My favorite problems

1. 2018 students stand in a circle. We call a student A as *excellent*, if both the left and the right neighbor of A are of the different gender of A. Let B and G be the number of excellent boys and excellent girls respectively, find the maximum value of $B^2 - G^2$.

2. Let $\triangle ABC$ be acute with circumcenter O. Let BO intersect AC at F, and CO intersect AB at E. The perpendicular bisector of EF intersects BC at D. Let ED intersect BF at M, FD intersect CE at N. Assume that the intersection of the perpendicular bisector of EM and that of DF is on EF. Prove that $\angle BAC = 60°$.

3. $A_1, A_2, ..., A_{2^n}$ is an array of all subsets of $\{1, 2, ..., n\}$, and $A_{2^n+1} = A_1$. Find the maximum of

$$\sum_{i=1}^{2^n} |A_i \cap A_{i+1}||A_i \cup A_{i+1}|.$$

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4. Show that for every positive integer n, there exists $k \in \mathbb{N}$ such that exactly n different tuples (x, y, m) satisfies

$$x^m - y^m = k$$

with $x, y, m \in \mathbb{N}$ and $m \geqslant 2$.

5. In the Euclidean plane, we call a line *good*, if it passes through infinitely many lattice points.

Now we color all the lattice points, such that every good line parallel to x-axis or y-axis has infinitely many different colored lattice points on it. Prove or disprove that there exists a good line not parallel to x-axis and y-axis has infinitely many different colored lattice.

6. Is there a sequence of positive real numbers $a_1, a_2, \cdots$, such that for any distinct positive integers i, j, k, there is some triangle with side lengths

$$(i + j)a_k, (j + k)a_i, (k + i)a_j?$$