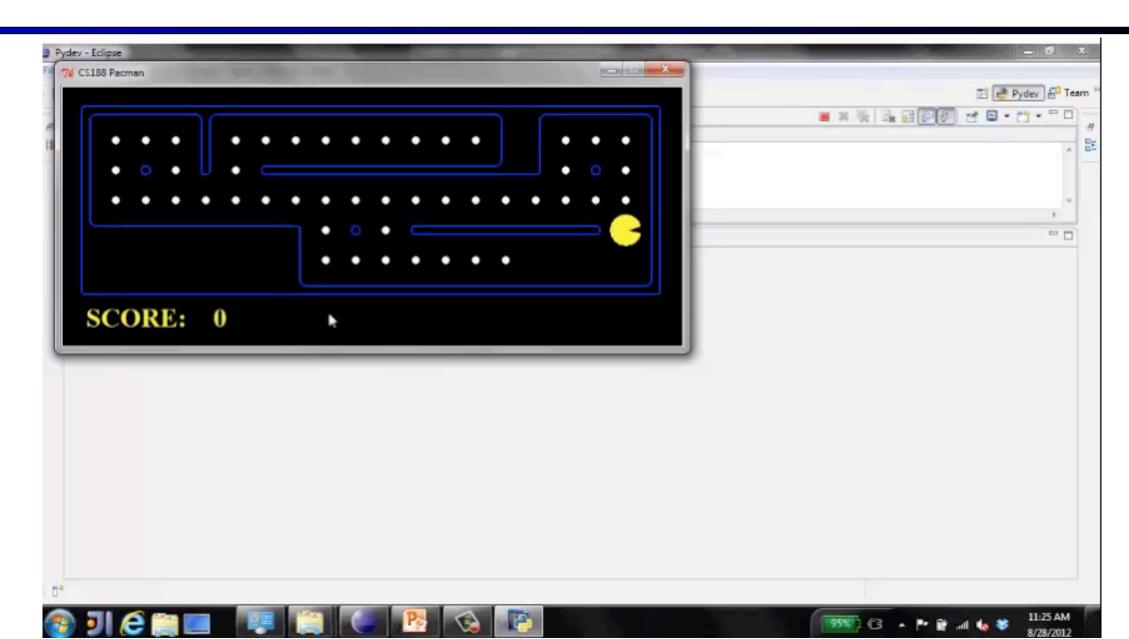
# Video of Demo Replanning



#### Video of Demo Mastermind



#### Video of Demo Mastermind





A search problem consists of:

- A search problem consists of:
  - A state space













- A search problem consists of:
  - A state space











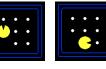


A successor function (with actions, costs)

- A search problem consists of:
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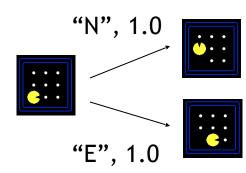








 A successor function (with actions, costs)



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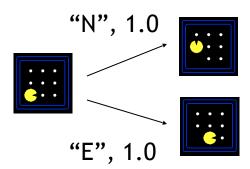








A successor function (with actions, costs)



A start state and a goal test

- A search problem consists of:
  - A state space





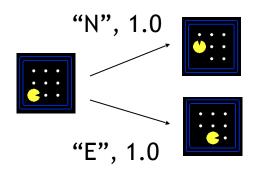








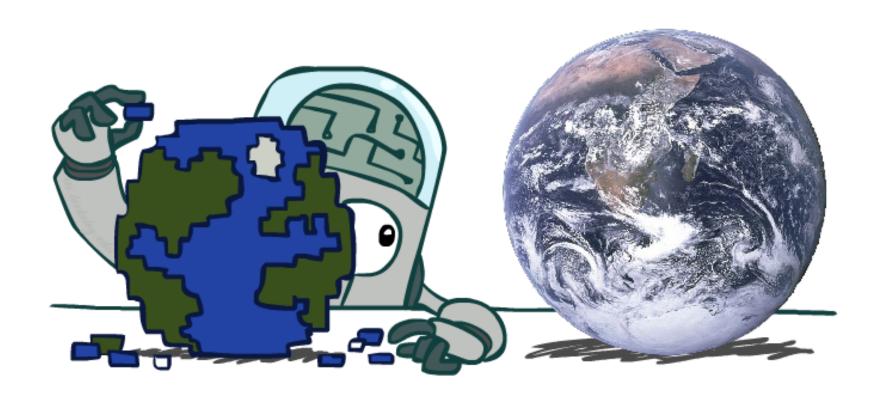
A successor function (with actions, costs)

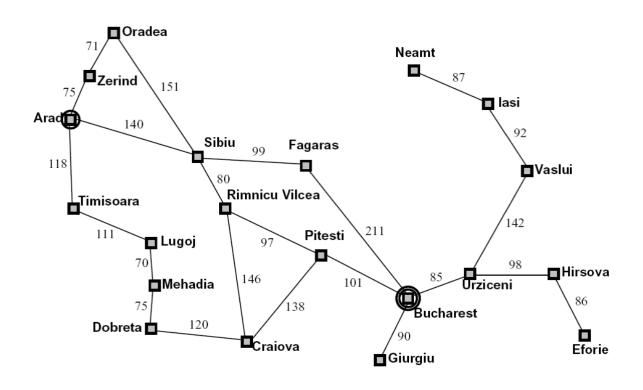


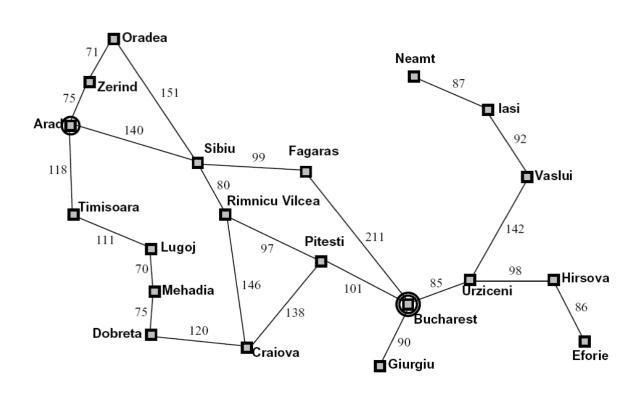
- A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

#### Search Problems Are Models

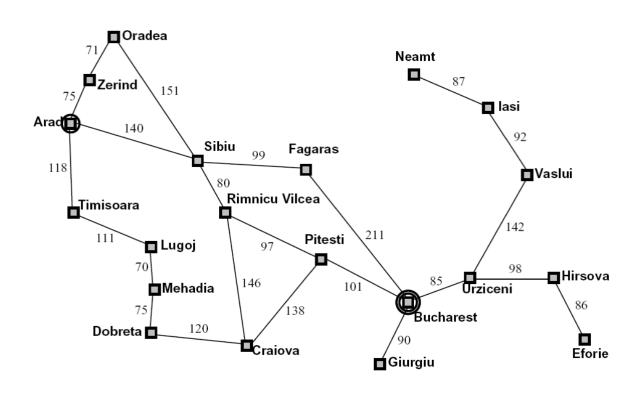
#### Search Problems Are Models



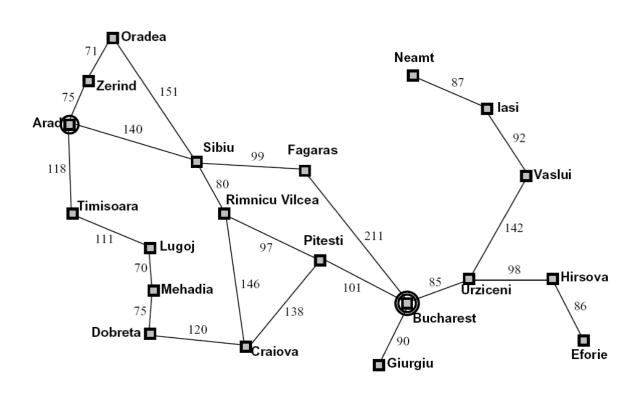




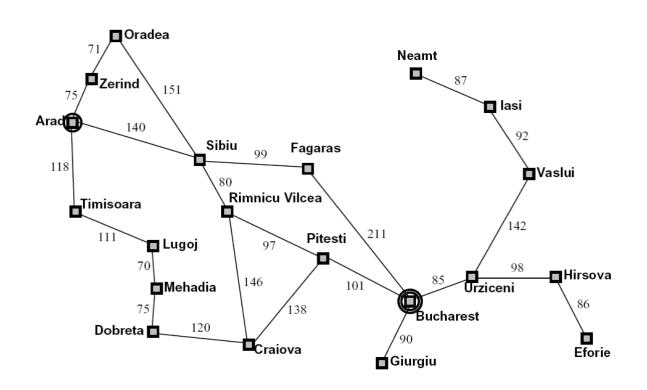
- State space:
  - Cities



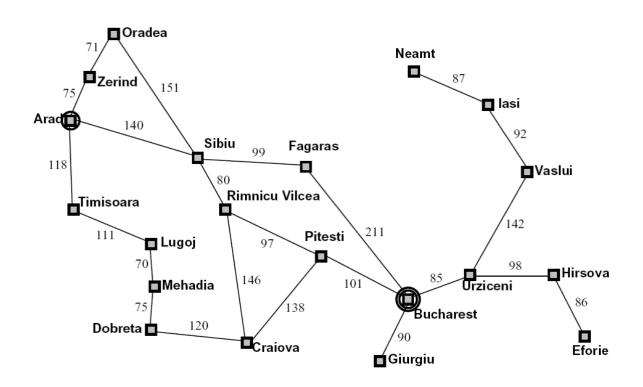
- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad

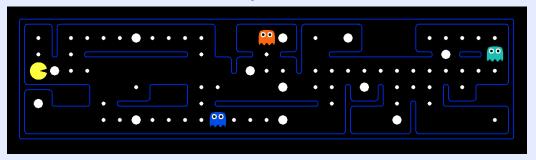


- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?

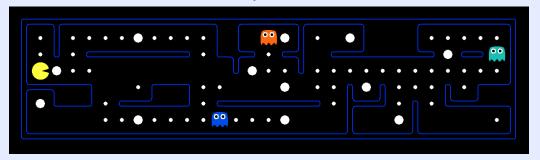


- State space:
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- Successor function:
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- Goal test:
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- Solution?

The world state includes every last detail of the environment

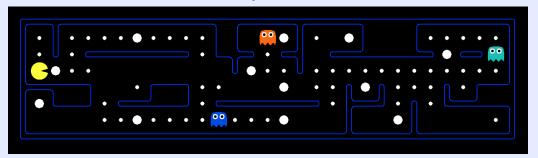


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A search state keeps only the details needed for planning (abstraction)

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Problem: Pathing

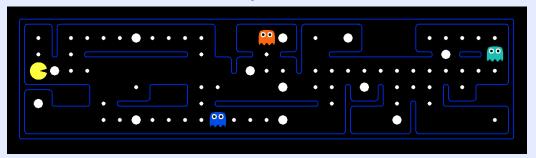
States: (x,y) location

Actions: NSEW

Successor: update location only

■ Goal test: is (x,y)=END

The world state includes every last detail of the environment



A search state keeps only the details needed for planning (abstraction)

- Problem: Pathing
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is (x,y)=END

- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}
  - Actions: NSEW
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false

# State Space Sizes?

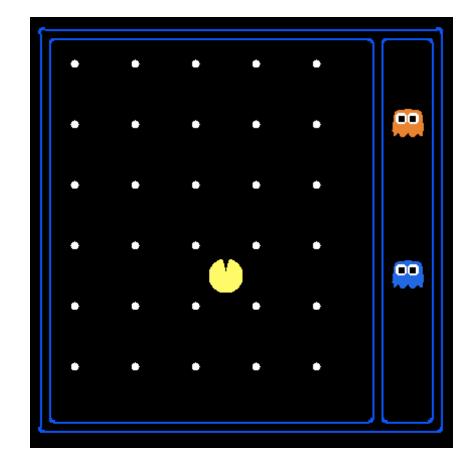
#### World state:

Agent positions: 120

• Food count: 30

Ghost positions: 12

Agent facing: NSEW



#### State Space Sizes?

#### World state:

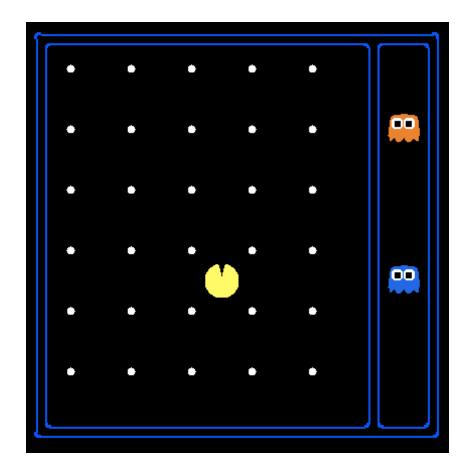
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# State Space Sizes?

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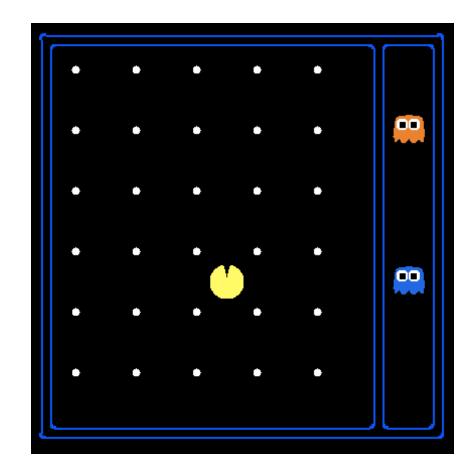
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#### How many

World states?



#### World state:

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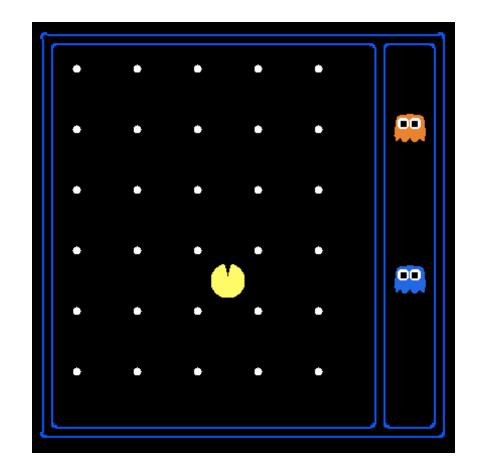
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#### How many

World states?120x(2<sup>30</sup>)x(12<sup>2</sup>)x4



#### World state:

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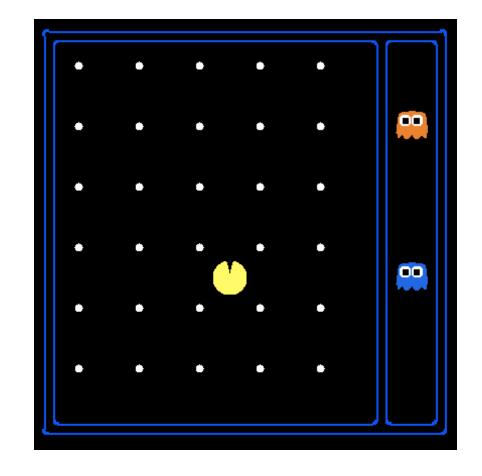
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#### World state:

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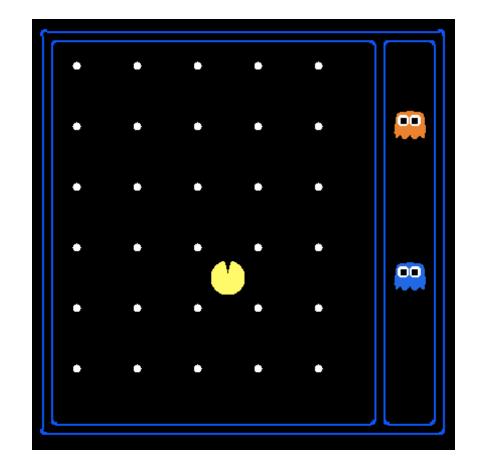
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#### World state:

Agent positions: 120

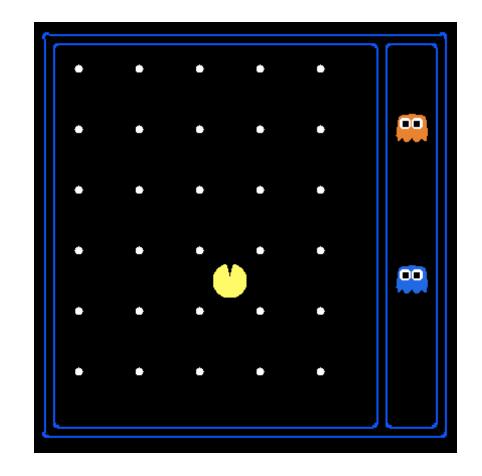
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- States for eat-all-dots?



