

Ch. 1 - Model comparisons

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Model Comparisons

Model 1 - CFAR ~ GSH

1. brms model

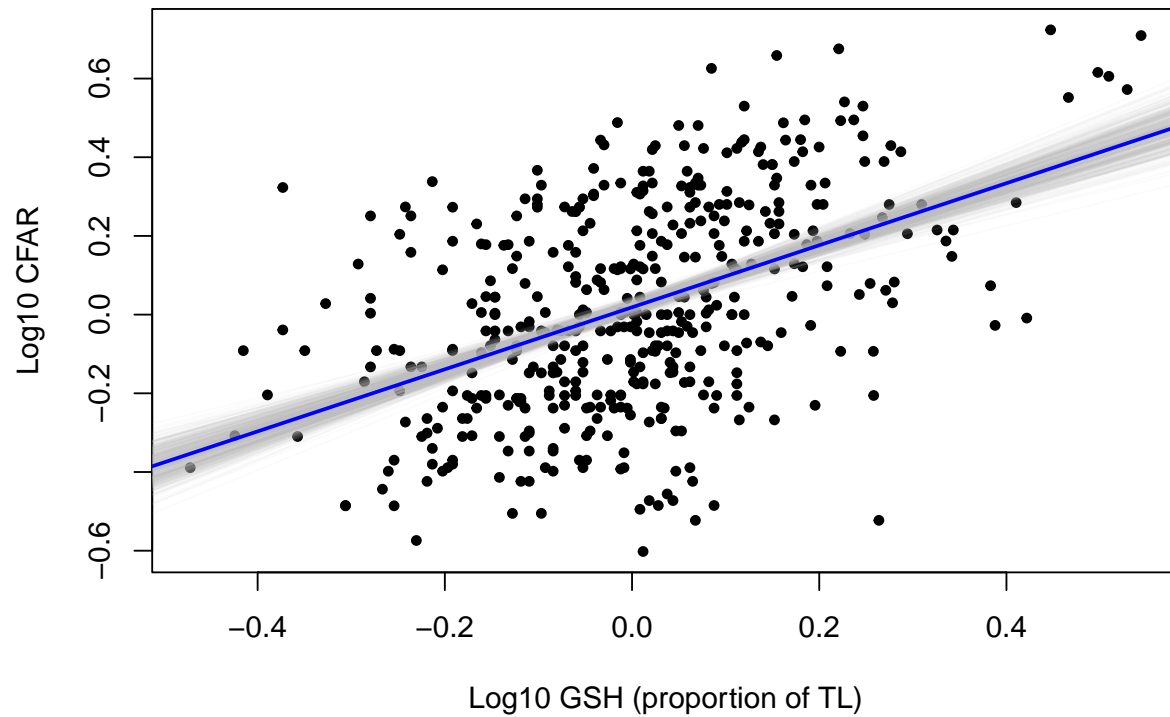
```
Model_plain <- brm(Log10CFAR ~ Log10MeanGSH_centered,  
  data = GSH_df_pglis,  
  family = gaussian(),  
  prior = Plain_prior  
)
```

```
knitr::kable(fixef(Model_plain, summary = T))
```

	Estimate	Est.Error	Q2.5	Q97.5
Intercept	0.0186040	0.0114242	-0.003649	0.0401433

	Estimate	Est.Error	Q2.5	Q97.5
Log10MeanGSH_centered	0.7884163	0.0697968	0.651545	0.9267644

Correlation= 0.471640472505464



2. STAN model

Data considered

```
data {
  int <lower=1> N;
  vector[N] x;
  vector[N] y;
}
```

The parameters accepted by the model

```
parameters {
  real alpha;
  real beta;
  real<lower=0> sigma;
}
```

Model form:

```

model {

  sigma ~ student_t(3, 0, 10);

  y ~ normal(alpha + x * beta , sigma);
}

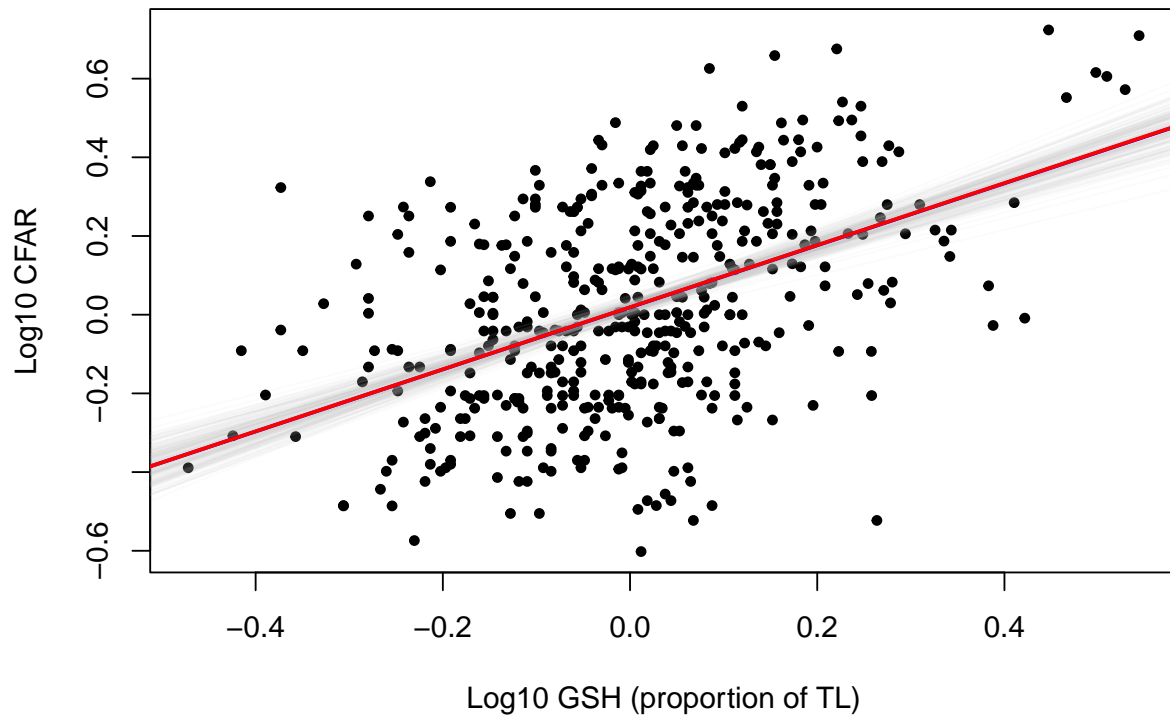
knitr::kable(fit1_summary, caption = 'STAN model summary')

```

Table 2: STAN model summary

	mean	se_mean	sd	2.5%	97.5%	n_eff	Rhat
alpha	0.0190087	0.0002460	0.0111142	-0.0023408	0.0404493	2041.046	1.0004828
beta	0.7889437	0.0017258	0.0719480	0.6467919	0.9279862	1738.128	0.9988377
sigma	0.2357984	0.0001759	0.0077345	0.2210283	0.2509776	1934.501	0.9998605

Correlation= 0.471640472505464



Model 2 - CFAR ~ GSH * Max Size

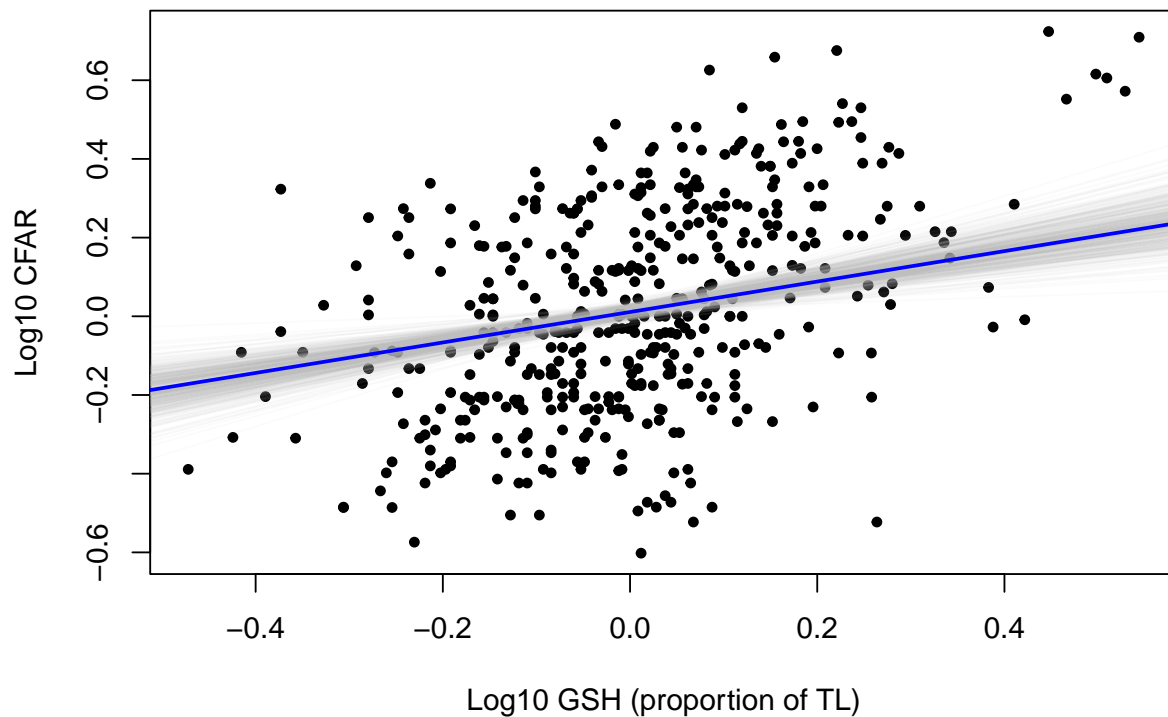
1. brms model

```
Model_size <- brm(Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered,  
  data = GSH_df_pgls,  
  family = gaussian(),  
  prior = Size_prior  
)
```

```
fixef(Model_size, summary = T)
```

##	Estimate	Est.Error	Q2.5
## Intercept	0.01086126	0.01162880	-0.01166550
## Log10MeanGSH_centered	0.38704145	0.08131335	0.22822665
## Log10MaxSize_centered	0.30527108	0.04319407	0.21824142
## Log10MeanGSH_centered:Log10MaxSize_centered	0.26337560	0.16627278	-0.06814918
##	Q97.5		
## Intercept	0.03383283		
## Log10MeanGSH_centered	0.54790906		
## Log10MaxSize_centered	0.38913950		
## Log10MeanGSH_centered:Log10MaxSize_centered	0.58814963		

Correlation= 0.471640472505464



2. STAN model

Data considered

```
data {  
  int <lower=0> N; // number of data points  
  int <lower=0> K; // number of predictors  
  matrix[N,K] x; // predictor matrix  
  vector[N] y; // CFAR  
}
```

The parameters accepted by the model.

```
parameters {  
  real alpha;  
  vector[K] beta;  
  real<lower=0> sigma;  
}
```

The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

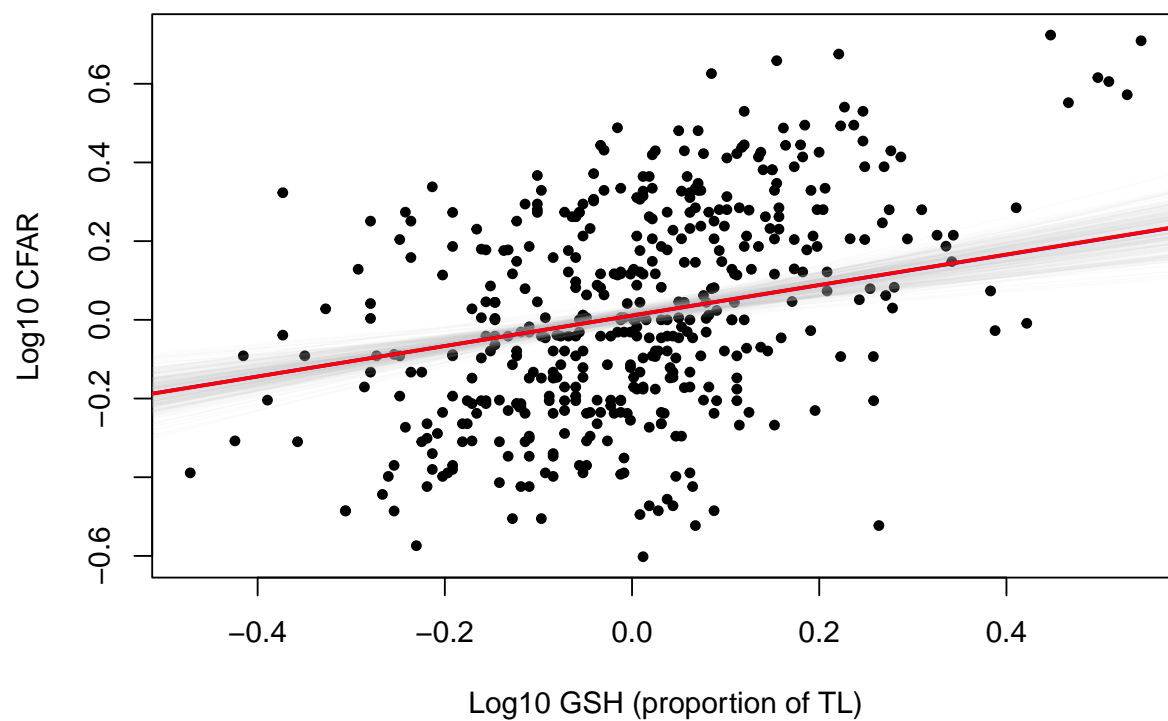
```
model {  
  
  sigma ~ student_t(3, 0, 10);  
  
  y ~ normal(alpha + x * beta , sigma);  
}
```

```
knitr::kable(fit2_summary, caption = 'STAN model summary')
```

