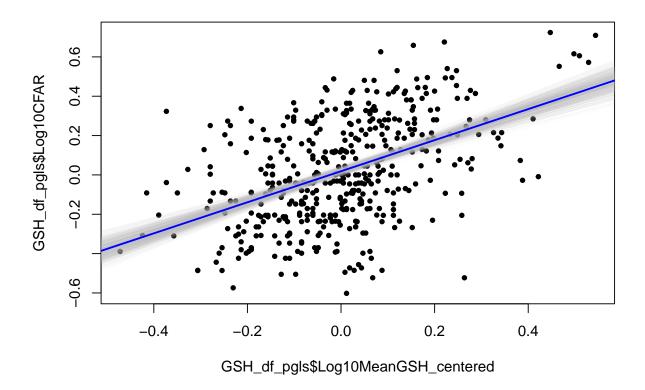
Ch. 1 - Model comaprisons

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2022-02-25

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| Model Comparisons Model 1 - CFAR \sim GSH | |
| 1. brms model | |
| <pre>Model_plain <- brm(Log10CFAR ~ Log10MeanGSH_centered,</pre> | |
| <pre>fixef(Model_plain, summary = T)</pre> | |
| ## Estimate Est.Error Q2.5 Q97.5 ## Intercept 0.01878724 0.01121401 -0.003048758 0.04068312 ## Log10MeanGSH_centered 0.78943177 0.06909079 0.653733169 0.92180824 | |



```
Data considered

data {
   int <lower=1> N;
   vector[N] x;
   vector[N] y;
}

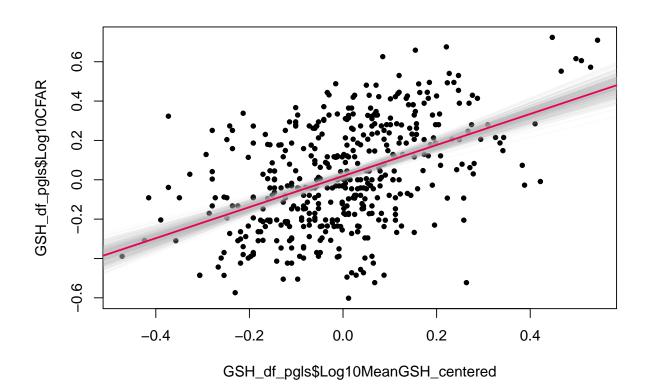
The parameters accepted by the model

parameters {
   real alpha;
   real beta;
   real<lower=0> sigma;
}

Model form:

model {
   sigma ~ student_t(3, 0, 10);
```

```
y ~ normal(alpha + x * beta , sigma);
```

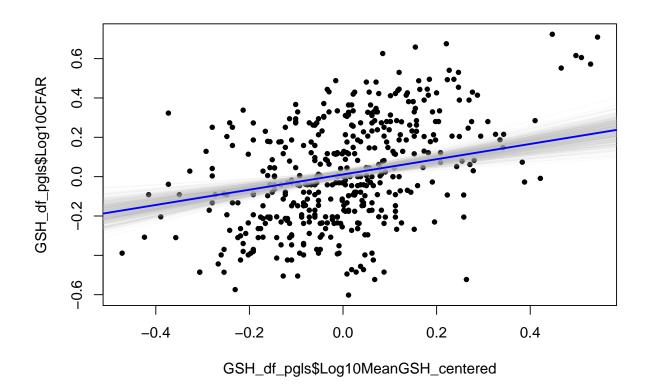


Model 2 - CFAR \sim GSH * Max Size

1. brms model

```
fixef(Model_size, summary = T)
```

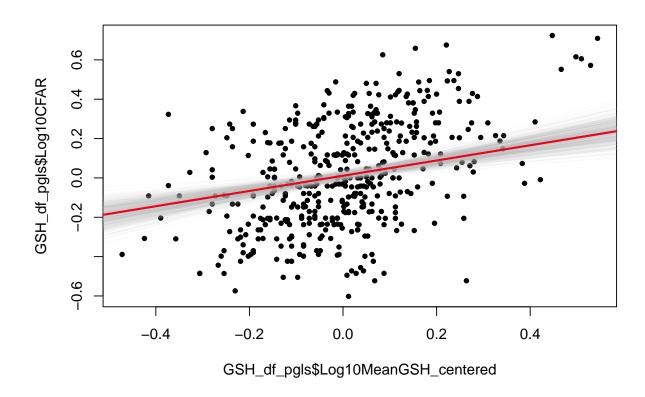
```
##
                                                 Estimate Est.Error
                                                                             Q2.5
## Intercept
                                               0.01107801 0.01158262 -0.01069365
## Log10MeanGSH_centered
                                               0.38761087 0.08262037 0.22932179
## Log10MaxSize_centered
                                               0.30460627 0.04281846 0.22222482
## Log10MeanGSH_centered:Log10MaxSize_centered 0.26628115 0.16406086 -0.05284798
##
                                                    Q97.5
## Intercept
                                               0.03436049
## Log10MeanGSH_centered
                                               0.55212619
## Log10MaxSize_centered
                                               0.38716498
## Log10MeanGSH_centered:Log10MaxSize_centered 0.58368860
```