#### World state:

Agent positions: 120

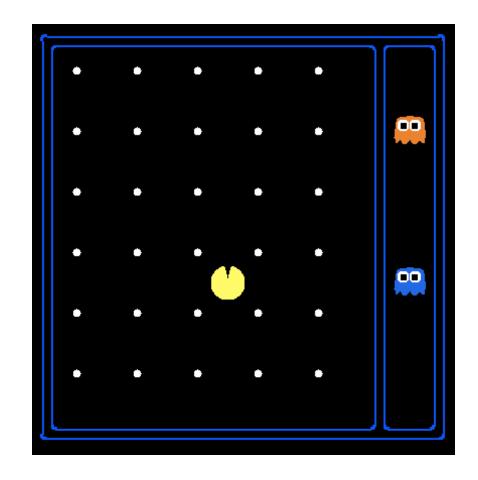
• Food count: 30

Ghost positions: 12

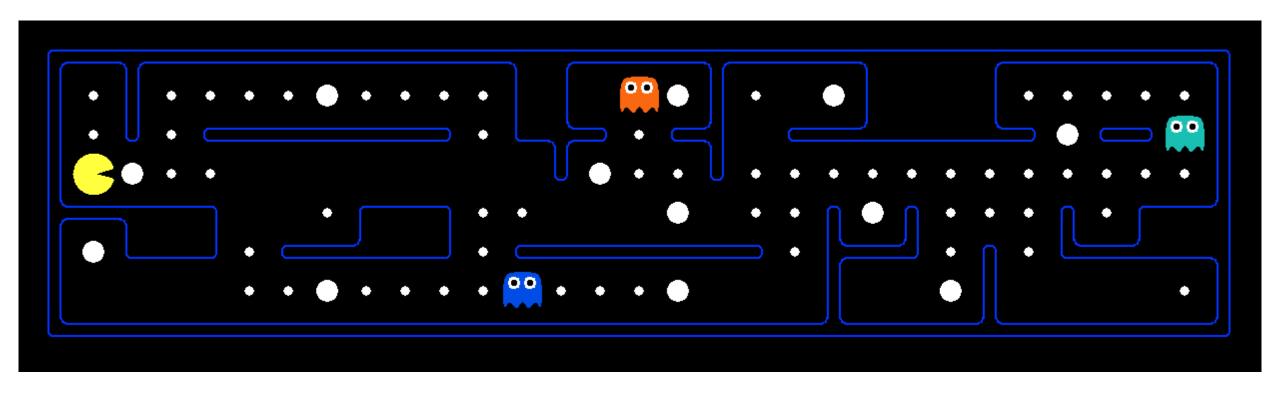
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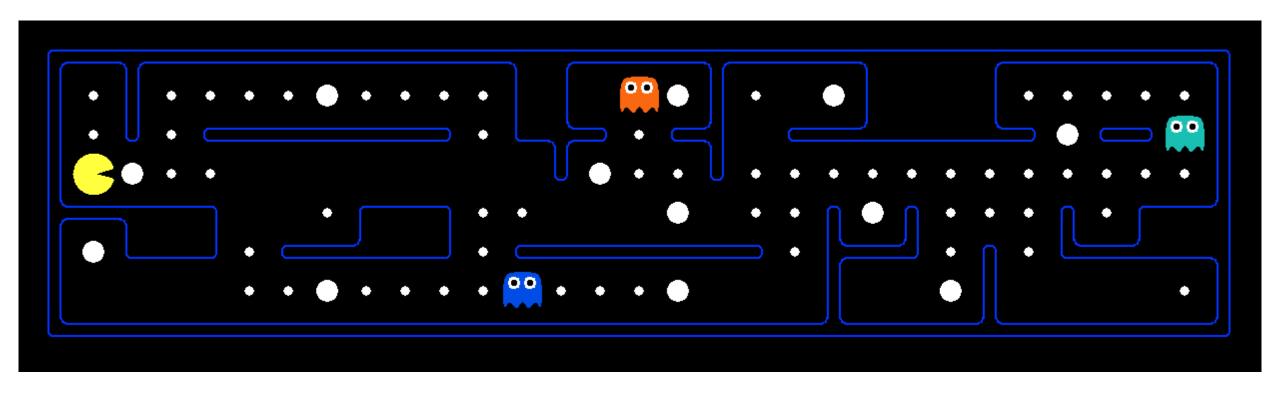


## Quiz: Safe Passage



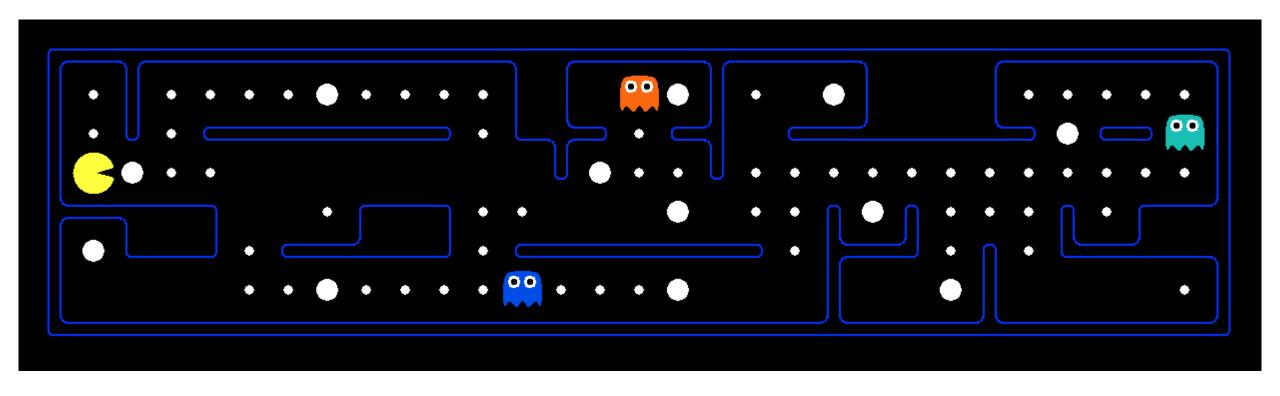
Problem: eat all dots while keeping the ghosts perma-scared

#### Quiz: Safe Passage



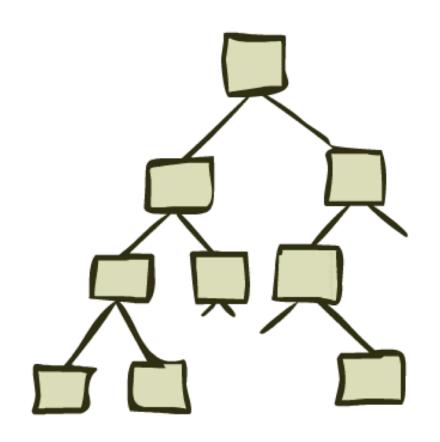
- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?

#### Quiz: Safe Passage



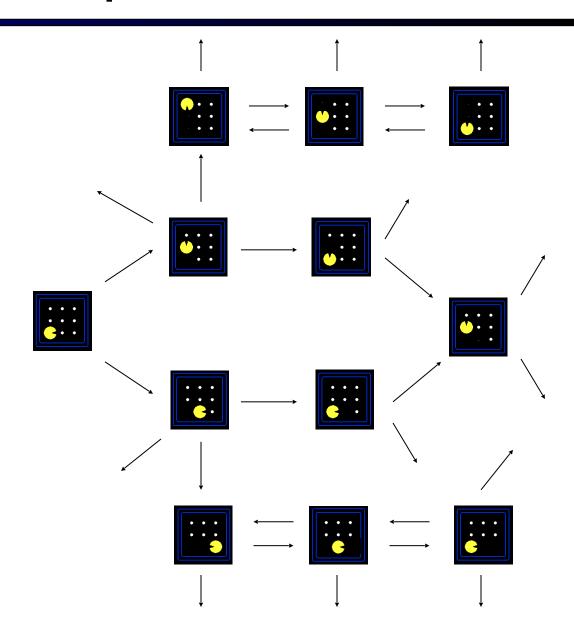
- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)

# State Space Graphs and Search Trees

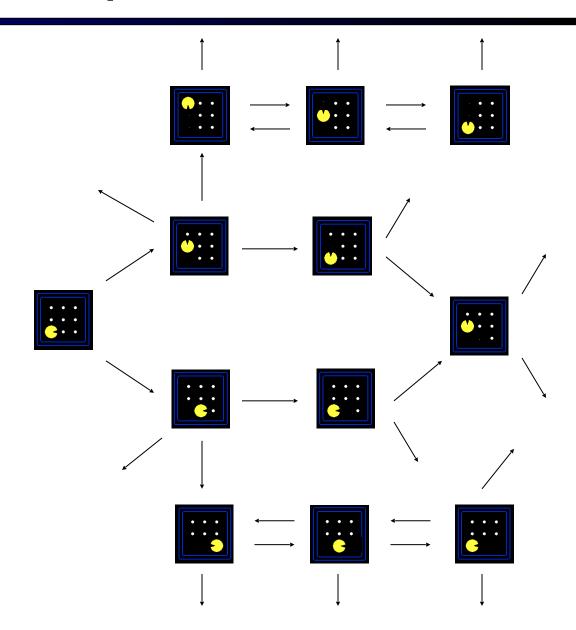


 State space graph: A mathematical representation of a search problem

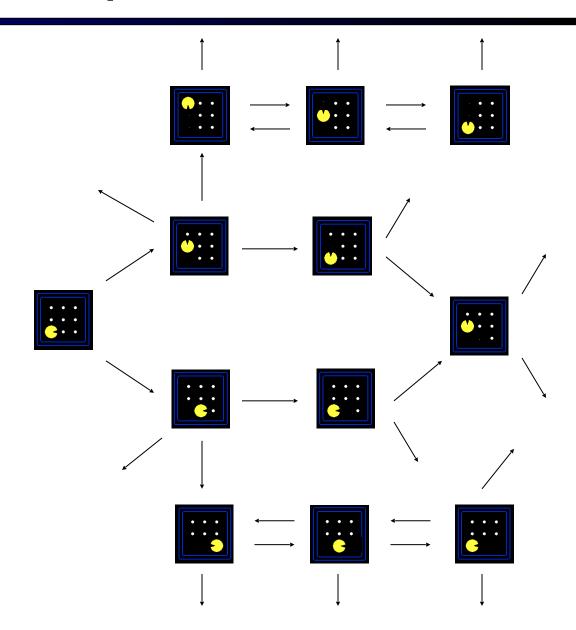
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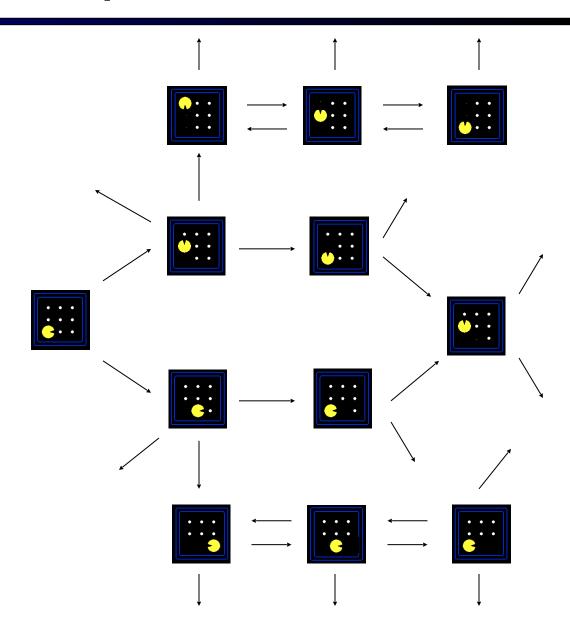
- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations



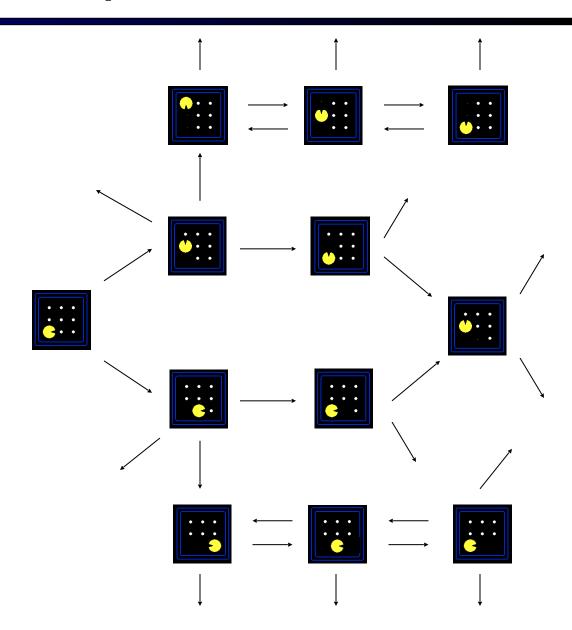
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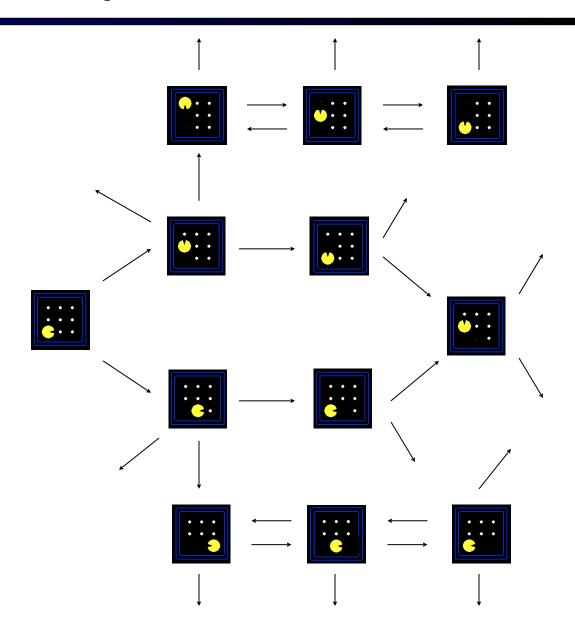
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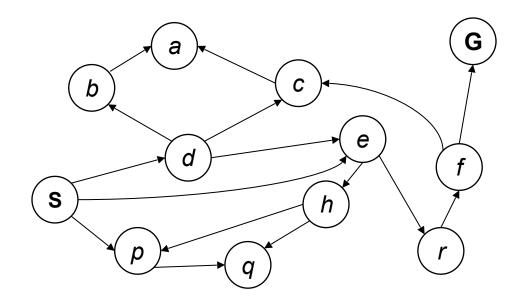
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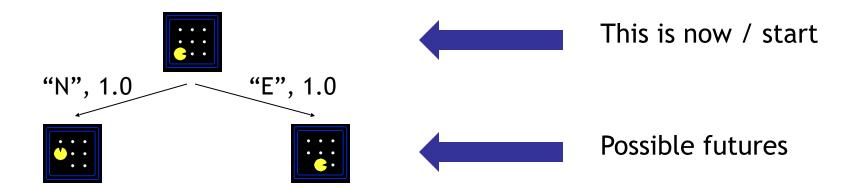


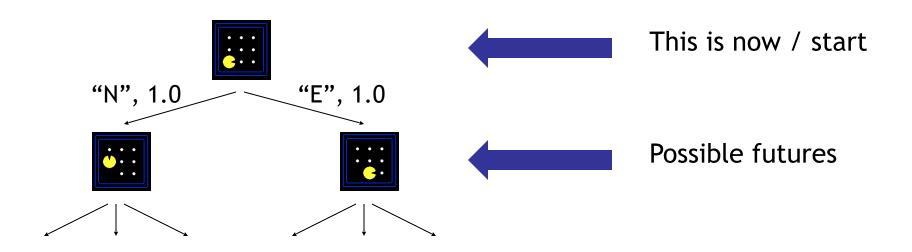
Tiny search graph for a tiny search problem

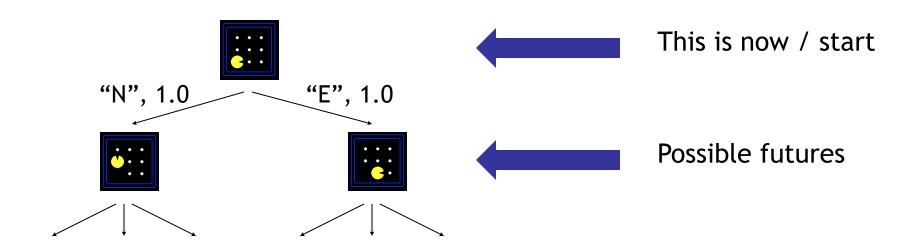




This is now / start

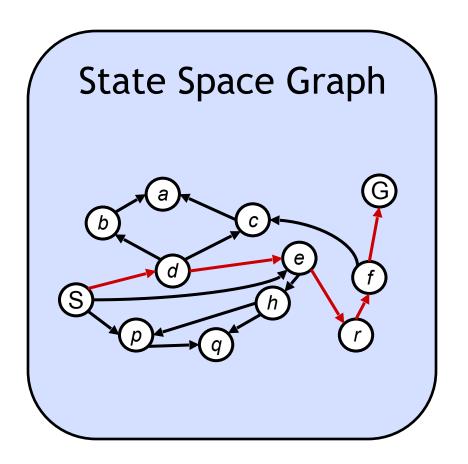






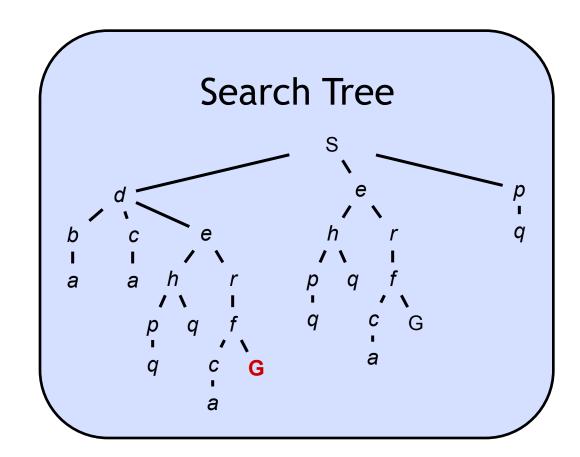
#### A search tree:

- A "what if" tree of plans and their outcomes
- The start state is the root node
- Children correspond to successors
- Nodes show states, but correspond to PLANS that achieve those states
- For most problems, we can never actually build the whole tree

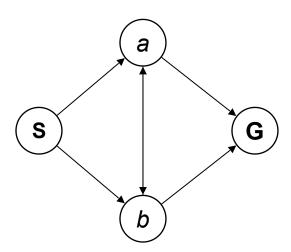


Each NODE in in the search tree is an entire PATH in the state space graph.

We construct both on demand - and we construct as little as possible.

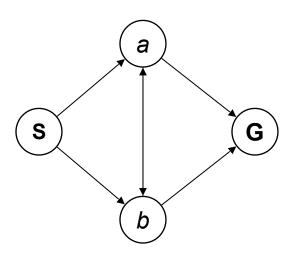


Consider this 4-state graph:



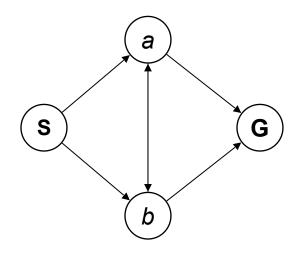
Consider this 4-state graph:

How big is its search tree (from S)?



Consider this 4-state graph:

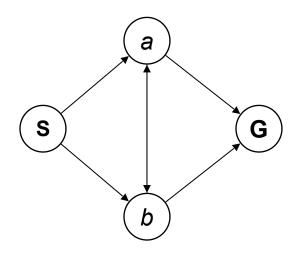
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Consider this 4-state graph:

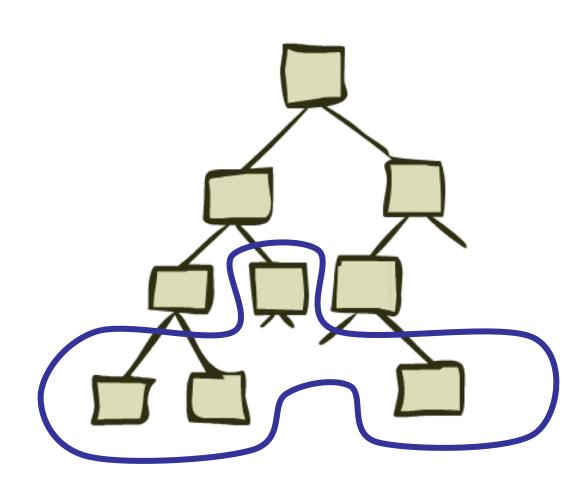
How big is its search tree (from S)?



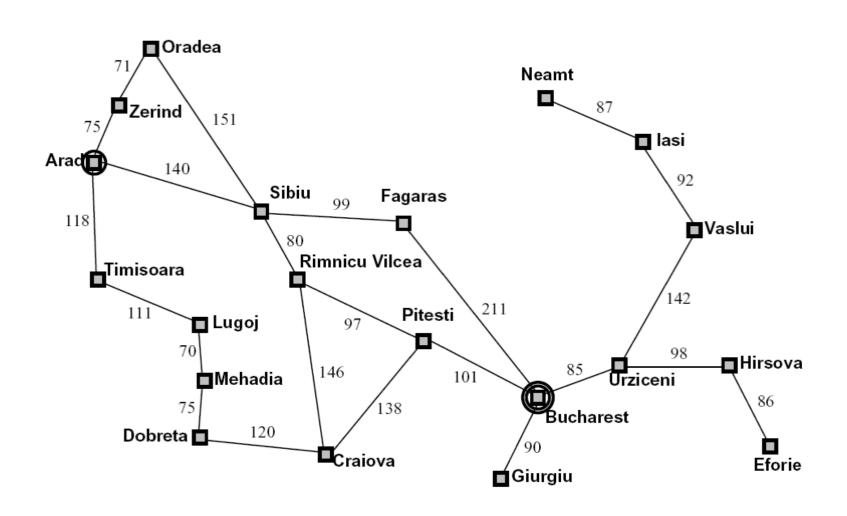


Important: Lots of repeated structure in the search tree!