Table 3: STAN model summary

	mean	se_mean	sd	2.5%	97.5%	n_eff	Rhat
beta[1]	0.0110471	0.0002538	0.0113730	-0.0103597	0.0340287	2008.373	0.9984008
beta[2]	0.3877210	0.0019674	0.0823295	0.2299960	0.5541936	1751.211	0.9999482
beta[3]	0.3051307	0.0010166	0.0433365	0.2186471	0.3898286	1817.070	1.0004711
beta[4]	0.2555854	0.0037876	0.1580119	-0.0597708	0.5605782	1740.451	0.9990472
sigma	0.2211473	0.0001653	0.0075782	0.2075152	0.2370725	2100.645	0.9996704

Correlation= 0.471640472505464



Model 3 - CFAR ~ GSH + Max Size (No interaction)

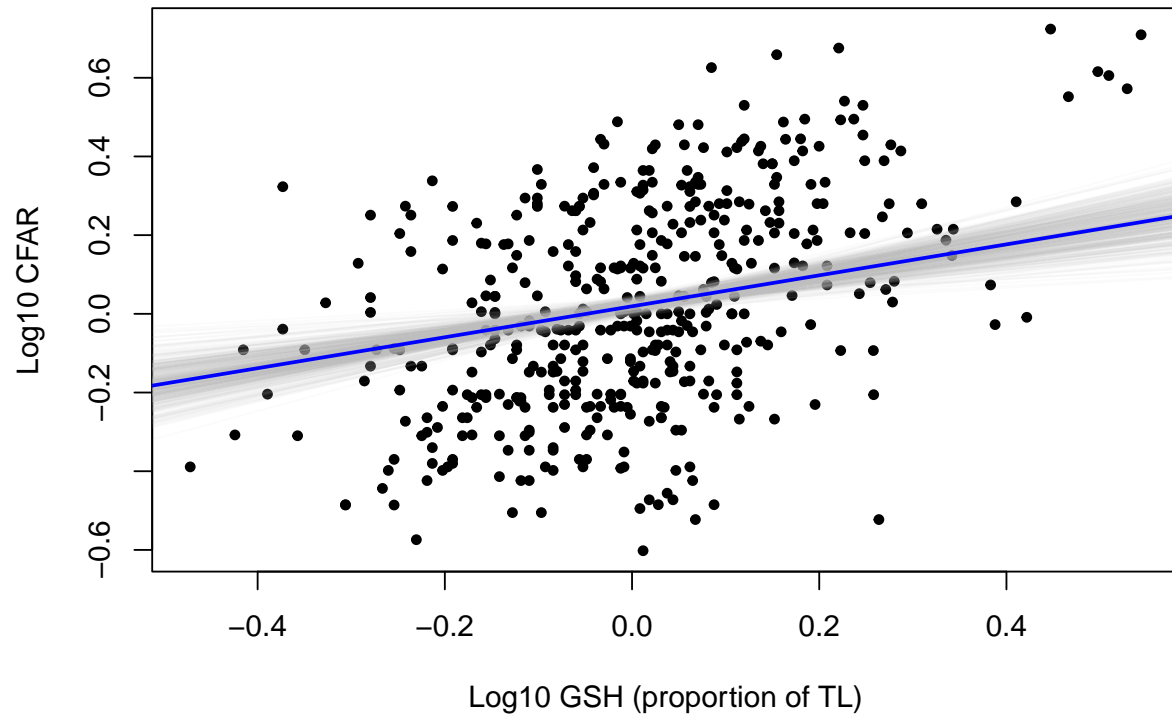
1. brms model

```
Model_size_2 <- brm(Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered,  
  data = GSH_df_pgls,  
  family = gaussian(),  
  prior = Size_prior_2)
```

```
summary(Model_size_2)
```

```
## Family: gaussian  
## Links: mu = identity; sigma = identity  
## Formula: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered  
## Data: GSH_df_pgls (Number of observations: 456)  
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;  
## total post-warmup draws = 4000  
##  
## Population-Level Effects:  
##               Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS  
## Intercept           0.02      0.01  -0.00   0.04 1.00    4082  
## Log10MeanGSH_centered 0.39      0.08   0.23   0.56 1.00    3089  
## Log10MaxSize_centered 0.33      0.04   0.25   0.41 1.00    3140  
##               Tail_ESS  
## Intercept           2916  
## Log10MeanGSH_centered 3017  
## Log10MaxSize_centered 3214  
##  
## Family Specific Parameters:  
##               Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS  
## sigma          0.22      0.01   0.21   0.24 1.00    3911    2661  
##  
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS  
## and Tail_ESS are effective sample size measures, and Rhat is the potential  
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



2. STAN model

Data considered

```
data {  
  int <lower=0> N; // number of data points  
  int <lower=0> K; // number of predictors  
  matrix[N,K] x; // predictor matrix  
  vector[N] y; // CFAR  
}
```

The parameters accepted by the model.

```
parameters {  
  real alpha;  
  vector[K] beta;  
  real<lower=0> sigma;  
}
```

The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

```
model {
```



```

sigma ~ student_t(3, 0, 10);

y ~ normal(alpha + x * beta , sigma);
}

fit3_summary <- summary(fit3, pars = c("beta[1]", "beta[2]", "sigma"), probs = c(0.025, 0.975))$summary

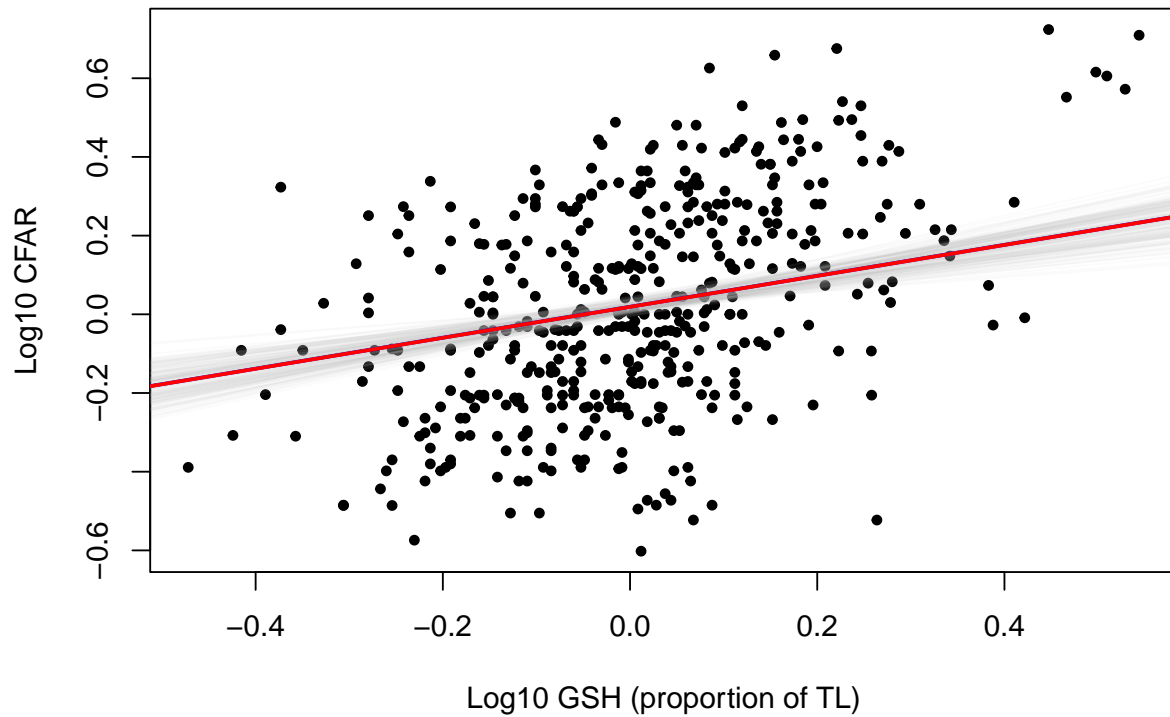
knitr::kable(fit3_summary, caption = 'STAN model summary')

```

Table 4: STAN model summary

	mean	se_mean	sd	2.5%	97.5%	n_eff	Rhat
beta[1]	0.3933554	0.0020241	0.0814803	0.2277471	0.5490099	1620.408	0.9990436
beta[2]	0.3242394	0.0010396	0.0409238	0.2412206	0.4017377	1549.592	1.0000786
sigma	0.2214438	0.0001766	0.0074746	0.2068589	0.2368011	1790.440	1.0004027

Correlation= 0.471640472505464



Model 4 - CFAR ~ GSH + PrimaryHabitat (No interaction)

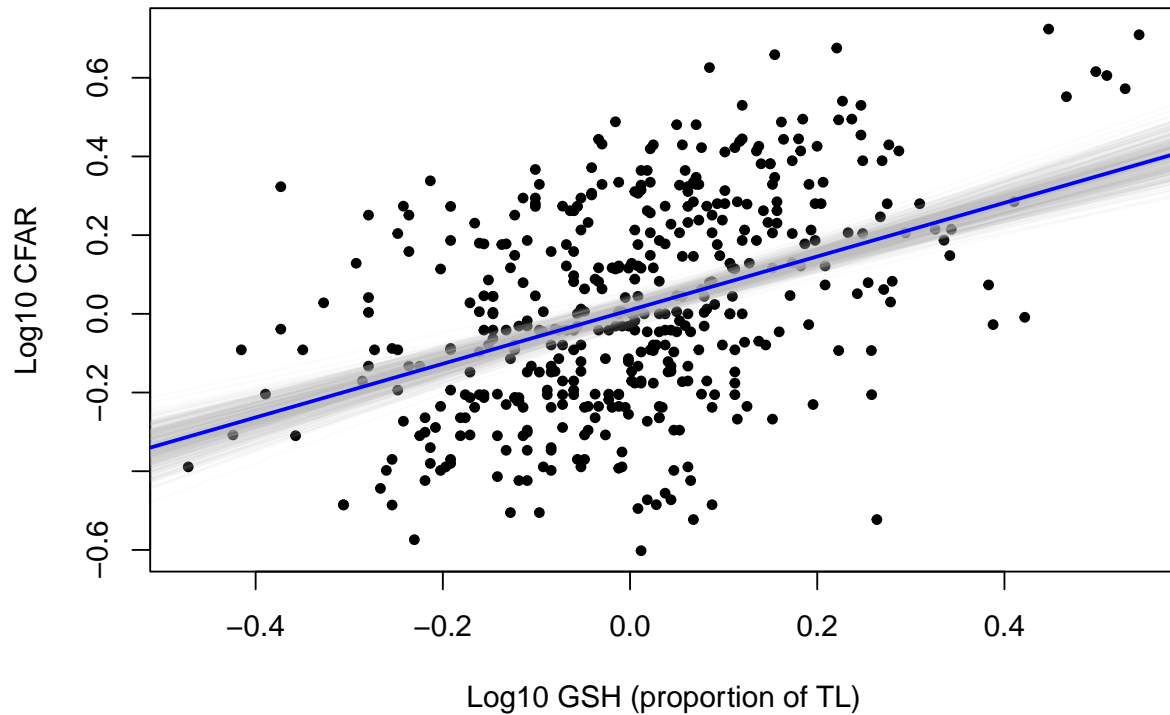
1. brms model

```
habitat_model <- brm(Log10CFAR ~ Log10MeanGSH_centered + PrimaryHabitat,
  data = GSH_df_pgls,
  family = gaussian(),
  prior = habitat_prior,
  cores = 2)
```