2. STAN model

```
Data considered
```

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // CFAR
}
The parameters accepted by the model.
parameters {
  real alpha;
  vector[K] beta;
  real<lower=0> sigma;
The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard
deviation 'sigma'.
model {
  sigma ~ student_t(3, 0, 10);
 y ~ normal(alpha + x * beta , sigma);
fit2
## Inference for Stan model: StanModel2.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##
             mean se_mean
                             sd
                                  2.5%
                                           25%
                                                  50%
                                                          75%
                                                               97.5% n_eff Rhat
## alpha
             0.01
                      0.00 0.01
                                 -0.01
                                          0.00
                                                 0.01
                                                         0.02
                                                                0.04
                                                                      1841
## beta[1]
             0.39
                      0.00 0.08
                                  0.23
                                          0.33
                                                 0.39
                                                        0.44
                                                                0.55
                                                                      1758
                                                                               1
## beta[2]
             0.31
                      0.00 0.04
                                  0.22
                                          0.28
                                                 0.31
                                                        0.33
                                                                0.39
                                                                      1742
                                                                               1
## beta[3]
             0.26
                      0.00 0.16
                                 -0.06
                                          0.16
                                                 0.27
                                                         0.37
                                                                0.59
                                                                      1975
                                                                               1
## sigma
             0.22
                      0.00 0.01
                                  0.21
                                         0.22
                                                 0.22
                                                        0.23
                                                                0.24
                                                                      2516
                                                                               1
## lp__
           459.42
                      0.05 1.62 455.40 458.60 459.75 460.60 461.57
                                                                               1
##
## Samples were drawn using NUTS(diag_e) at Wed Mar 02 11:20:54 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

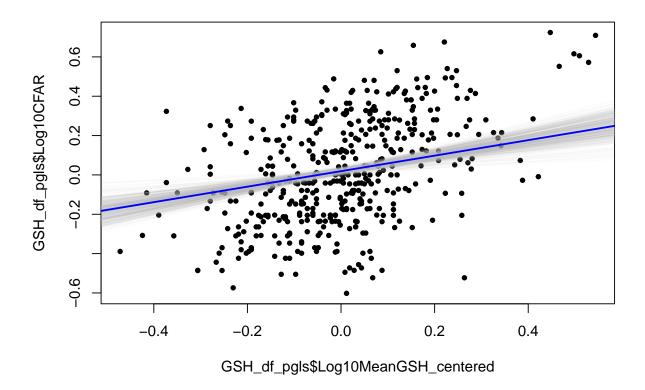


Model 3 - CFAR ~ GSH + Max Size (No interaction)

1. brms model

summary(Model_size_2)

```
Family: gaussian
    Links: mu = identity; sigma = identity
##
## Formula: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
            total post-warmup draws = 4000
##
##
## Population-Level Effects:
##
                         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS
                             0.02
                                       0.01
                                               -0.00
                                                         0.04 1.00
                                                                        3526
## Intercept
                                                                        3567
## Log10MeanGSH_centered
                             0.39
                                       0.08
                                                0.24
                                                          0.55 1.00
## Log10MaxSize_centered
                             0.33
                                       0.04
                                                0.25
                                                         0.40 1.00
                                                                        3528
                         Tail_ESS
                             2556
## Intercept
## Log10MeanGSH_centered
                             2768
## Log10MaxSize_centered
                             3043
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma
             0.22
                       0.01
                                0.21
                                         0.24 1.00
                                                        3652
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // CFAR
}
```

The parameters accepted by the model.

```
parameters {
  real alpha;
  vector[K] beta;
  real<lower=0> sigma;
}
```