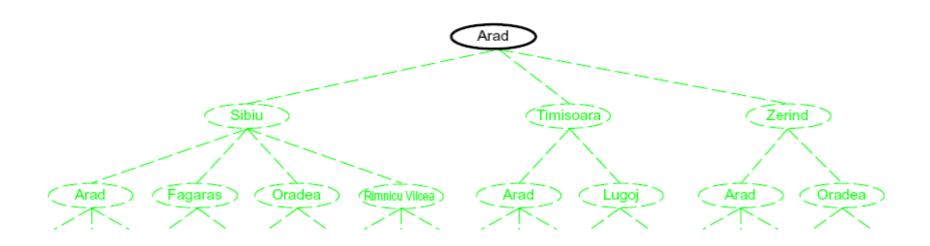
### Tree Search



# Search Example: Romania



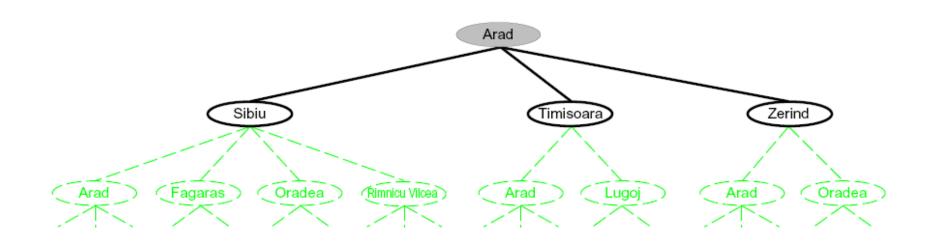
#### Searching with a Search Tree



#### Search:

- Expand out potential plans (tree nodes)
- Maintain a fringe of partial plans under consideration
- Try to expand as few tree nodes as possible

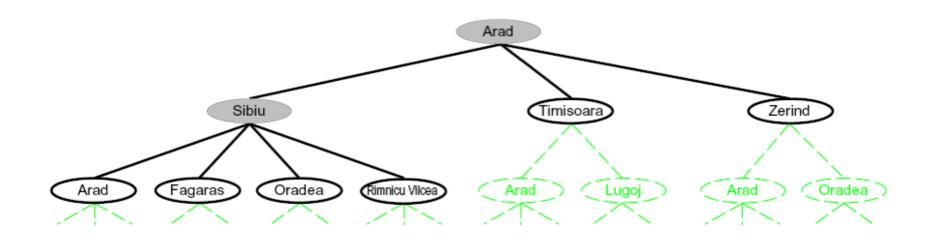
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#### Searching with a Search Tree



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#### General Tree Search

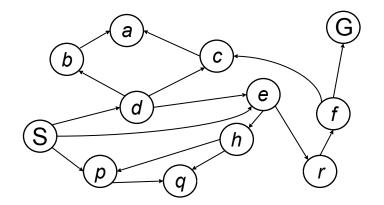
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure initialize the search tree using the initial state of problem loop do

if there are no candidates for expansion then return failure choose a leaf node for expansion according to strategy

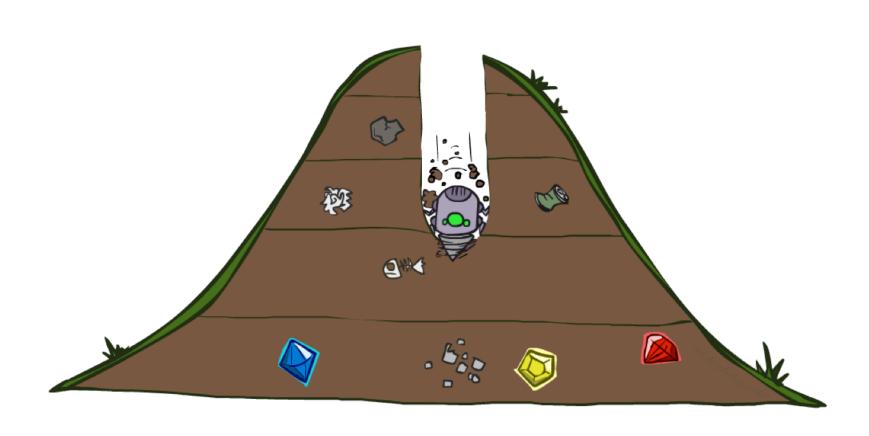
if the node contains a goal state then return the corresponding solution else expand the node and add the resulting nodes to the search tree end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Example: Tree Search



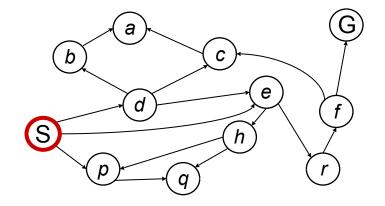
# Depth-First Search

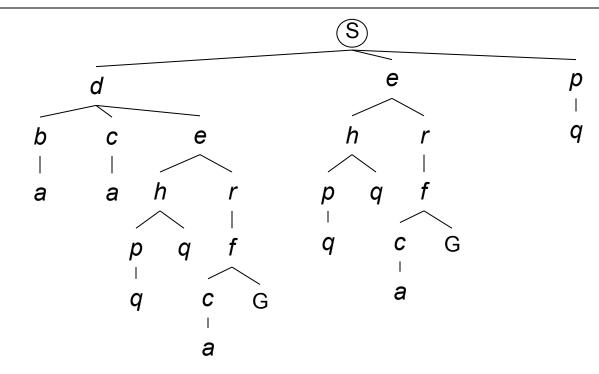


# Depth-First Search

Strategy: expand a deepest node first

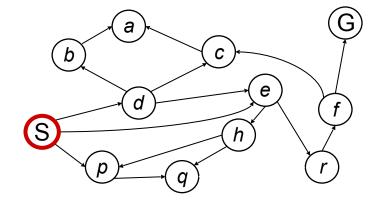
Implementation: Fringe is a LIFO stack

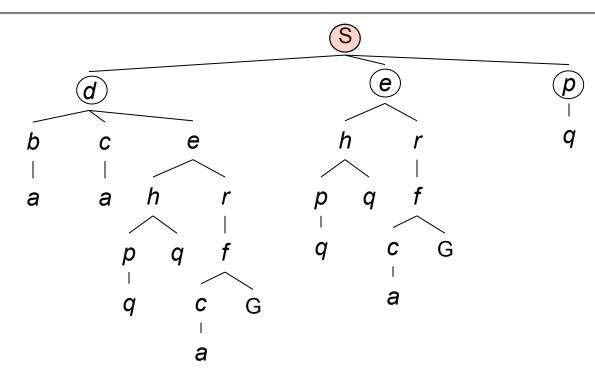




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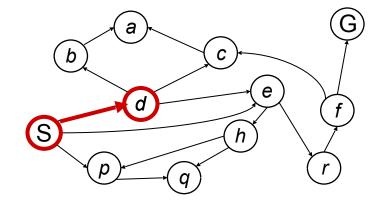
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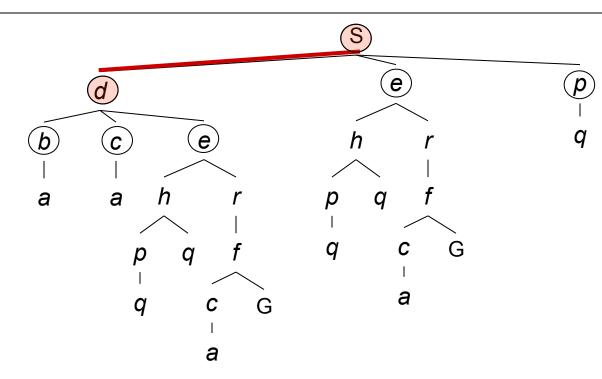




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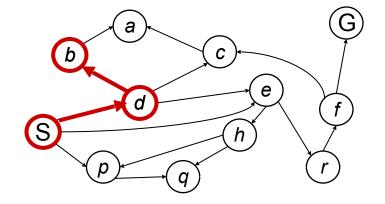
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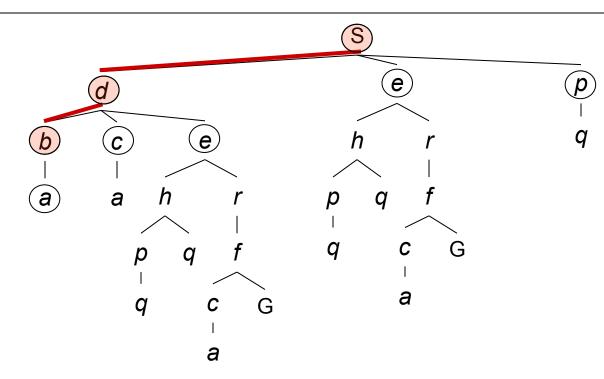




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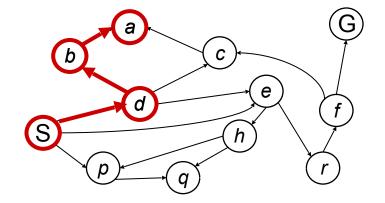
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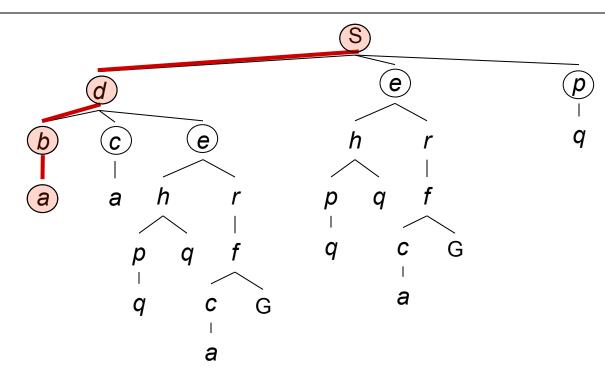




Strategy: expand a deepest node first

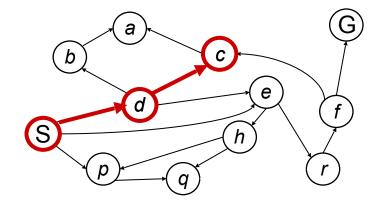
Implementation:

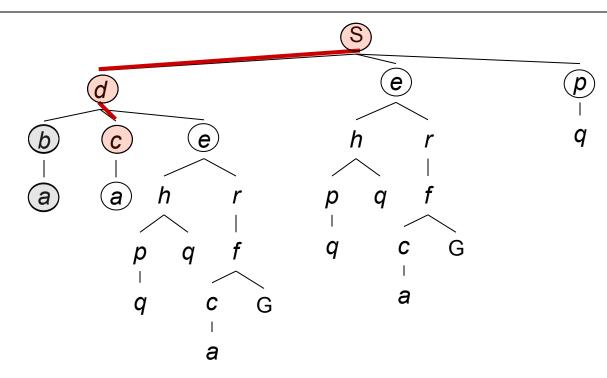




Strategy: expand a deepest node first

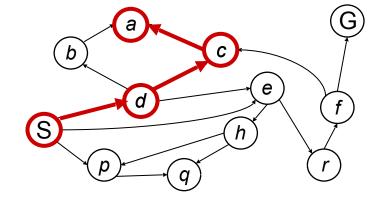
Implementation:

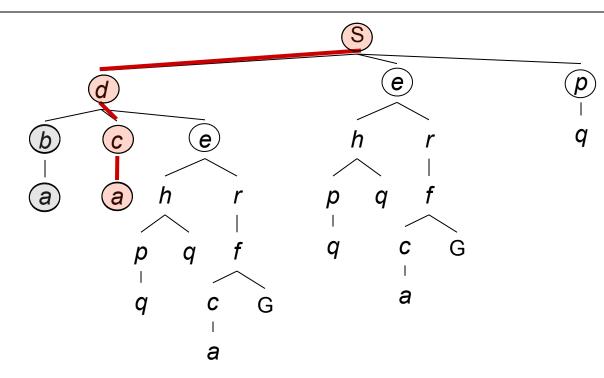




Strategy: expand a deepest node first

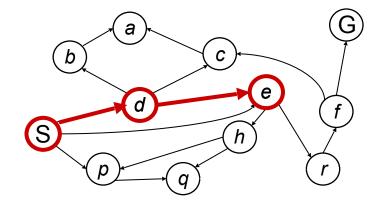
Implementation: Fringe is a LIFO stack

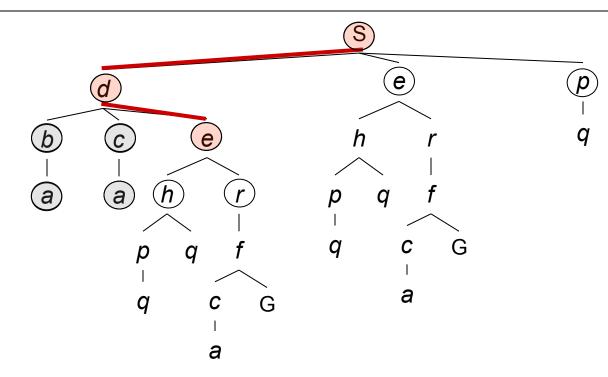




Strategy: expand a deepest node first

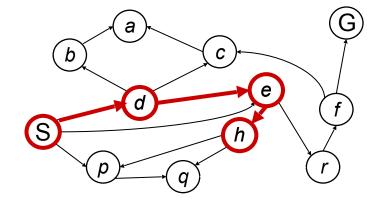
Implementation:

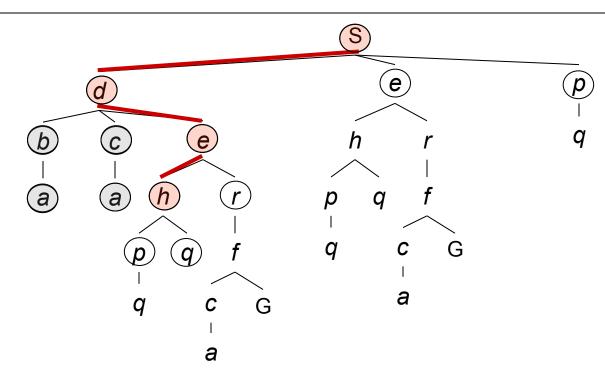




Strategy: expand a deepest node first

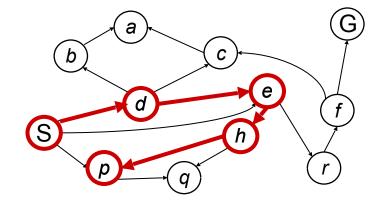
Implementation: Fringe is a LIFO stack

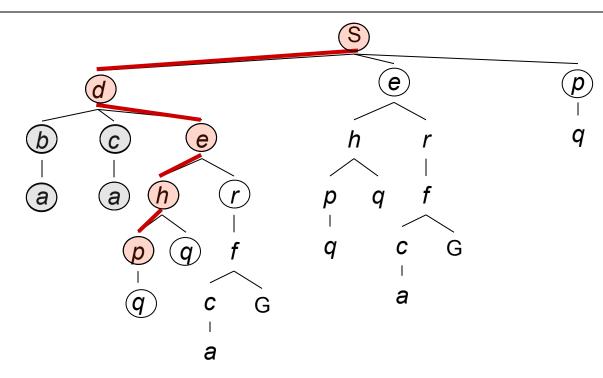




Strategy: expand a deepest node first

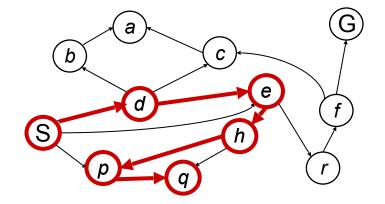
Implementation:

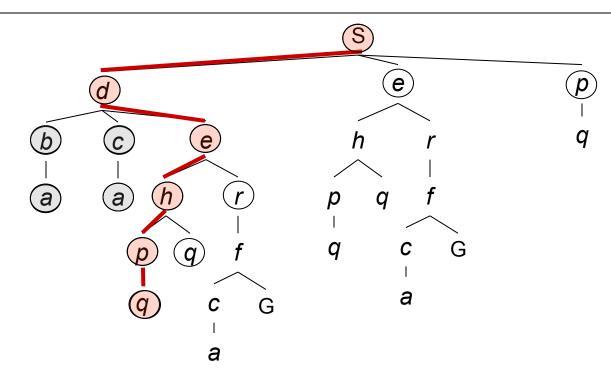




Strategy: expand a deepest node first

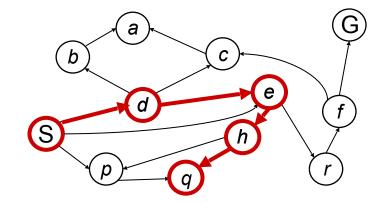
Implementation:

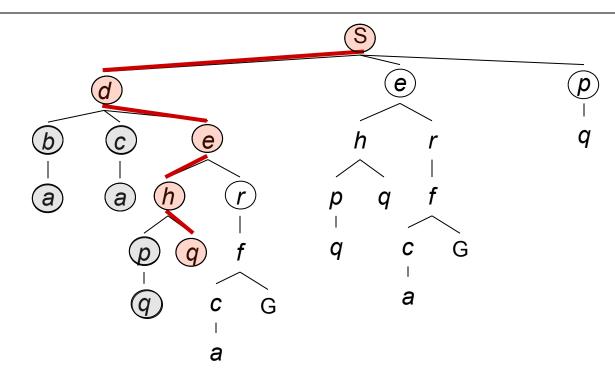




Strategy: expand a deepest node first

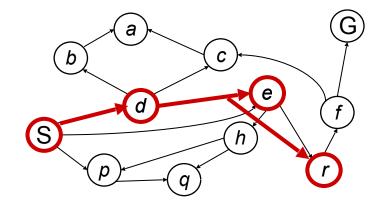
Implementation:

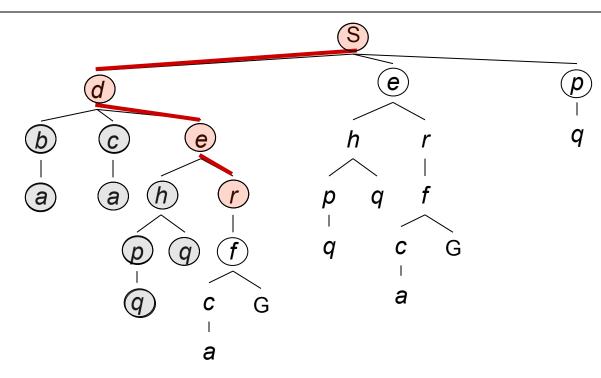




Strategy: expand a deepest node first

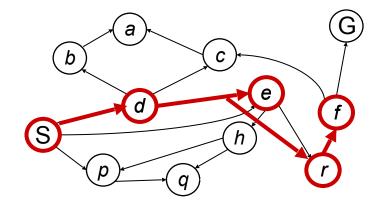
Implementation:

