



Emittance blow-up studies at the transfer from the PSB to the PS

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Outline

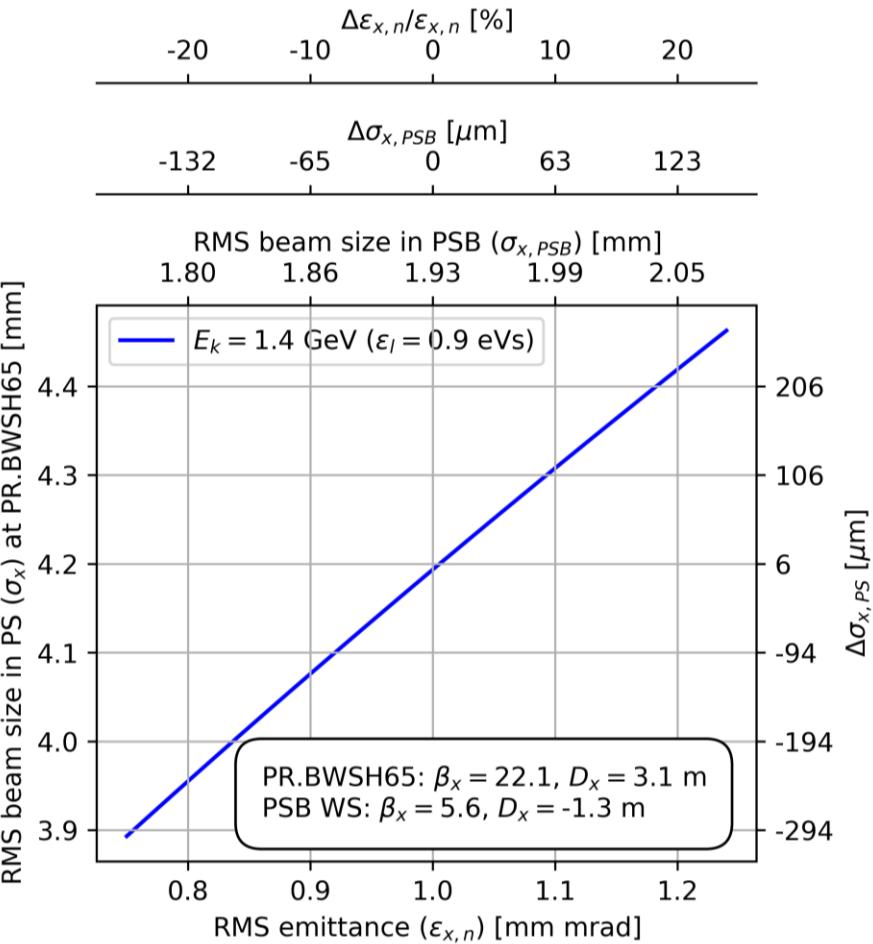
- Introduction
 - *A quick aside on systematic errors*
- Turn-by-turn measurements vs. simulation
 - Dispersion, chromaticity, beam size
- Emittance blow-up estimations
 - Analytic and tracking results
 - Measurements of filamented beam
- Conclusions

Introduction

- A **larger than expected** emittance blow-up is measured by comparing wire-scanner beam profiles measured in the **PSB** and **PS**
- Our understanding is that the dispersion mismatch should be the dominant source of blow-up:
...but we are plagued by systematic errors
- Relative beam size measurements in PS only saw a very small effect of re-matching the transfer line:
 - **Operational (OP) transfer line optics** was compared to a **rematched (ReM) optics**
 - **Turn-by-turn beam profile measurements** carried out to quantify the level of mismatch

Systematic errors

- Assuming a systematic error on beam size of $\pm 100 \mu\text{m}$:
 - Error of $\pm 15\%$ on ε_x in PSB
 - Error of $\pm 10\%$ on ε_x in PS
- In addition, we also have to consider other sources errors, for example:
 - Measured momentum spread (used to deconvolute)
 - Treatment of profile (fitting), e.g. typically Gaussian distributions are assumed
 - Measured optics functions
 - Even statistical - lots of shots needed f(intensity)

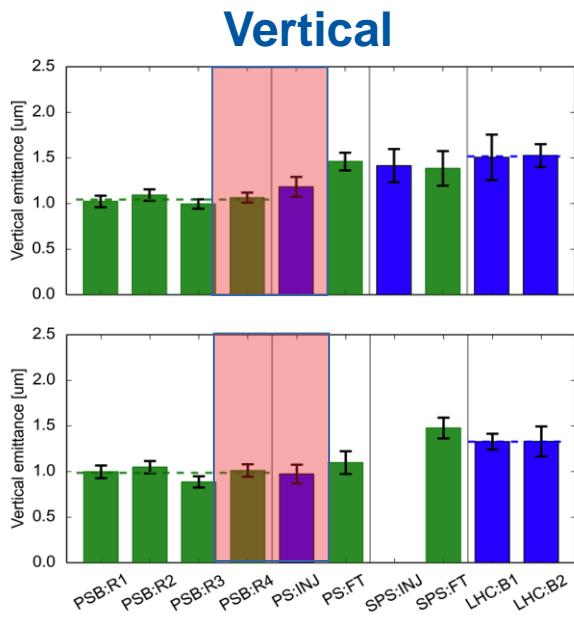
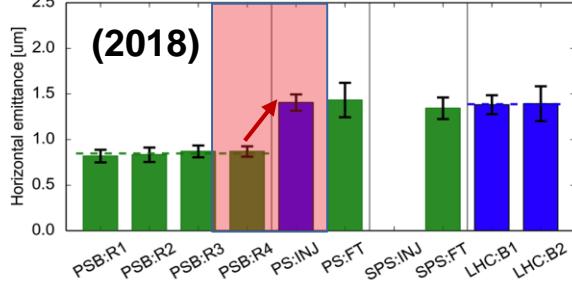
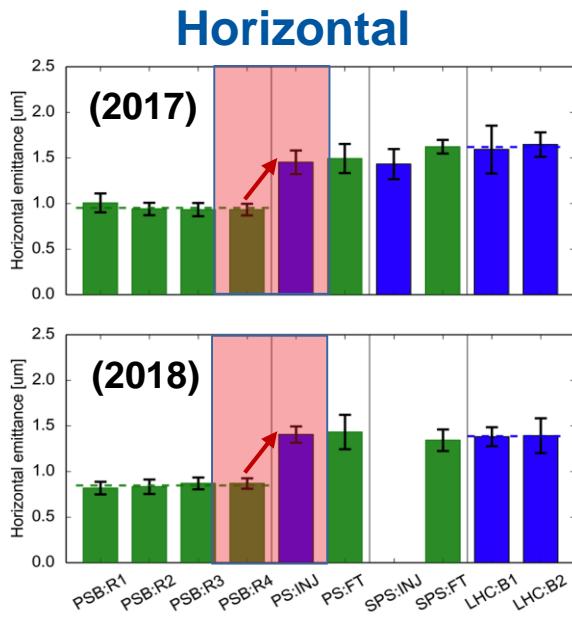


(1.4 GeV OP BCMS beam parameters)

See extra slides for similar plots with LIU beam parameters

Emittance blow-up

- No known source can explain such a large blow-up:
systematic errors between different WS devices are important especially with the large dispersion component



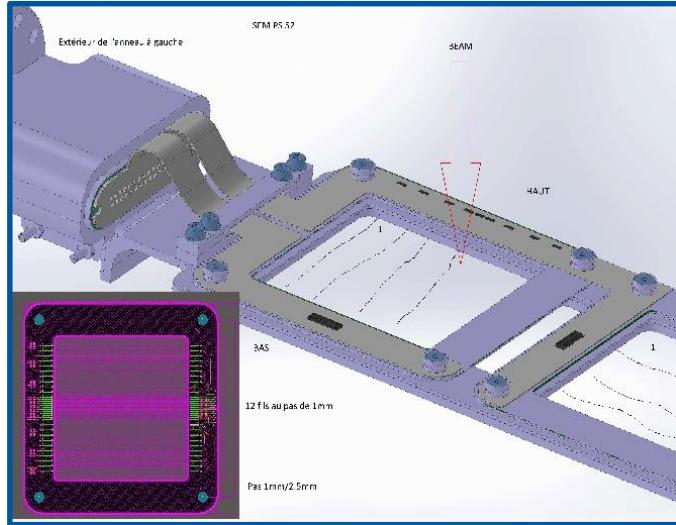
- Detailed studies in 2018 showed ~25% transverse emittance in the horizontal plane
- We expect ~10% from dispersion mismatch

F. Antoniou *et al.*

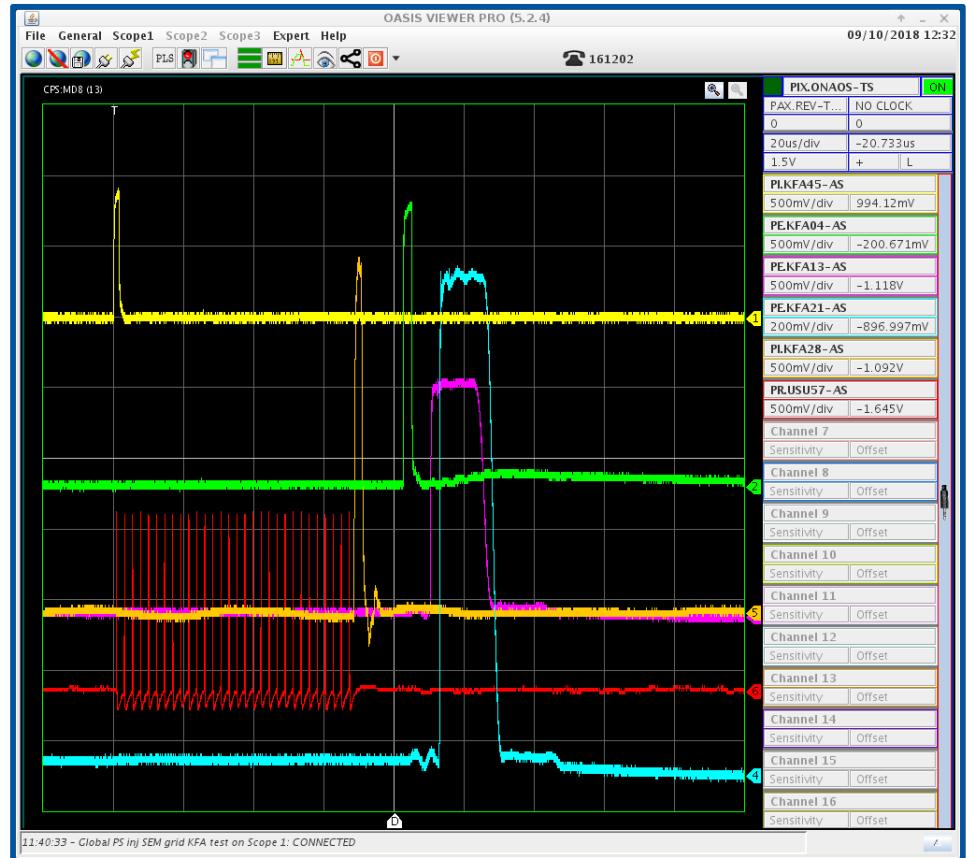
Turn-by-turn MD conditions

- Beam injected on BCMS cycle with low chroma, coupling corrected:
 - $Q_x \sim 6.21$, $Q_y \sim 6.23$
 - **RF OFF** and TFB ON
 - Single bunch from Ring 3 with $50 - 90 \times 10^{10}$ ppb
 - **Results for $\sim 65 \times 10^{10}$ ppp presented**
- Transfer line optics (**ReM**) designed and tested by V. Forte:
 - See *HB2018 paper for details*
 - **Significantly improved matching on Ring 3** used in PPM MD operation
- Beam profiles measured turn-by-turn using a wire-grid in SS52:
 - 30 turns acquired before beam removed by kickers (to protect grid)
- Limited beam time because inserting the BSG blocks the complex:
 - A couple of hours on the last day of dedicated proton MD

Turn-by-turn measurements



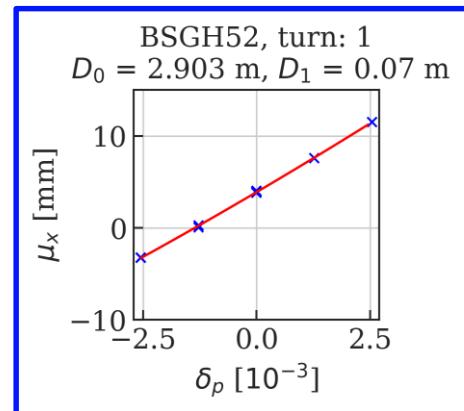
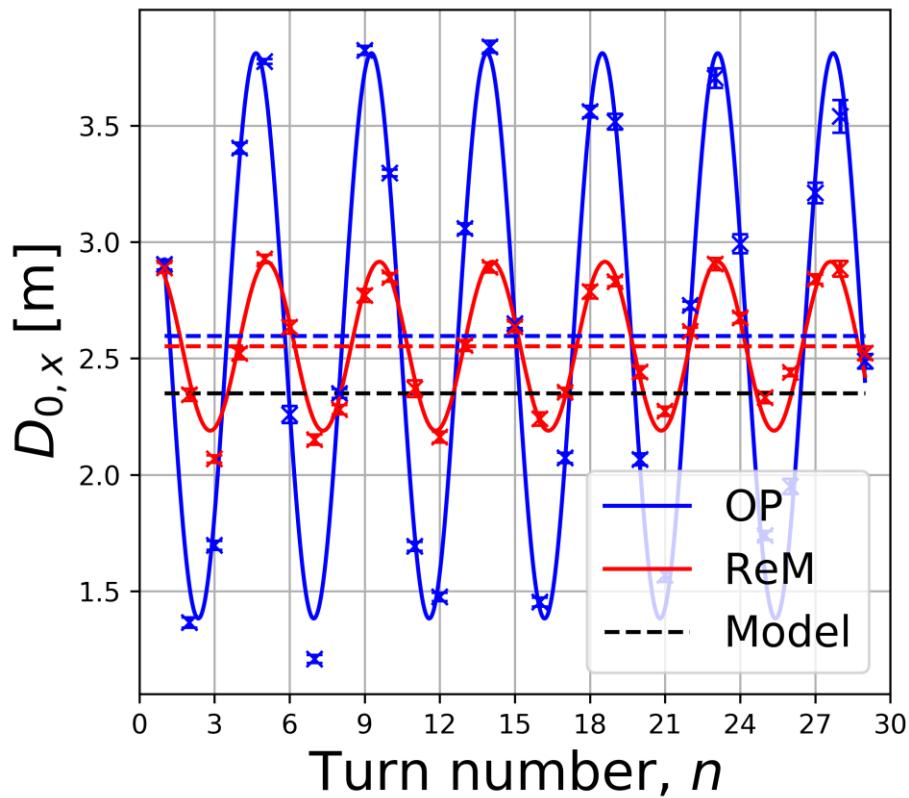
SSGH52:
Central 12 wires: 1 mm pitch
External 20 wires: 2.5 mm pitch