```
summary(habitat_model)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + PrimaryHabitat
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Population-Level Effects:
##           Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
## Intercept           0.01      0.02   -0.02    0.04 1.00     4378
## Log10MeanGSH_centered 0.68      0.07    0.54    0.82 1.00     4023
## PrimaryHabitatDeepwater -0.01     0.02   -0.05    0.04 1.00     4456
## PrimaryHabitatPelagic  0.25      0.05    0.15    0.35 1.00     4574
##           Tail_ESS
## Intercept          3214
## Log10MeanGSH_centered 3226
## PrimaryHabitatDeepwater 3080
## PrimaryHabitatPelagic  3175
##
## Family Specific Parameters:
##           Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma      0.23      0.01    0.21    0.25 1.00     5224     2998
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



2. STAN model

Data considered

```
data {  
  int <lower=0> N; // number of data points  
  int <lower=0> K; // number of predictors  
  matrix[N,K] x; // predictor matrix  
  vector[N] y; // CFAR  
}
```

The parameters accepted by the model.

```
parameters {  
  real alpha;  
  vector[K] beta;  
  real<lower=0> sigma;  
}
```

The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

```
model {
```

```

sigma ~ student_t(3, 0, 10);

y ~ normal(alpha + x * beta , sigma);
}

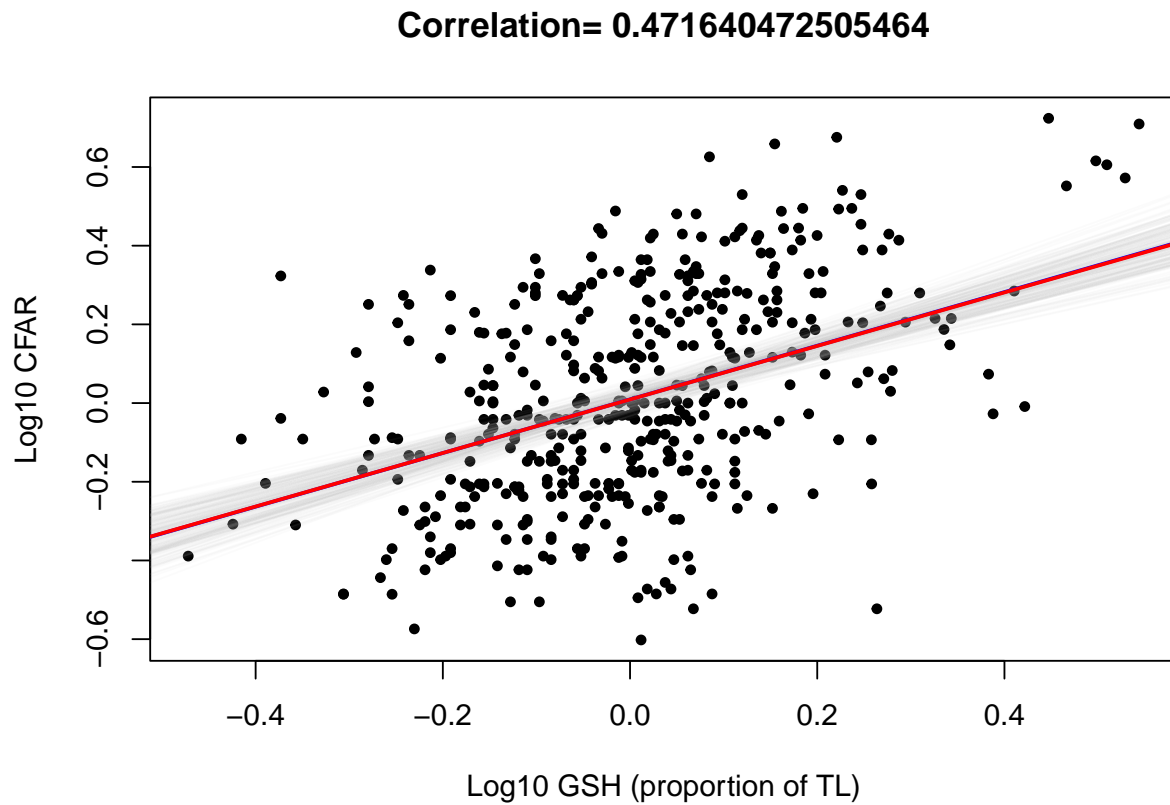
fit6_summary <- summary(fit6, pars = c("beta[1]", "beta[2]", "beta[3]", "beta[4]", "sigma"), probs = c(0.025, 0.975))

knitr::kable(fit6_summary, caption = 'STAN model summary')

```

Table 5: STAN model summary

	mean	se_mean	sd	5%	95%	n_eff	Rhat
beta[1]	0.0091105	0.0004736	0.0166776	-0.0187022	0.0361588	1239.848	1.0005826
beta[2]	0.6799882	0.0019090	0.0751355	0.5557578	0.8052186	1549.018	0.9991609
beta[3]	-0.0094880	0.0006463	0.0233858	-0.0482268	0.0309939	1309.108	1.0000710
beta[4]	0.2501865	0.0012114	0.0481924	0.1721314	0.3327457	1582.632	1.0007002
sigma	0.2293831	0.0001781	0.0075067	0.2174131	0.2416877	1775.682	1.0003352



Model 5 - CFAR ~ GSH * PrimaryHabitat

1. brms model

```
habitat_model_2 <- brm(Log10CFAR ~ Log10MeanGSH_centered * PrimaryHabitat,
  data = GSH_df_pgls,
  family = gaussian(),
  prior = habitat_prior2,
  cores = 2)
```

```
summary(habitat_model_2)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered * PrimaryHabitat
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Population-Level Effects:
##
```

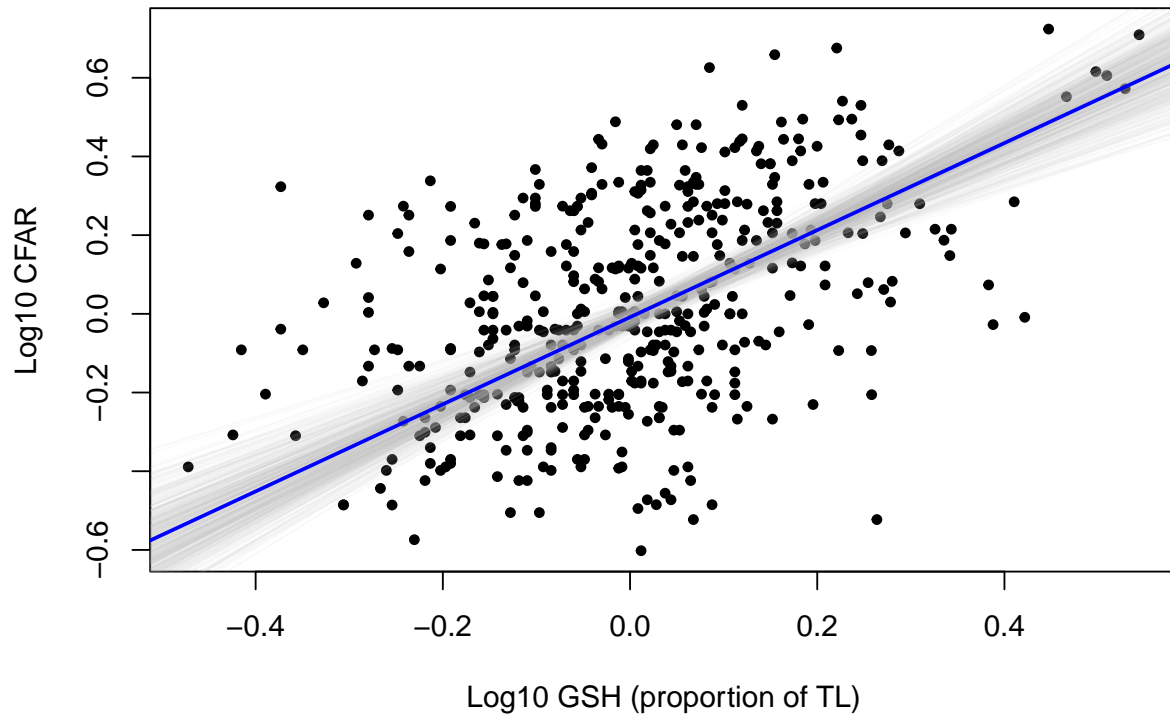
	Estimate	Est.Error	1-95% CI
## Intercept	-0.01	0.02	-0.04
## Log10MeanGSH_centered	1.11	0.13	0.85
## PrimaryHabitatDeepwater	-0.00	0.02	-0.05
## PrimaryHabitatPelagic	0.29	0.06	0.18
## Log10MeanGSH_centered:PrimaryHabitatDeepwater	-0.64	0.17	-0.97
## Log10MeanGSH_centered:PrimaryHabitatPelagic	-0.56	0.23	-1.01

```
##
```

	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
## Intercept	0.03	1.00	3454	2997
## Log10MeanGSH_centered	1.37	1.00	1919	2386
## PrimaryHabitatDeepwater	0.04	1.00	3794	2979
## PrimaryHabitatPelagic	0.40	1.00	3394	3042
## Log10MeanGSH_centered:PrimaryHabitatDeepwater	-0.30	1.00	2102	2147
## Log10MeanGSH_centered:PrimaryHabitatPelagic	-0.13	1.00	2258	2664

```
##
## Family Specific Parameters:
## Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma 0.23 0.01 0.21 0.24 1.00 3580 2618
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



2. STAN model

Data considered

```
data {  
  int <lower=0> N; // number of data points  
  int <lower=0> K; // number of predictors  
  matrix[N,K] x; // predictor matrix  
  vector[N] y; // CFAR  
}
```

The parameters accepted by the model.

```
parameters {  
  real alpha;  
  vector[K] beta;  
  real<lower=0> sigma;  
}
```

The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

```
model {
```

```

sigma ~ student_t(3, 0, 10);

y ~ normal(alpha + x * beta , sigma);
}

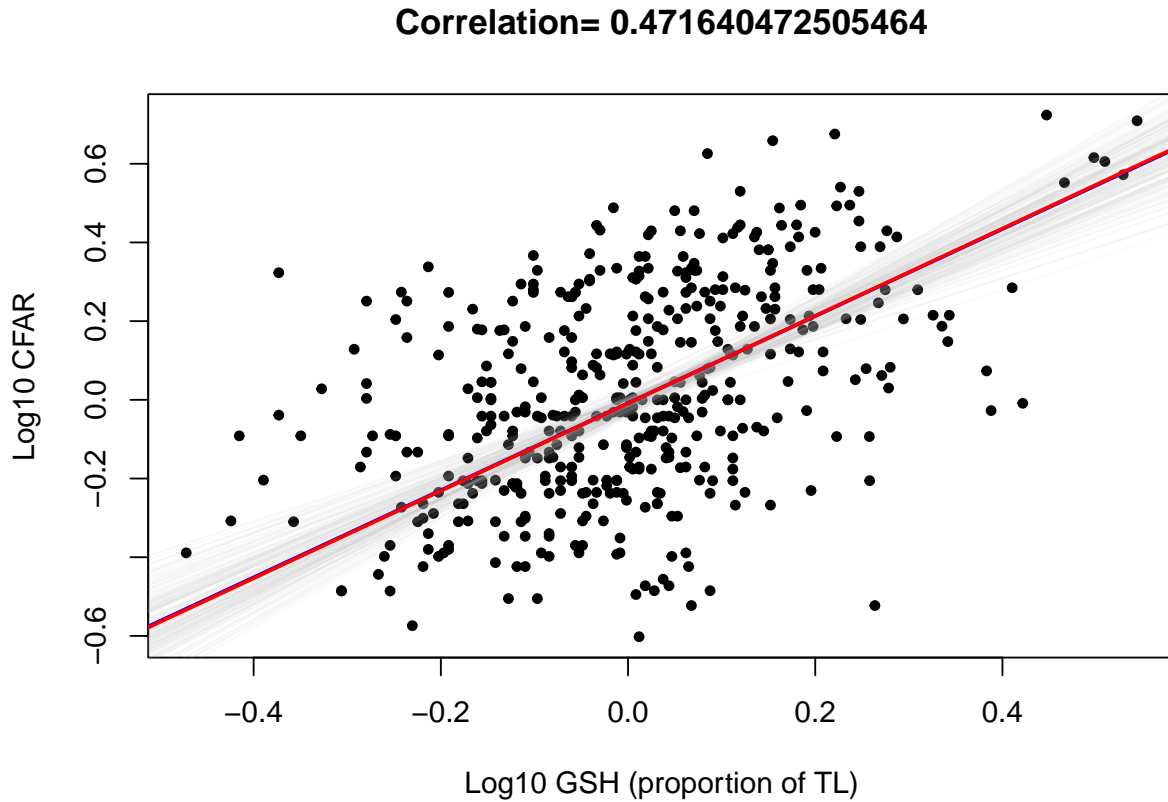
fit7_summary <- summary(fit7, pars = c("beta[1]", "beta[2]", "beta[3]", "beta[4]", "beta[5]", "beta[6]", "sigma"))