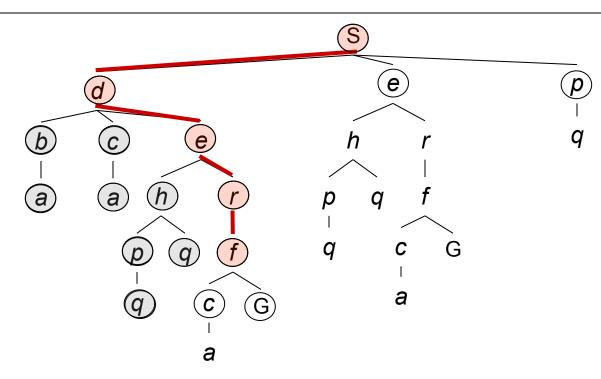




Strategy: expand a deepest node first

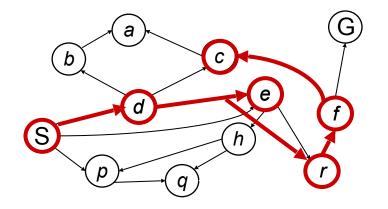
Implementation:

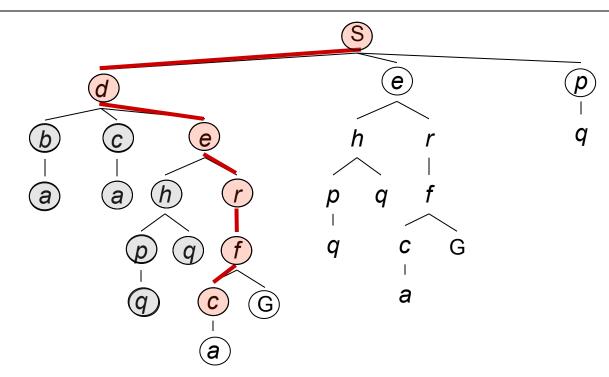




Strategy: expand a deepest node first

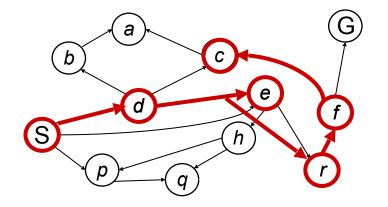
Implementation: Fringe is a LIFO stack

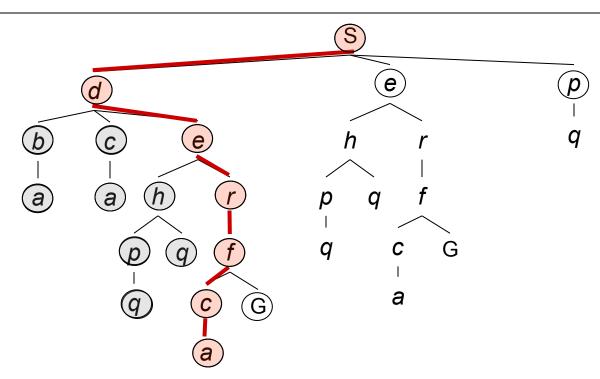




Strategy: expand a deepest node first

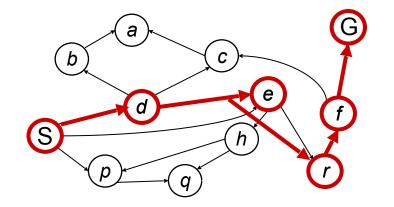
Implementation:

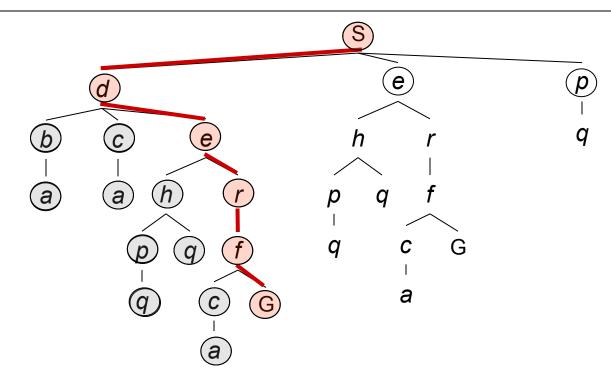


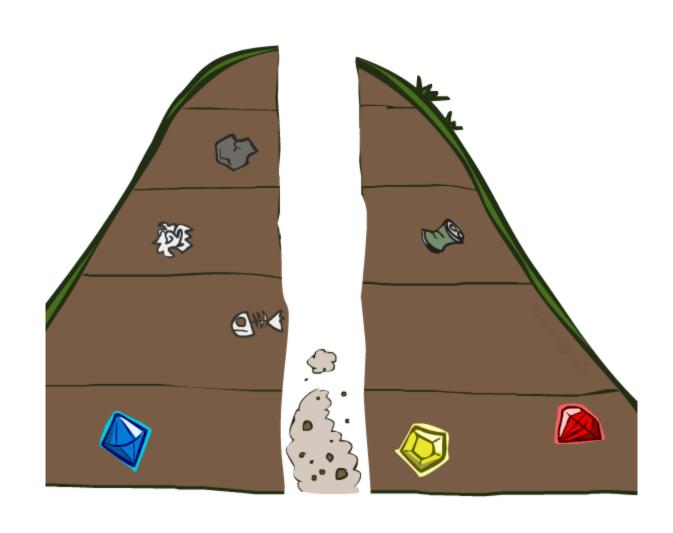


Strategy: expand a deepest node first

*Implementation:* 







Complete: Guaranteed to find a solution if one exists?

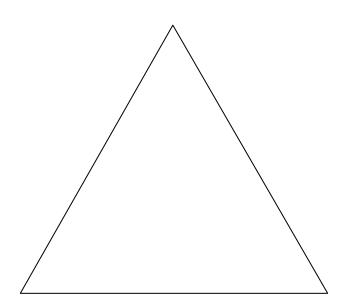
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?

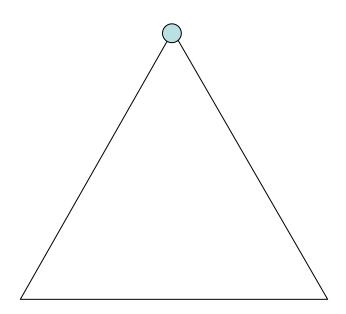
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:

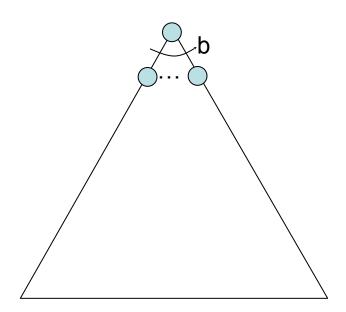
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:



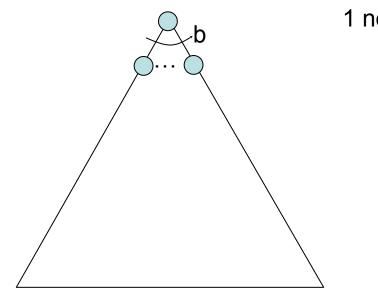
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:



- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor

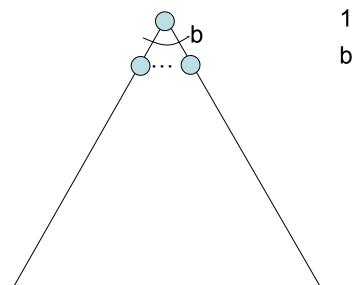


- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor



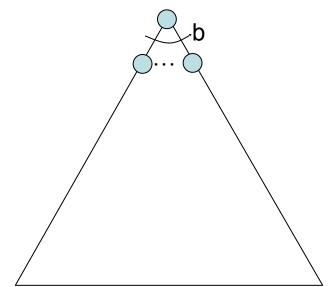
1 node

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor



1 node b nodes

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor

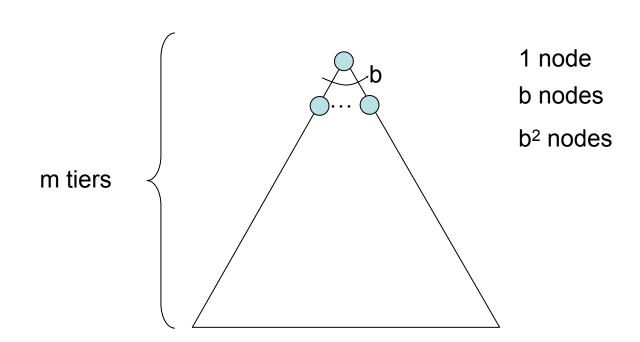


1 node

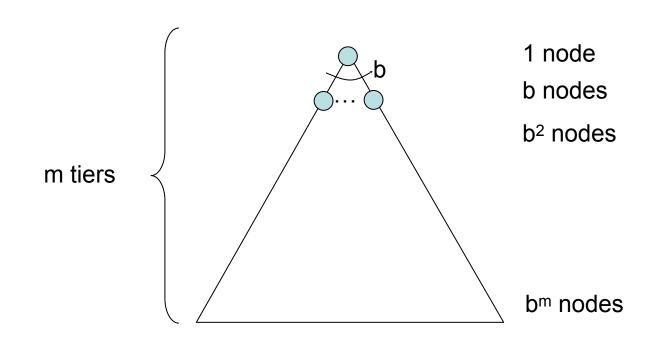
b nodes

b<sup>2</sup> nodes

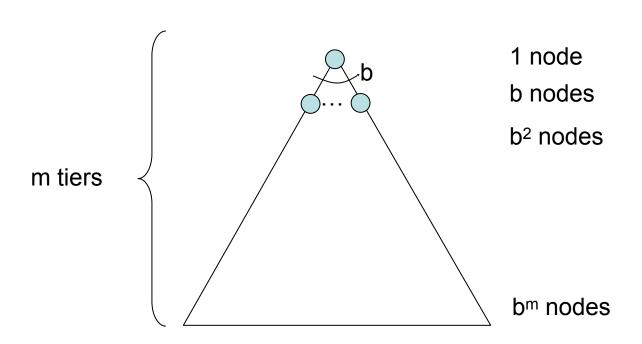
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth



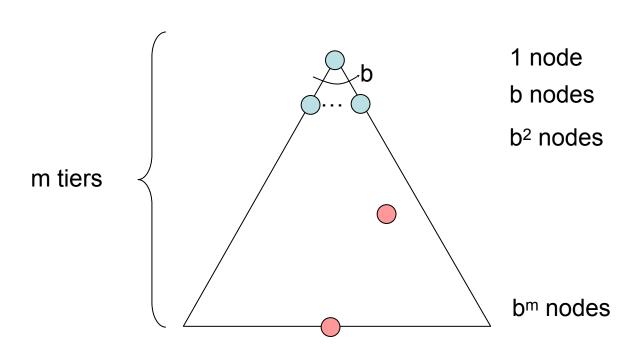
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth



- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths

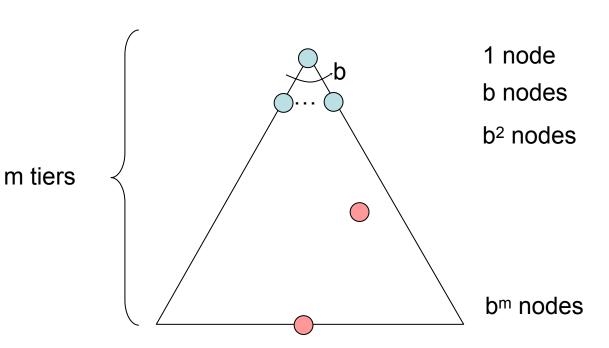


- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths

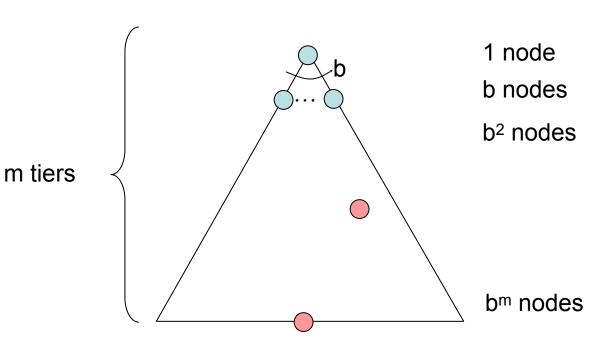


- Complete: Guaranteed to find a solution if one exists?
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- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths

Number of nodes in entire tree?

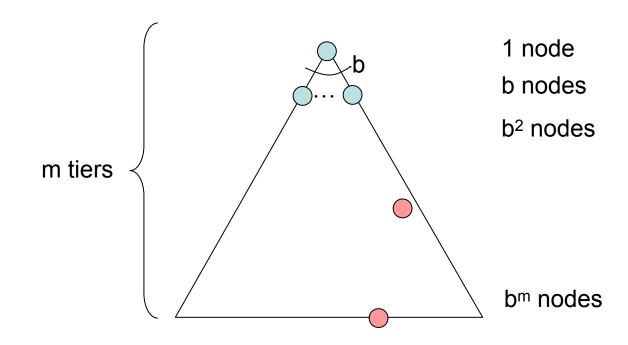


- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths
- Number of nodes in entire tree?
  - $1 + b + b^2 + .... b^m = O(b^m)$



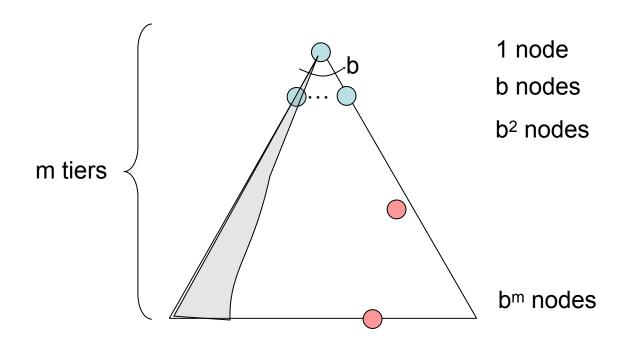
# Depth-First Search (DFS) Properties

What nodes DFS expand?

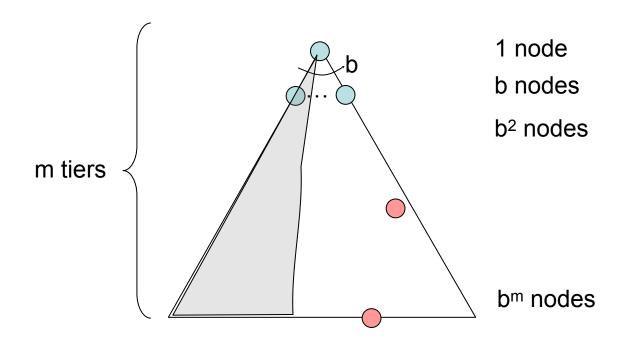


# Depth-First Search (DFS) Properties

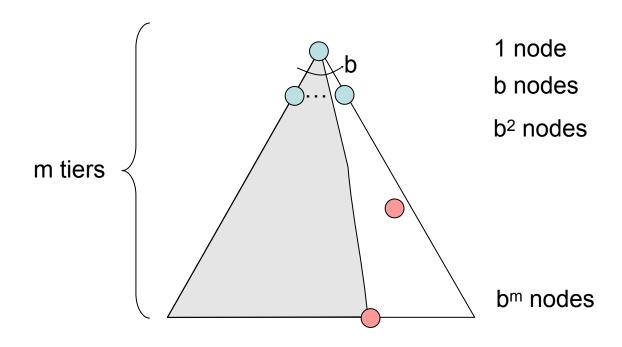
What nodes DFS expand?



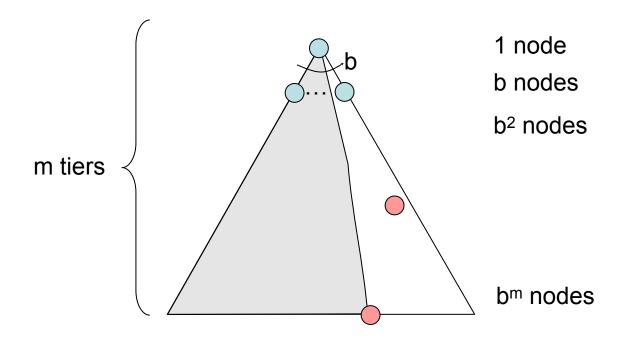
What nodes DFS expand?



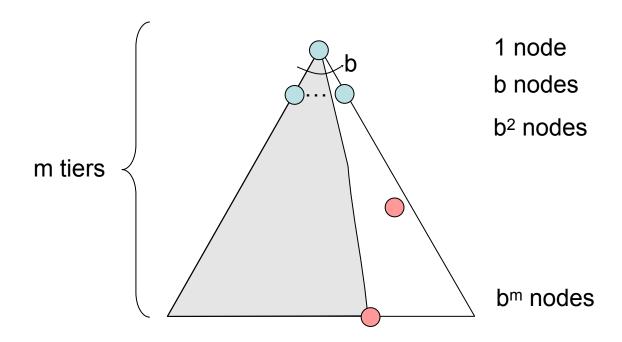
What nodes DFS expand?



- What nodes DFS expand?
  - Some left prefix of the tree.

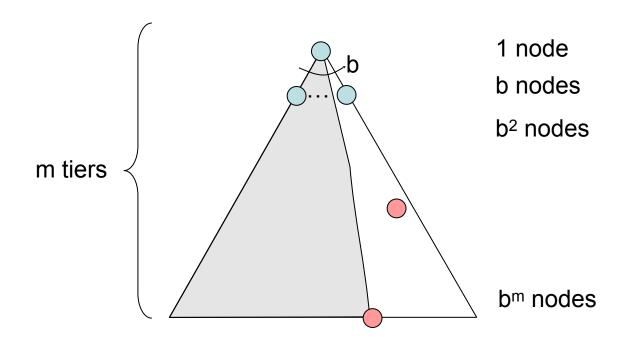


- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!

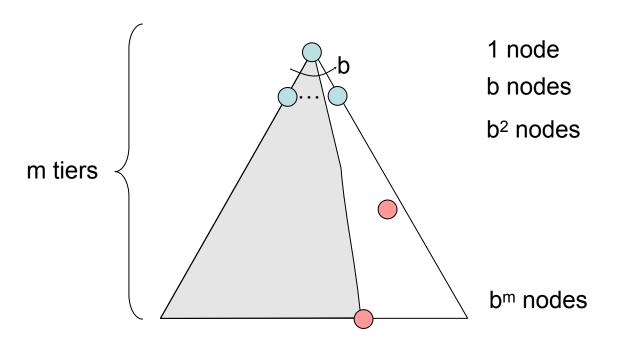


#### What nodes DFS expand?

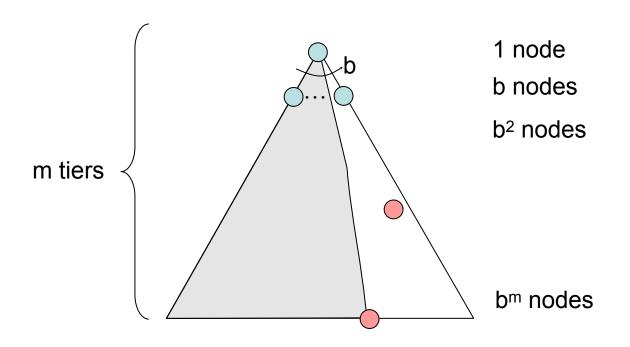
- Some left prefix of the tree.
- Could process the whole tree!
- If m is finite, takes time O(b<sup>m</sup>)



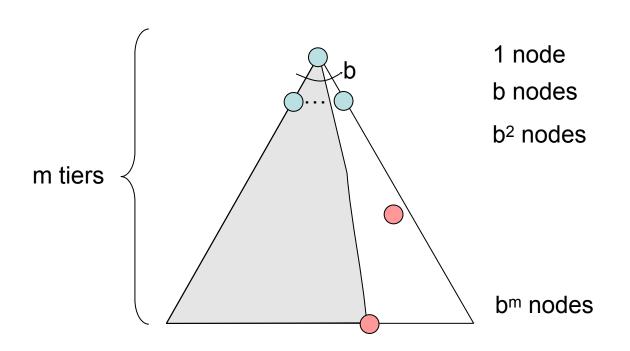
- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time O(b<sup>m</sup>)
- How much space does the fringe take?



- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time O(b<sup>m</sup>)
- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)

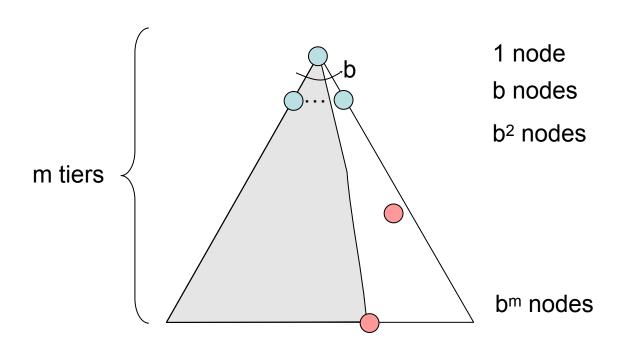


- What nodes DFS expand?
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  - Could process the whole tree!
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- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)
- Is it complete?

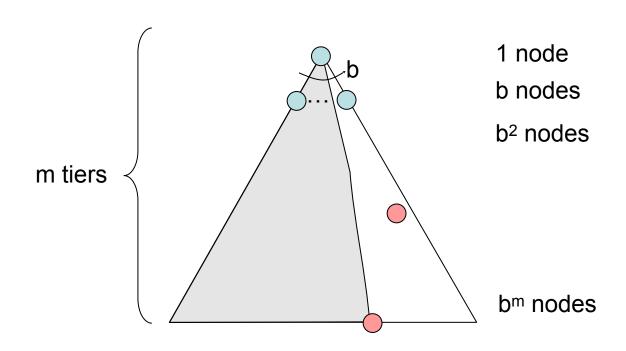


#### What nodes DFS expand?

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- If m is finite, takes time O(b<sup>m</sup>)
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  - Only has siblings on path to root, so O(bm)
- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)

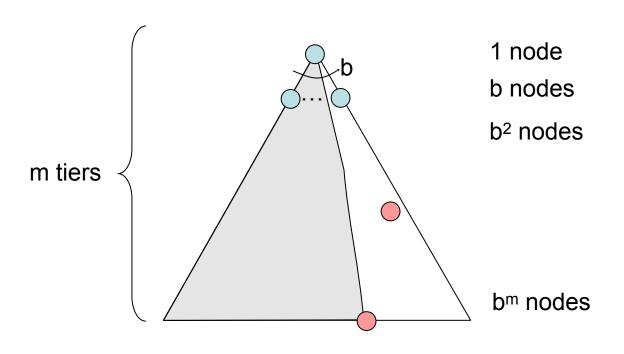


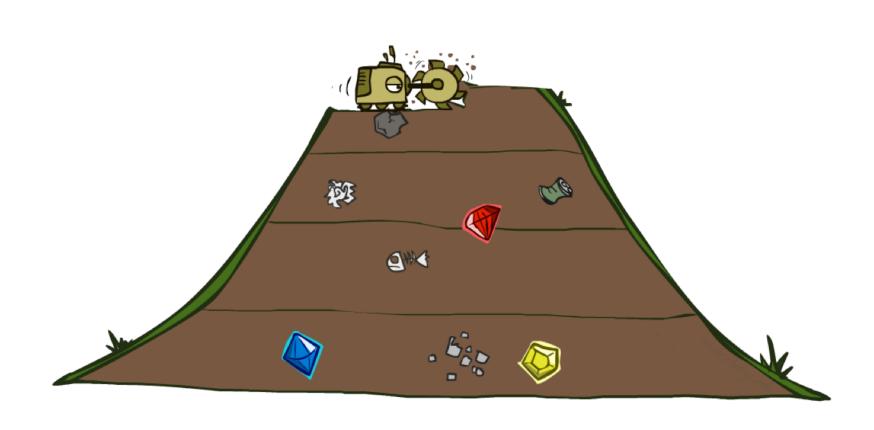
- What nodes DFS expand?
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- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)
- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?



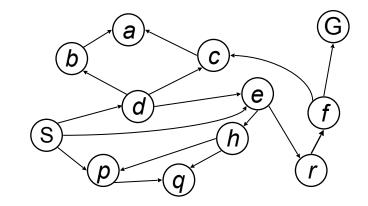
#### What nodes DFS expand?

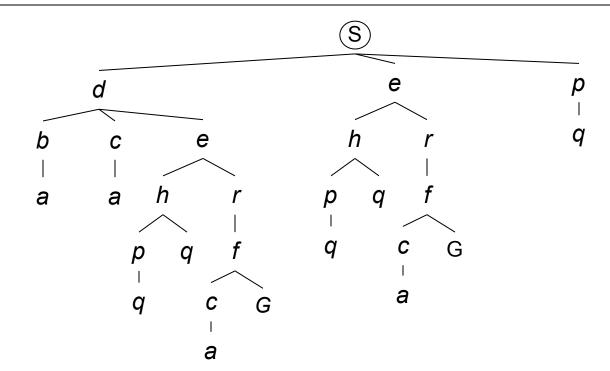
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- Could process the whole tree!
- If m is finite, takes time O(b<sup>m</sup>)
- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)
- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost



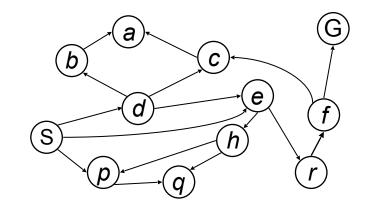


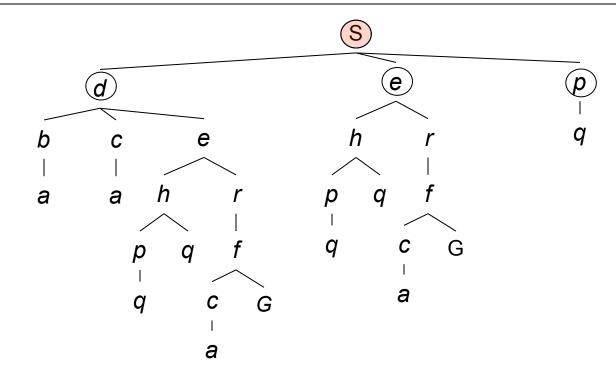
Strategy: expand a shallowest node first



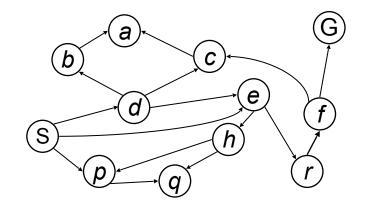


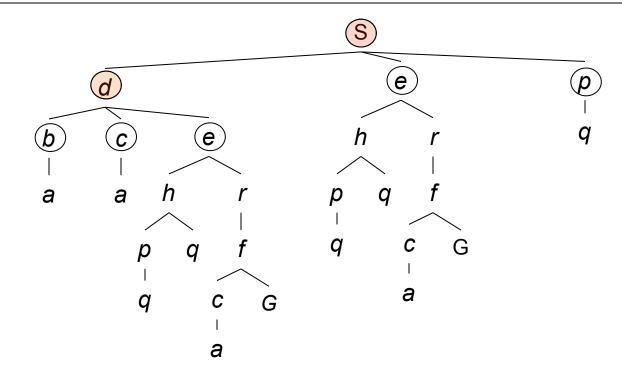
Strategy: expand a shallowest node first



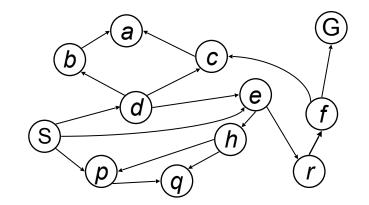


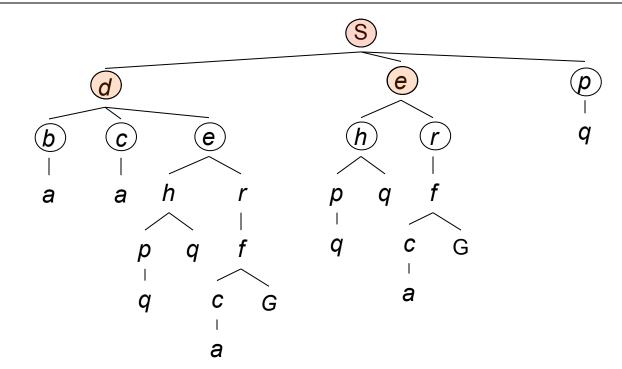
Strategy: expand a shallowest node first



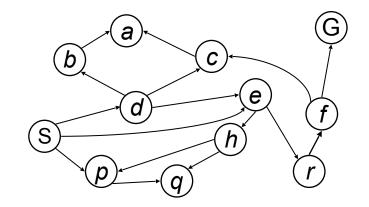


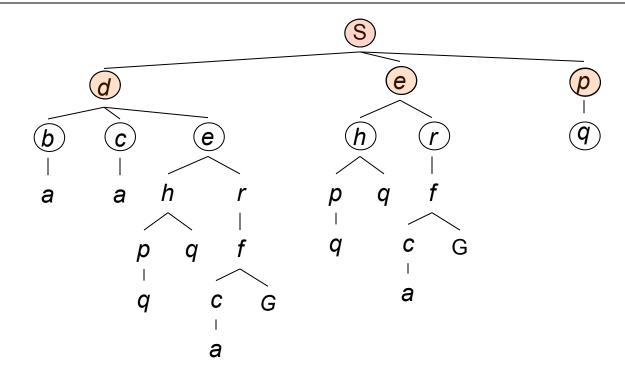
Strategy: expand a shallowest node first



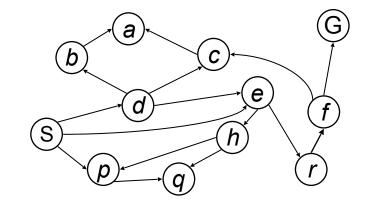


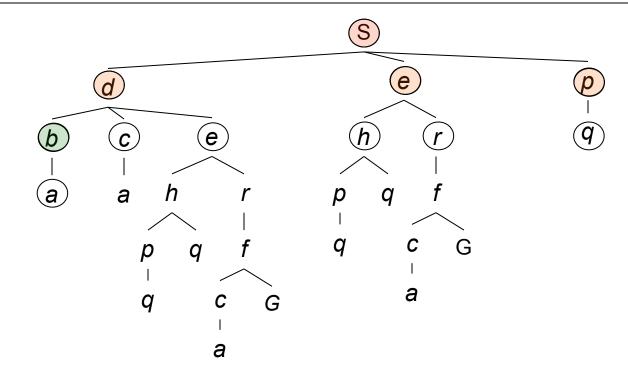
Strategy: expand a shallowest node first



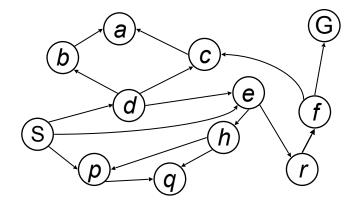


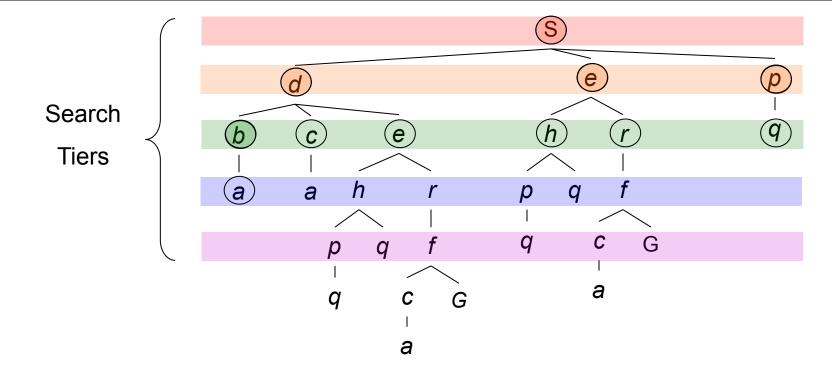
Strategy: expand a shallowest node first

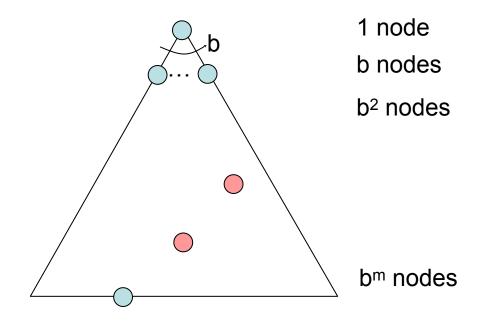


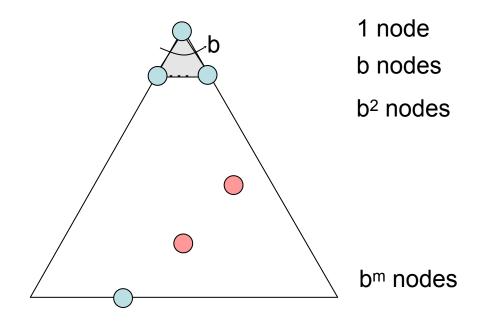


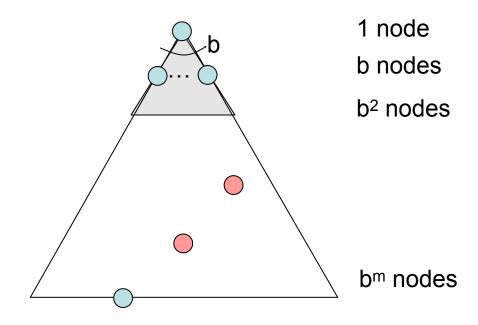
Strategy: expand a shallowest node first

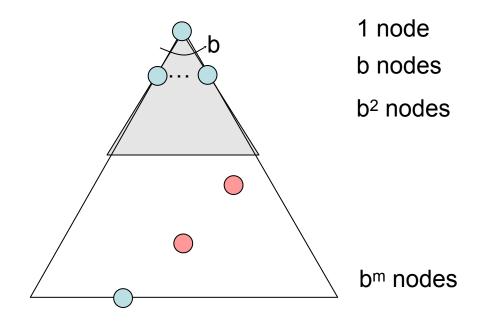


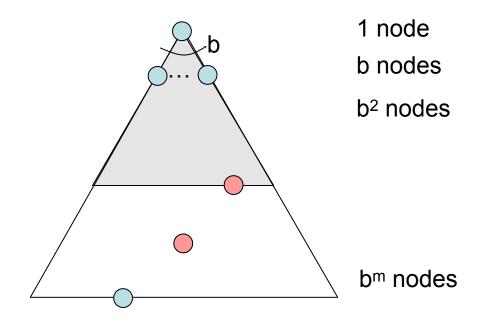




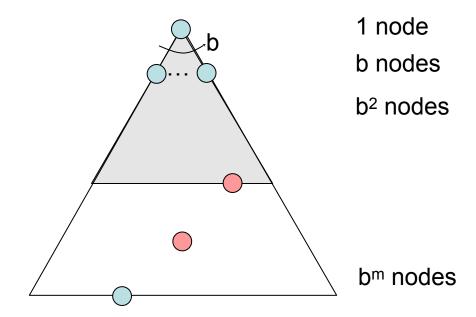




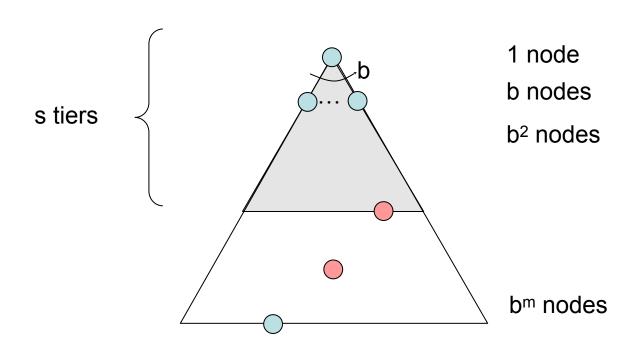




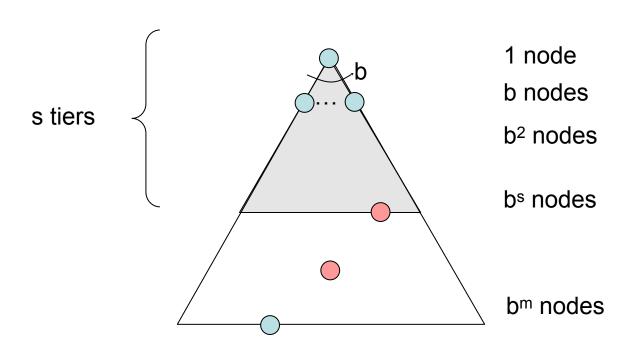
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution



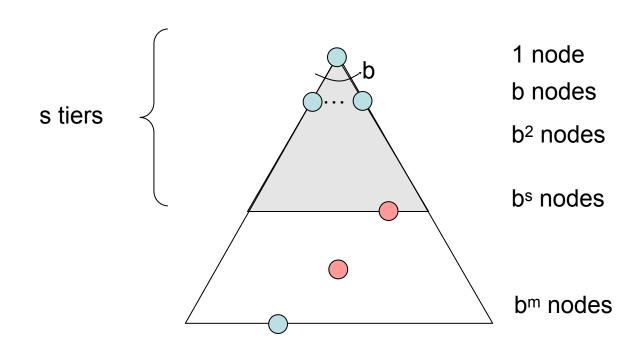
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s



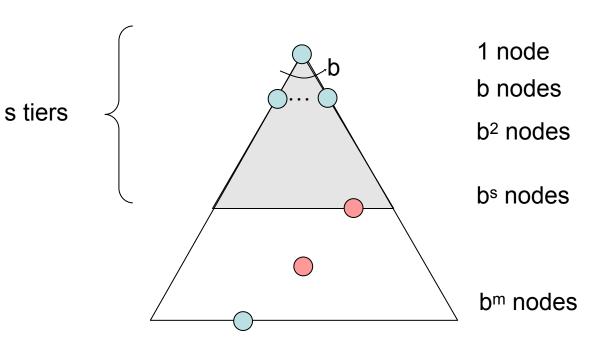
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s



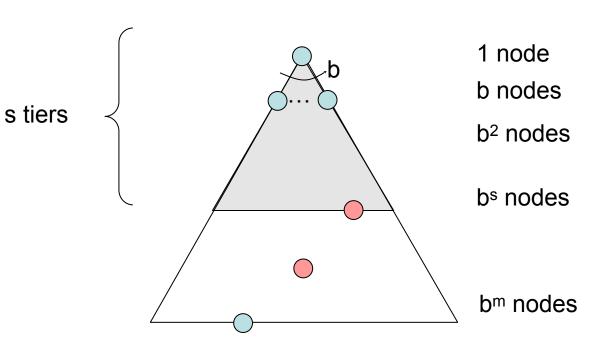
- Processes all nodes above shallowest solution
- Let depth of shallowest solution be s
- Search takes time O(bs)



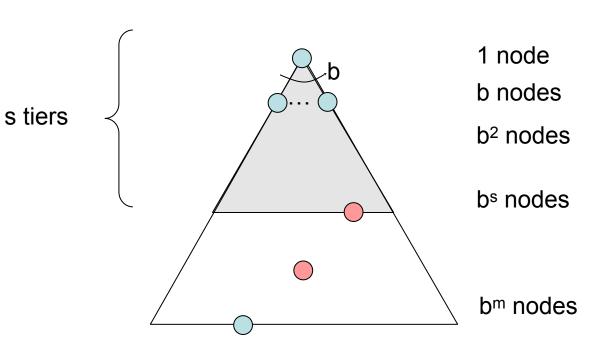
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time O(bs)
- How much space does the fringe take?



