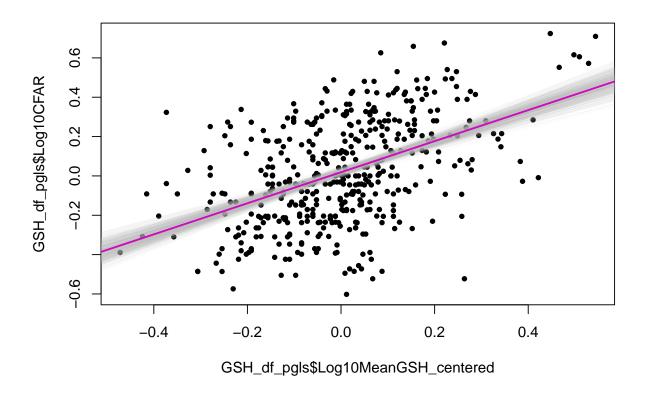
# Ch. 1 - Model comaprisons

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$\label{eq:model-6-CFAR} \mbox{Model 6 - CFAR} \sim \mbox{GSH} + \mbox{Max Size} + \mbox{Phylogeny}  .  .  .  .  .  .  .  .  .  $	17
Model Comparisons  Model 1 - CFAR ~ GSH  brms model  Model_plain <- brm(Log10CFAR ~ Log10MeanGSH_centered, data = GSH_df_pgls,	
<pre>family = gaussian(), prior = Plain_prior )</pre>	
<pre>fixef(Model_plain, summary = T)</pre>	
## Estimate Est.Error Q2.5 Q97.5 ## Intercept 0.01878724 0.01121401 -0.003048758 0.04068312 ## Log10MeanGSH_centered 0.78943177 0.06909079 0.653733169 0.92180824	



```
Data considered

data {
   int <lower=1> N;
   vector[N] x;
   vector[N] y;
}

The parameters accepted by the model

parameters {
   real alpha;
   real beta;
   real<lower=0> sigma;
}

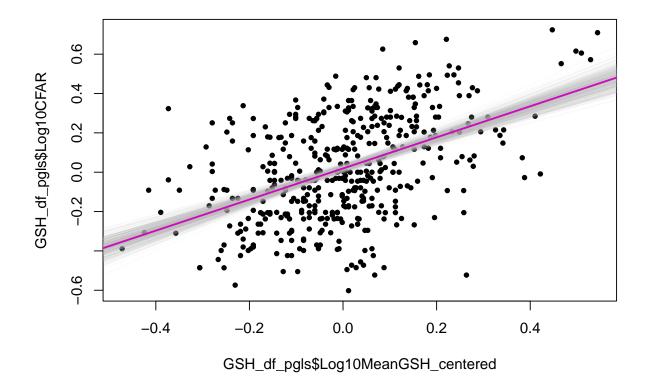
Model form:

model {
   sigma ~ student_t(3, 0, 10);
```

```
y ~ normal(alpha + x * beta , sigma);
}
```