knitr::kable(fit7_summary, caption = 'STAN model summary')

```

Table 6: STAN model summary

	mean	se_mean	sd	5%	95%	n_eff	Rhat
beta[1]	-0.0090254	0.0004529	0.0161585	-0.0355365	0.0178685	1272.809	1.000057
beta[2]	1.1110802	0.0037731	0.1225481	0.9043711	1.3095872	1054.926	1.002573
beta[3]	-0.0026746	0.0006063	0.0221765	-0.0390670	0.0330421	1337.945	1.001677
beta[4]	0.2947284	0.0014465	0.0560107	0.2003516	0.3865111	1499.447	1.000261
beta[5]	-0.6441106	0.0047495	0.1571603	-0.8969147	-0.3792308	1094.938	1.002106
beta[6]	-0.5689295	0.0062937	0.2160958	-0.9398260	-0.2136299	1178.923	1.002095
sigma	0.2260172	0.0001645	0.0075462	0.2138852	0.2383921	2105.021	1.000974



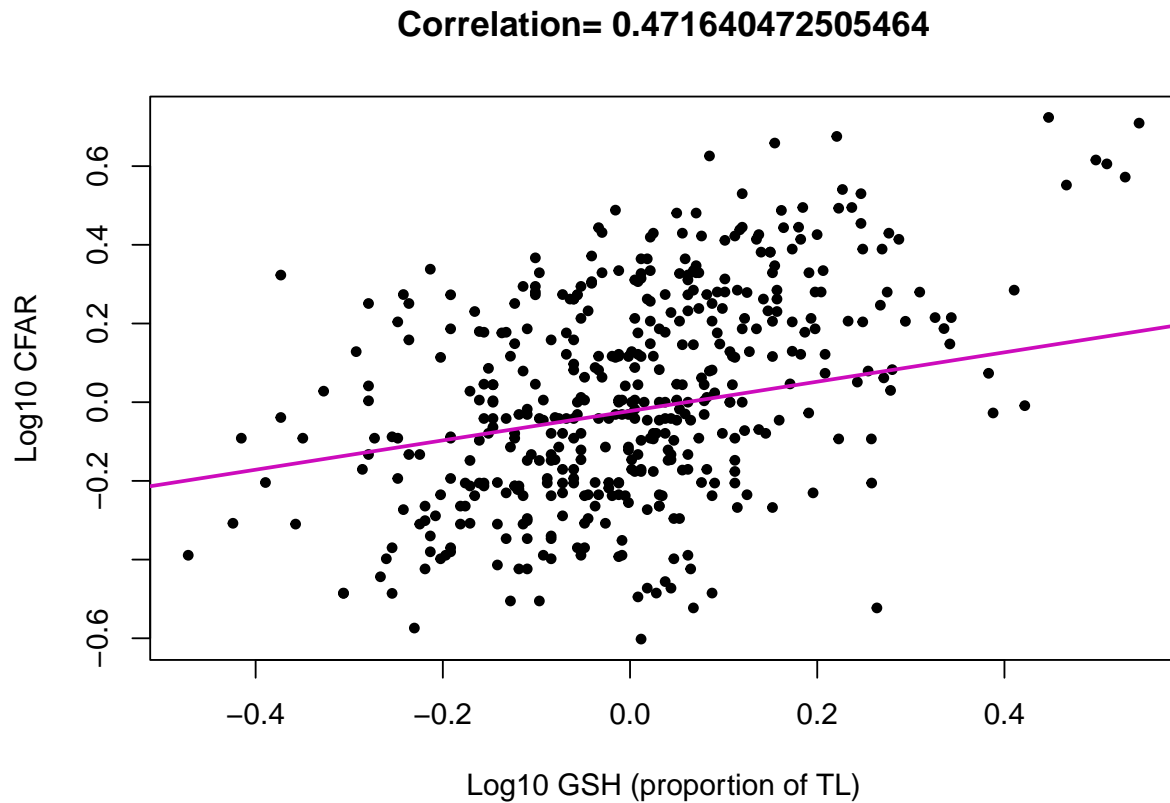
Model 6 - CFAR ~ GSH + Phylogeny

1. PGLS model

```
pglsMod1 <- gls(Log10CFAR ~ Log10MeanGSH_centered, correlation = corPagel(phy = SampleTree, value = 0.5
```

Table 7: PGLS summary

	Value	Std.Error	t-value	p-value	2.5 %	97.5 %
(Intercept)	-0.0224754	0.0628286	-0.3577261	0.7207146	-0.1456172	0.1006663
Log10MeanGSH_centered	0.3728837	0.0595042	6.2665128	0.0000000	0.2562576	0.4895097



2. brms model

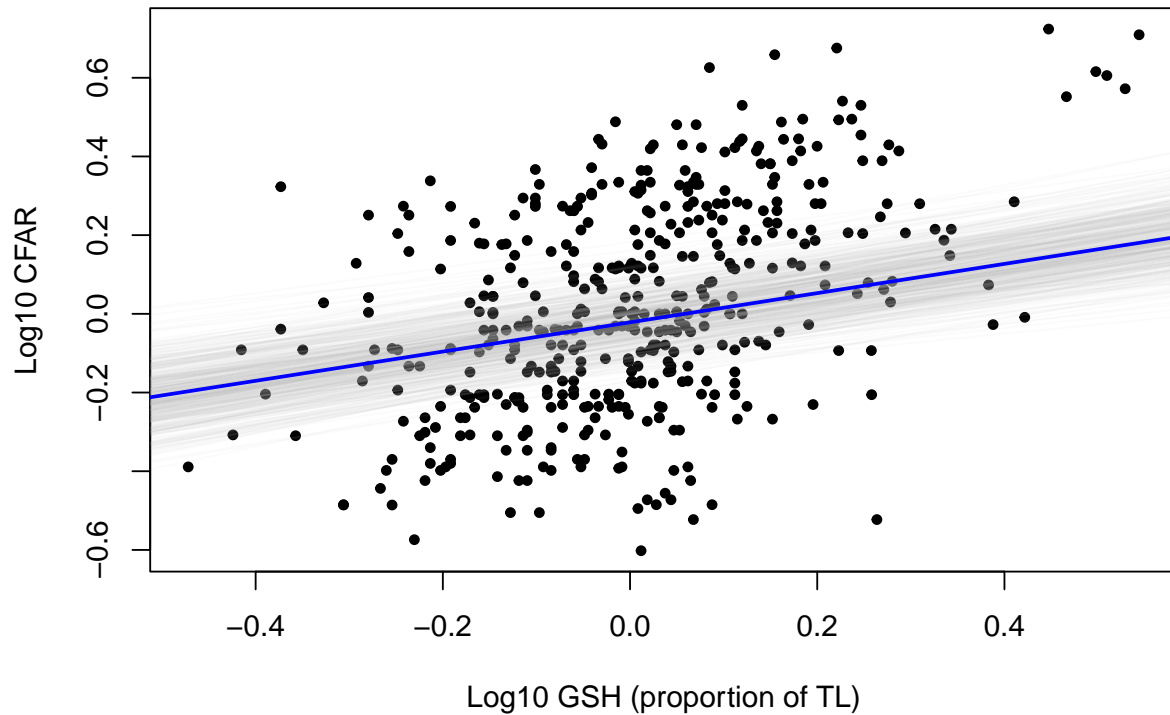
```
Model_simple <- brm(Log10CFAR ~ Log10MeanGSH_centered + (1|gr(Binomial, cov = A)),  
  data = GSH_df_pgls,  
  family = gaussian(),  
  data2 = list(A = A),  
  prior = Simple_prior,  
  cores = 2)
```



```
summary(Model_simple)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + (1 | gr(Binomial, cov = A))
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Binomial (Number of levels: 456)
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)    0.18     0.01    0.16    0.22 1.00    1340    2231
##
## Population-Level Effects:
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## Intercept          -0.02     0.06   -0.15    0.11 1.00    1435
## Log10MeanGSH_centered  0.37     0.06    0.25    0.49 1.00    4722
##      Tail_ESS
## Intercept          2029
## Log10MeanGSH_centered 3623
##
## Family Specific Parameters:
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma    0.12     0.01    0.11    0.13 1.00    2140    2875
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



3. STAN model

Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // outcome (CFAR)
  matrix[N, N] d_mat; // sigma matrix
  matrix[N, N] A; // vcov matrix
}
```

The parameters accepted by the model.

```
parameters {
  real alpha;
  vector[K] beta; // coefficients
  real<lower=0> sigma; // error
  real<lower=0,upper=1> lambda; // phylogenetic signal
}
```

```
transformed parameters {
```

```

matrix[N, N] sigma_mat;
matrix[N, N] sigma_total;

vector[N] mu_y;

sigma_mat = (1-lambda)*d_mat + lambda*A;
sigma_total = sigma*sigma_mat;

}

```

The model to be estimated. We model the output ‘y’ to be normally distributed with mean ‘mu’ and standard deviation ‘sigma’.

```

model {

  beta ~ student_t(3, 0, 10);
  lambda ~ uniform(0,1);
  sigma ~ student_t(3, 0, 2.5);

  y ~ multi_normal(alpha + x * beta, sigma_total);
}

```

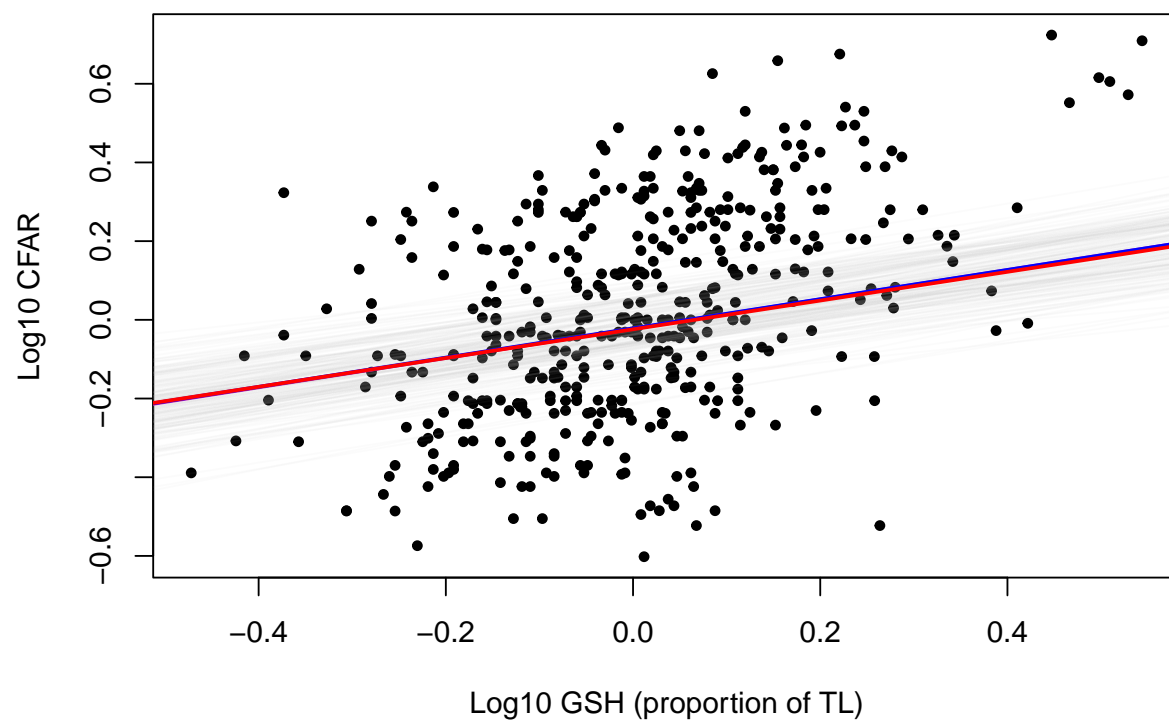
```
fit4
```