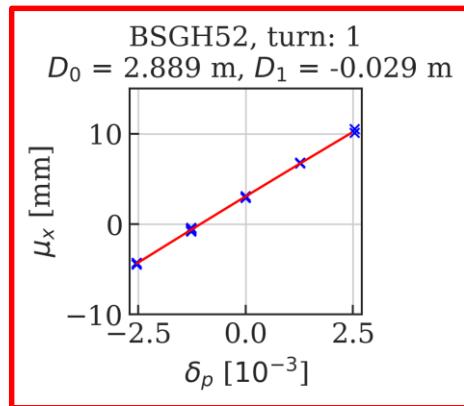
Multiple (redundant) kicker systems timed to remove the beam

D measurements (BSGH52) (1)

- Closed dispersion at BSGH52 differs from the model by 10%:



OP transfer line optics



ReM transfer line optics

Model $D_x = 2.35$ m, measurements $D_x = 2.60$ m (OP) and 2.55 m (ReM)

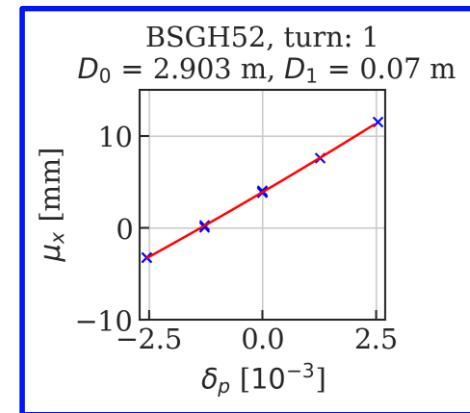
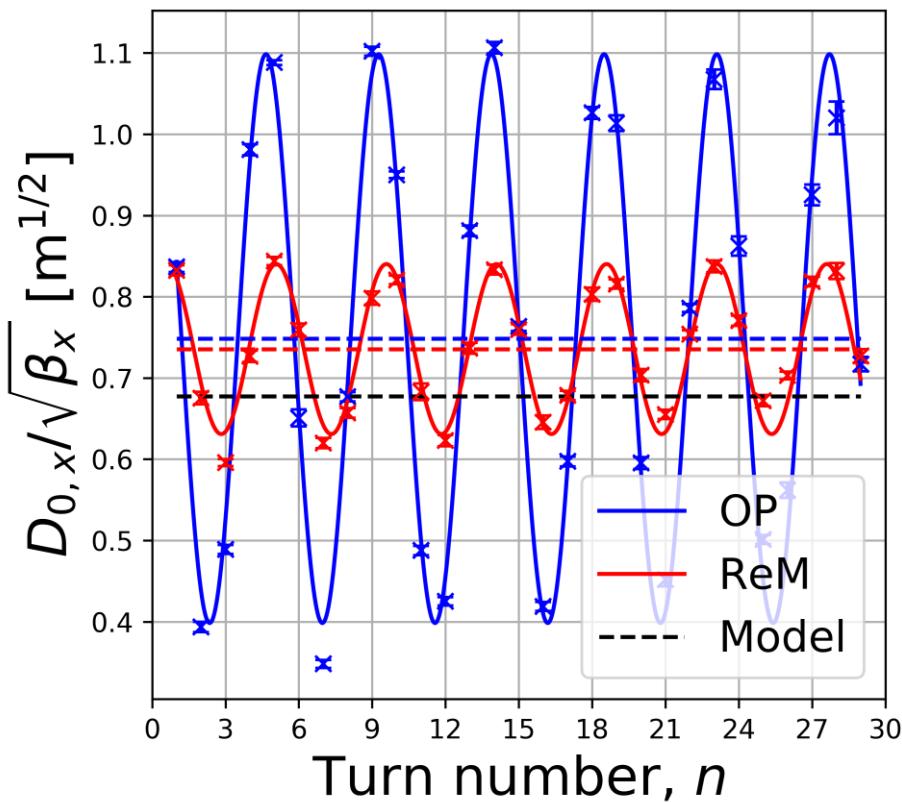
M.A. Fraser, 4th ICFA Mini-Workshop on Space Charge, 4 - 6 November 2019

D measurements (BSGH52) (2)

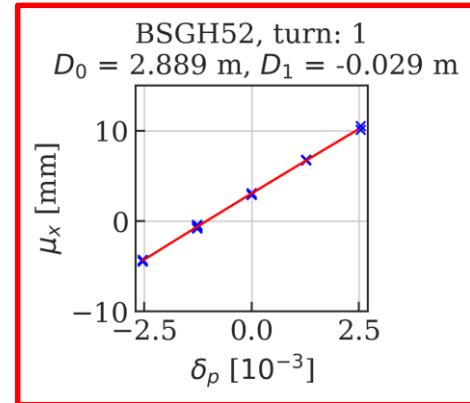
$$\bar{D}(n) = \bar{D}_0(n) + M_D \cos(\theta + 2\pi(n-1)q_x)$$



Amplitude of mismatch



OP transfer line optics



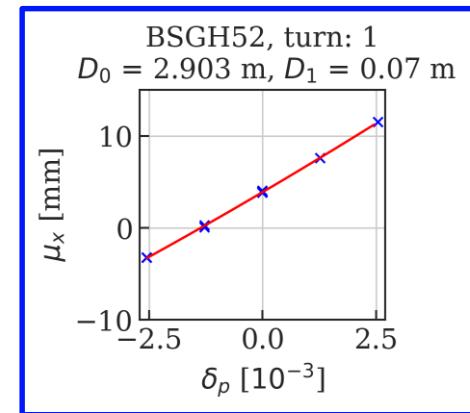
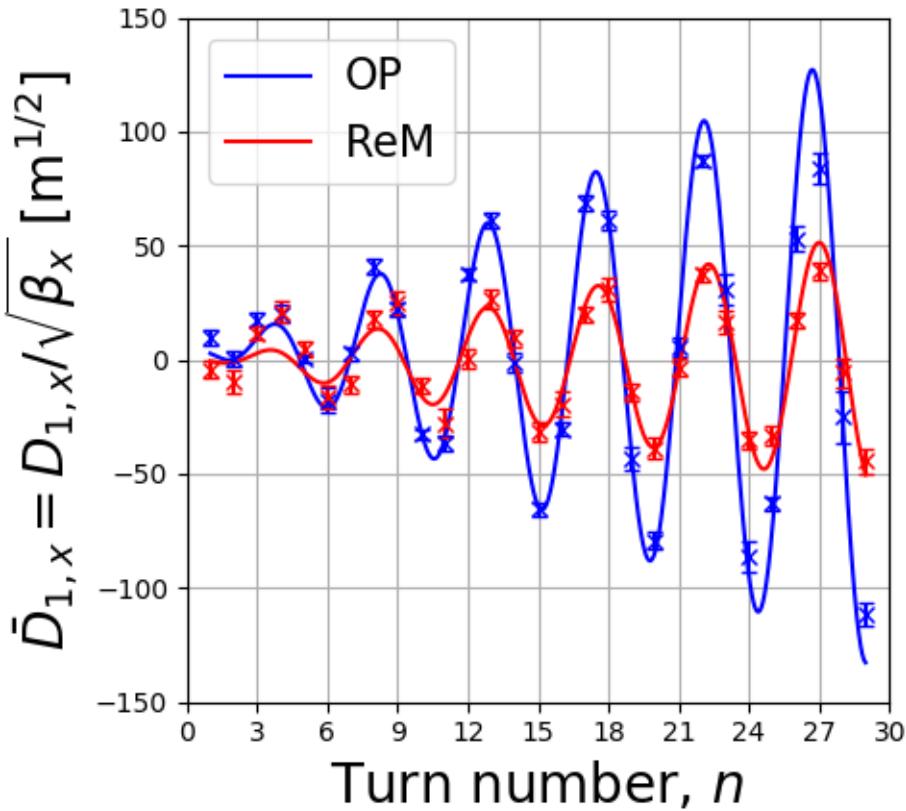
ReM transfer line optics

Normalisation made with model $\beta_x = 12.04 \text{ m}$, measurements (P. Skowronski) $\beta_x = 12.02 - 12.07 \text{ m}$

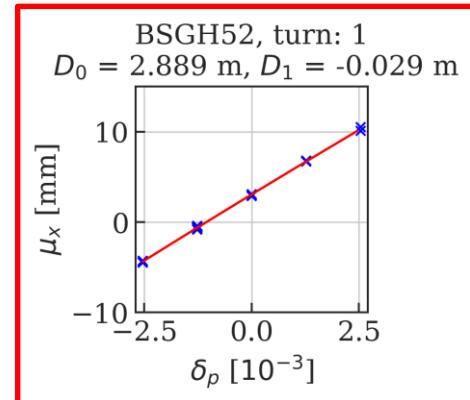
M.A. Fraser, 4th ICFA Mini-Workshop on Space Charge, 4 - 6 November 2019

Non-linear component (BSGH52)

$$\bar{D}(n) = \bar{D}_0(n) + \bar{D}_1(n)\delta_p$$



OP transfer line optics

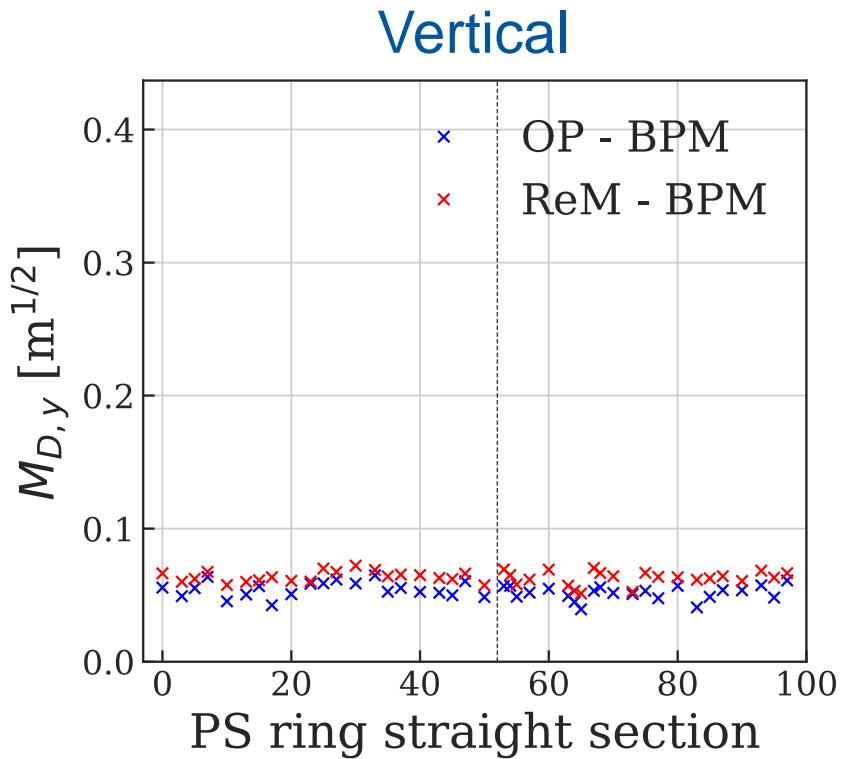
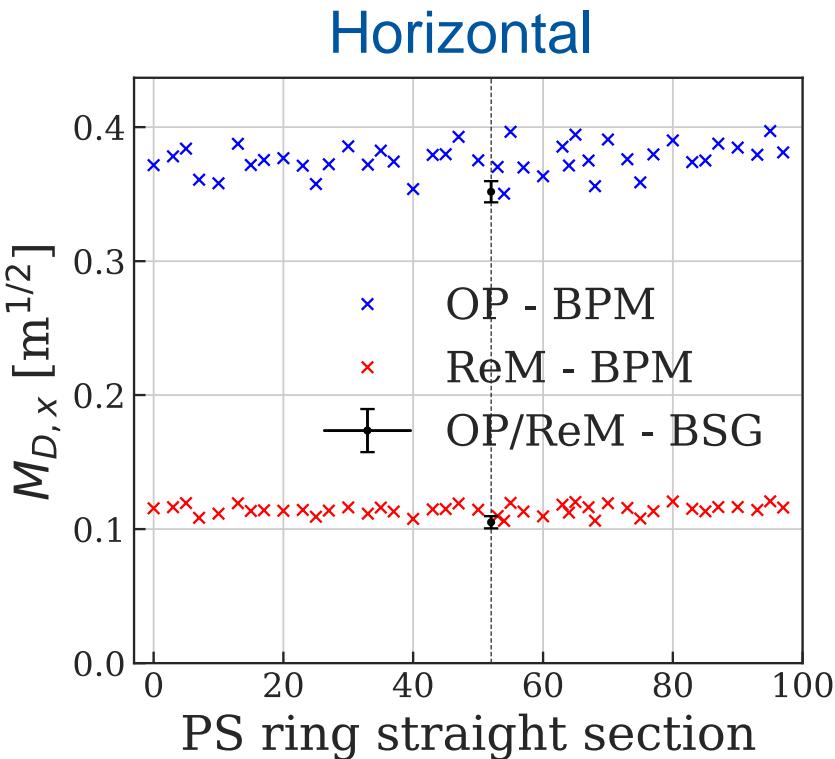