The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

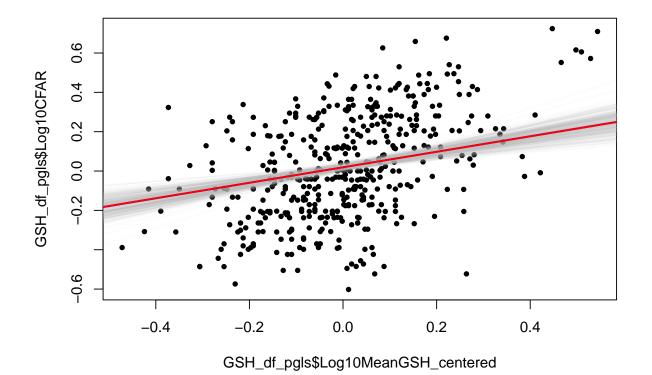
```
model {
```

```
sigma ~ student_t(3, 0, 10);

y ~ normal(alpha + x * beta , sigma);
}
```

fit3

```
## Inference for Stan model: StanModel2.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##
                                  2.5%
                                                              97.5% n_eff Rhat
             mean se_mean
                             sd
                                          25%
                                                  50%
                                                         75%
             0.02
                     0.00 0.01
                                  0.00
                                          0.01
                                                 0.02
                                                        0.03
                                                               0.04
                                                                      2075
## alpha
                     0.00 0.08
                                                 0.39
## beta[1]
             0.39
                                  0.23
                                         0.34
                                                        0.45
                                                               0.55
                                                                      1620
                                                                              1
             0.32
                     0.00 0.04
                                  0.24
                                         0.30
                                                 0.32
                                                        0.35
## beta[2]
                                                                0.40
                                                                      1550
## sigma
             0.22
                     0.00 0.01
                                  0.21
                                         0.22
                                                 0.22
                                                        0.23
                                                                0.24
                                                                      1790
                                                                              1
## lp__
           458.67
                     0.05 1.41 455.06 458.03 458.99 459.70 460.42
                                                                       948
##
## Samples were drawn using NUTS(diag_e) at Wed Mar 02 11:22:15 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

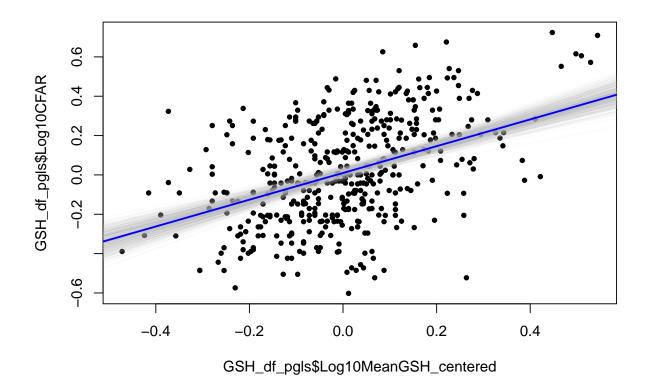


Model 4 - CFAR ~ GSH + PrimaryHabitat (No interaction)

1. brms model

summary(habitat_model)

```
## Family: gaussian
    Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered + PrimaryHabitat
      Data: GSH_df_pgls (Number of observations: 456)
##
    Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
            total post-warmup draws = 4000
##
##
## Population-Level Effects:
##
                           Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
## Intercept
                                                 -0.02
                               0.01
                                         0.02
                                                            0.04 1.00
                                                                          4027
## Log10MeanGSH_centered
                               0.68
                                         0.07
                                                  0.54
                                                            0.83 1.00
                                                                          4122
## PrimaryHabitatDeepwater
                              -0.01
                                         0.02
                                                 -0.06
                                                            0.04 1.00
                                                                          3851
## PrimaryHabitatPelagic
                                         0.05
                                                            0.34 1.00
                                                                          4558
                               0.25
                                                  0.16
                           Tail_ESS
## Intercept
                               2871
## Log10MeanGSH_centered
                               2943
## PrimaryHabitatDeepwater
                               3151
## PrimaryHabitatPelagic
                               3159
##
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
             0.23
                       0.01
                                         0.25 1.00
## sigma
                                0.21
                                                        3991
                                                                 2793
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // CFAR
}
```

The parameters accepted by the model.

```
parameters {
  real alpha;
  vector[K] beta;
  real<lower=0> sigma;
}
```

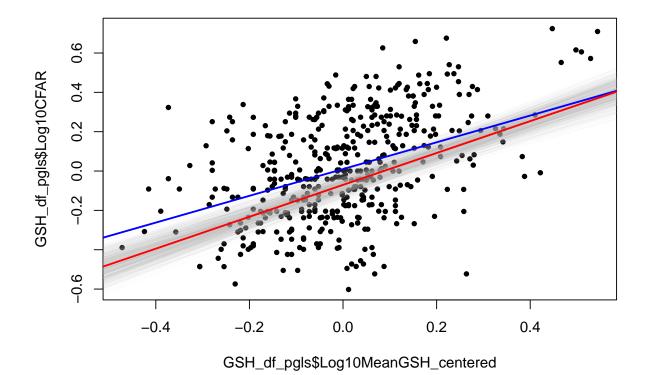
The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard deviation 'sigma'.