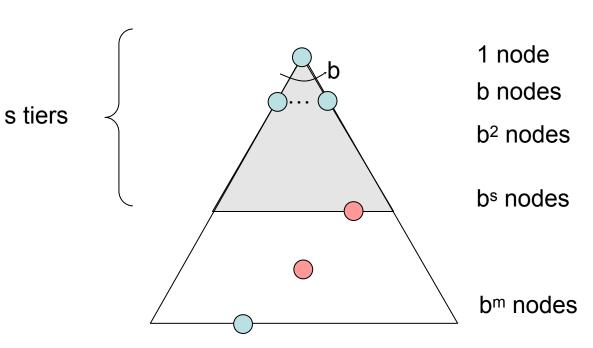
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time O(bs)
- How much space does the fringe take?
  - Has roughly the last tier, so O(bs)



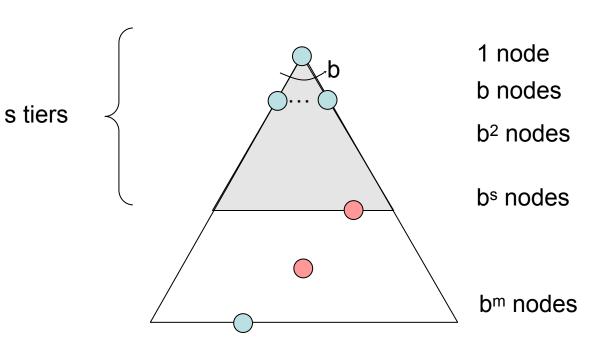
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time O(bs)
- How much space does the fringe take?
  - Has roughly the last tier, so O(bs)
- Is it complete?



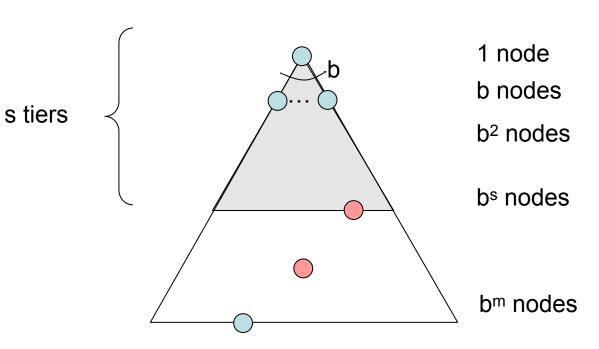
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time O(bs)
- How much space does the fringe take?
  - Has roughly the last tier, so O(bs)
- Is it complete?
  - s must be finite if a solution exists, so yes!



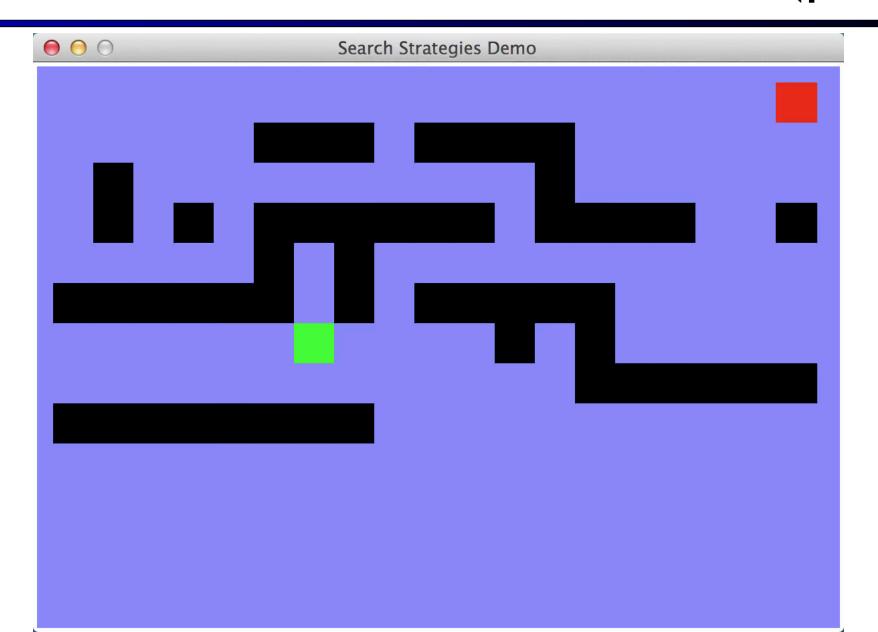
- What nodes does BFS expand?
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  - Let depth of shallowest solution be s
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- Is it optimal?



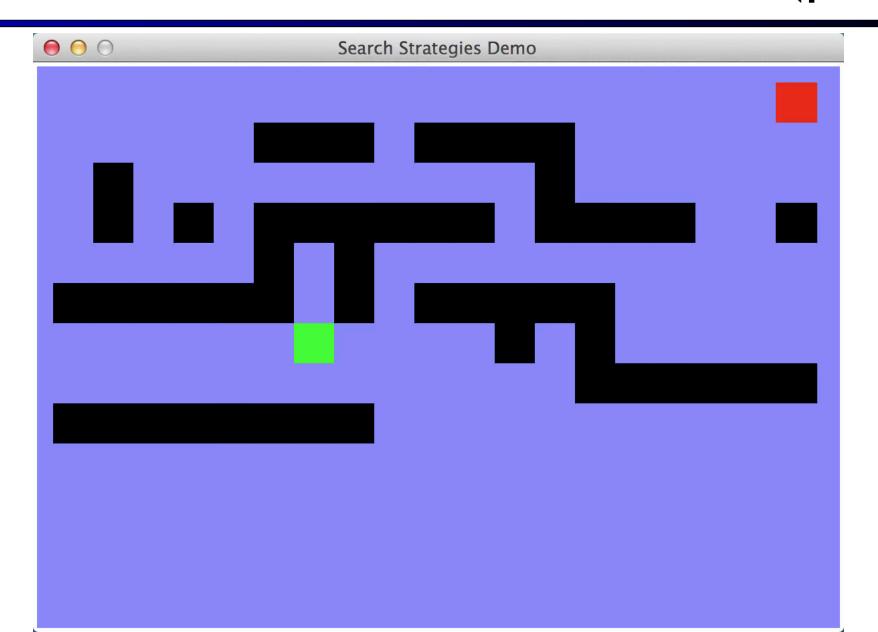
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- How much space does the fringe take?
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- Is it complete?
  - s must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



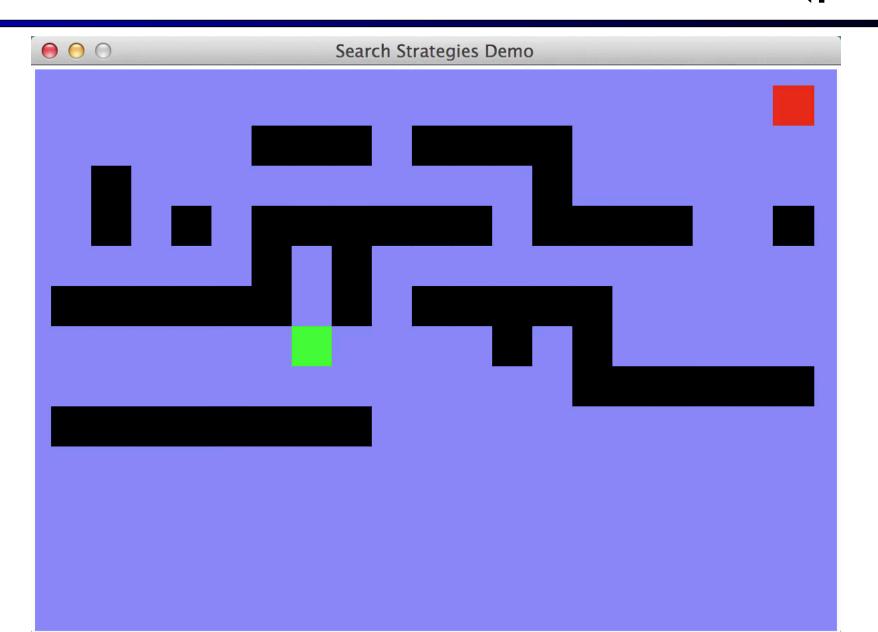
### Video of Demo Maze Water DFS/BFS (part 1)



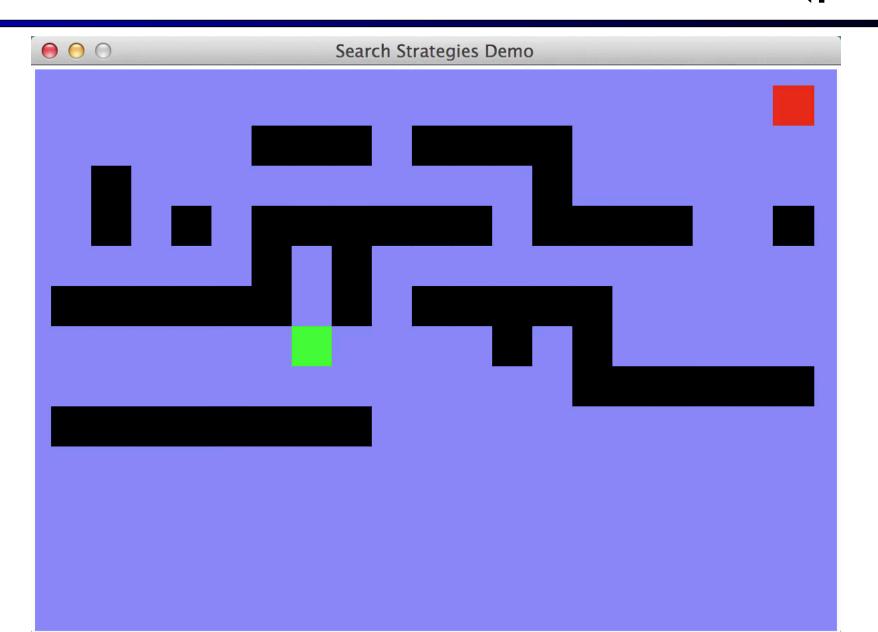
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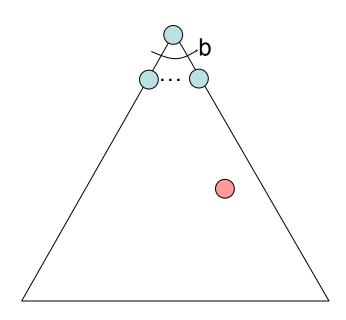
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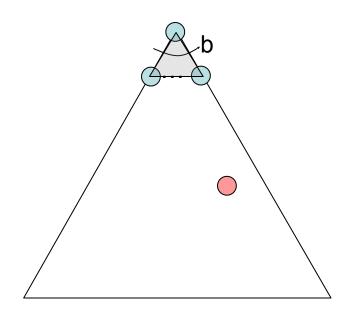
## Video of Demo Maze Water DFS/BFS (part 1)



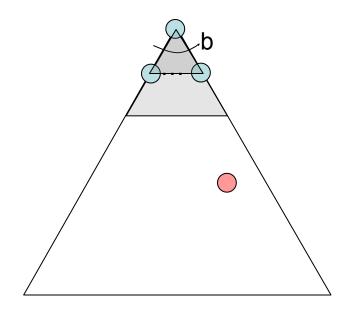
Idea: get DFS's space advantage with
 BFS's time / shallow-solution advantages



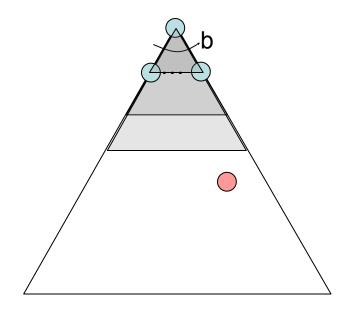
- Idea: get DFS's space advantage with
   BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...



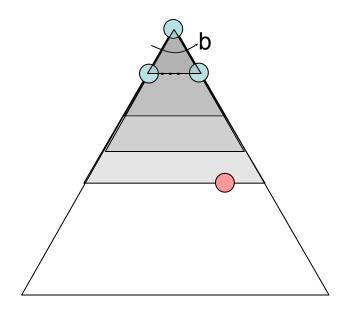
- Idea: get DFS's space advantage with
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  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...



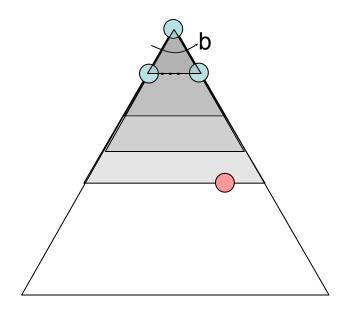
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  - Run a DFS with depth limit 1. If no solution...
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  - Run a DFS with depth limit 3. .....



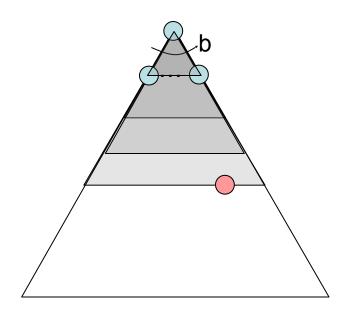
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- Isn't that wastefully redundant?



- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...
  - Run a DFS with depth limit 3. .....
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!



## Recap: Search

#### Search problem:

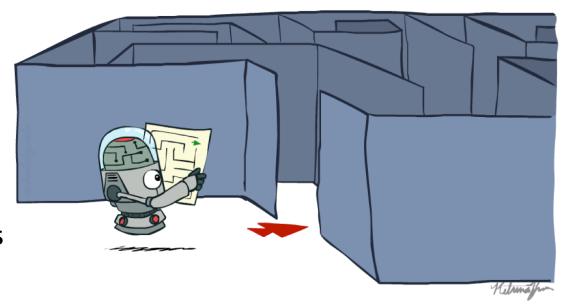
- States (configurations of the world)
- Actions and costs
- Successor function (world dynamics)
- Start state and goal test

#### Search tree:

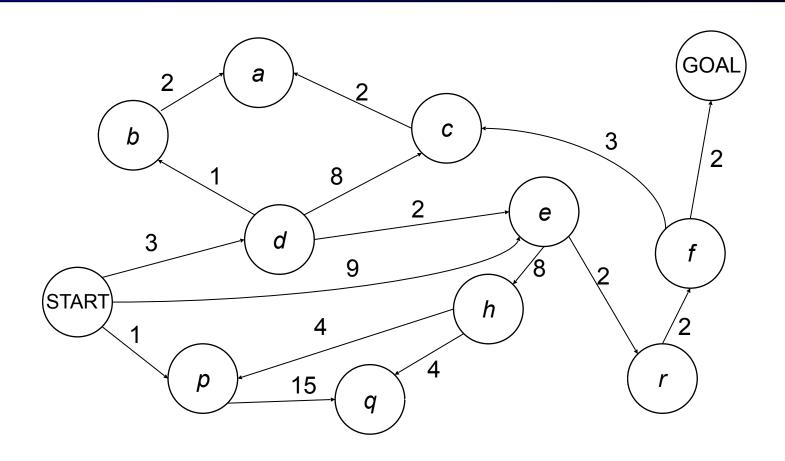
- Nodes: represent plans for reaching states
- Plans have costs (sum of action costs)

#### Search algorithm:

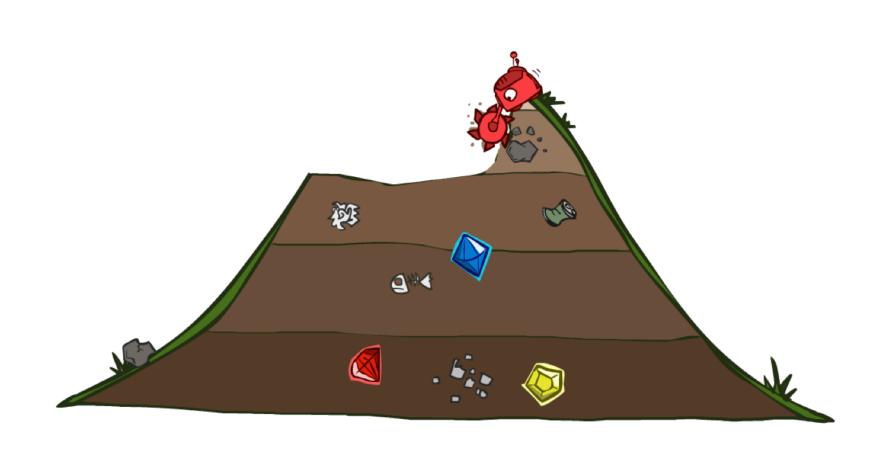
- Systematically builds a search tree
- Chooses an ordering of the fringe (unexplored nodes)
- Optimal: finds least-cost plans



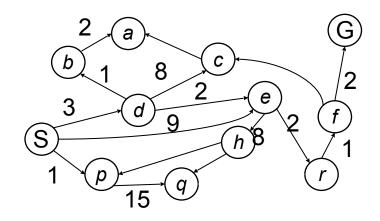
#### Cost-Sensitive Search

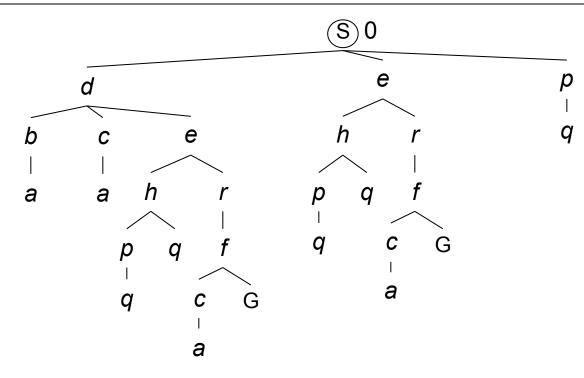


BFS finds the shortest path in terms of number of actions. It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.