ReM transfer line optics

Dispersion mismatch summary

- Identical behaviour observed on all ring BPM's:

$$\bar{D}(n) = \bar{D}_0(n) + M_D \cos(\theta + 2\pi(n - 1)q_x)$$



See extra slides for example on BPM00

Tracking simulations

- Latest optics model of machine:
 - *acc-models* by A. Huschauer
 - <https://gitlab.cern.ch/ahuschau/acc-models>
- Maps computed using MADX-PTC and exported from tracking using python:
 - *maptrack* by F. Velotti <https://gitlab.cern.ch/fvelotti/maptrack>
 - Used and tested extensively for SX studies
- Conditions: **no space-charge**, injection bump on but static (no collapse), 5D (RF off as in measurements)
- Transverse distribution is Gaussian and longitudinal distribution is parabolic:

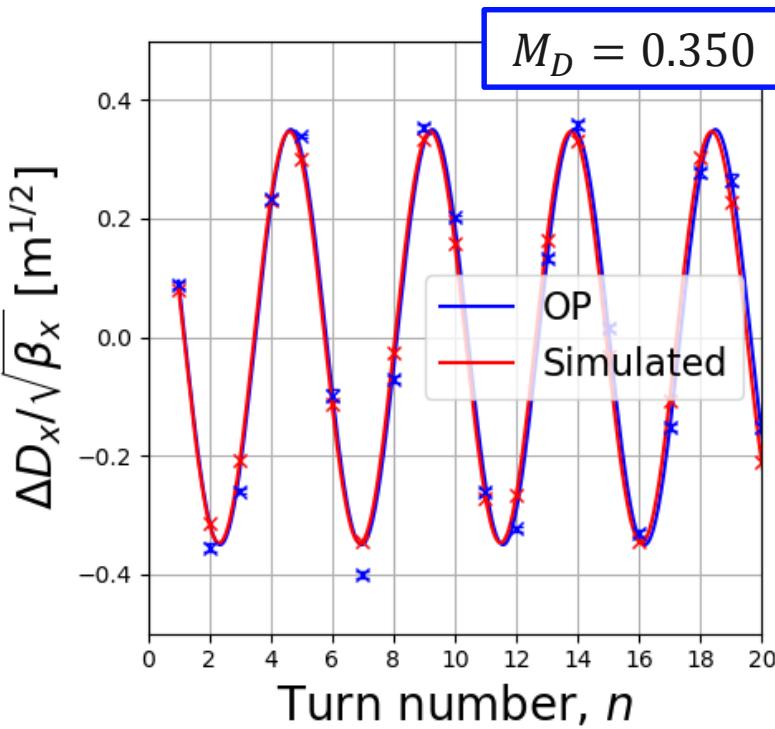
$$\propto (1 - \Delta p^2) \text{ where } \Delta p_{\max} = \sqrt{5} \Delta p_{\text{rms}}$$

See extra slides for benchmarking of maptrack vs. PTC tracking

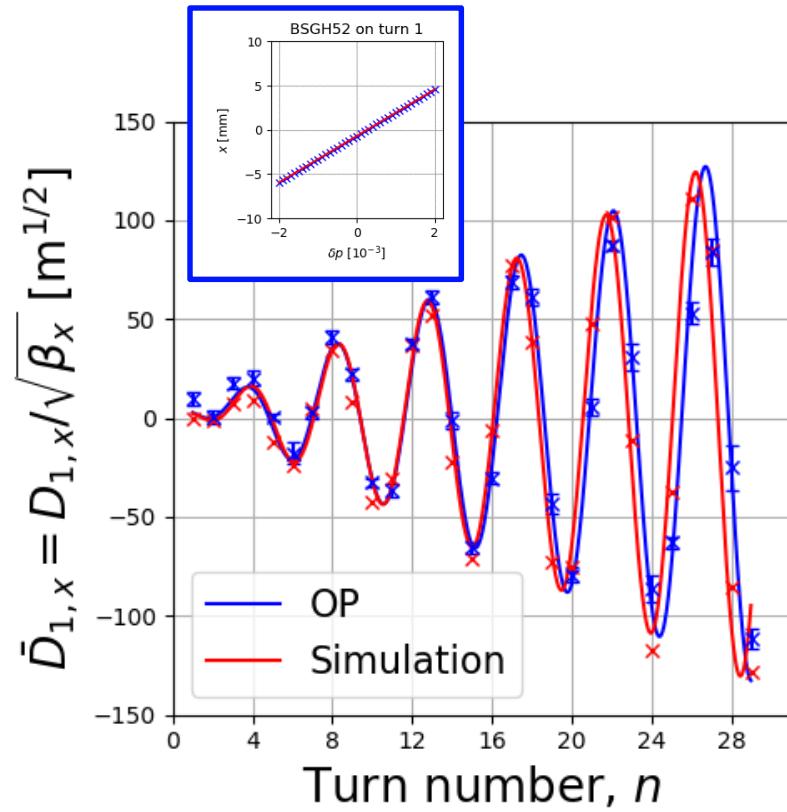


Dispersion: OP optics vs model

- Relative mismatch fitted in simulation by adjusting the unknown D' :

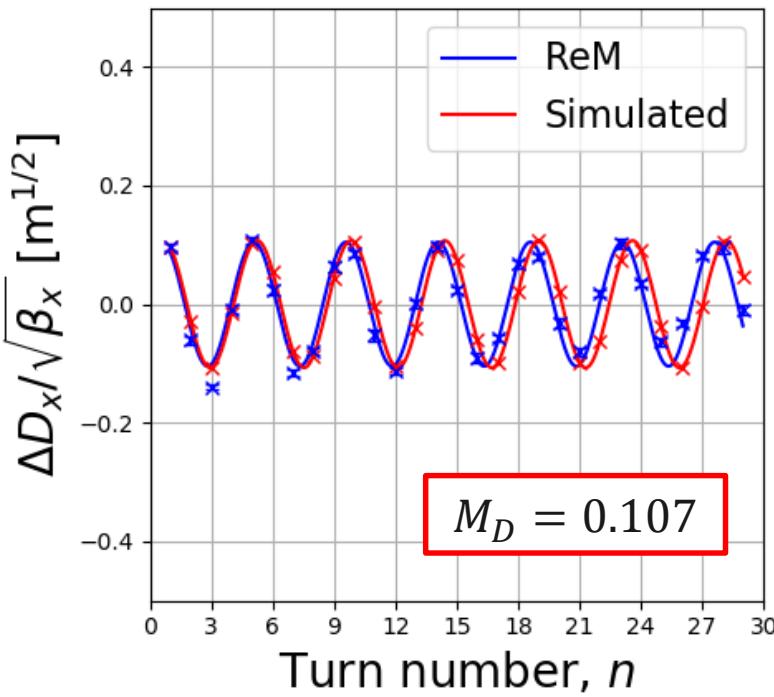


$$D_{x,0} = 2.633$$
$$D'_{x,0} = -0.104$$

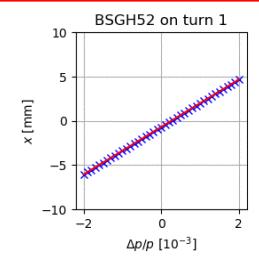
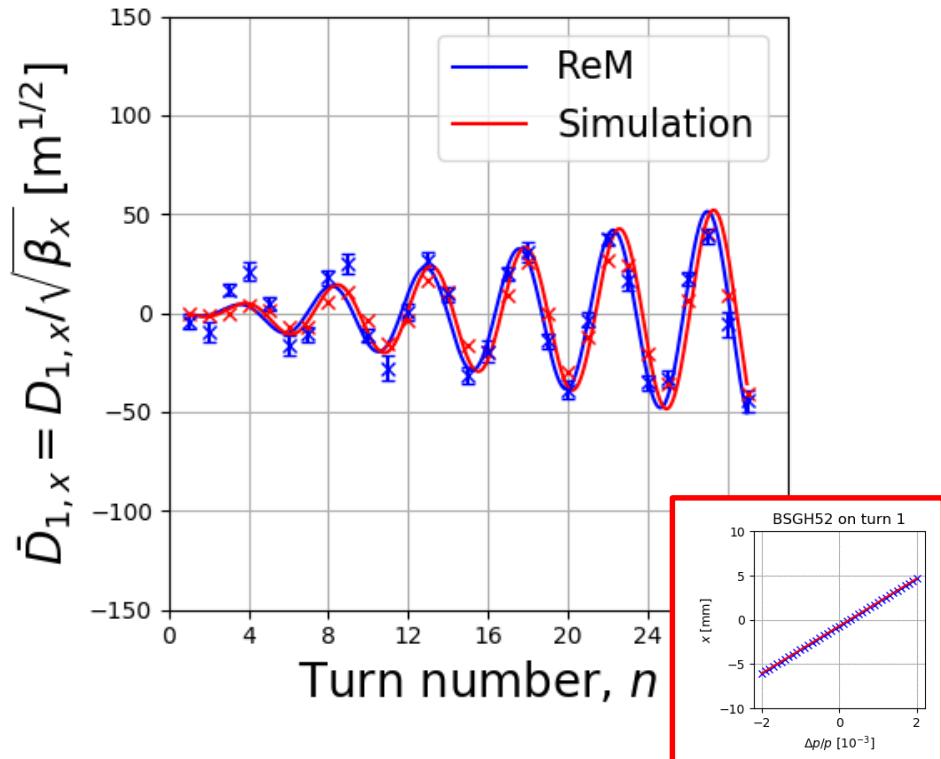


Dispersion: ReM optics vs model

- Relative mismatch fitted in simulation by adjusting the unknown D' :

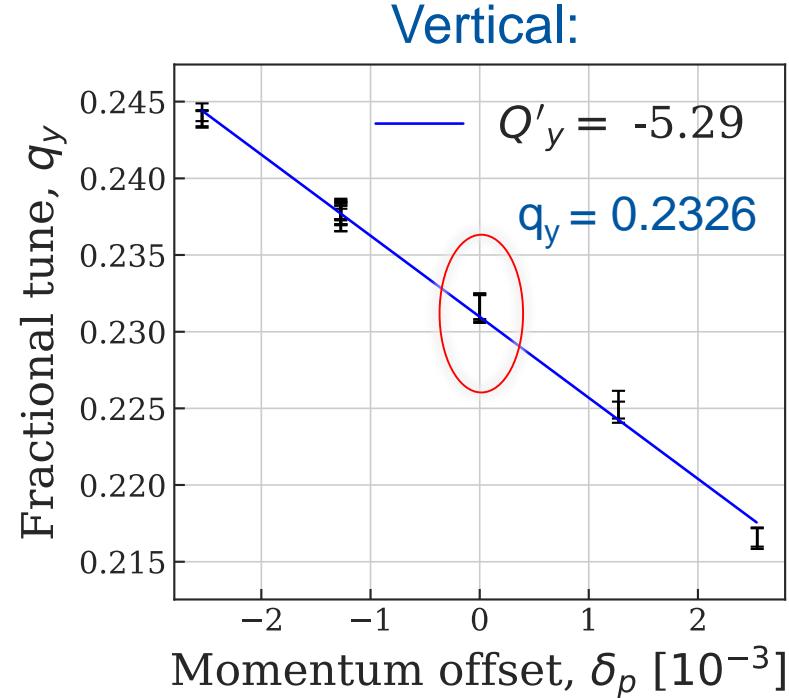
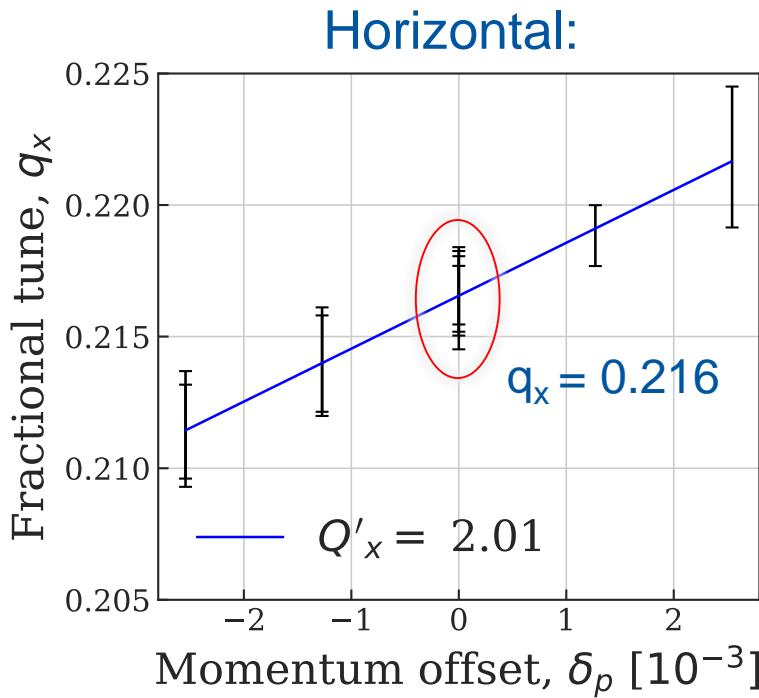