```
model {
```

```
sigma ~ student_t(3, 0, 10);

y ~ normal(alpha + x * beta , sigma);
}
fit6
```

```
## Inference for Stan model: StanModel2.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##
             mean se_mean
                             sd
                                  2.5%
                                          25%
                                                  50%
                                                         75%
                                                              97.5% n_eff Rhat
            -0.07
                     0.00 0.03
                                 -0.13
                                        -0.09
                                                -0.07
                                                              -0.01
                                                                      903 1.00
## alpha
                                                       -0.05
## beta[1]
             0.81
                     0.00 0.07
                                  0.67
                                         0.76
                                                 0.81
                                                        0.86
                                                               0.94
                                                                      1396 1.00
             0.05
                     0.00 0.02
                                  0.02
                                         0.04
                                                 0.05
                                                        0.07
                                                               0.09
## beta[2]
                                                                       979 1.00
## sigma
             0.23
                     0.00 0.01
                                  0.22
                                         0.23
                                                 0.23
                                                        0.24
                                                               0.25
                                                                      1463 1.00
## lp__
           433.52
                     0.05 1.43 430.05 432.77 433.84 434.58 435.32
                                                                       725 1.01
##
## Samples were drawn using NUTS(diag_e) at Fri Mar 18 14:02:20 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

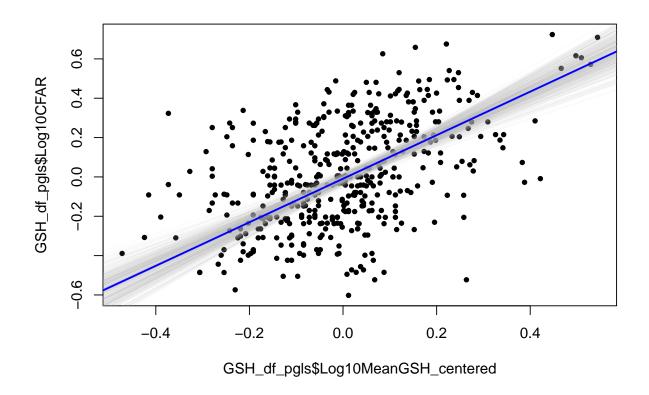


Model 5 - CFAR ~ GSH * PrimaryHabitat

1. brms model

summary(habitat_model_2)

```
Family: gaussian
##
    Links: mu = identity; sigma = identity
## Formula: Log10CFAR ~ Log10MeanGSH_centered * PrimaryHabitat
     Data: GSH_df_pgls (Number of observations: 456)
##
    Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
            total post-warmup draws = 4000
##
##
## Population-Level Effects:
                                                 Estimate Est.Error 1-95% CI
##
## Intercept
                                                    -0.01 0.02 -0.04
## Log10MeanGSH_centered
                                                     1.11
                                                               0.13
                                                                       0.85
## PrimaryHabitatDeepwater
                                                    -0.00
                                                               0.02
                                                                       -0.05
                                                               0.06
                                                                       0.18
## PrimaryHabitatPelagic
                                                     0.29
## Log10MeanGSH_centered:PrimaryHabitatDeepwater
                                                    -0.64
                                                               0.16
                                                                       -0.96
## Log10MeanGSH_centered:PrimaryHabitatPelagic
                                                    -0.56
                                                               0.22
                                                                       -1.00
                                                 u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept
                                                                             3045
                                                     0.02 1.00
                                                                   3731
## Log10MeanGSH_centered
                                                                   2427
                                                                             2696
                                                     1.36 1.00
## PrimaryHabitatDeepwater
                                                     0.04 1.00
                                                                   3866
                                                                             2861
## PrimaryHabitatPelagic
                                                                   3449
                                                                             3073
                                                     0.40 1.00
## Log10MeanGSH_centered:PrimaryHabitatDeepwater
                                                    -0.32 1.00
                                                                   2630
                                                                             2589
## Log10MeanGSH_centered:PrimaryHabitatPelagic
                                                    -0.10 1.00
                                                                   2539
                                                                             2375
## Family Specific Parameters:
##
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS
## sigma
                       0.01
                                0.21
                                         0.24 1.00
             0.23
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



2. STAN model

TBD

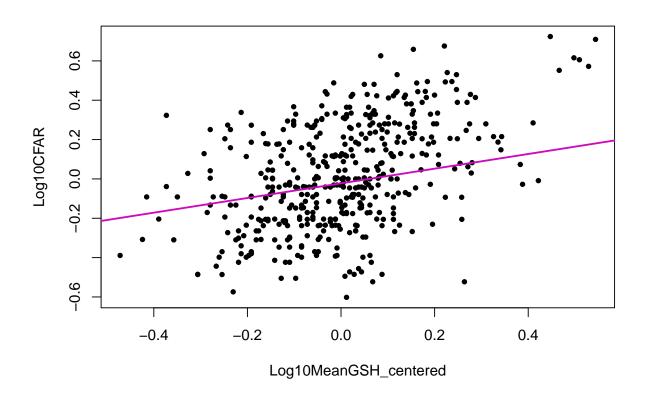
Model 6 - CFAR \sim GSH + Phylogeny

1. PGLS model

pglsMod1 <- gls(Log10CFAR ~ Log10MeanGSH_centered, correlation = corPagel(phy = SampleTree, value = 0.5