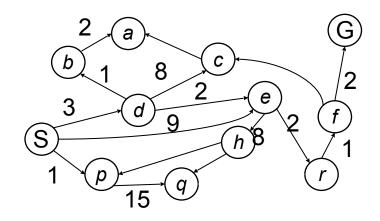


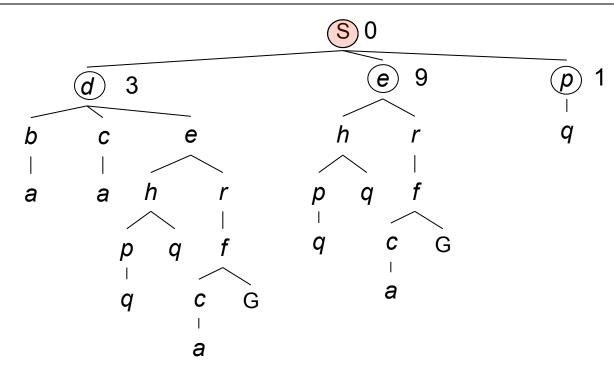
Strategy: expand a cheapest node first:



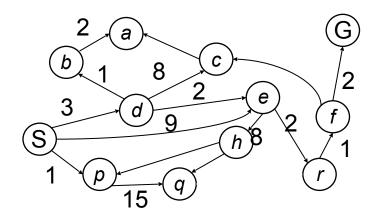


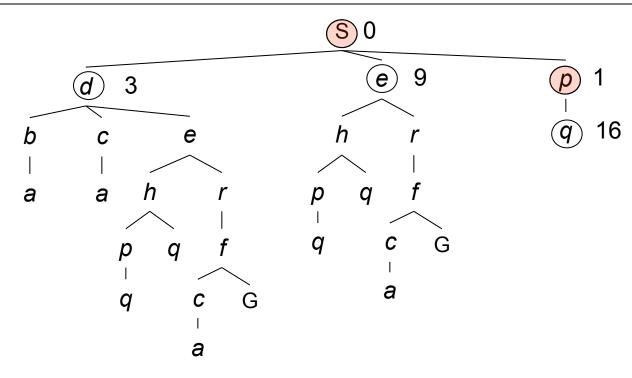
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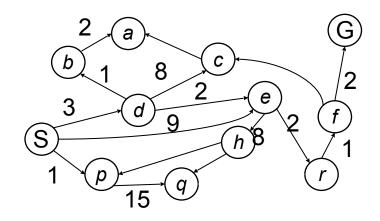


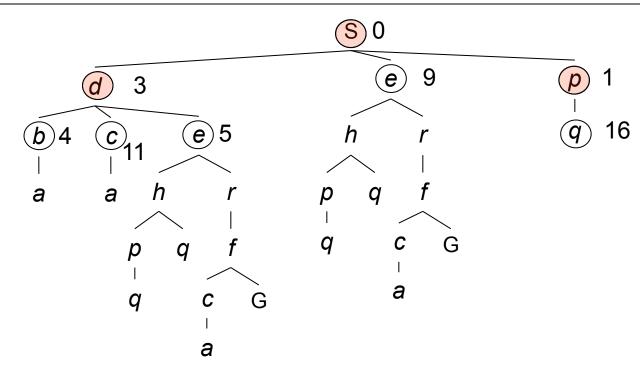
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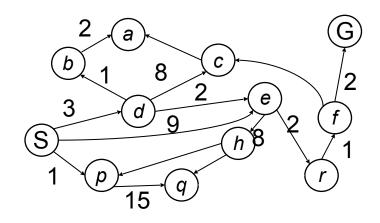


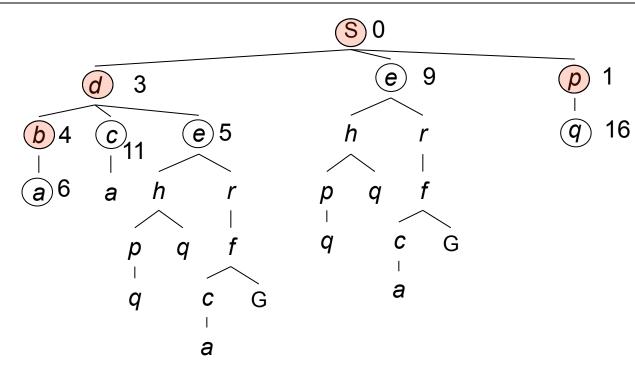
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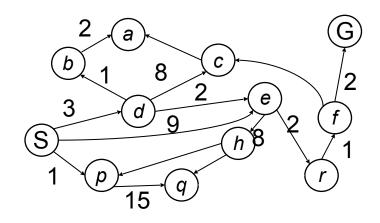


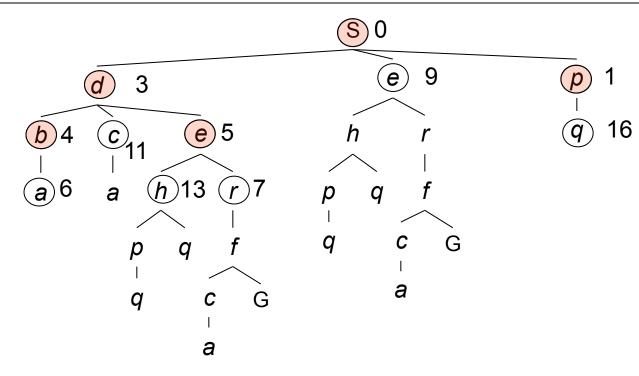
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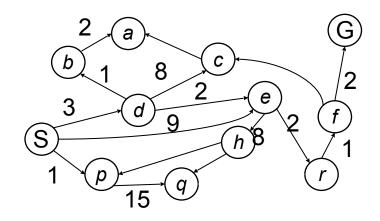


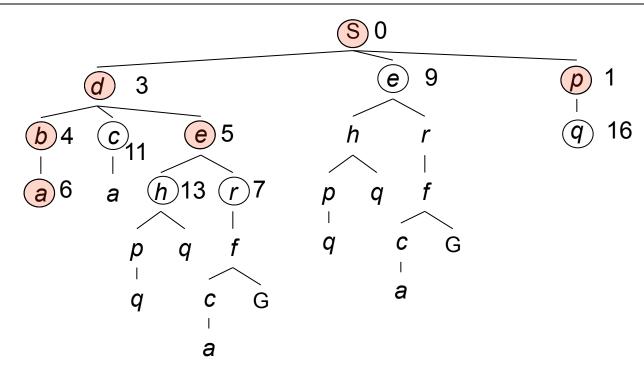
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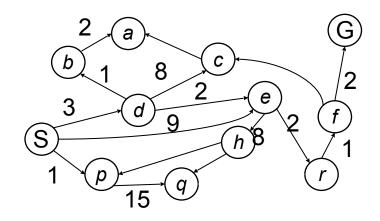


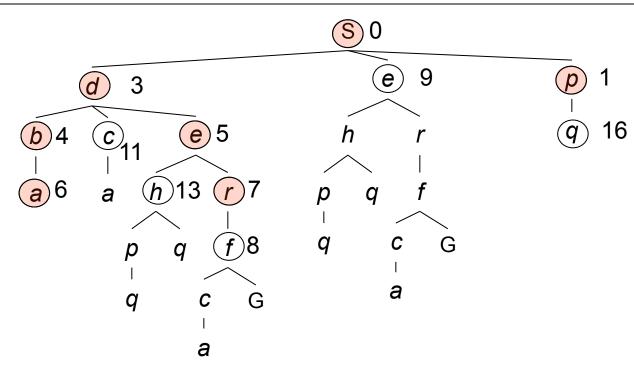
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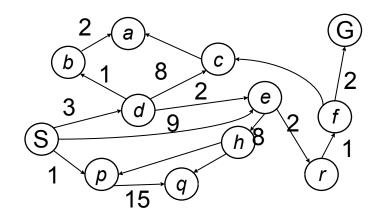


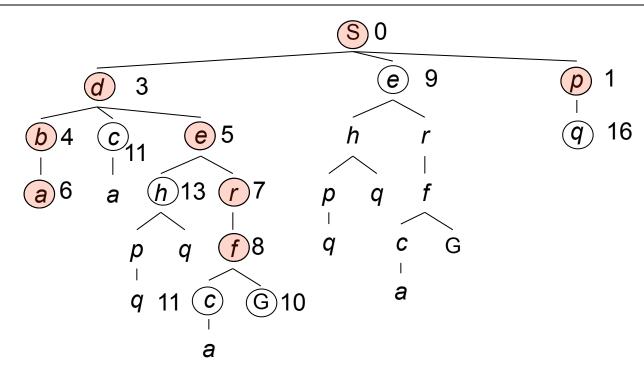
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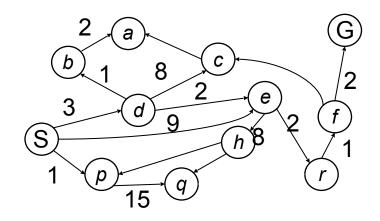


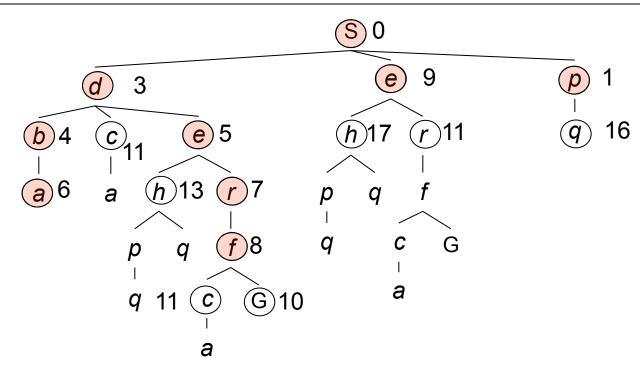
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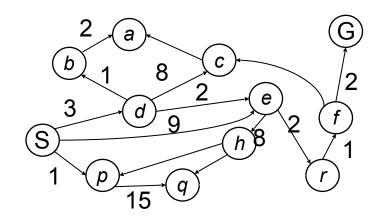


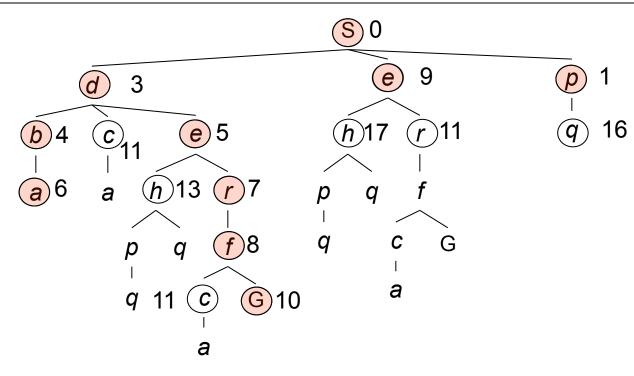
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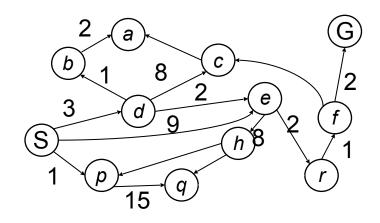


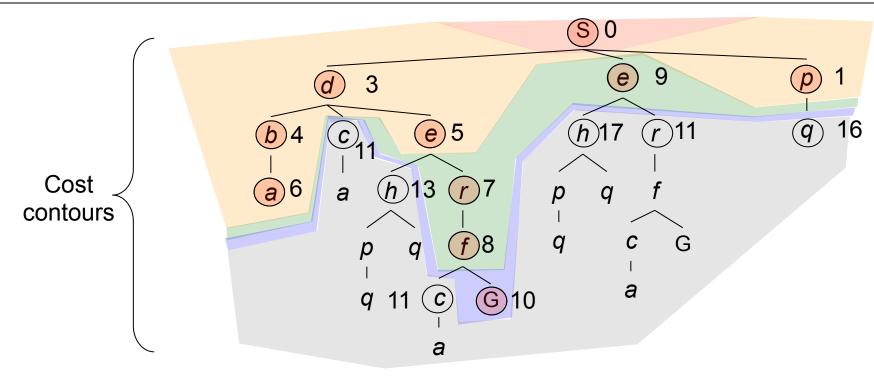
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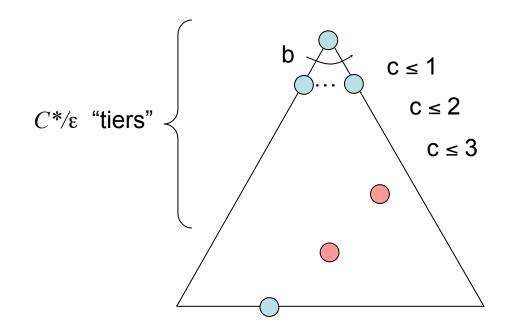


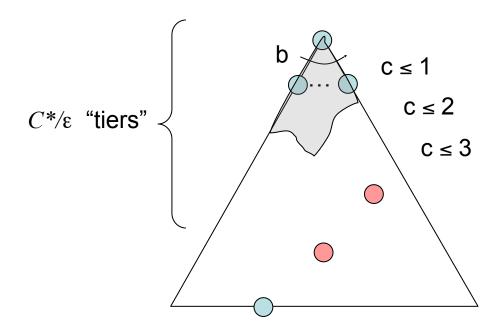


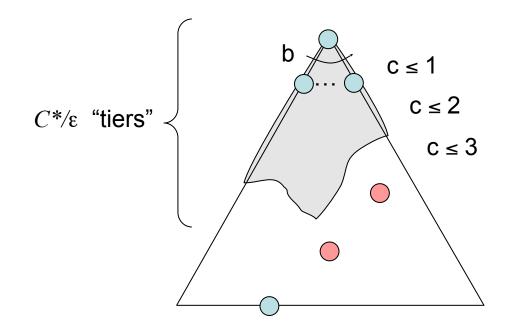
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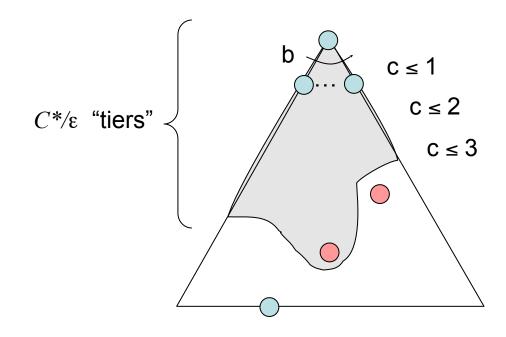




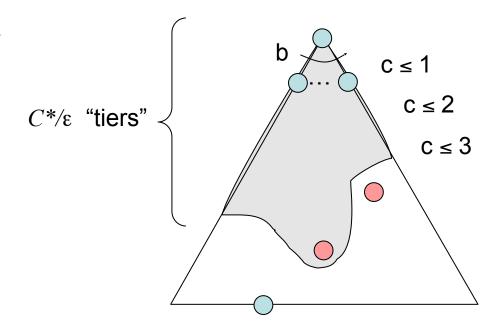




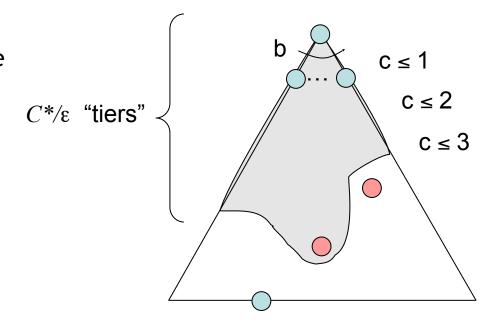




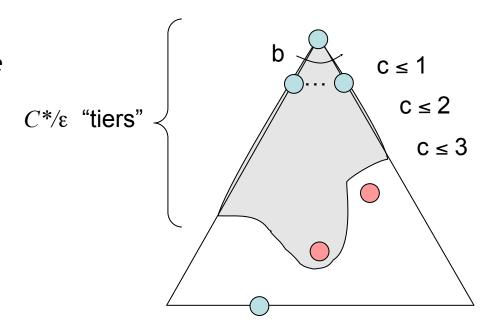
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!



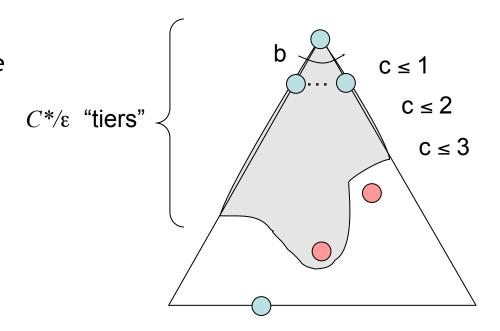
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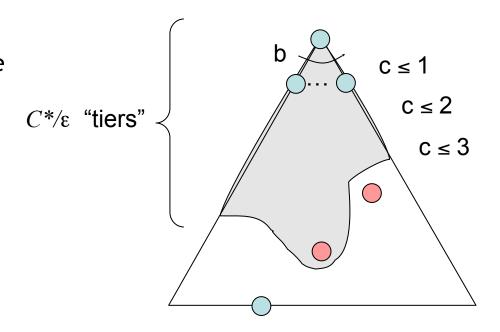
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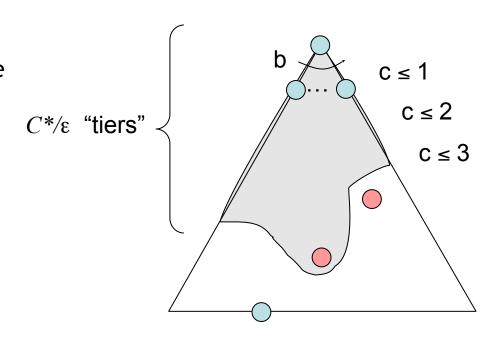
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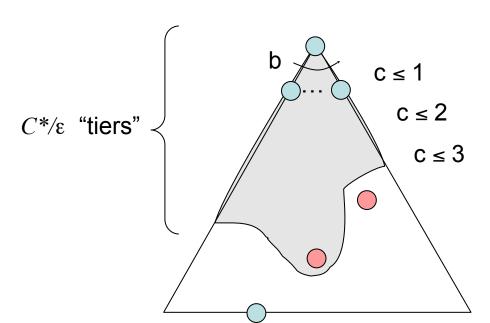
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#### Is it complete?

Assuming best solution has a finite cost and minimum arc cost is positive, yes!



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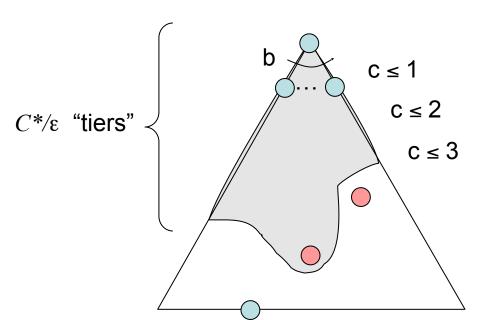
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# Uniform Cost Search (UCS) Properties

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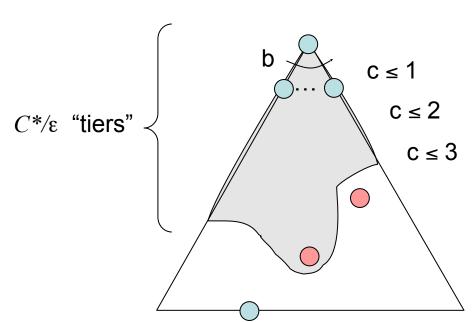
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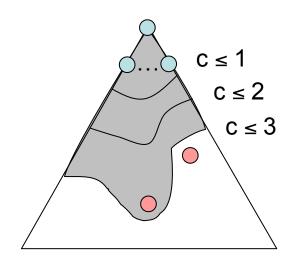
#### Is it optimal?