#### fit1

```
## Inference for Stan model: StanModel1.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##
                                               50%
                                                           97.5% n_eff Rhat
           mean se_mean
                           sd
                                2.5%
                                        25%
                                                      75%
           0.02
                   0.00 0.01
                                0.00
                                       0.01
                                              0.02
                                                     0.03
                                                            0.04 2098
## alpha
           0.79
                   0.00 0.07
                                0.66
                                       0.75
                                              0.79
                                                     0.84
                                                            0.93
                                                                  1707
## beta
                                0.22
                                       0.23
                                                     0.24
           0.24
                   0.00 0.01
                                              0.24
                                                            0.25
                                                                  1951
## sigma
## lp__
                   0.03 1.19 426.78 429.34 430.19 430.76 431.27
         429.89
                                                                   1176
##
## Samples were drawn using NUTS(diag_e) at Wed Mar 02 11:19:28 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

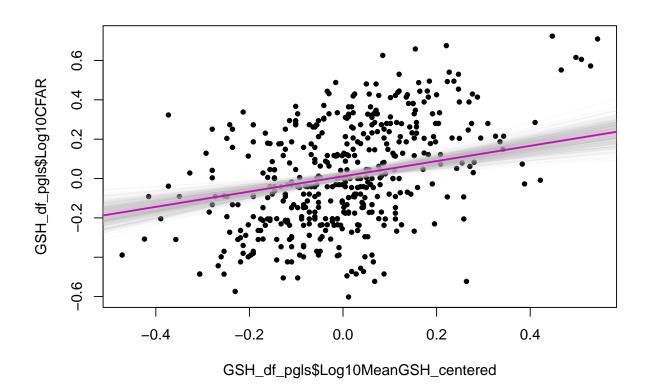


## Model 2 - CFAR $\sim$ GSH \* Max Size

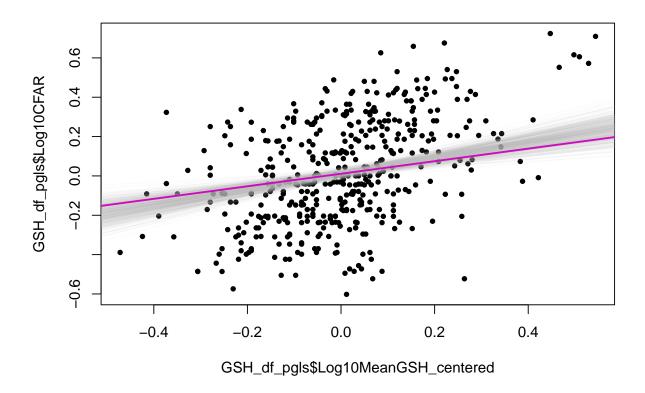
#### brms model

```
fixef(Model_size, summary = T)
```

```
##
                                                 Estimate Est.Error
                                                                             Q2.5
## Intercept
                                               0.01107801 0.01158262 -0.01069365
## Log10MeanGSH_centered
                                               0.38761087 0.08262037 0.22932179
## Log10MaxSize_centered
                                               0.30460627 0.04281846 0.22222482
## Log10MeanGSH_centered:Log10MaxSize_centered 0.26628115 0.16406086 -0.05284798
##
                                                    Q97.5
## Intercept
                                               0.03436049
## Log10MeanGSH_centered
                                               0.55212619
## Log10MaxSize_centered
                                               0.38716498
## Log10MeanGSH_centered:Log10MaxSize_centered 0.58368860
```



```
Data considered
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // CFAR
}
The parameters accepted by the model.
parameters {
  real alpha;
  vector[K] beta;
  real<lower=0> sigma;
}
The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard
deviation 'sigma'.
model {
  sigma ~ student_t(3, 0, 10);
  y ~ normal(alpha + x * beta , sigma);
fit2
## Inference for Stan model: StanModel2.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
                              \operatorname{sd}
                                                                97.5% n_eff Rhat
             mean se_mean
                                   2.5%
                                            25%
                                                   50%
                                                          75%
             0.01
                      0.00 0.01
                                  -0.01
                                          0.00
                                                  0.01
                                                          0.02
                                                                 0.04
                                                                      1841
## alpha
## beta[1]
             0.39
                      0.00 0.08
                                   0.23
                                          0.33
                                                  0.39
                                                         0.44
                                                                 0.55
                                                                       1758
                                                                                1
## beta[2]
             0.31
                      0.00 0.04
                                   0.22
                                          0.28
                                                  0.31
                                                          0.33
                                                                 0.39
                                                                       1742
                                                                                1
                                                  0.27
## beta[3]
             0.26
                      0.00 0.16
                                  -0.06
                                          0.16
                                                          0.37
                                                                 0.59
                                                                       1975
                                                                                1
## sigma
             0.22
                                   0.21
                                           0.22
                                                  0.22
                                                          0.23
                                                                        2516
                      0.00 0.01
                                                                 0.24
                                                                                1
           459.42
                      0.05 1.62 455.40 458.60 459.75 460.60 461.57
                                                                                1
## lp__
## Samples were drawn using NUTS(diag_e) at Wed Mar 02 11:20:54 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