$$\begin{aligned} D_{x,0} &= 2.683 \\ D'_{x,0} &= -0.021 \end{aligned}$$



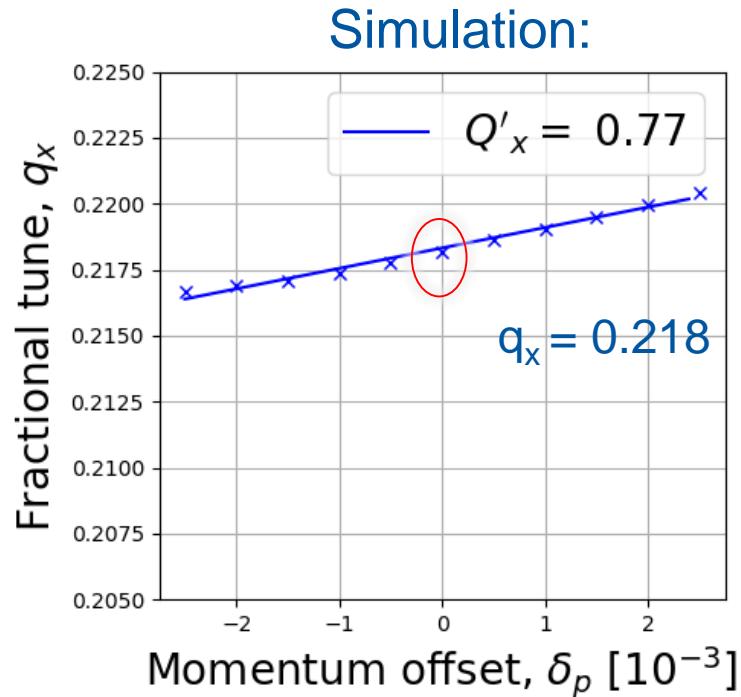
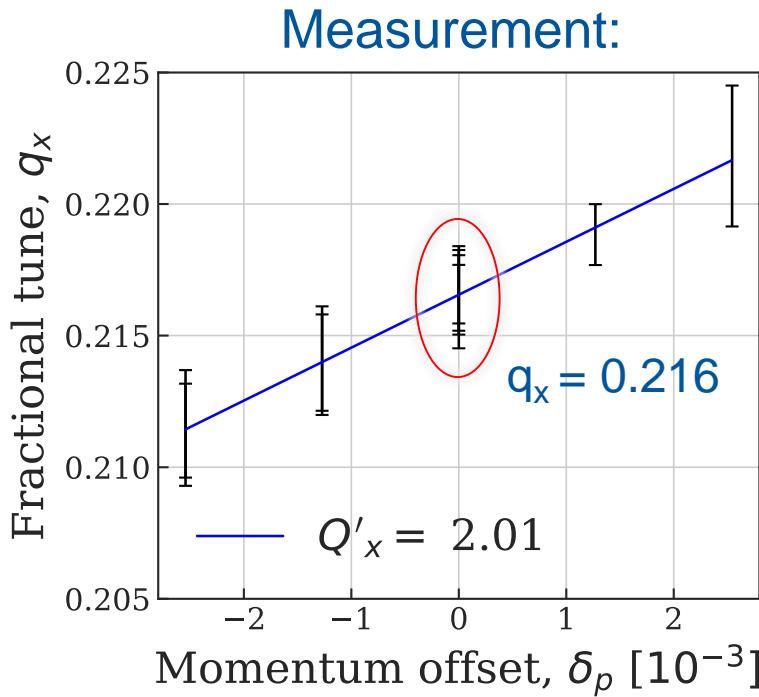
Chromaticity (BPM data)

- During first 30 turns Q' measured **x2 larger** than expected:
 - FFT (Hanning window) vs. simple harmonic fit gives same result
 - Measurements using Q-meter a few ms later $Q'_x \sim 0.8$ and $Q'_y \sim -2.9$



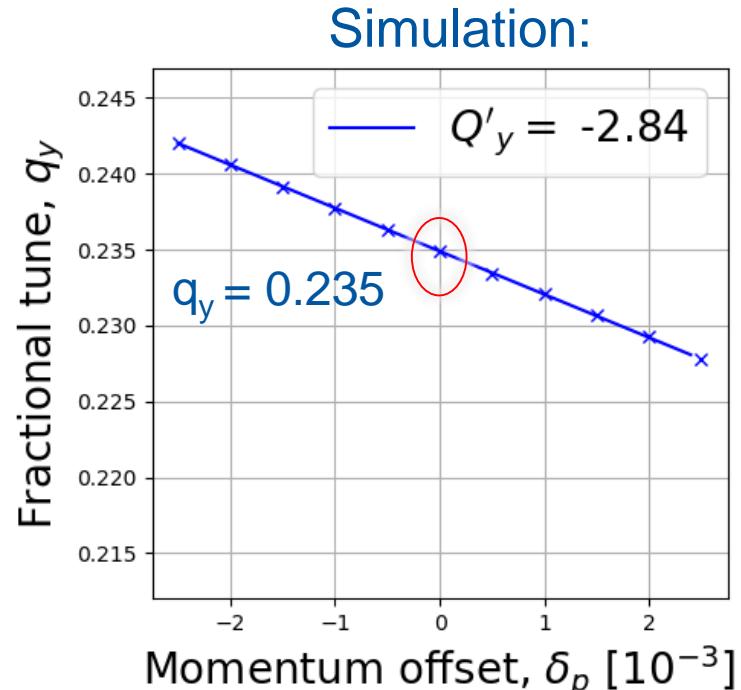
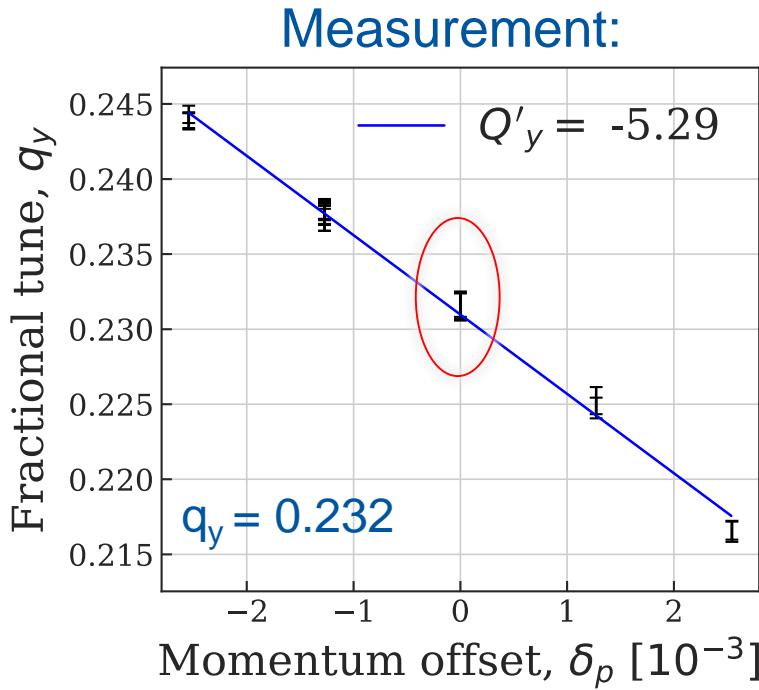
H chromaticity vs. model

- During first 30 turns Q' measured **x2 larger** than expected:
 - Simulating the measurement gives the nominal chromaticity $Q'_x \sim 0.8$



V chromaticity vs. model

- During first 30 turns Q' measured **x2 larger** than expected:
 - Simulating the measurement gives the nominal chromaticity $Q'_y \sim -2.9$



Beam profile measurements - H (1)

- Fitting with a 5-parameter Gaussian with linear baseline:

$$\frac{\sigma_x^2(n)}{\beta_x} = \frac{\sigma_\beta^2(n)}{\beta_x} + \frac{\sigma_D^2(n)}{\beta_x} + \frac{\sigma_{BSG}^2(n)}{\beta_x}$$

Betatronic component

Dispersive component

blow-up from wire grid

The diagram illustrates the decomposition of the normalized beam size variance $\sigma_x^2(n)/\beta_x$ into its components. It shows three terms: $\sigma_\beta^2(n)/\beta_x$, $\sigma_D^2(n)/\beta_x$, and $\sigma_{BSG}^2(n)/\beta_x$. Arrows point from the labels 'Betatronic component' and 'Dispersive component' to their respective terms. A double-headed arrow points between the second and third terms, indicating they are added together. A final arrow points from the label 'blow-up from wire grid' to the third term.

See extra slides for V plane measurements

Beam profile measurements - H (1)

- Fitting with a 5-parameter Gaussian with linear baseline:

$$\frac{\sigma_x^2(n)}{\beta_x} = \frac{\epsilon_x}{2} \left(\left(M_g + \frac{1}{M_g} \right) + \left(M_g - \frac{1}{M_g} \right) \cos(\phi + 4\pi(n-1)q_x) \right) + \left(\frac{\sigma_p}{p} \underbrace{\frac{(\bar{D}_{0,x} + M_{D,x} \cos(\theta + 2\pi(n-1)q_x))}{\bar{D}_{0,x}(n)=D_{0,x}(n)/\sqrt{\beta_x}}}_{} \right)^2 + \frac{\sigma_{BSG}^2}{\beta_x}$$

envelope oscillating at $2q_x$

envelope oscillation at q_x for $M_D \ll 1$

blow-up from wire grid

Betatron mismatch: $M_g > 1$

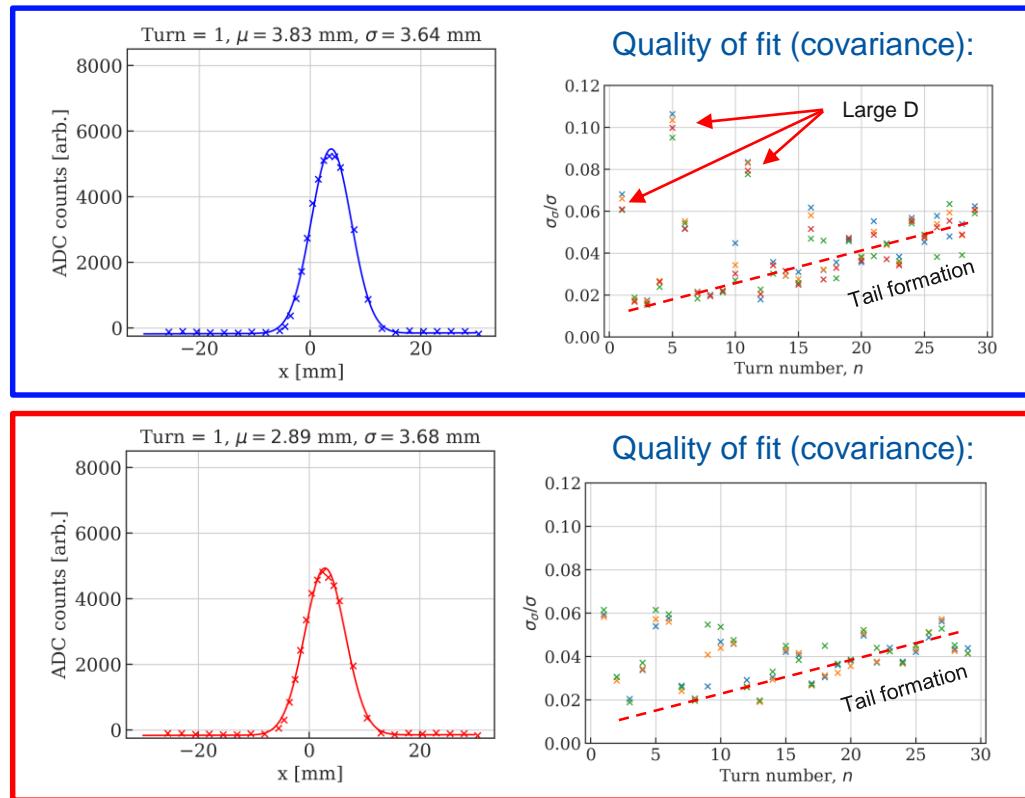
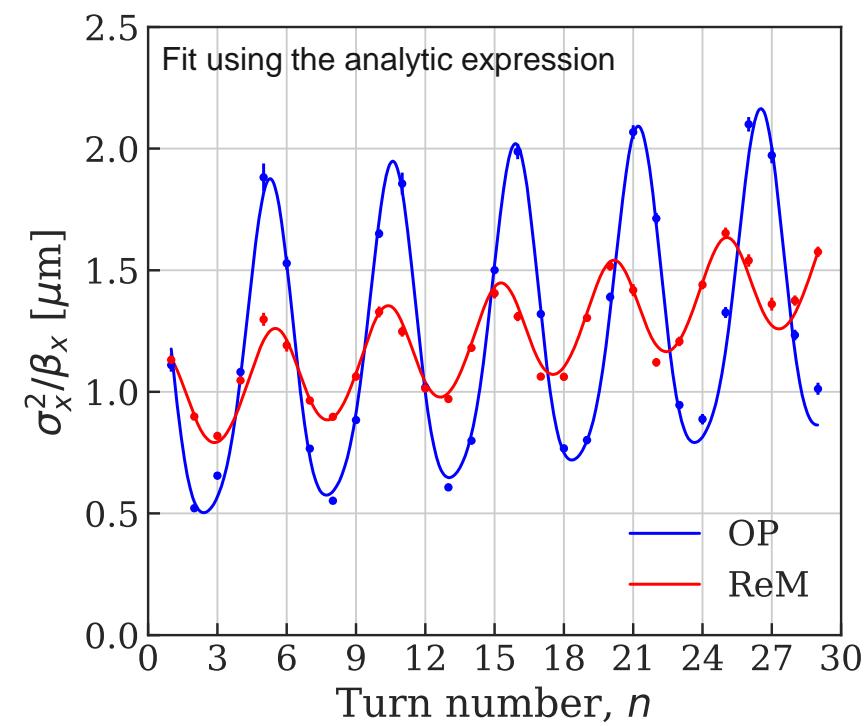
Dispersion mismatch: $M_D \neq 0$