```

## Inference for Stan model: StanModel3.
## 4 chains, each with iter=1000; warmup=500; thin=10;
## post-warmup draws per chain=50, total post-warmup draws=200.
##
##           mean se_mean   sd  2.5%   25%   50%   75%  97.5% n_eff Rhat
## alpha    -0.02     0.0 0.06  -0.15  -0.06  -0.03   0.02   0.09  219 0.99
## beta[1]   0.37     0.0 0.06   0.25   0.33   0.36   0.41   0.47  216 1.01
## lp__      625.81     0.1 1.36 622.82 625.03 626.10 626.84 627.47  193 1.02
##
## Samples were drawn using NUTS(diag_e) at Thu Mar 03 14:30:50 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).

```

Correlation= 0.471640472505464



Model 7 - CFAR ~ GSH * Max Size + Phylogeny

1. PGLS model

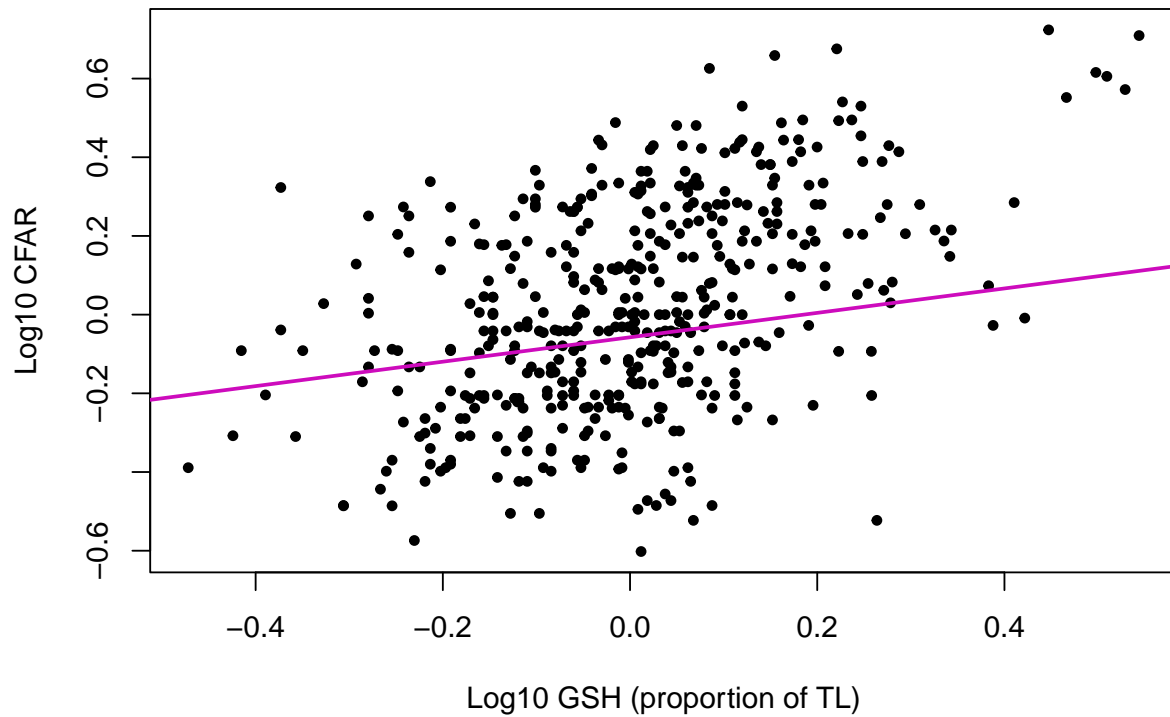
```
pglsMod2 <- gls(Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered, correlation = corPagel(phy =
```

```
Table_fun(pglsMod2)
```

Table 8: PGLS summary

	Value	Std.Error	t-value	p-value	2.5 %	97.5 %
(Intercept)	-	0.0576548	-	0.3184390	-	0.0554173
	0.0575841		0.998773		0.1705854	
Log10MeanGSH_centered	0.3102797	0.0596610	5.200711	0.0000003	0.1933462	0.4272131
Log10MaxSize_centered	0.1272174	0.0352490	3.609111	0.0003416	0.0581307	0.1963041
Log10MeanGSH_centered:Log10MaxSize_centered	0.4391177	0.1232626	3.516191	0.0004820	0.1918245	0.6750050

Correlation= 0.471640472505464



2. brms model

```
Model_BS <- brm(Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + (1|gr(Binomial, cov = A)),
  data = GSH_df_pgls,
  family = gaussian(),
```

```

data2 = list(A = A),
prior = BS_prior,
sample_prior = TRUE, chains = 4, cores = 2)

```

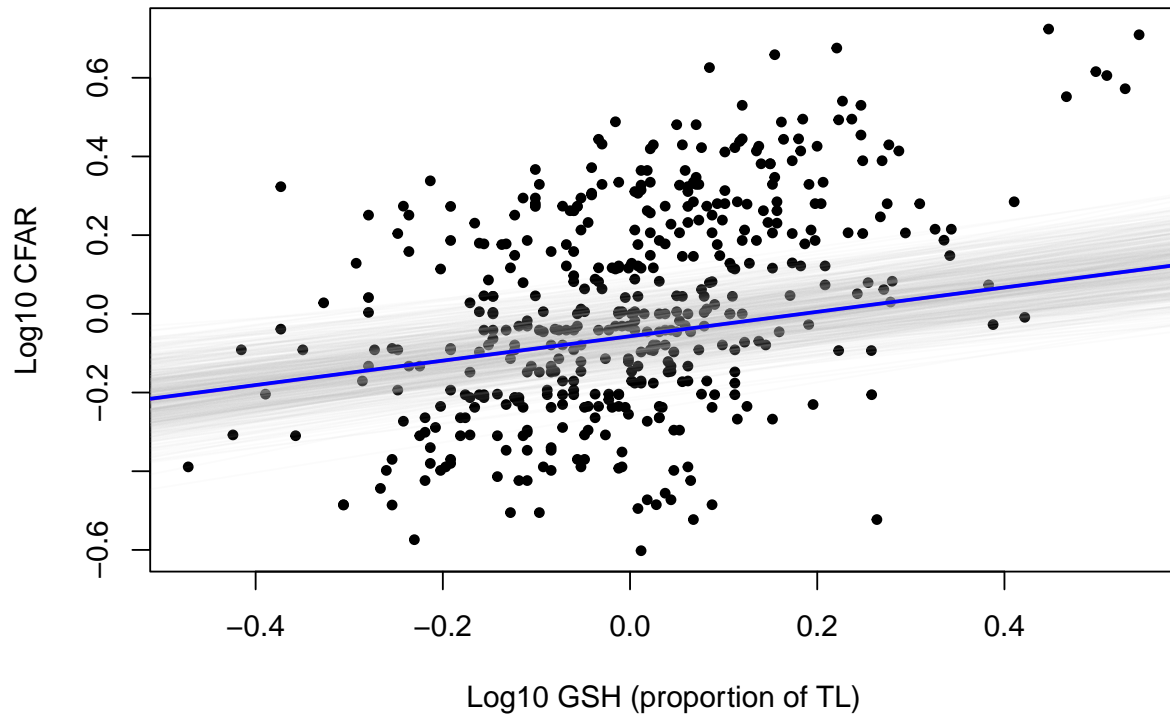
```
summary(Model_BS)
```

```

## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + (1 | gr(Binomial, cov = A))
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Binomial (Number of levels: 456)
##      Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)      0.17      0.01      0.14      0.20 1.00      1331      2195
##
## Population-Level Effects:
##      Estimate Est.Error 1-95% CI
## Intercept      -0.06      0.06     -0.17
## Log10MeanGSH_centered      0.31      0.06      0.19
## Log10MaxSize_centered      0.13      0.04      0.06
## Log10MeanGSH_centered:Log10MaxSize_centered      0.43      0.12      0.19
##      u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept      0.06 1.00      1706      2225
## Log10MeanGSH_centered      0.43 1.00      5377      3443
## Log10MaxSize_centered      0.20 1.00      3787      3425
## Log10MeanGSH_centered:Log10MaxSize_centered      0.68 1.00      5000      3365
##
## Family Specific Parameters:
##      Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma      0.12      0.01      0.11      0.13 1.00      2213      2863
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).

```

Correlation= 0.471640472505464



3. STAN model

Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // outcome (CFAR)
  matrix[N, N] d_mat; // sigma matrix
  matrix[N, N] A; // vcov matrix
}
```

The parameters accepted by the model.

```
parameters {
  real alpha;
  vector[K] beta; // coefficients
  real<lower=0> sigma; // error
  real<lower=0,upper=1> lambda; // phylogenetic signal
}
```

```
transformed parameters {
```

```

matrix[N, N] sigma_mat;
matrix[N, N] sigma_total;

vector[N] mu_y;

sigma_mat = (1-lambda)*d_mat + lambda*A;
sigma_total = sigma*sigma_mat;

}

```

The model to be estimated. We model the output ‘y’ to be normally distributed with mean ‘mu’ and standard deviation ‘sigma’.

```

model {

  beta ~ student_t(3, 0, 10);
  lambda ~ uniform(0,1);
  sigma ~ student_t(3, 0, 2.5);

  y ~ multi_normal(alpha + x * beta, sigma_total);
}

```

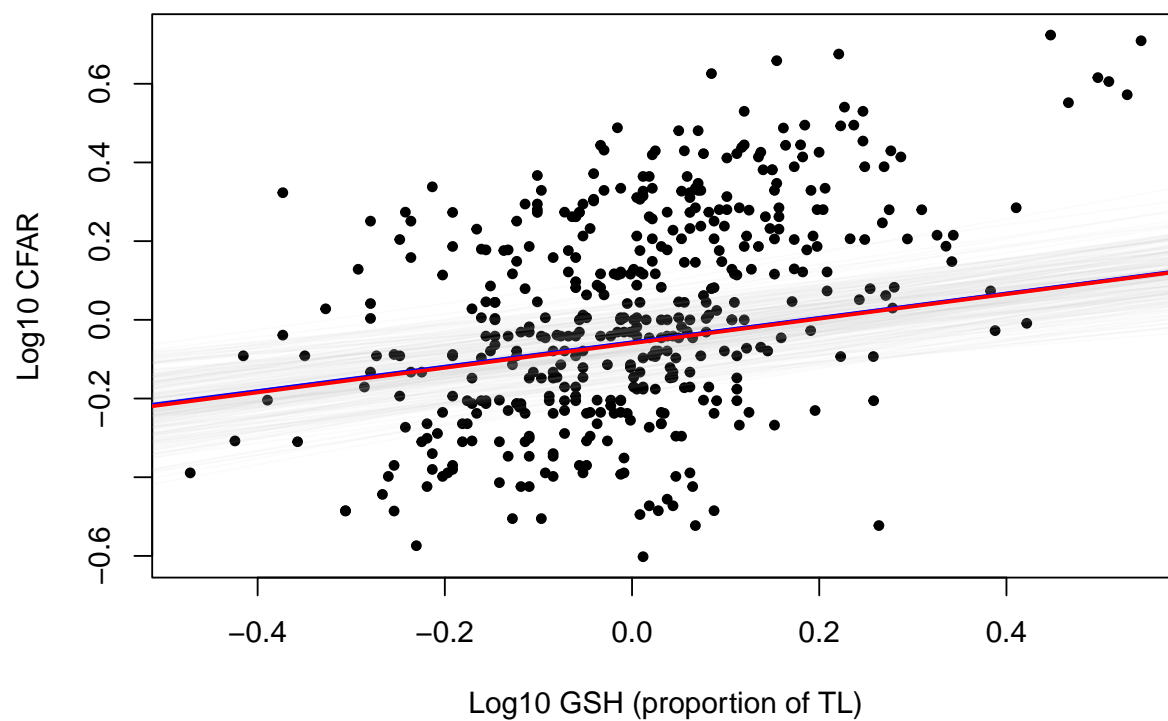
```
fit5
```

```

## Inference for Stan model: StanModel3.
## 4 chains, each with iter=1000; warmup=500; thin=10;
## post-warmup draws per chain=50, total post-warmup draws=200.
##
##          mean se_mean   sd  2.5%  25%   50%   75%  97.5% n_eff Rhat
## alpha    -0.06    0.00 0.06  -0.17 -0.10  -0.06  -0.02   0.05  154 1.01
## beta[1]   0.31    0.00 0.06   0.20  0.26   0.32   0.35   0.44  272 0.99
## beta[2]   0.12    0.00 0.04   0.06  0.10   0.12   0.15   0.20  194 0.99
## beta[3]   0.43    0.01 0.12   0.22  0.35   0.44   0.51   0.65  210 0.99
## lp__      641.68    0.12 1.76 637.09 640.81 642.00 643.00 643.99  199 1.00
##
## Samples were drawn using NUTS(diag_e) at Mon Mar 07 12:09:07 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).