Yes! (Proof next lecture via A\*)



Remember: UCS explores increasing cost contours

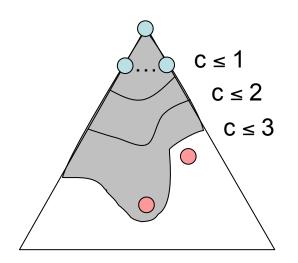
The good: UCS is complete and optimal!



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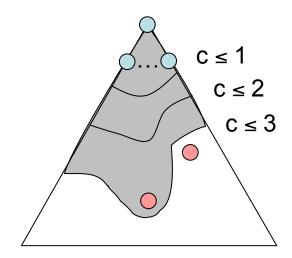
- The bad:
  - Explores options in every "direction"
  - No information about goal location

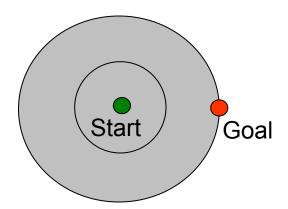


Remember: UCS explores increasing cost contours

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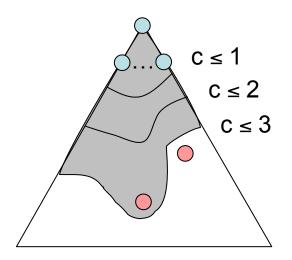


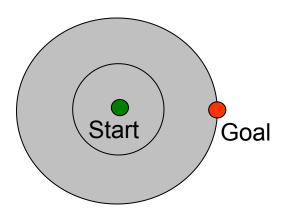
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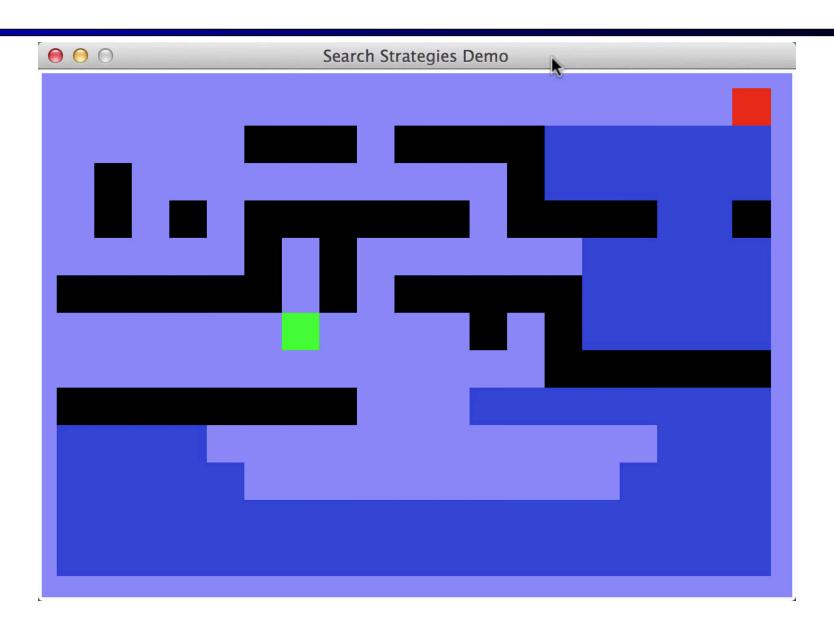
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We'll fix that soon!

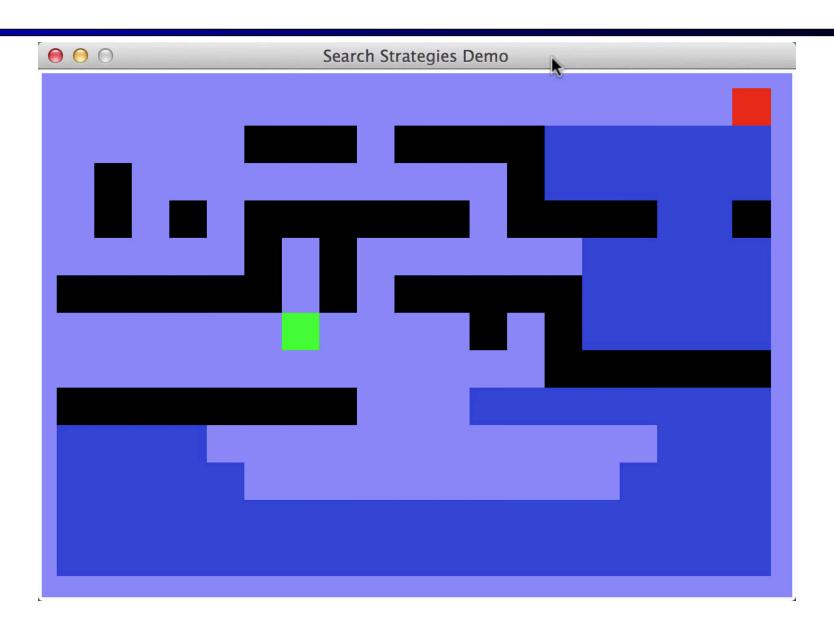




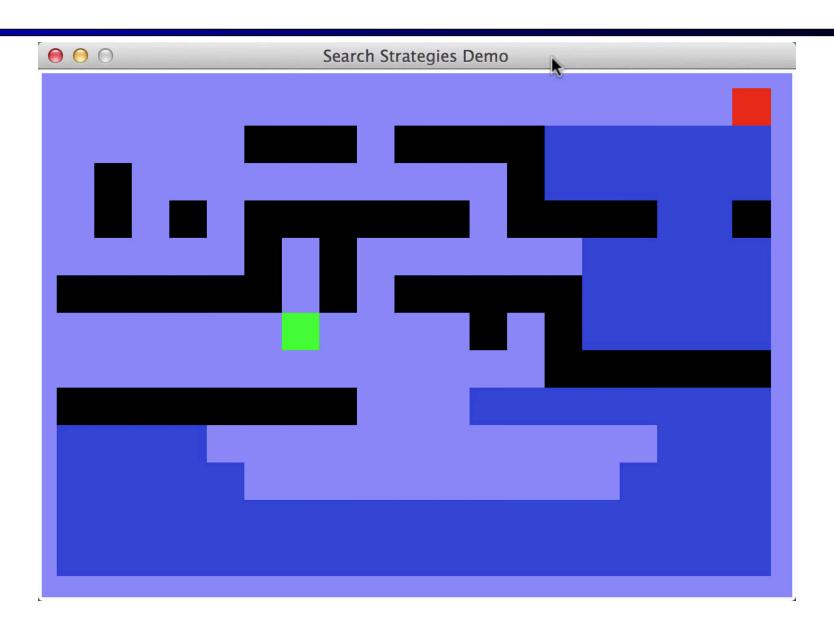
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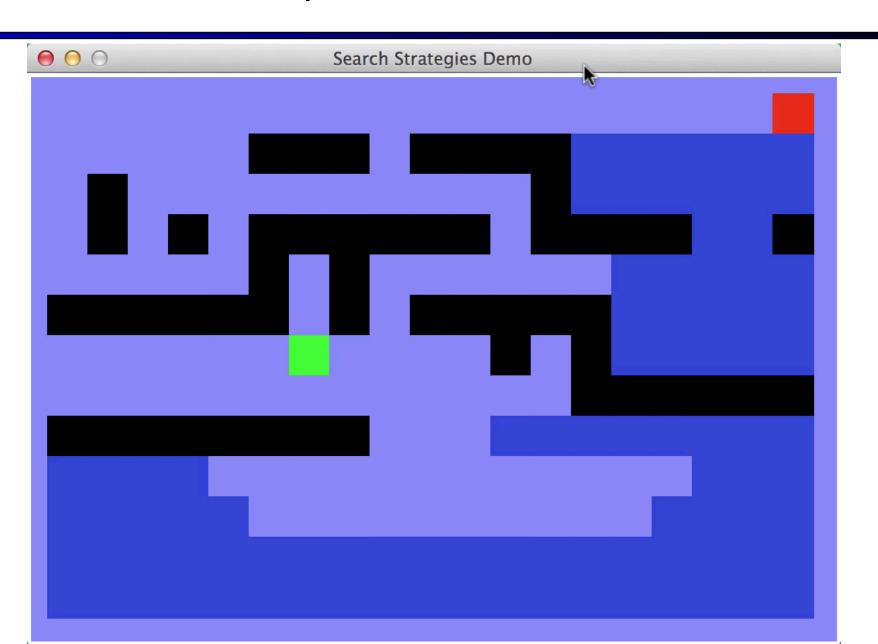
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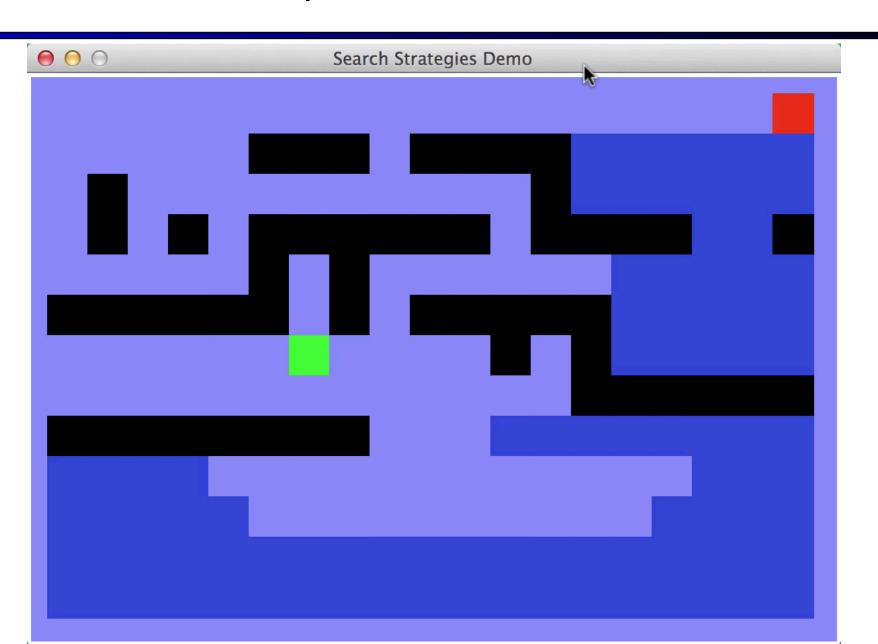
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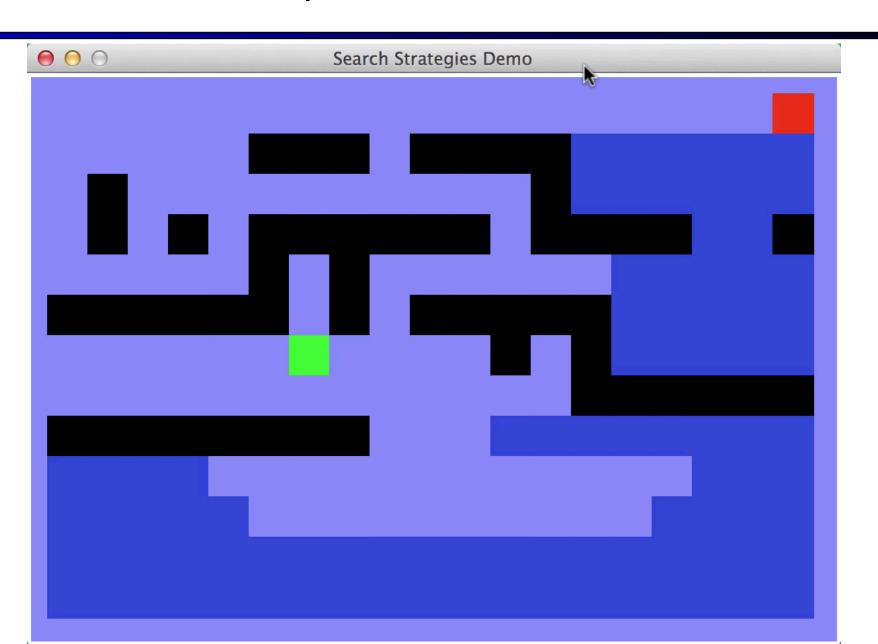
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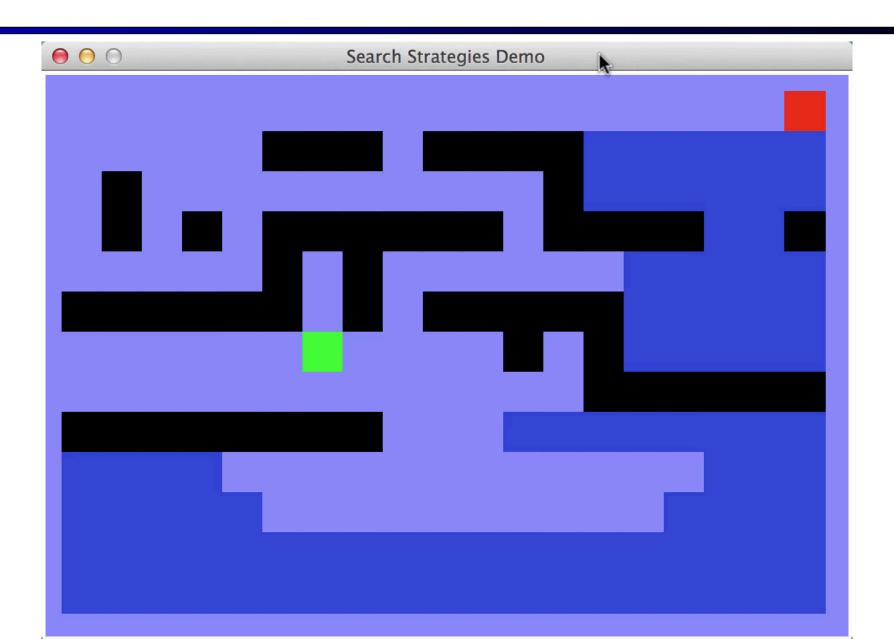
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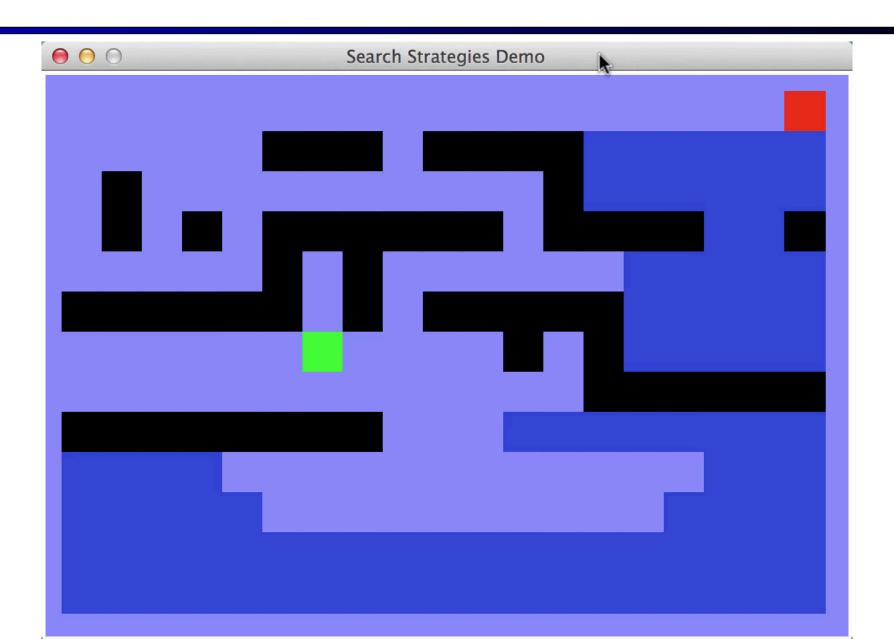
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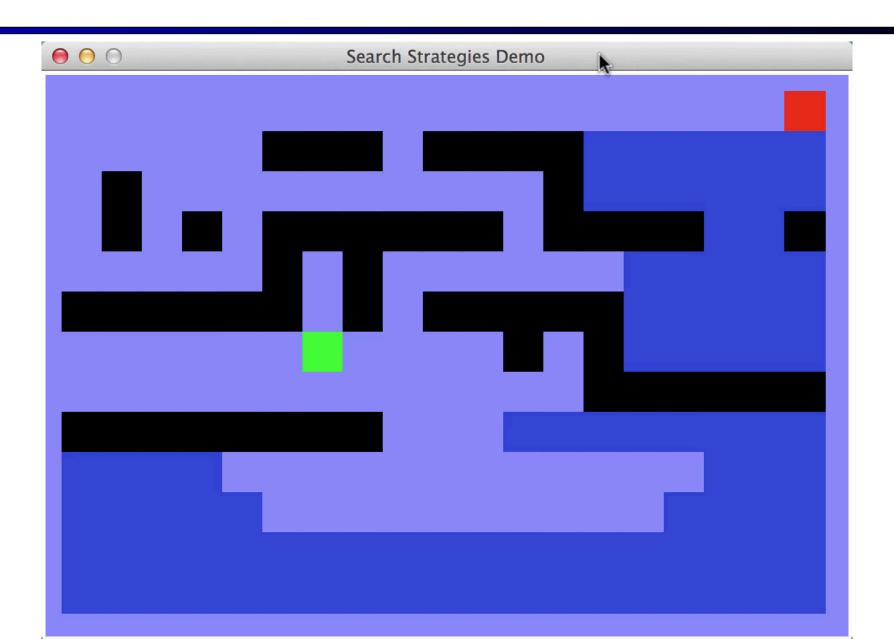
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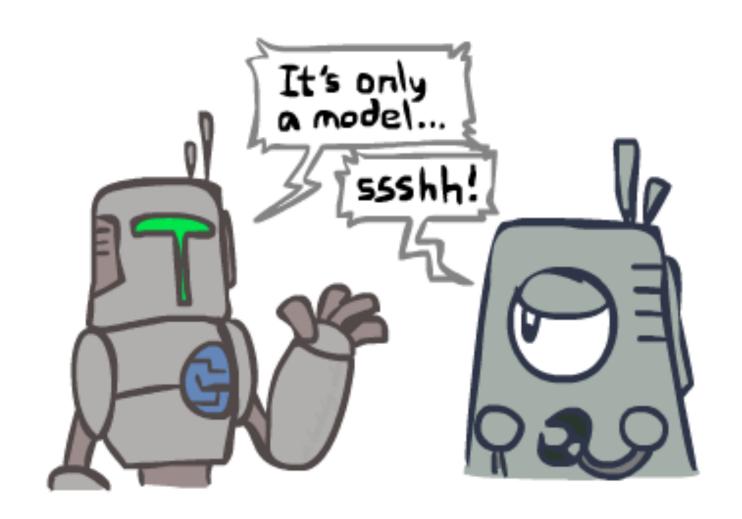


## Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models...

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