```
summary(pglsMod1)
```

```
## Generalized least squares fit by maximum likelihood
##
     Model: Log10CFAR ~ Log10MeanGSH_centered
##
     Data: GSH_df_pgls
           AIC
##
                     BIC
                           logLik
##
     -427.4206 -410.9306 217.7103
##
## Correlation Structure: corPagel
## Formula: ~Binomial
## Parameter estimate(s):
      lambda
##
## 0.7030228
##
## Coefficients:
##
                              Value Std.Error
                                                 t-value p-value
## (Intercept)
                         -0.0224754 0.06282858 -0.357726 0.7207
## Log10MeanGSH_centered 0.3728837 0.05950418 6.266513 0.0000
##
##
   Correlation:
##
                         (Intr)
## Log10MeanGSH_centered -0.065
##
## Standardized residuals:
##
         Min
                      Q1
                                            QЗ
                                                      Max
                                Med
## -2.7884571 -0.6933649 0.1151372 1.0885641 2.9034051
## Residual standard error: 0.2147249
## Degrees of freedom: 456 total; 454 residual
```

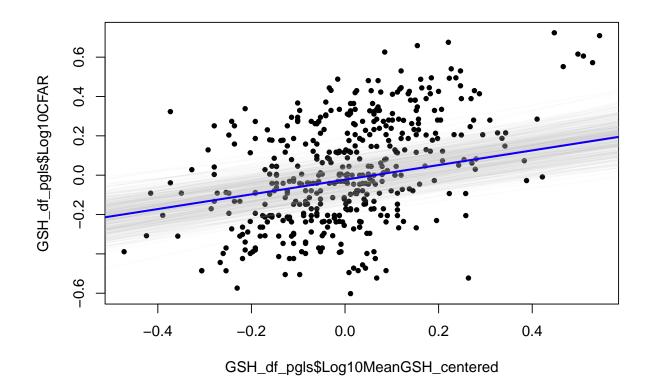


2. brms model

summary(Model_simple)

```
##
   Family: gaussian
     Links: mu = identity; sigma = identity
##
## Formula: Log10CFAR ~ Log10MeanGSH_centered + (1 | gr(Binomial, cov = A))
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 4000
##
## Group-Level Effects:
  ~Binomial (Number of levels: 456)
##
##
                 Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)
                     0.01
                               0.00
                                         0.01
                                                  0.01 1.00
                                                                1200
                                                                          1938
## Population-Level Effects:
```

```
Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
##
                                        0.06
                                                -0.15
                                                          0.10 1.00
## Intercept
                             -0.02
                                                                         1257
## Log10MeanGSH_centered
                              0.37
                                        0.06
                                                 0.25
                                                          0.49 1.00
                                                                         4238
##
                         Tail_ESS
## Intercept
                              2076
## Log10MeanGSH_centered
                              3483
## Family Specific Parameters:
##
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
             0.12
                       0.01
                                0.11
                                          0.13 1.00
                                                        1717
## sigma
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