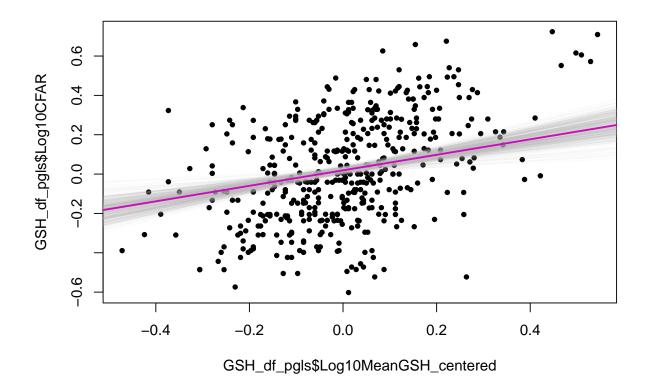


Model 3 - CFAR ~ GSH + Max Size (No interaction)

#### summary(Model\_size\_2)

```
##
   Family: gaussian
##
     Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
            total post-warmup draws = 4000
##
## Population-Level Effects:
                         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
##
                             0.02
                                       0.01
                                                -0.00
                                                          0.04 1.00
## Intercept
                                                                         3526
## Log10MeanGSH_centered
                             0.39
                                        0.08
                                                 0.24
                                                          0.55 1.00
                                                                         3567
## Log10MaxSize_centered
                             0.33
                                       0.04
                                                 0.25
                                                          0.40 1.00
                                                                         3528
```

```
Tail_ESS
##
                             2556
## Intercept
## Log10MeanGSH_centered
                             2768
## Log10MaxSize_centered
                             3043
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
                       0.01
                                0.21
                                         0.24 1.00
             0.22
## sigma
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // CFAR
}
```

The parameters accepted by the model.

```
parameters {
    real alpha;
    vector[K] beta;
    real<lower=0> sigma;
}

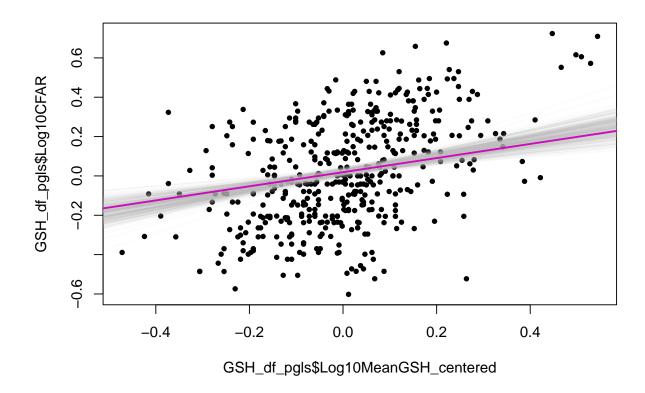
The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

model {
    sigma ~ student_t(3, 0, 10);
    y ~ normal(alpha + x * beta , sigma);
}

fit3

## Inference for Stan model: StanModel2.
```

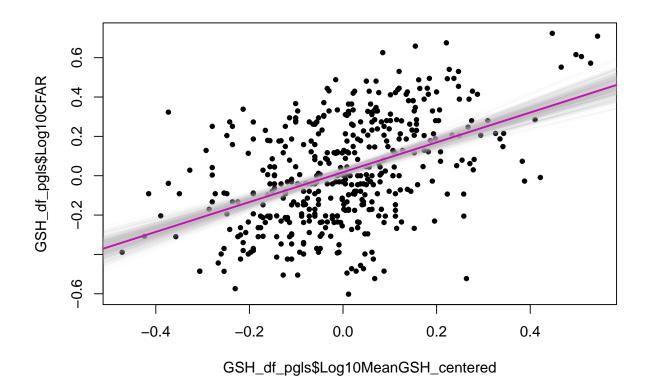
```
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##
             mean se_mean
                                  2.5%
                                          25%
                                                 50%
                                                         75%
                                                              97.5% n_eff Rhat
                            sd
             0.02
                                  0.00
                                                0.02
                                                               0.04
## alpha
                     0.00 0.01
                                         0.01
                                                       0.03
                                                                    2075
## beta[1]
             0.39
                     0.00 0.08
                                  0.23
                                         0.34
                                                0.39
                                                       0.45
                                                               0.55
                                                                     1620
                                                                             1
                                  0.24
## beta[2]
             0.32
                     0.00 0.04
                                         0.30
                                                0.32
                                                       0.35
                                                               0.40
                                                                     1550
                                                                             1
                     0.00 0.01
                                                               0.24
                                                                     1790
## sigma
             0.22
                                  0.21
                                         0.22
                                                0.22
                                                       0.23
                                                                             1
                     0.05 1.41 455.06 458.03 458.99 459.70 460.42
## lp__
           458.67
                                                                      948
                                                                             1
##
## Samples were drawn using NUTS(diag_e) at Wed Mar 02 11:22:15 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```



Model 4 - CFAR  $\sim$  GSH + (1|Max Size) (No interaction)