```


Correlation= 0.471640472505464



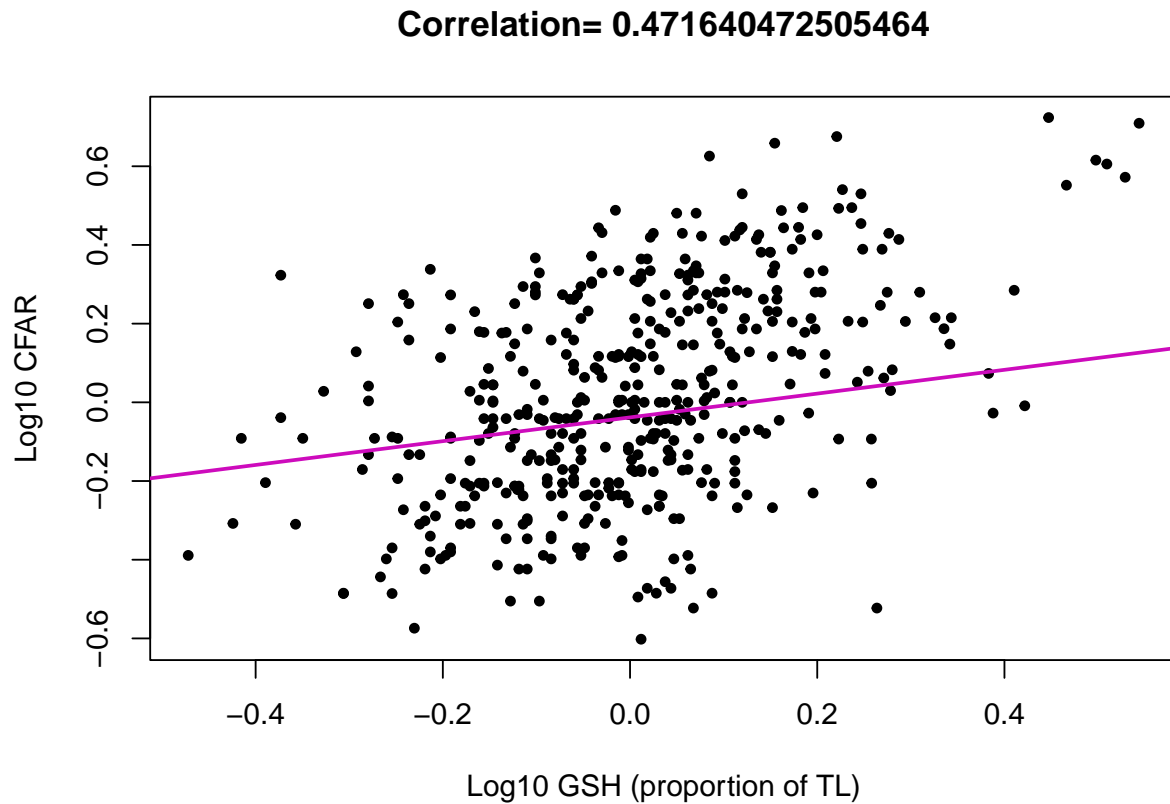
Model 8 - CFAR ~ GSH + Max Size + Phylogeny

1. PGLS model

```
pglsMod3 <- gls(Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered, correlation = corPagel(phy =  
Table_fun(pglsMod3)
```

Table 9: PGLS summary

	Value	Std.Error	t-value	p-value	2.5 %	97.5 %
(Intercept)	-0.0382795	0.0591546	-0.6471096	0.5178888	-0.1542204	0.0776614
Log10MeanGSH_centered	0.3022271	0.0603415	5.0086129	0.0000008	0.1839600	0.4204942
Log10MaxSize_centered	0.1626095	0.0341242	4.7652331	0.0000025	0.0957274	0.2294917



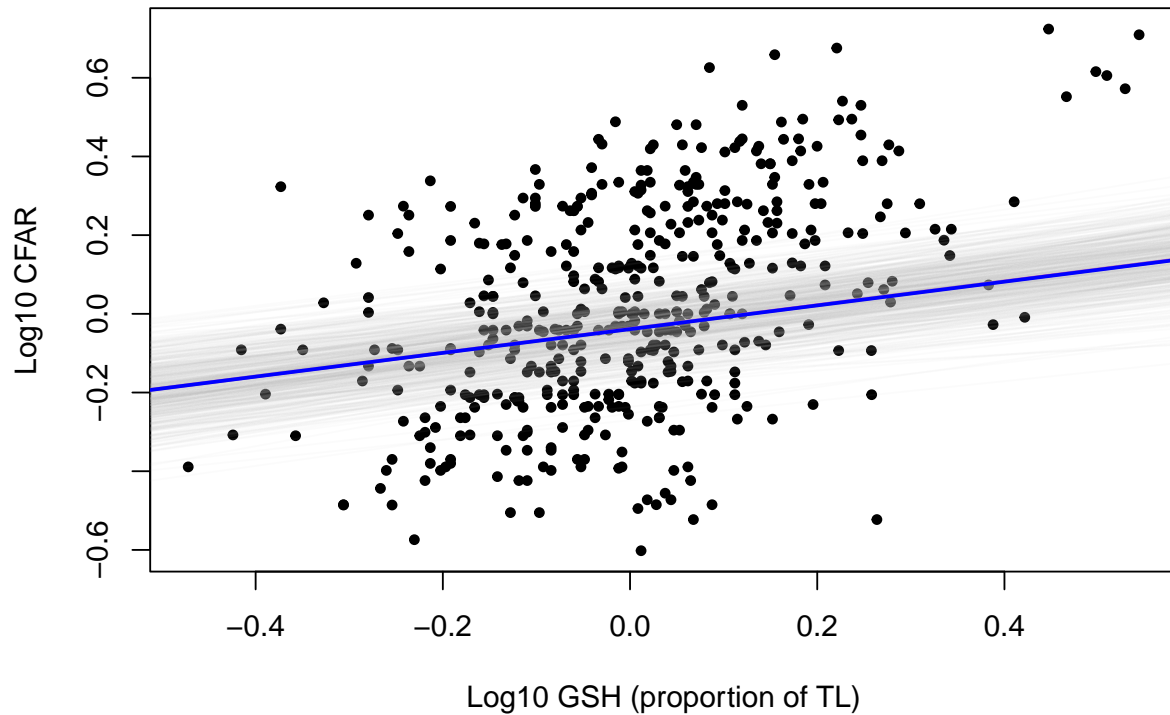
2. brms model

```
Model_BS <- brm(Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered + (1|gr(Binomial, cov = A)),  
data = GSH_df_pgls,  
family = gaussian(),  
data2 = list(A = A),  
prior = BS_prior,  
sample_prior = TRUE, chains = 4, cores = 2)
```

```
summary(Model_BS2)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered + (1 | gr(Binomial, cov = A))
## Data: GSH_df_pglis (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Binomial (Number of levels: 456)
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)      0.17      0.01      0.14      0.20 1.00      1229      2139
##
## Population-Level Effects:
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## Intercept           -0.04      0.06     -0.16      0.08 1.00      1628
## Log10MeanGSH_centered  0.30      0.06      0.19      0.42 1.00      4203
## Log10MaxSize_centered  0.16      0.03      0.10      0.23 1.00      4125
##
##      Tail_ESS
## Intercept           2083
## Log10MeanGSH_centered  3130
## Log10MaxSize_centered  3208
##
## Family Specific Parameters:
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma      0.12      0.01      0.11      0.13 1.00      1845      2597
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



3. STAN model

Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // outcome (CFAR)
  matrix[N, N] d_mat; // sigma matrix
  matrix[N, N] A; // vcov matrix
}
```

The parameters accepted by the model.

```
parameters {
  real alpha;
  vector[K] beta; // coefficients
  real<lower=0> sigma; // error
  real<lower=0,upper=1> lambda; // phylogenetic signal
}
```

```
transformed parameters {
```

```

matrix[N, N] sigma_mat;
matrix[N, N] sigma_total;

vector[N] mu_y;

sigma_mat = (1-lambda)*d_mat + lambda*A;
sigma_total = sigma*sigma_mat;

}

```

The model to be estimated. We model the output ‘y’ to be normally distributed with mean ‘mu’ and standard deviation ‘sigma’.

```

model {

  beta ~ student_t(3, 0, 10);
  lambda ~ uniform(0,1);
  sigma ~ student_t(3, 0, 2.5);

  y ~ multi_normal(alpha + x * beta, sigma_total);
}

```

WRONG STAN MODEL WILL FIX SOON :3

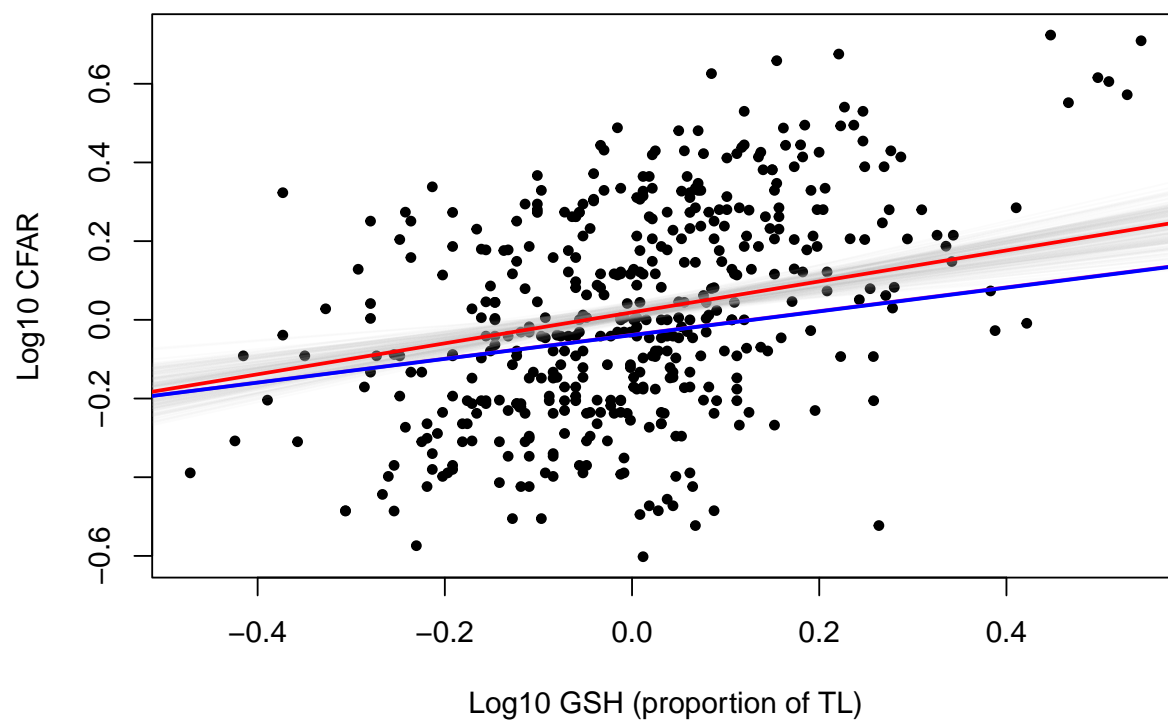
```
fit3
```

```

## Inference for Stan model: StanModel2.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##               mean se_mean   sd  2.5%   25%   50%   75%  97.5% n_eff Rhat
## alpha         0.02    0.00 0.01   0.00   0.01   0.02   0.03   0.04  2075   1
## beta[1]       0.39    0.00 0.08   0.23   0.34   0.39   0.45   0.55  1620   1
## beta[2]       0.32    0.00 0.04   0.24   0.30   0.32   0.35   0.40  1550   1
## sigma         0.22    0.00 0.01   0.21   0.22   0.22   0.23   0.24  1790   1
## lp__         458.67    0.05 1.41 455.06 458.03 458.99 459.70 460.42   948   1
##
## Samples were drawn using NUTS(diag_e) at Wed Mar 02 11:22:15 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).