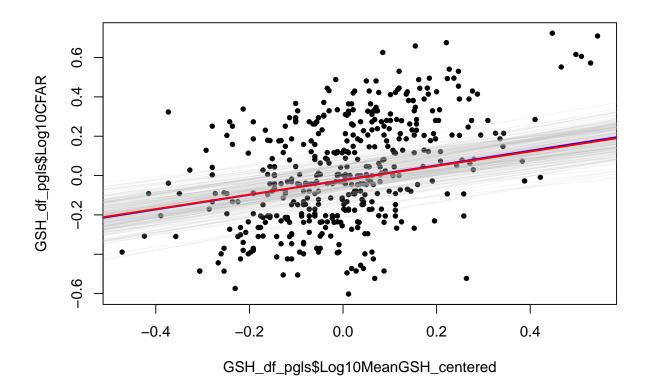
Data considered

```
data {
  int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // outcome (CFAR)
```

```
matrix[N, N] d_mat; // sigma matrix
  matrix[N, N] A; // vcov matrix
      }
The parameters accepted by the model.
parameters {
        real alpha;
        vector[K] beta; // coefficients
        real<lower=0> sigma; // error
        real<lower=0,upper=1> lambda; // phylogenetic signal
      }
      transformed parameters {
        matrix[N, N] sigma_mat;
        matrix[N, N] sigma_total;
        vector[N] mu_y;
        sigma_mat = (1-lambda)*d_mat + lambda*A;
        sigma_total = sigma*sigma_mat;
      }
The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard
deviation 'sigma'.
model {
  beta ~ student_t(3, 0, 10);
  lambda ~ uniform(0,1);
  sigma ~ student_t(3, 0, 2.5);
    ~ multi_normal(alpha + x * beta, sigma_total);
}
fit4
## Inference for Stan model: StanModel3.
## 4 chains, each with iter=1000; warmup=500; thin=10;
## post-warmup draws per chain=50, total post-warmup draws=200.
##
                                                         75% 97.5% n_eff Rhat
##
             mean se_mean
                             sd
                                  2.5%
                                          25%
                                                 50%
## alpha
            -0.02
                      0.0 0.06 -0.15
                                       -0.06
                                               -0.03
                                                        0.02
                                                               0.09
                                                                      219 0.99
             0.37
                      0.0 0.06
                                  0.25
                                         0.33
                                                0.36
                                                        0.41
                                                               0.47
                                                                      216 1.01
## beta[1]
                      0.1 1.36 622.82 625.03 626.10 626.84 627.47
## lp__
           625.81
                                                                      193 1.02
```

##

```
## Samples were drawn using NUTS(diag_e) at Thu Mar 03 14:30:50 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```



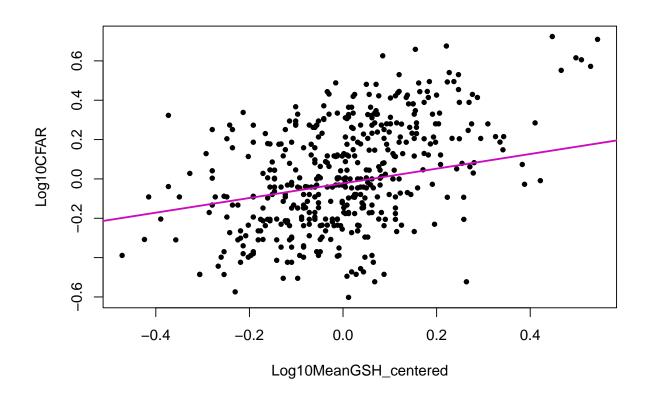
Model 7 - CFAR \sim GSH * Max Size + Phylogeny

1. PGLS model

pglsMod2 <- gls(Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered, correlation = corPagel(phy =

summary(pglsMod2)

```
## Generalized least squares fit by maximum likelihood
##
     Model: y ~ x
     Data: GSH_df_pgls
                    BIC
##
           AIC
                         logLik
##
     -427.4206 -410.9306 217.7103
##
## Correlation Structure: corPagel
## Formula: ~Binomial
## Parameter estimate(s):
     lambda
##
## 0.7030228
##
## Coefficients:
                    Value Std.Error t-value p-value
## (Intercept) -0.0224754 0.06282858 -0.357726 0.7207
               0.3728837 0.05950418 6.266513 0.0000
##
## Correlation:
##
     (Intr)
## x -0.065
##
## Standardized residuals:
         Min
                     Q1
                               Med
                                           QЗ
                                                     Max
## -2.7884572 -0.6933649 0.1151373 1.0885641 2.9034051
## Residual standard error: 0.2147249
## Degrees of freedom: 456 total; 454 residual
```

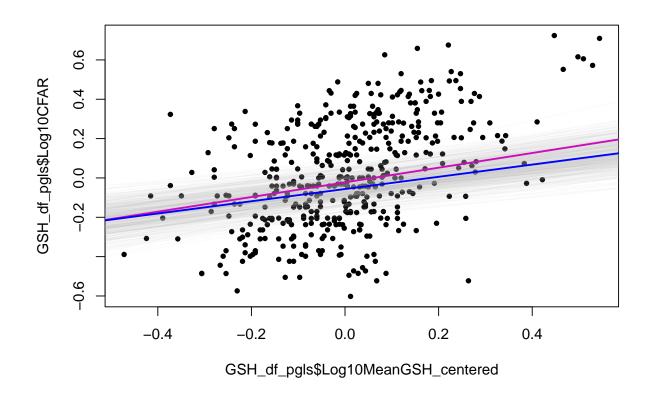


2. brms model

summary(Model_BS)