- Gaussian works well as a quick and easy fitting method:
 - Aim here is not to deconvolute, yet... it's really tricky
- Blow-up from interaction with grid:
$$\frac{\sigma_{BSG}^2}{\beta_x} = \beta_x \Delta x'_{\text{rms}}^2 \left(\frac{n-1/2}{2} - \frac{\sin(2\pi(2n-1)q_x)}{4 \sin(2\pi q_x)} \right)$$

See extra slides for V plane measurements

Beam profile measurements - H (2)

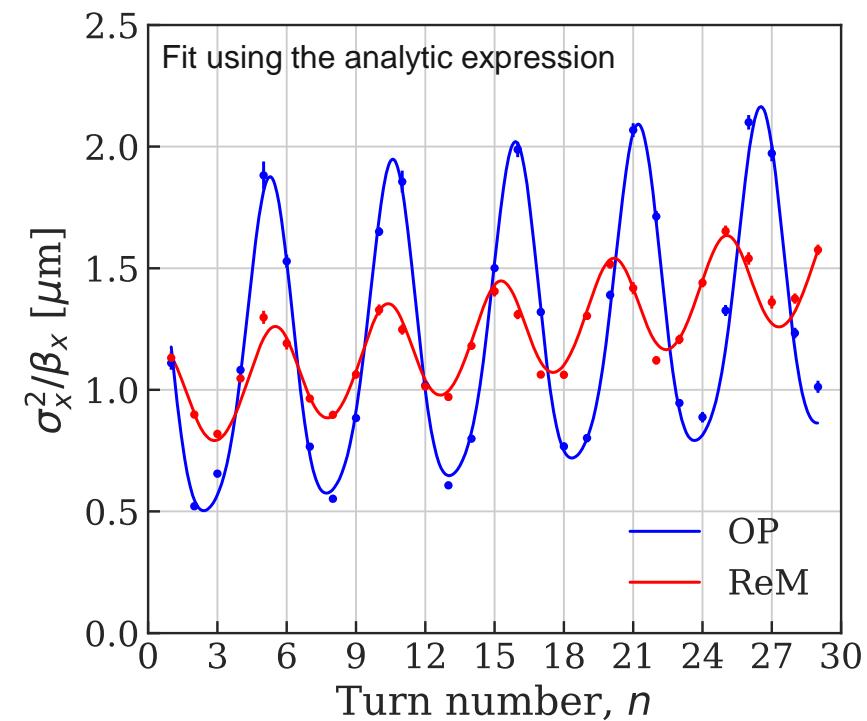
- Beam size computed from profiles at BSGH52 by fitting a 5-parameter Gaussian (with linear baseline)



Don't forget that the grid is scattering and blowing up the beam!

Beam profile measurements - H (3)

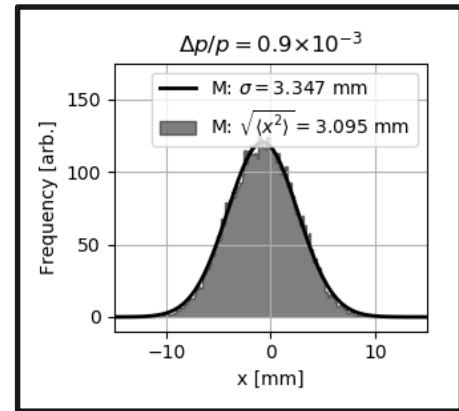
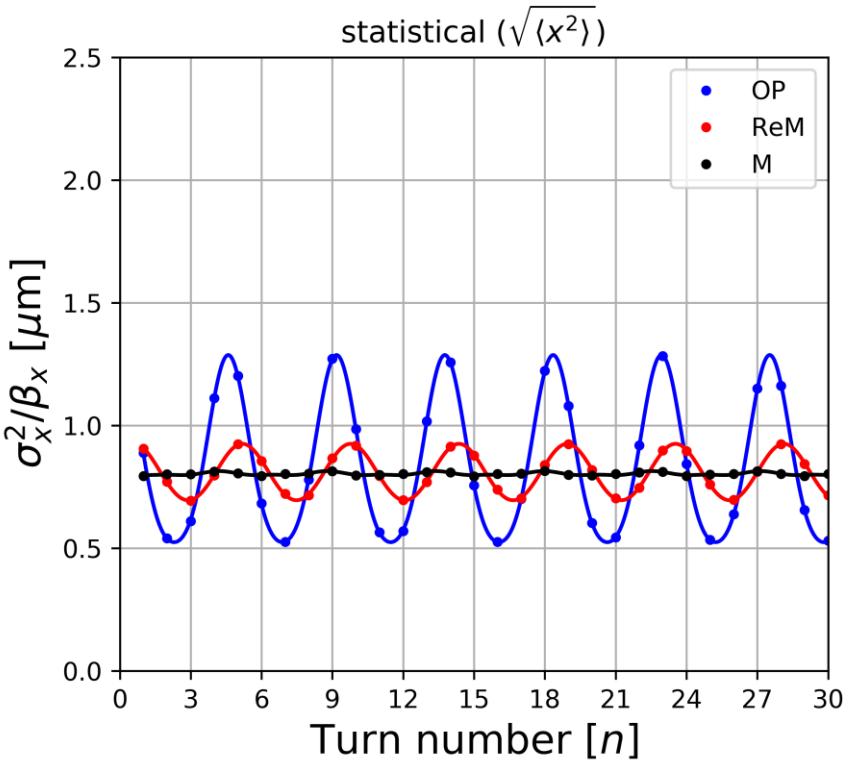
- Oscillation frequency of beam size is **clearly depressed** and depends on the mismatch



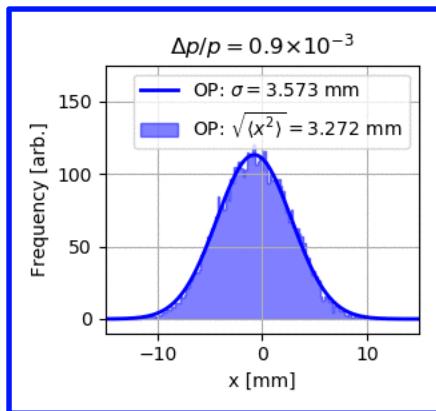
Free parameters	OP optics	ReM optics
q_x	0.188	0.204
ϵ [mm mrad]	0.69	0.74
$\sigma_p/p [10^{-3}]$	1.09	1.09
ϕ	1.38	-0.29
M_g	1.09	1.05
$\beta_x \Delta x'^2_{\text{rms}}$ [μm]	0.027	0.038
Constrained parameters	OP optics	ReM optics
M_D	0.38	0.12
\bar{D}_0 [$\text{m}^{1/2}$]	0.72	0.74
θ	1.27	0.59

Beam size simulations

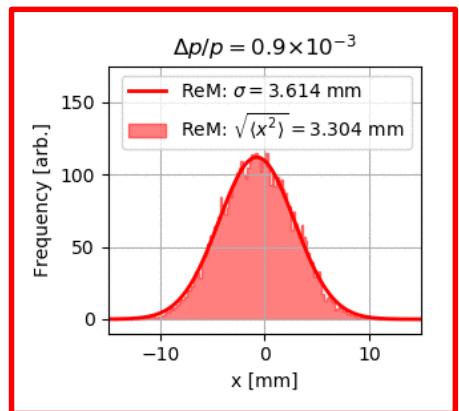
- Gaussian fitting plays a role where D is large (parabolic distribution)



M transfer line optics



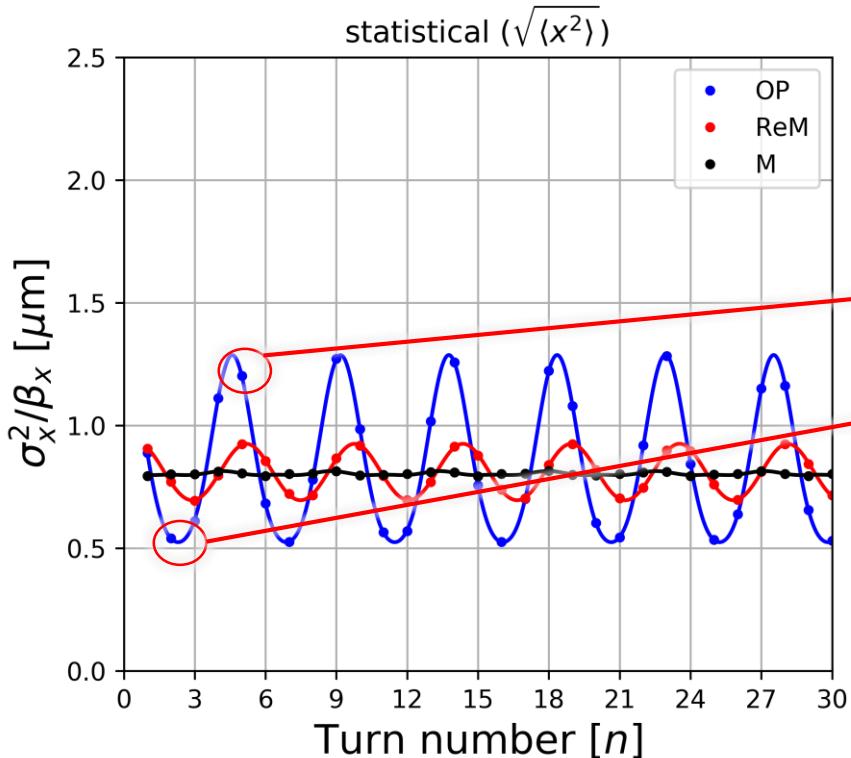
OP transfer line optics



ReM transfer line optics

Beam size simulations

- Gaussian fitting plays a role in systematic errors where D is large (parabolic distribution):