```

Correlation= 0.471640472505464



Model 9 - CFAR ~ GSH + PrimaryHabitat + Phylogeny

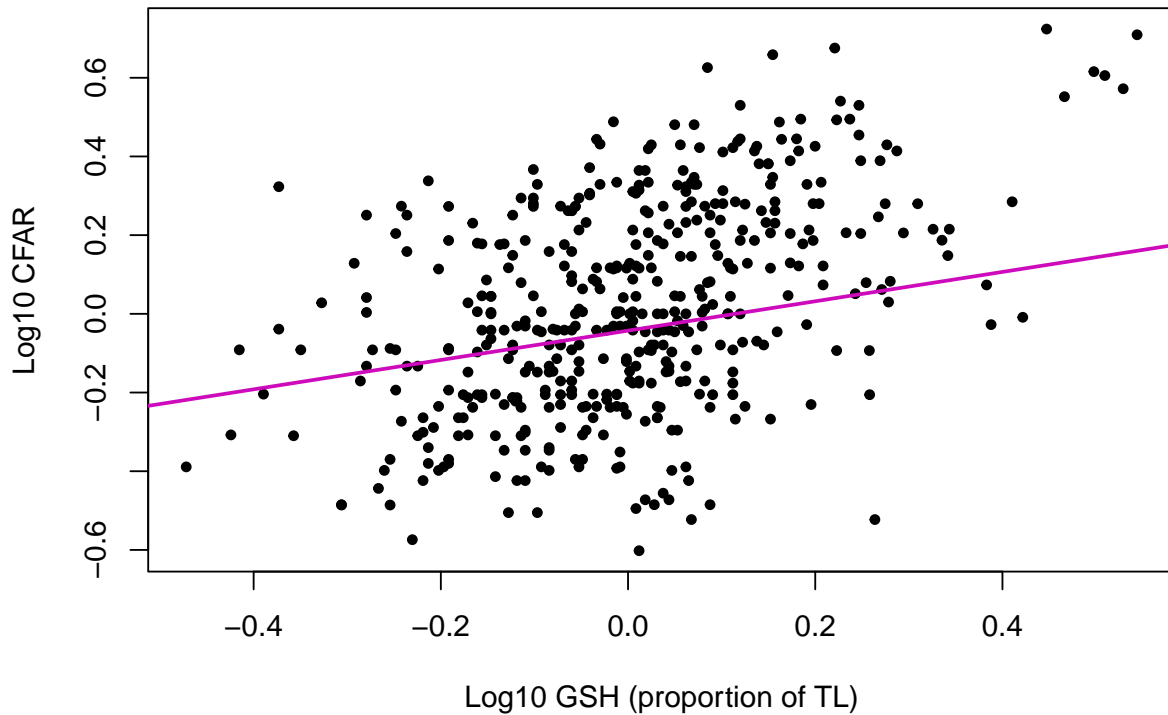
1. PGLS model

```
pglsMod5 <- gls(Log10CFAR ~ Log10MeanGSH_centered + PrimaryHabitat,  
               correlation = corPagel(phy = SampleTree, value = 0.5, fixed = F, form = ~Binomial),  
               data = GSH_df_pgls, method = "ML")
```

```
summary(pglsMod5)
```

```
## Generalized least squares fit by maximum likelihood  
## Model: Log10CFAR ~ Log10MeanGSH_centered + PrimaryHabitat  
## Data: GSH_df_pgls  
##      AIC      BIC    logLik  
## -438.0882 -413.3532 225.0441  
##  
## Correlation Structure: corPagel  
## Formula: ~Binomial  
## Parameter estimate(s):  
##      lambda  
## 0.6779675  
##  
## Coefficients:  
##              Value Std. Error  t-value p-value  
## (Intercept)    -0.0426521 0.06109477 -0.698131 0.4855  
## Log10MeanGSH_centered 0.3729626 0.05874881 6.348428 0.0000  
## PrimaryHabitatDeepwater 0.0139319 0.02640938 0.527538 0.5981  
## PrimaryHabitatPelagic 0.1500620 0.03902742 3.845041 0.0001  
##  
## Correlation:  
##              (Intr) L10MGS PrmrHD  
## Log10MeanGSH_centered -0.072  
## PrimaryHabitatDeepwater -0.217 0.040  
## PrimaryHabitatPelagic -0.113 -0.022 0.274  
##  
## Standardized residuals:  
##      Min      Q1      Med      Q3      Max  
## -2.8717036 -0.6717345 0.1624002 1.1635186 3.1192192  
##  
## Residual standard error: 0.206333  
## Degrees of freedom: 456 total; 452 residual
```

Correlation= 0.471640472505464



2. brms model

```
HabitatPhylo <- brm(Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered + (1|gr(Binomial, cov = A)),
  data = GSH_df_pgls,
  family = gaussian(),
  data2 = list(A = A),
  prior = BS_prior2,
  sample_prior = TRUE, chains = 4, cores = 2)
```

```
summary(HabitatPhylo)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + PrimaryHabitat + (1 | gr(Binomial, cov = A))
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Binomial (Number of levels: 456)
## Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept) 0.17 0.01 0.15 0.21 1.00 1198 1742
##
## Population-Level Effects:
## Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
```

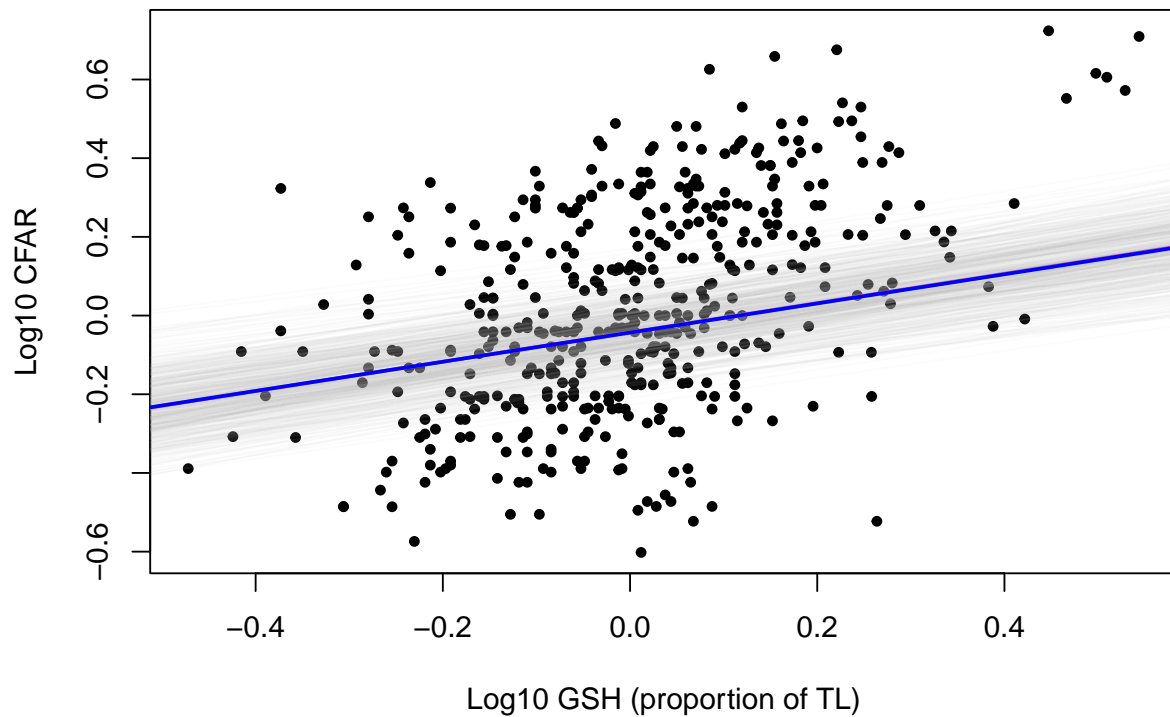


```

## Intercept                -0.04      0.06    -0.16     0.08 1.00     1629
## Log10MeanGSH_centered    0.37      0.06     0.25     0.49 1.00     4286
## PrimaryHabitatDeepwater   0.01      0.03    -0.04     0.07 1.00     3703
## PrimaryHabitatPelagic     0.15      0.04     0.08     0.23 1.00     4367
##                               Tail_ESS
## Intercept                2644
## Log10MeanGSH_centered    3543
## PrimaryHabitatDeepwater   3320
## PrimaryHabitatPelagic     3260
##
## Family Specific Parameters:
##      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma      0.12      0.01     0.11     0.13 1.00     1892     2841
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).

```

Correlation= 0.471640472505464



3. STAN Model

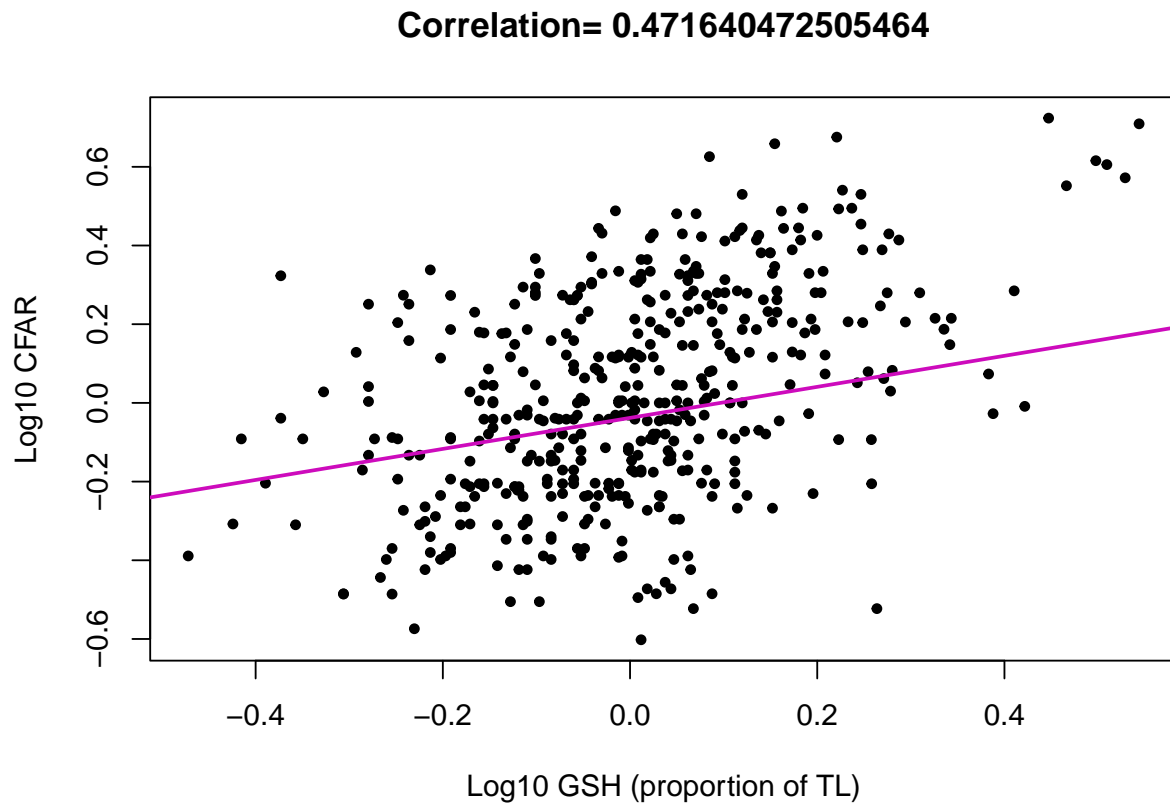
Model 10 - CFAR ~ GSH * PrimaryHabitat + Phylogeny

1. PGLS model

```
pglsMod6 <- gls(Log10CFAR ~ Log10MeanGSH_centered * PrimaryHabitat,
  correlation = corPagel(phy = SampleTree, value = 0.5, fixed = F, form = ~Binomial),
  data = GSH_df_pgls, method = "ML")
```

Table 10: PGLS summary

	Value	Std.Error	t-value	p-value	2.5 %	97.5 %
(Intercept)	-	0.0600813	-	0.5267004	-	0.0796931
	0.0380642		0.6335439		0.1558215	
Log10MeanGSH_centered	0.3942985	0.1149440	3.4303530	0.0006584	0.1690124	0.6195846
PrimaryHabitatDeepwater	-	0.0270931	-	0.9628340	-	0.0518382
	0.0012632		0.0466234		0.0543646	
PrimaryHabitatPelagic	0.1089660	0.0431700	2.5241114	0.0119421	0.0243543	0.1935777
Log10MeanGSH_centered:PrimaryHabitatDeepwater	0.1297042		-	0.4659136	-	0.1595620
	0.0946536		0.7297649		0.3488692	
Log10MeanGSH_centered:PrimaryHabitatPelagic	0.1147980	0.1808652	1.7681563	0.0777123	-	0.6742874
					0.0346913	