```
Family: gaussian
     Links: mu = identity; sigma = identity
##
## Formula: Log10CFAR ~ Log10MeanGSH_centered * Log10MaxSize_centered + (1 | gr(Binomial, cov = A))
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 4000
##
## Group-Level Effects:
  ~Binomial (Number of levels: 456)
##
                 Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)
                     0.01
                               0.00
                                         0.01
                                                  0.01 1.00
                                                                1303
                                                                          2376
## Population-Level Effects:
```

```
Estimate Est.Error 1-95% CI
##
## Intercept
                                                    -0.06
                                                               0.06
                                                                       -0.17
## Log10MeanGSH_centered
                                                               0.06
                                                     0.31
                                                                        0.19
## Log10MaxSize_centered
                                                     0.13
                                                               0.04
                                                                        0.06
## Log10MeanGSH_centered:Log10MaxSize_centered
                                                     0.43
                                                               0.12
                                                                        0.20
                                                u-95% CI Rhat Bulk_ESS Tail_ESS
##
## Intercept
                                                     0.05 1.00
                                                                   1824
                                                                             2402
## Log10MeanGSH_centered
                                                     0.43 1.00
                                                                            3478
                                                                   6595
## Log10MaxSize_centered
                                                     0.20 1.00
                                                                   4866
                                                                             3342
## Log10MeanGSH_centered:Log10MaxSize_centered
                                                     0.67 1.00
                                                                   5734
                                                                            3590
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
                       0.01
                                          0.13 1.00
             0.12
                                 0.11
                                                         2164
                                                                  3283
## sigma
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

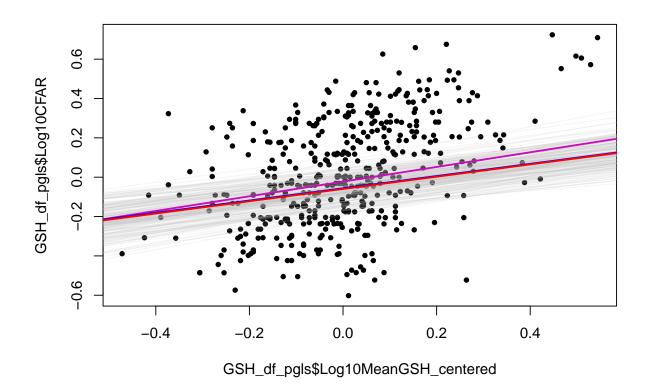


Data considered

data {

```
int <lower=0> N; // number of data points
  int <lower=0> K; // number of predictors
  matrix[N,K] x; // predictor matrix
  vector[N] y; // outcome (CFAR)
  matrix[N, N] d_mat; // sigma matrix
  matrix[N, N] A; // vcov matrix
      }
The parameters accepted by the model.
parameters {
        real alpha;
        vector[K] beta; // coefficients
        real<lower=0> sigma; // error
        real<lower=0,upper=1> lambda; // phylogenetic signal
      }
      transformed parameters {
        matrix[N, N] sigma_mat;
        matrix[N, N] sigma_total;
        vector[N] mu_y;
        sigma_mat = (1-lambda)*d_mat + lambda*A;
        sigma_total = sigma*sigma_mat;
      }
The model to be estimated. We model the output 'y' to be normally distributed with mean 'mu' and standard
deviation 'sigma'.
model {
  beta ~ student_t(3, 0, 10);
  lambda ~ uniform(0,1);
  sigma ~ student_t(3, 0, 2.5);
 y ~ multi_normal(alpha + x * beta, sigma_total);
fit5
## Inference for Stan model: StanModel3.
## 4 chains, each with iter=1000; warmup=500; thin=10;
## post-warmup draws per chain=50, total post-warmup draws=200.
##
                                                 50%
                                                         75% 97.5% n_eff Rhat
##
             mean se mean
                             sd
                                  2.5%
                                          25%
```

```
## alpha
            -0.06
                     0.00 0.06 -0.17 -0.10 -0.06 -0.02
                                                               0.05
                                                                      154 1.01
## beta[1]
             0.31
                     0.00 0.06
                                  0.20
                                         0.26
                                                0.32
                                                       0.35
                                                              0.44
                                                                      272 0.99
             0.12
                     0.00 0.04
                                                0.12
                                                                      194 0.99
## beta[2]
                                  0.06
                                         0.10
                                                       0.15
                                                               0.20
## beta[3]
             0.43
                     0.01 0.12
                                  0.22
                                         0.35
                                                0.44
                                                       0.51
                                                               0.65
                                                                      210 0.99
           641.68
                     0.12 1.76 637.09 640.81 642.00 643.00 643.99
                                                                      199 1.00
## lp__
##
## Samples were drawn using NUTS(diag_e) at Mon Mar 07 12:09:07 2022.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```



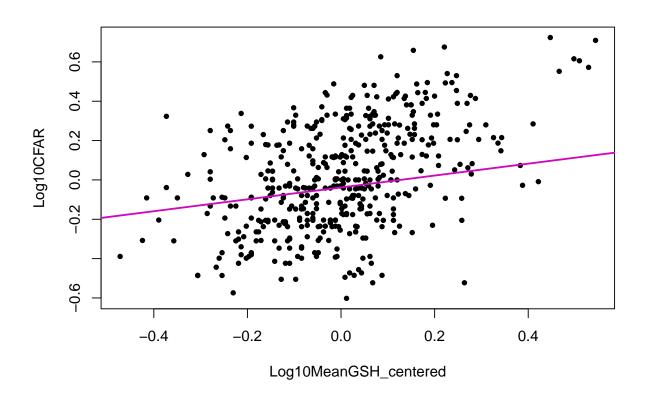
Model 8 - CFAR \sim GSH + Max Size + Phylogeny

1. PGLS model

pglsMod3 <- gls(Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered, correlation = corPagel(phy =