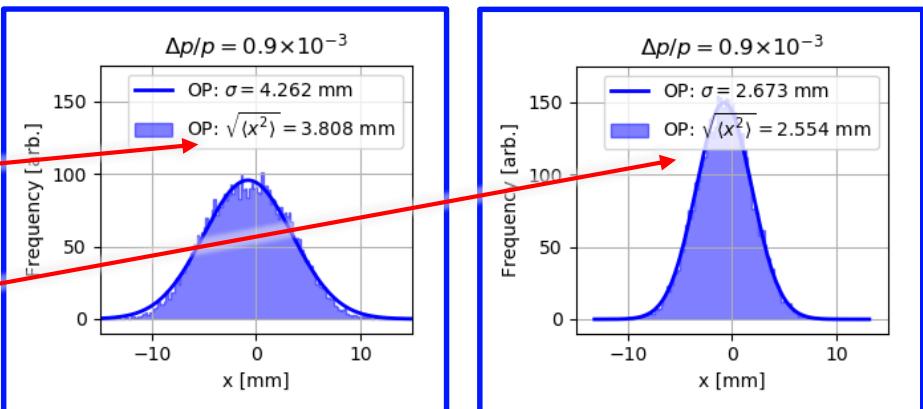


Turn 5

12% error

Turn 2

4% error

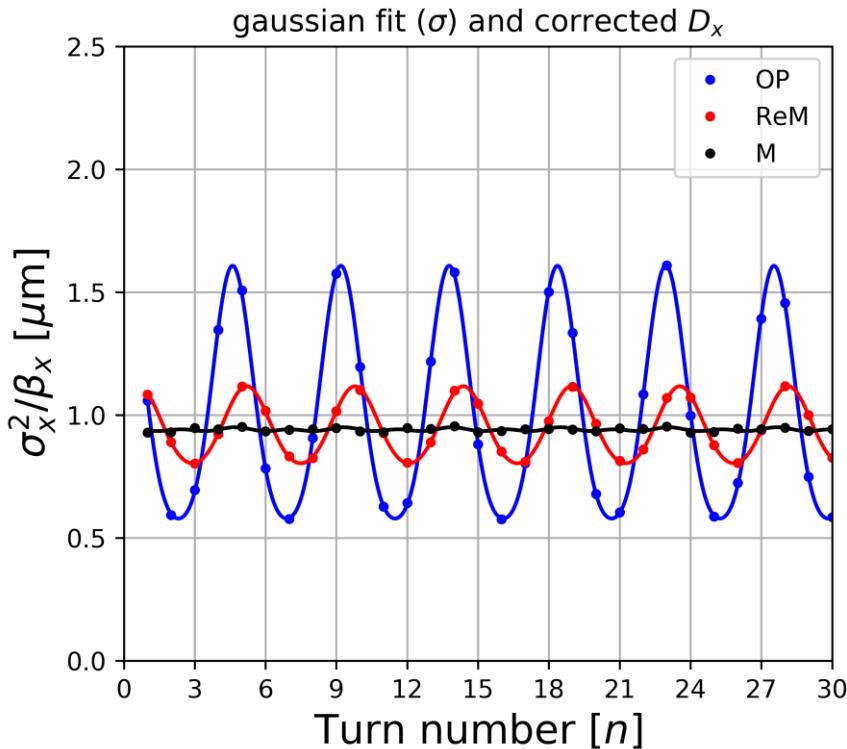


OP transfer line
optics

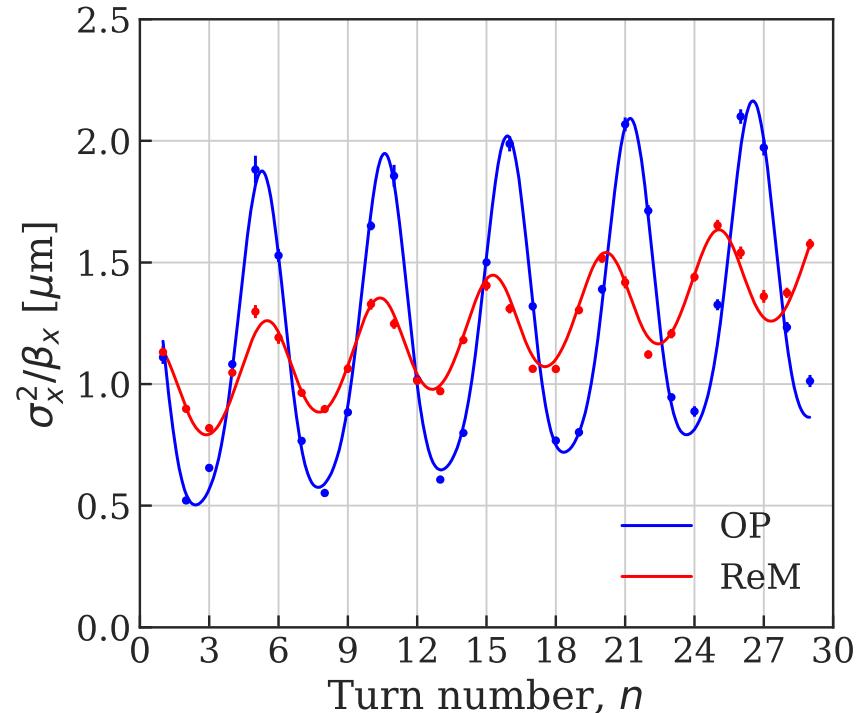
Beam size simulations vs model

- Detuning large and obvious:
 - One can simply count the different number of oscillations!

Simulation:



Measurement:



Scattering from grid not included in simulation

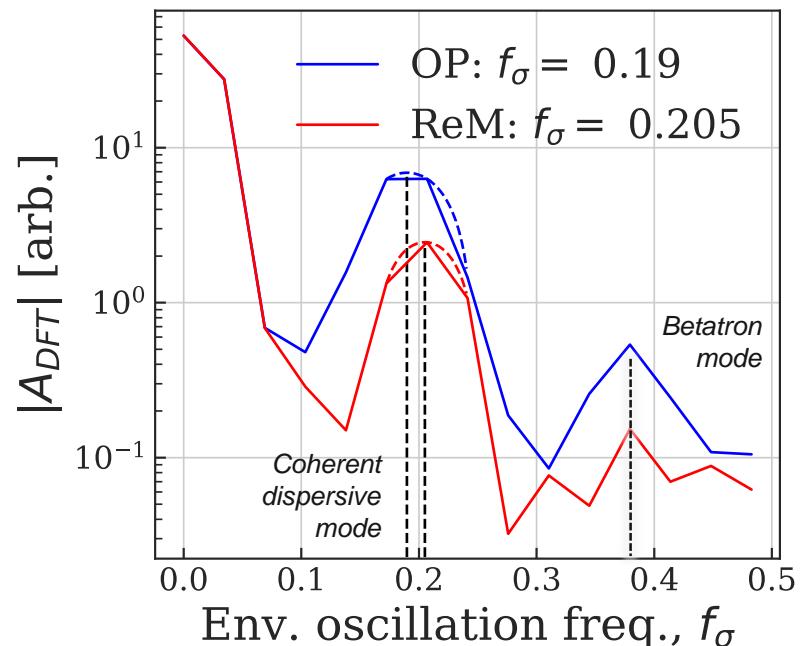
Beam size oscillation frequency

- FFT used to compute beam size oscillation frequency:
- Envelope oscillation depressed from q_x
- Dispersion dominated mismatch: *effect of optics rematch is clear*
- Frequency depends on the amount of mismatch
- Limited resolution from number of turns: parabolic interpolation of the spectral peaks employed
- Higher frequency mode also appears depressed from $2q_x$

Measurement:

$$(q_x = 0.216)$$

Hanning window function:



See extra slides for explanation of error propagation (extracted from FFT on analytic model)

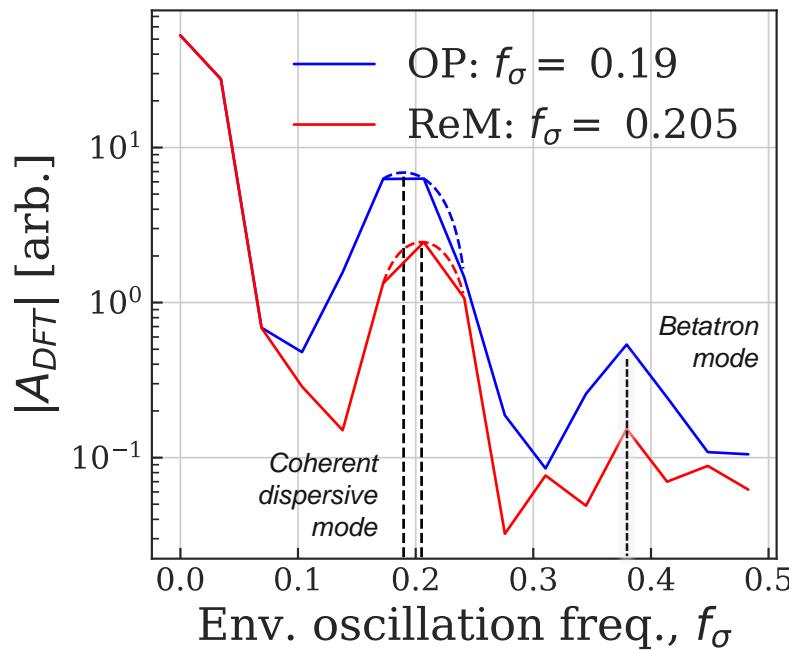
Beam size oscillation freq. vs. model

- As expected, no detuning in simulation without space-charge:

Measurement:

$$(q_x = 0.216)$$

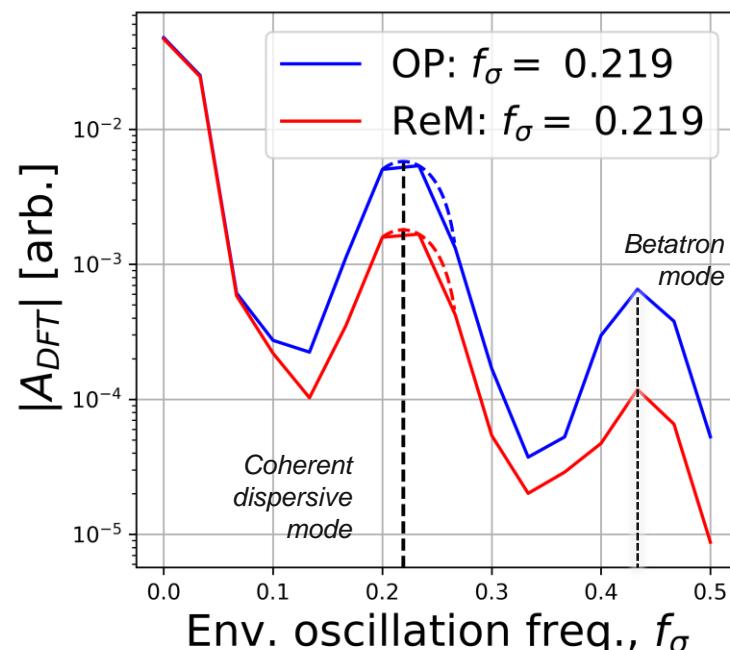
Hanning window function:



Simulation:

$$(q_x = 0.216)$$

Hanning window function:

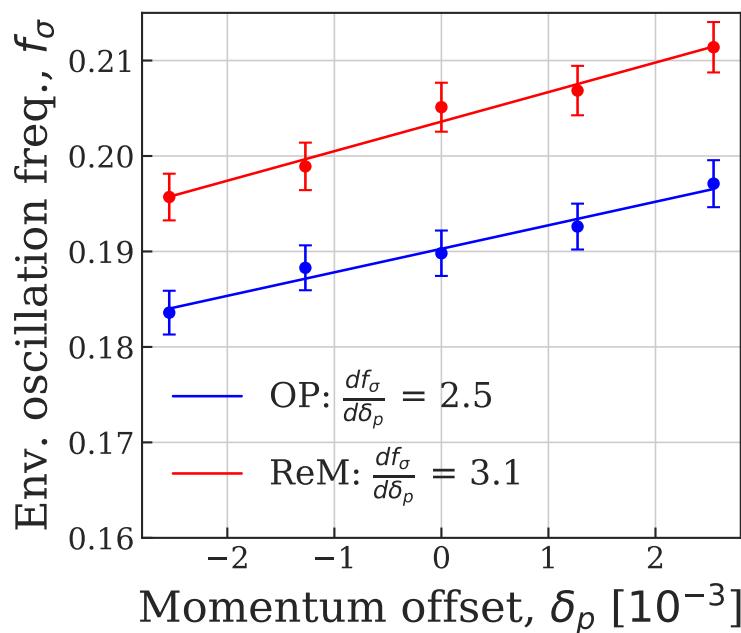


See extra slides for explanation of error propagation (extracted from FFT on analytic model)

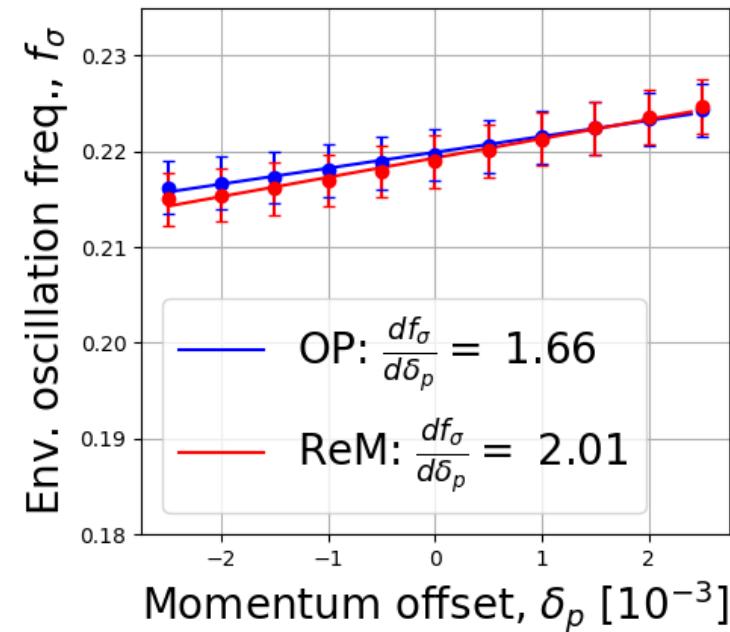
Beam size oscillation freq. vs. model

- Beam size oscillation frequency depends on momentum:
 - Hanning window applied to simulation and measurements
 - Larger than can be explained by chromaticity (but errors in FFT play a significant role)

Measurement:



Simulation:



See extra slides for explanation of error propagation (extracted from FFT on analytic model)

Emittance blow-up estimates

- Blow-up from dispersion mismatch can be written analytically as:

$$\Delta\epsilon = \frac{M_D^2}{2} \left(\frac{\sigma_p}{p} \right)^2 (\beta\gamma)_{\text{rel}} \sim 0.11 \text{ mm mrad}$$

- From dispersion mismatch alone we would expect a factor 10 difference in the measured blow-up:

$$\frac{\Delta\epsilon_{\text{OP}}}{\Delta\epsilon_{\text{ReM}}} = \left(\frac{M_{D,\text{OP}}}{M_{D,\text{ReM}}} \right)^2 \sim 10$$