```
Model_size_3 <- brm(Log10CFAR ~ Log10MeanGSH_centered + (1|Log10MaxSize_centered),</pre>
                    data = GSH_df_pgls,
                    family = gaussian(),
                    prior = Size_prior_3,
                    control = list(adapt_delta = 0.9)) #needed to make model converge ...
"r
summary(Model_size_3)
    Family: gaussian
##
     Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + (1 | Log10MaxSize_centered)
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Log10MaxSize_centered (Number of levels: 222)
```

```
Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
                                         0.01
                                                  0.11 1.01
## sd(Intercept)
                     0.06
                                0.03
                                                                  435
                                                                           525
##
## Population-Level Effects:
                         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
##
## Intercept
                              0.02
                                        0.01
                                                -0.01
                                                          0.04 1.00
                                                                         5528
## Log10MeanGSH_centered
                              0.76
                                        0.07
                                                 0.62
                                                          0.90 1.00
                                                                         4392
                         Tail_ESS
##
## Intercept
                              2718
## Log10MeanGSH_centered
                              2985
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
                                          0.25 1.01
## sigma
             0.23
                       0.01
                                 0.21
                                                         917
                                                                  1516
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



TBD

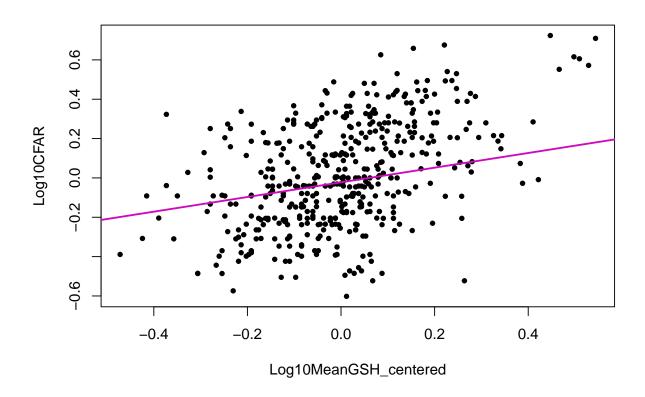
### Model 5 - CFAR $\sim$ GSH + Phylogeny

#### PGLS model

summary(pglsMod1)

pglsMod1 <- gls(Log10CFAR ~ Log10MeanGSH\_centered, correlation = corPagel(phy = SampleTree, value = 0.5

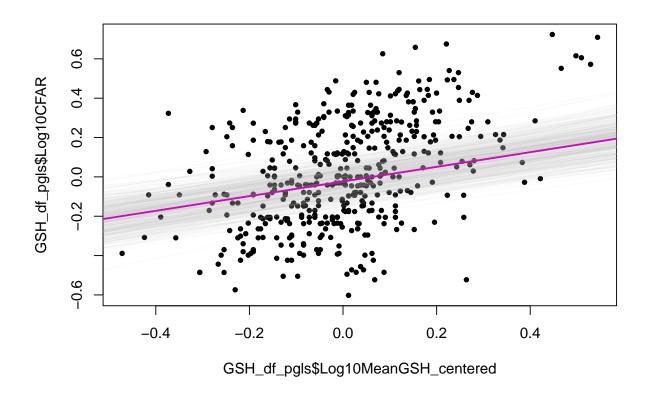
```
## Generalized least squares fit by maximum likelihood
##
     Model: Log10CFAR ~ Log10MeanGSH_centered
     Data: GSH_df_pgls
##
           AIC
##
                     BIC
                           logLik
     -427.4206 -410.9306 217.7103
##
##
## Correlation Structure: corPagel
##
   Formula: ~Binomial
   Parameter estimate(s):
##
      lambda
##
## 0.7030228
##
## Coefficients:
##
                              Value Std.Error
                                                 t-value p-value
## (Intercept)
                         -0.0224754 0.06282858 -0.357726 0.7207
## Log10MeanGSH_centered 0.3728837 0.05950418 6.266513 0.0000
##
   Correlation:
                         (Intr)
##
## Log10MeanGSH_centered -0.065
##
## Standardized residuals:
##
          Min
                      Q1
                                Med
                                            QЗ
                                                      Max
## -2.7884571 -0.6933649 0.1151372 1.0885641 2.9034051
## Residual standard error: 0.2147249
## Degrees of freedom: 456 total; 454 residual
```