```
summary(pglsMod3)
```

```
## Generalized least squares fit by maximum likelihood
##
     Model: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered
##
     Data: GSH_df_pgls
##
           AIC
                     BIC
                           logLik
     -447.4392 -426.8268 228.7196
##
##
## Correlation Structure: corPagel
## Formula: ~Binomial
## Parameter estimate(s):
      lambda
##
## 0.6786106
##
## Coefficients:
##
                               Value Std.Error
                                                  t-value p-value
## (Intercept)
                         -0.03827951 0.05915461 -0.647110 0.5179
## Log10MeanGSH_centered    0.30222706    0.06034147    5.008613    0.0000
## Log10MaxSize_centered 0.16260955 0.03412415 4.765233 0.0000
   Correlation:
##
                         (Intr) L10MGS
## Log10MeanGSH_centered -0.050
## Log10MaxSize_centered -0.056 -0.268
##
## Standardized residuals:
##
         Min
                      Q1
                                            QЗ
                                Med
                                                       Max
## -2.8353741 -0.5472167 0.2440250 1.1919410 2.8367324
## Residual standard error: 0.2047976
## Degrees of freedom: 456 total; 453 residual
```

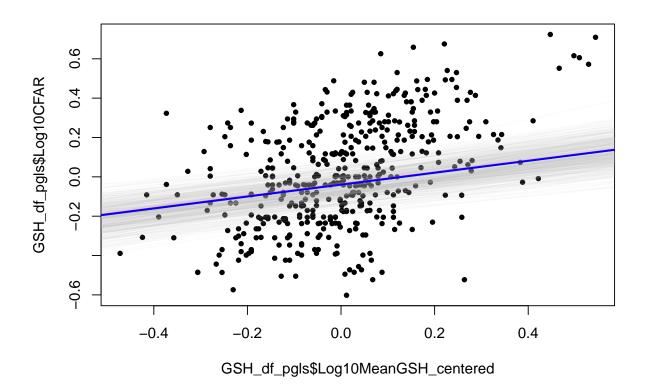


2. brms model

summary(Model_BS2)

```
Family: gaussian
     Links: mu = identity; sigma = identity
##
## Formula: Log10CFAR ~ Log10MeanGSH_centered + Log10MaxSize_centered + (1 | gr(Binomial, cov = A))
      Data: GSH_df_pgls (Number of observations: 456)
##
     Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##
##
            total post-warmup draws = 4000
##
## Group-Level Effects:
  ~Binomial (Number of levels: 456)
##
                 Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)
                     0.01
                               0.00
                                         0.01
                                                  0.01 1.00
                                                                1186
                                                                          1678
## Population-Level Effects:
```

```
Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS
##
                                        0.06
                                                -0.16
                                                          0.08 1.00
## Intercept
                             -0.04
                                                                         1283
                                                 0.19
## Log10MeanGSH_centered
                              0.30
                                        0.06
                                                          0.42 1.00
                                                                         3622
## Log10MaxSize_centered
                              0.16
                                        0.03
                                                 0.10
                                                          0.23 1.00
                                                                         3812
                         Tail_ESS
## Intercept
                              2054
## Log10MeanGSH_centered
                              3227
## Log10MaxSize_centered
                              3414
##
## Family Specific Parameters:
         Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
             0.12
                       0.01
                                 0.11
                                          0.13 1.00
                                                        1686
                                                                  2672
## sigma
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```



 TBD

Model 9 - CFAR \sim GSH + PrimaryHabitat + Phylogeny

Model 10 - CFAR \sim GSH * PrimaryHabitat + Phylogeny

 ${\bf Model~11 - CFAR \sim GSH~*~MaxSize~+~PrimaryHabitat~+~Phylogeny}$