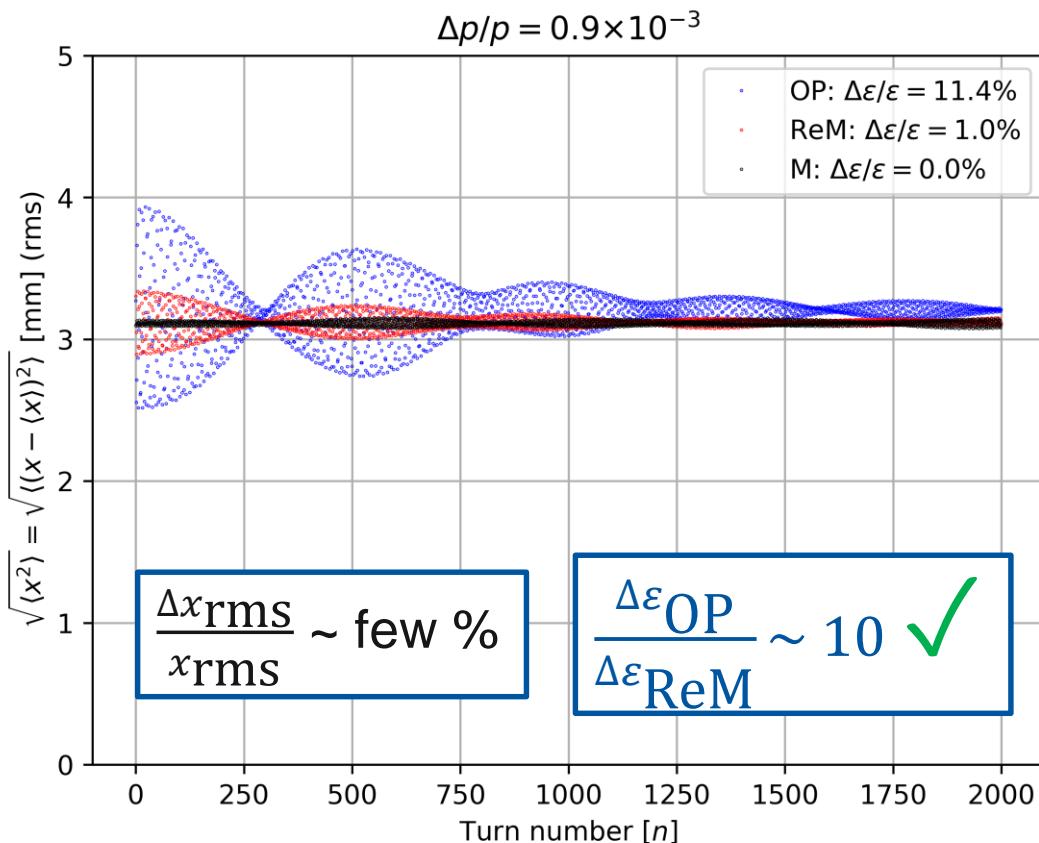
- Analytic estimates have been confirmed with tracking studies (independent of distribution using RMS)

See extra slides for comparison to analytic estimates and associated errors introduced by assuming Gaussian distributions



Tracking studies: $E_i = 0.9$ eVs

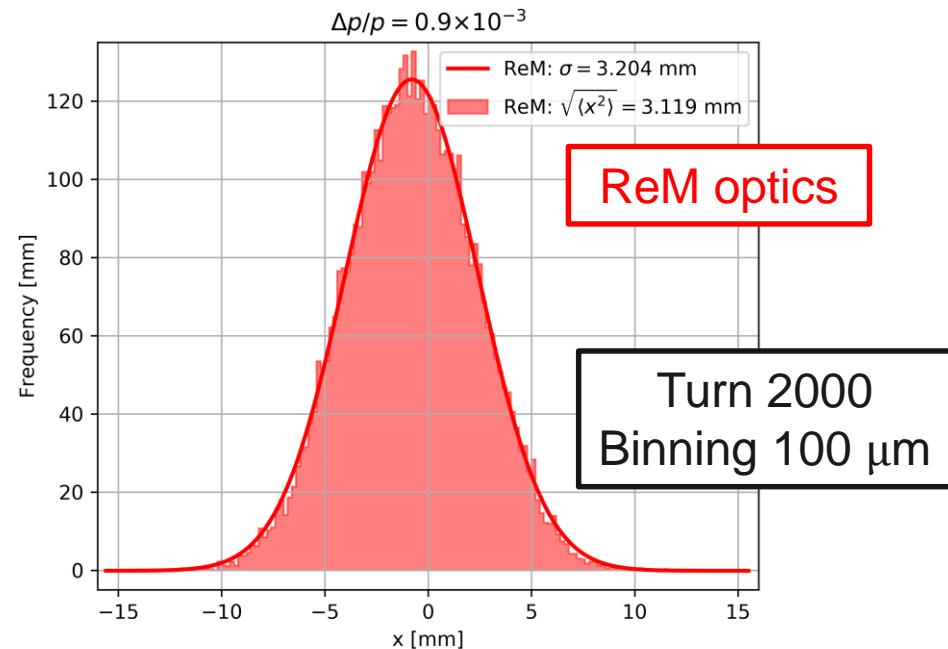
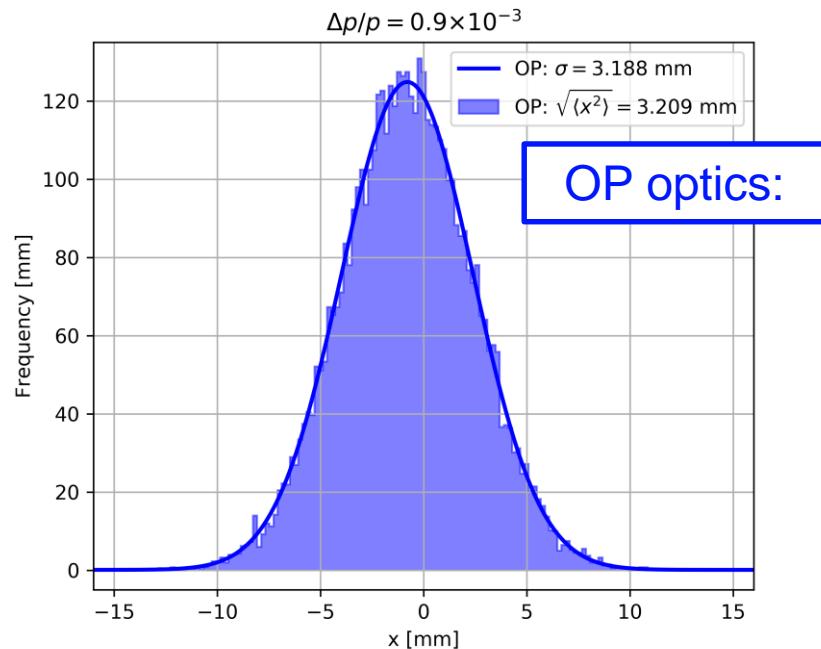
RMS horizontal beam size:



- Comments:
 - Beam is still filamenting at 2000 turns
 - $\Delta\epsilon/\epsilon \sim 11\%$ corresponds to 0.1 mm in the beam size
 - Challenging to measure!
 - Analytic results verified**

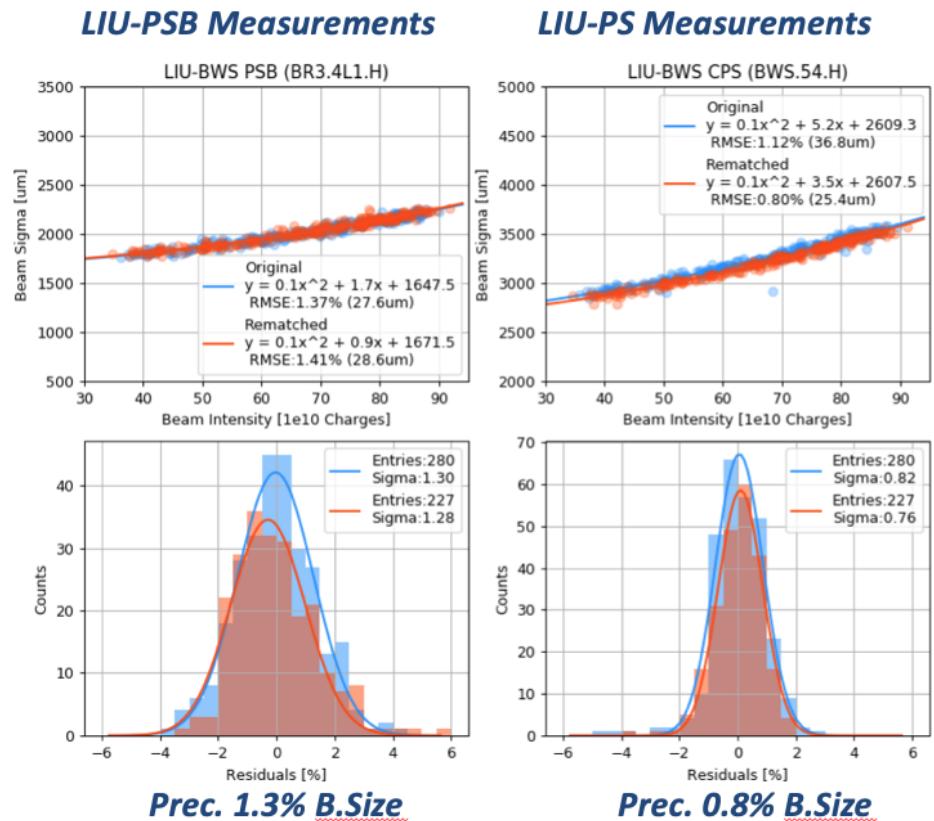
Tracking studies: $E_I = 0.9$ eVs

- Horizontal profiles compared at turn 2000:
 - Such small differences in RMS ($\sqrt{\langle x^2 \rangle}$) will be very hard to discern using the wire-scanner
 - Fitting a Gaussian function (σ) reduces the difference



Emittance measurements

- Prototype LIU WS used and transfer line optics rematched:
- Large dataset (> 500 shots) observed a small beam size reduction of $\sim 30 \text{ um}$ at $65 \times 10^{10} \text{ ppp}$
- Details of analysis now being checked by BI for fit quality / tail population



Conclusions

- Non-linear behaviour of PS is **well characterised** by the MADX-PTC model:
 - “filamentation” of dispersion well described
- **Chromaticity measured a factor 2 higher** than expected during first 30 turns
- **Significant detuning of beam size (envelope) oscillation frequency observed:**
 - Shift of -0.026 for OP transfer line optics
 - Frequency is a function of injection mismatch

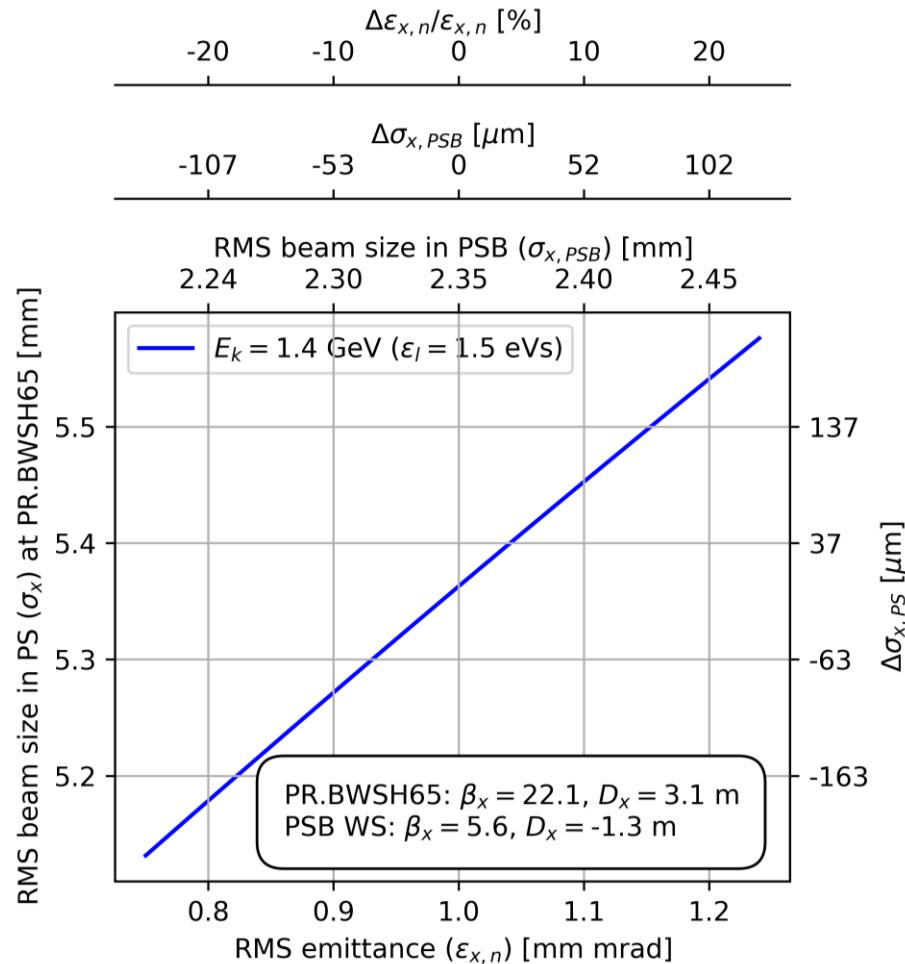
Next steps...