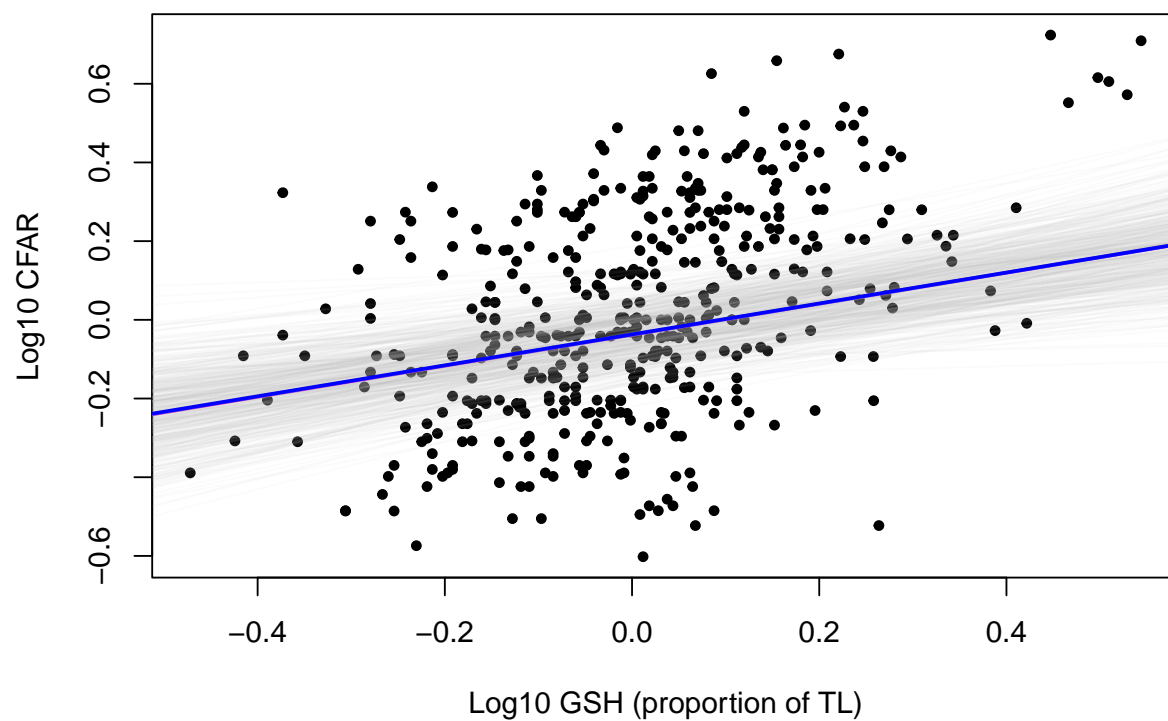
2. BRMS Model

```
HabitatPhylo2 <- brm(Log10CFAR ~ Log10MeanGSH_centered * PrimaryHabitat + (1|gr(Binomial, cov = A)),
  data = GSH_df_pgls,
  family = gaussian(),
  data2 = list(A = A),
  prior = HabitatPhylo_prior2,
  sample_prior = TRUE, chains = 4, cores = 4)
```

```
summary(HabitatPhylo2)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered * PrimaryHabitat + (1 | gr(Binomial, cov = A))
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Binomial (Number of levels: 456)
##      Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)      0.17      0.01      0.14      0.20 1.00      1449      2455
##
## Population-Level Effects:
##      Estimate Est.Error 1-95% CI
## Intercept          -0.04      0.06     -0.16
## Log10MeanGSH_centered      0.39      0.12      0.15
## PrimaryHabitatDeepwater     -0.00      0.03     -0.05
## PrimaryHabitatPelagic       0.11      0.04      0.03
## Log10MeanGSH_centered:PrimaryHabitatDeepwater     -0.09      0.13     -0.35
## Log10MeanGSH_centered:PrimaryHabitatPelagic       0.32      0.18     -0.04
##      u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept          0.08 1.00      1969      2137
## Log10MeanGSH_centered      0.62 1.00      2271      2797
## PrimaryHabitatDeepwater     0.05 1.00      3382      2870
## PrimaryHabitatPelagic       0.19 1.00      3712      2991
## Log10MeanGSH_centered:PrimaryHabitatDeepwater     0.17 1.00      2587      2969
## Log10MeanGSH_centered:PrimaryHabitatPelagic       0.69 1.00      2373      3261
##
## Family Specific Parameters:
##      Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma      0.12      0.01      0.11      0.13 1.00      2236      2630
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



3. STAN Model

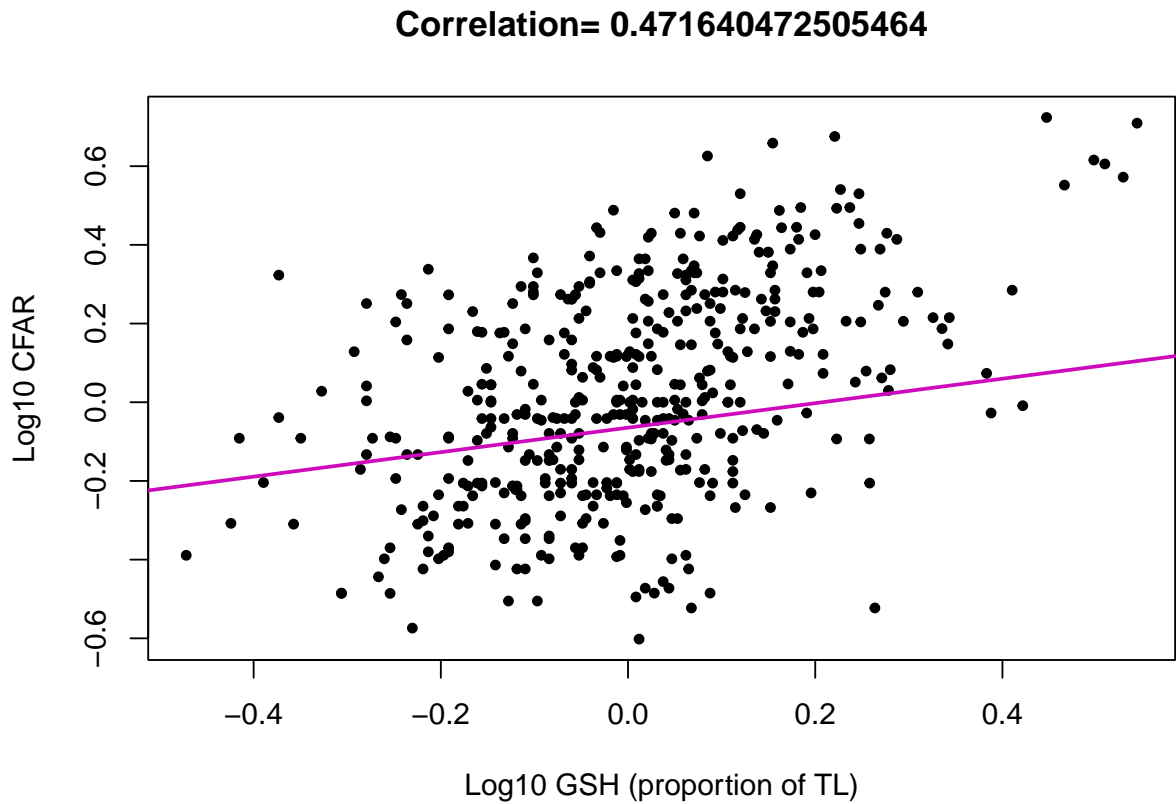
Model 11 - CFAR ~ GSH * MaxSize + PrimaryHabitat + Phylogeny

1. PGLS model

```
pglsMod8 <- gls(Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + PrimaryHabitat,
  correlation = corPagel(phy = SampleTree, value = 0.5, fixed = F, form = ~Binomial),
  data = GSH_df_pgls, method = "ML")
```

Table 11: PGLS summary

	Value	Std.Error	t-value	p-value	2.5 %	97.5 %
(Intercept)	-	0.0577730	-	0.2639488	-	0.0486137
	0.0646193		1.1185034		0.1778522	
Log10MeanGSH_centered	0.3111825	0.0594286	5.2362390	0.0000003	0.1947045	0.4276605
Log10MaxSize_centered	0.1234860	0.0351974	3.5083817	0.0004962	0.0545003	0.1924717
PrimaryHabitatDeepwater	0.0069539	0.0258660	0.2688422	0.7881743	-	0.0576502
					0.0437425	
PrimaryHabitatPelagic	0.0990615	0.0399968	2.4767337	0.0136256	0.0206692	0.1774539
Log10MeanGSH_centered:Log10MaxSize_centered	0.3443064	0.1284790	2.6798657	0.0076353	0.0924922	0.5961206



2. BRMS Model

```
SizeHabitatModel <- brm(Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + PrimaryHabitat + (1 | gr(Binomial)),
  data = GSH_df_pgls,
  family = gaussian(),
  data2 = list(A = A),
  prior = SizeHabitatMod_prior,
  sample_prior = TRUE, chains = 4, cores = 4)
```

```
summary(SizeHabitatModel)
```

```
## Family: gaussian
## Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + PrimaryHabitat + (1 | gr(Binomial))
## Data: GSH_df_pgls (Number of observations: 456)
## Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
## total post-warmup draws = 4000
##
## Group-Level Effects:
## ~Binomial (Number of levels: 456)
##
```

	Estimate	Est.Error	1-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
sd(Intercept)	0.16	0.01	0.14	0.19	1.00	1425	2319

```
##
## Population-Level Effects:
##
```

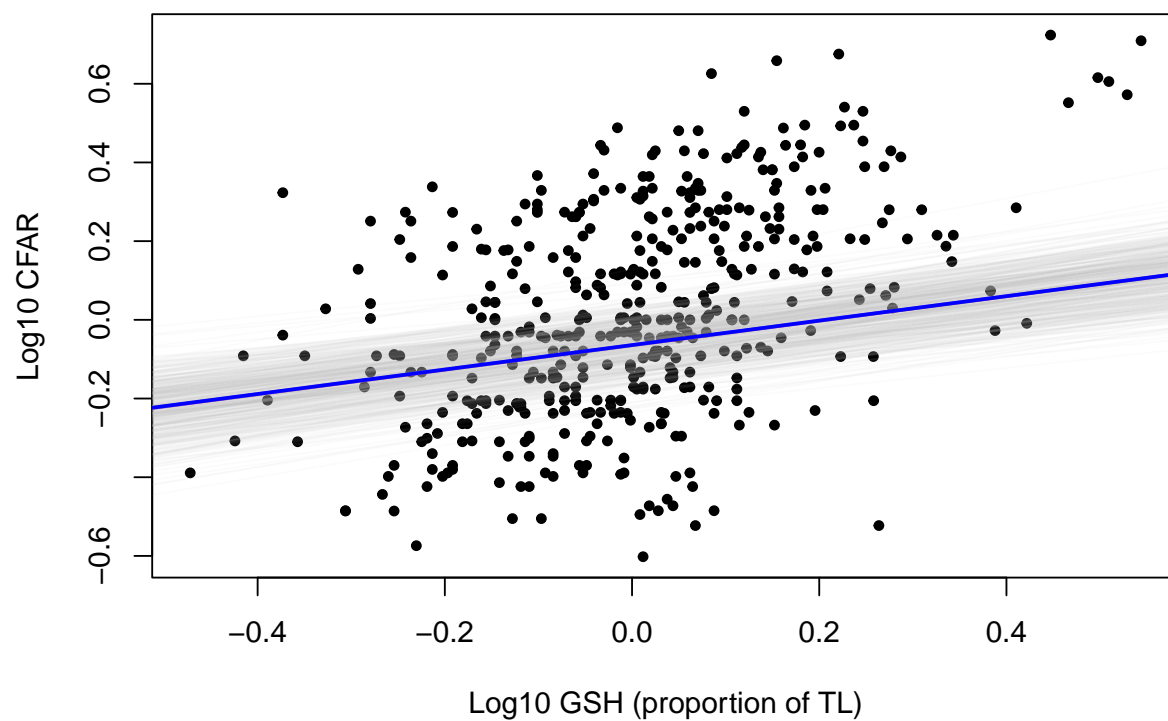
	Estimate	Est.Error	1-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
Intercept	-0.06	0.06	-0.18	0.05	1.00	2087	2622
Log10MeanGSH_centered	0.31	0.06	0.19	0.43	1.00	5727	3001
Log10MaxSize_centered	0.12	0.04	0.05	0.19	1.00	5879	3334
PrimaryHabitatDeepwater	0.01	0.03	-0.05	0.06	1.00	3704	3118
PrimaryHabitatPelagic	0.10	0.04	0.02	0.18	1.00	4987	3454
Log10MeanGSH_centered:Log10MaxSize_centered	0.34	0.13	0.09	0.59	1.00	5446	3659

```
##
## Family Specific Parameters:
##
```

	Estimate	Est.Error	1-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
sigma	0.12	0.01	0.11	0.13	1.00	2008	2798

```
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Correlation= 0.471640472505464



3. STAN Model