#### summary(Model\_simple)

```
##
    Family: gaussian
     Links: mu = identity; sigma = identity
##
## Formula: Log10CFAR ~ Log10MeanGSH_centered + (1 | gr(Binomial, cov = A))
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 4000
##
## Group-Level Effects:
  ~Binomial (Number of levels: 456)
##
##
                 Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)
                     0.01
                               0.00
                                         0.01
                                                  0.01 1.00
                                                                1200
                                                                          1938
## Population-Level Effects:
```

```
Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
##
                                                -0.15
## Intercept
                            -0.02
                                        0.06
                                                          0.10 1.00
                                                                         1257
## Log10MeanGSH_centered
                              0.37
                                        0.06
                                                 0.25
                                                          0.49 1.00
                                                                         4238
                         Tail_ESS
##
## Intercept
                              2076
## Log10MeanGSH_centered
                              3483
## Family Specific Parameters:
##
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
             0.12
                       0.01
                                0.11
                                          0.13 1.00
                                                        1717
## sigma
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



TBD

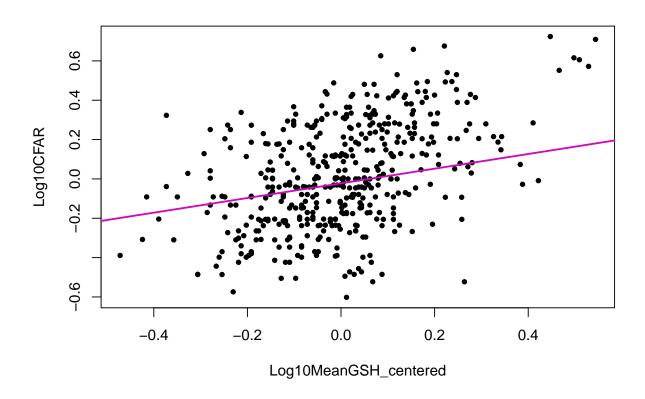
Model 5 - CFAR  $\sim$  GSH \* Max Size + Phylogeny

#### PGLS model

pglsMod2 <- gls(Log10CFAR ~ Log10MeanGSH\_centered \* Log10MaxSize\_centered, correlation = corPagel(phy =

#### summary(pglsMod2)

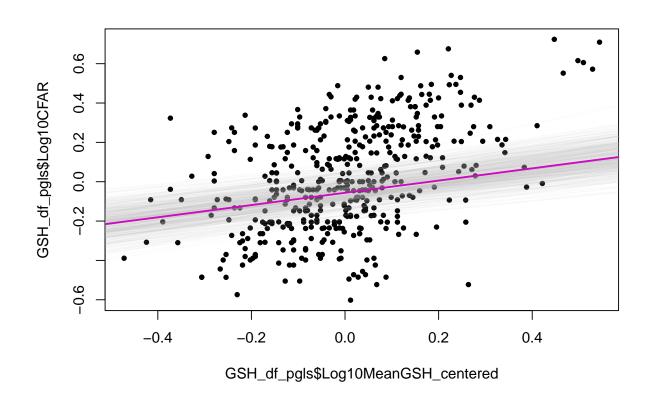
```
## Generalized least squares fit by maximum likelihood
##
    Model: y ~ x
    Data: GSH_df_pgls
##
##
          AIC
                    BIC logLik
##
     -427.4206 -410.9306 217.7103
##
## Correlation Structure: corPagel
## Formula: ~Binomial
## Parameter estimate(s):
##
     lambda
## 0.7030228
##
## Coefficients:
                   Value Std.Error t-value p-value
## (Intercept) -0.0224754 0.06282858 -0.357726 0.7207
## x
              0.3728837 0.05950418 6.266513 0.0000
##
## Correlation:
##
    (Intr)
## x -0.065
##
## Standardized residuals:
                     Q1
         Min
                               Med
                                          QЗ
                                                     Max
## -2.7884571 -0.6933649 0.1151372 1.0885641 2.9034051
##
## Residual standard error: 0.2147249
## Degrees of freedom: 456 total; 454 residual
```



#### summary(Model\_BS)

```
##
   Family: gaussian
     Links: mu = identity; sigma = identity
##
## Formula: Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + (1 | gr(Binomial, cov = A))
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 4000
##
## Group-Level Effects:
  ~Binomial (Number of levels: 456)
##
##
                 Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)
                     0.01
                               0.00
                                         0.01
                                                  0.01 1.00
                                                                1303
                                                                          2376
## Population-Level Effects:
```

```
Estimate Est.Error 1-95% CI
##
## Intercept
                                                    -0.06
                                                               0.06
                                                                       -0.17
## Log10MeanGSH_centered
                                                     0.31
                                                               0.06
                                                                        0.19
## Log10MaxSize_centered
                                                     0.13
                                                               0.04
                                                                        0.06
## Log10MeanGSH_centered:Log10MaxSize_centered
                                                     0.43
                                                               0.12
                                                                        0.20
                                                u-95% CI Rhat Bulk_ESS Tail_ESS
##
## Intercept
                                                     0.05 1.00
                                                                   1824
                                                                            2402
## Log10MeanGSH_centered
                                                     0.43 1.00
                                                                   6595
                                                                            3478
## Log10MaxSize_centered
                                                     0.20 1.00
                                                                   4866
                                                                            3342
## Log10MeanGSH_centered:Log10MaxSize_centered
                                                     0.67 1.00
                                                                   5734
                                                                            3590
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma
             0.12
                       0.01
                                          0.13 1.00
                                                         2164
                                 0.11
                                                                  3283
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



TBD

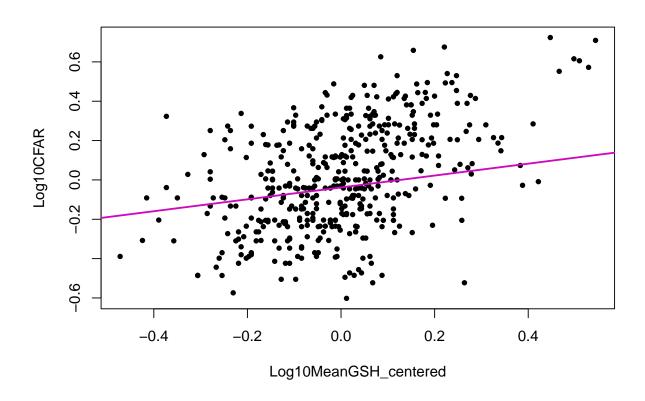
### Model 6 - CFAR $\sim$ GSH + Max Size + Phylogeny

#### PGLS model

summary(pglsMod3)

pglsMod3 <- gls(Log10CFAR ~ Log10MeanGSH\_centered + Log10MaxSize\_centered, correlation = corPagel(phy =

```
## Generalized least squares fit by maximum likelihood
##
    Model: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered
##
     Data: GSH_df_pgls
           AIC
##
                     BIC
                           logLik
     -447.4392 -426.8268 228.7196
##
##
## Correlation Structure: corPagel
## Formula: ~Binomial
  Parameter estimate(s):
      lambda
##
## 0.6786106
##
## Coefficients:
##
                                                  t-value p-value
                               Value Std.Error
## (Intercept)
                         -0.03827951 0.05915461 -0.647110 0.5179
## Log10MeanGSH_centered    0.30222706    0.06034147    5.008613    0.0000
## Log10MaxSize_centered 0.16260955 0.03412415 4.765233 0.0000
##
##
   Correlation:
                         (Intr) L10MGS
##
## Log10MeanGSH_centered -0.050
## Log10MaxSize_centered -0.056 -0.268
##
## Standardized residuals:
##
          Min
                      Q1
                                             QЗ
                                Med
                                                       Max
## -2.8353741 -0.5472167 0.2440250 1.1919410 2.8367324
## Residual standard error: 0.2047976
## Degrees of freedom: 456 total; 453 residual
```



TBD

STAN model

TBD