- Comparison with simulations including space-charge in PyORBIT:
 - H. Rafique to implement dispersion mismatch
 - **We cannot rely on beam size measurements after filamentation for...**
 1. absolute measurements of betatronic emittance
 2. injection matching optimization
 - Transfer line matching will be significantly improved with LIU upgrade:
 - However, we will need to develop an optimizer working to **minimise the turn-by-turn beating** after injection

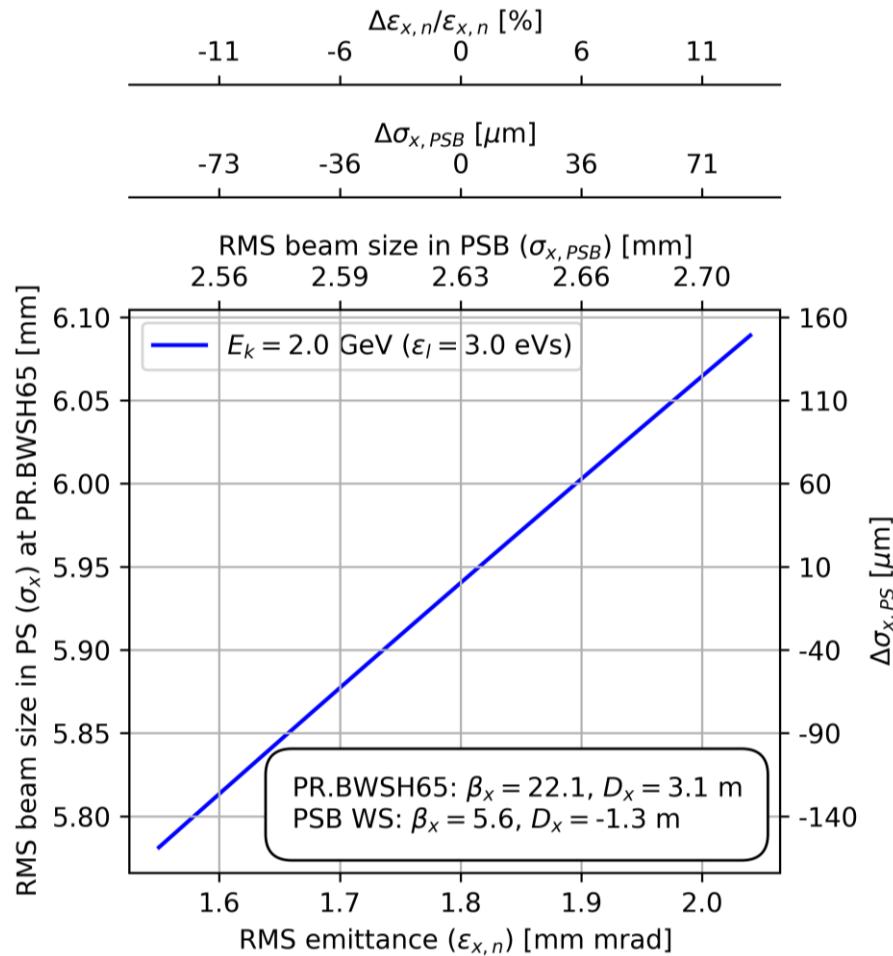
Extra slides



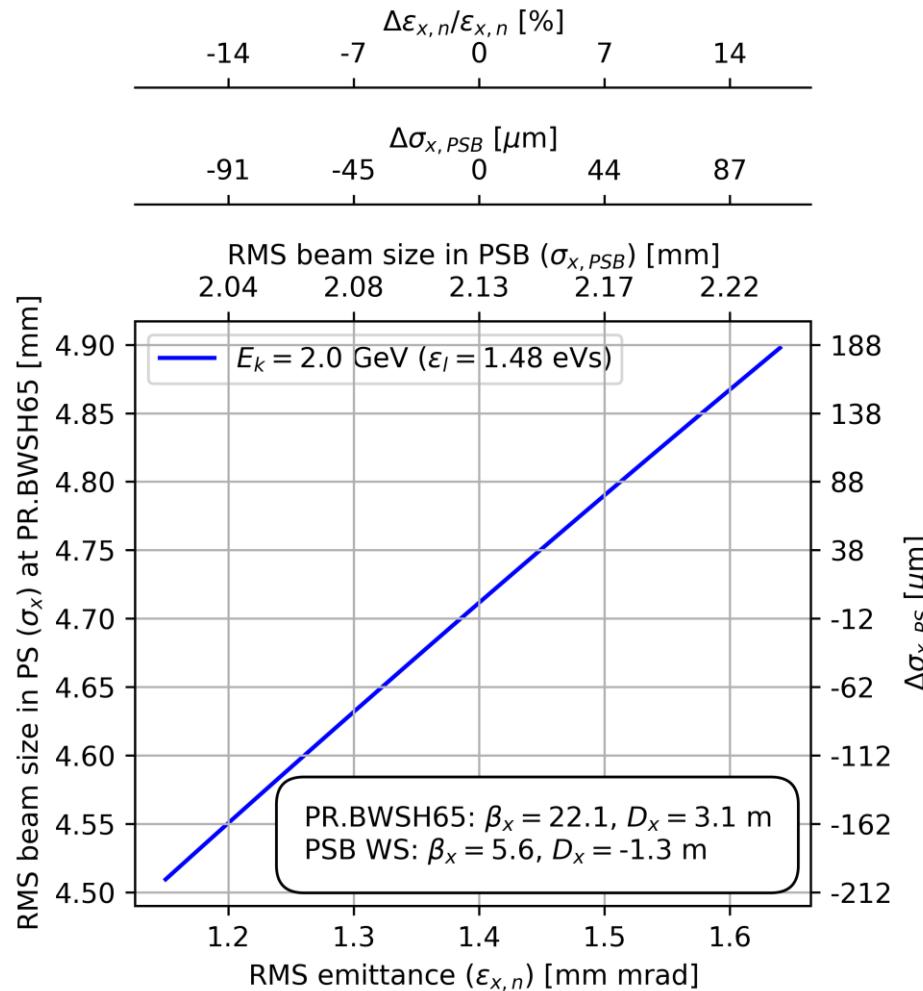
Systematic errors (1.4 GeV, 1.5 eVs)



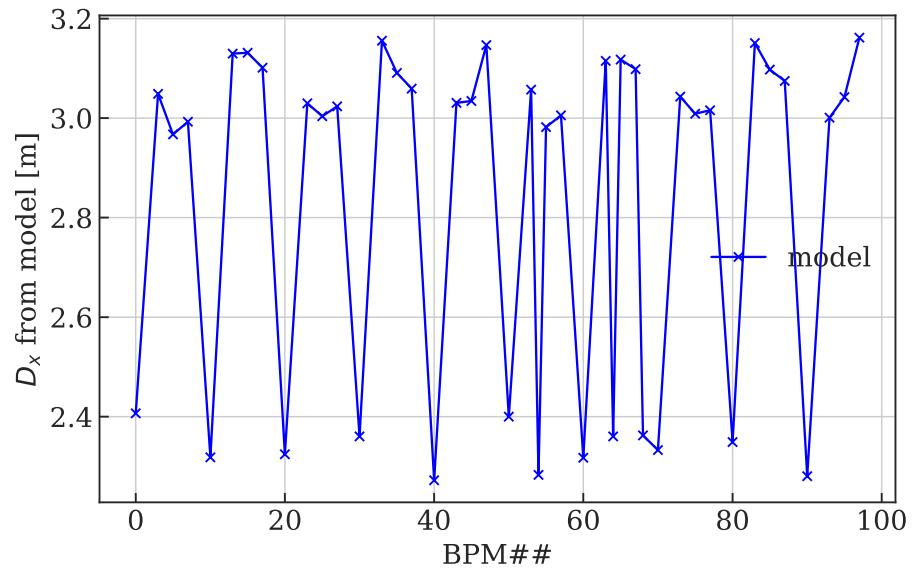
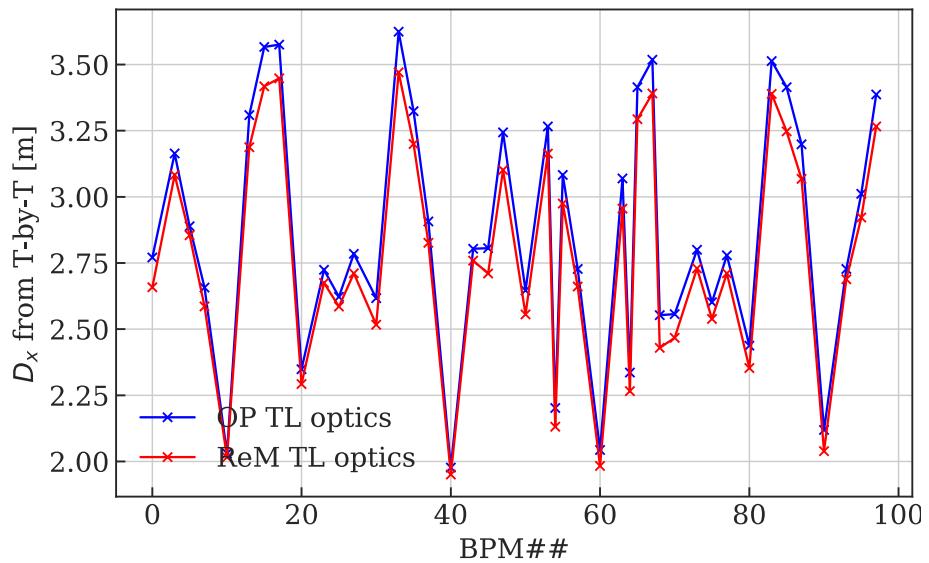
Systematic errors (LIU STD)



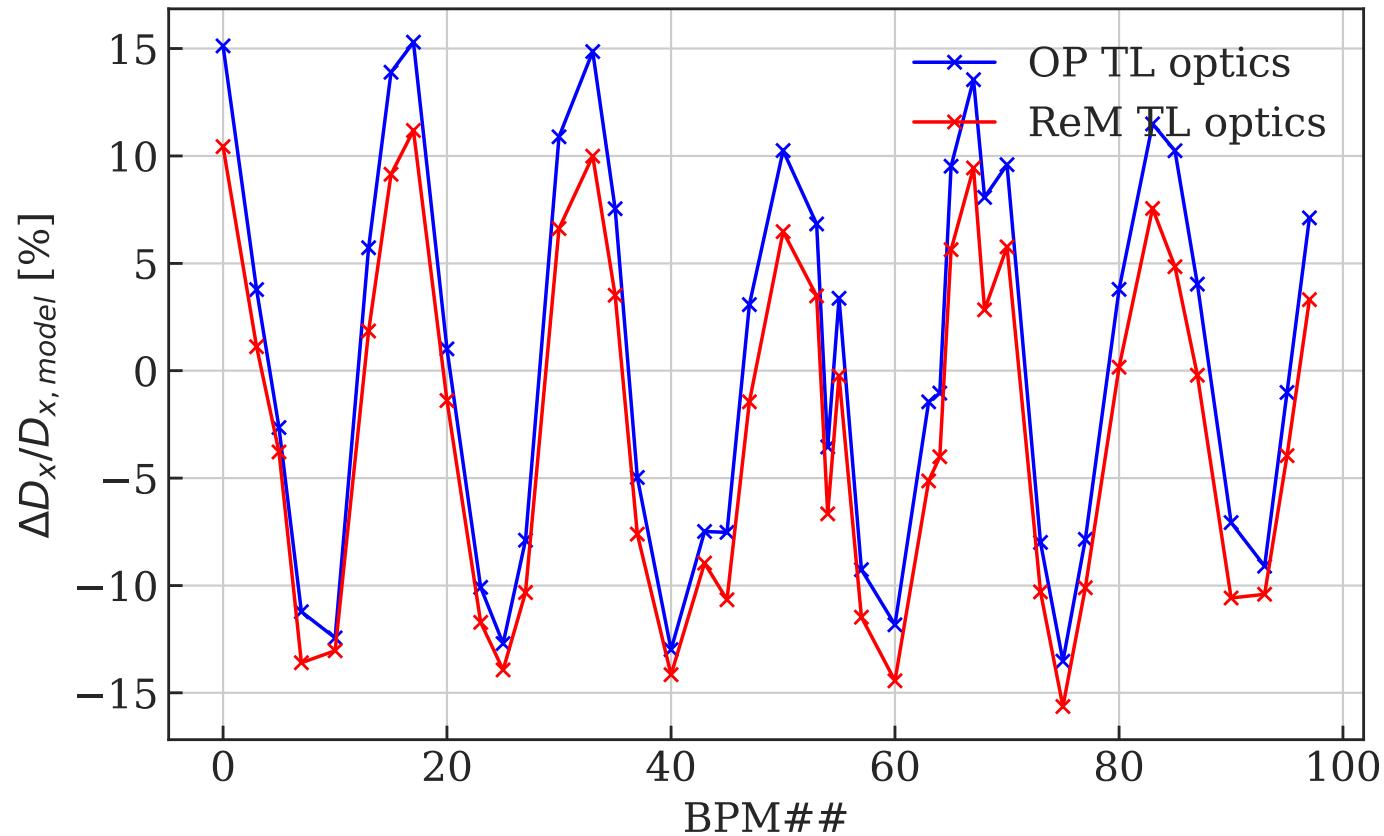
Systematic errors (LIU BCMS)



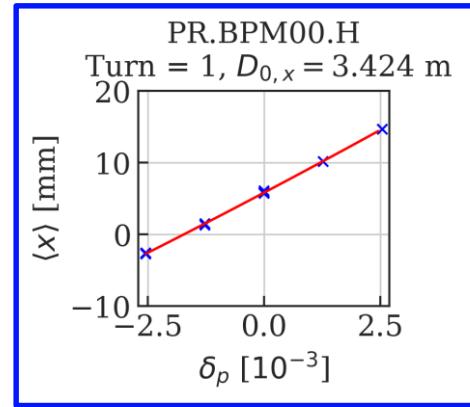
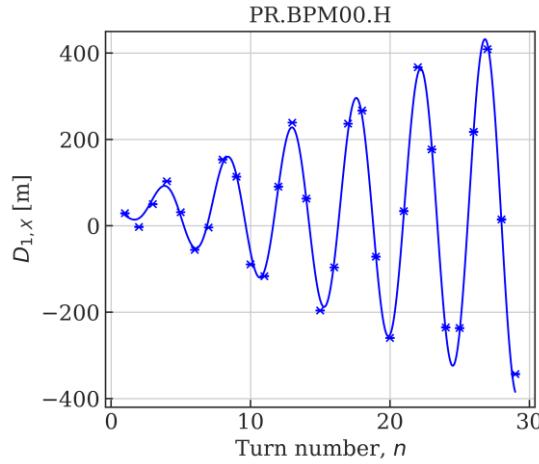
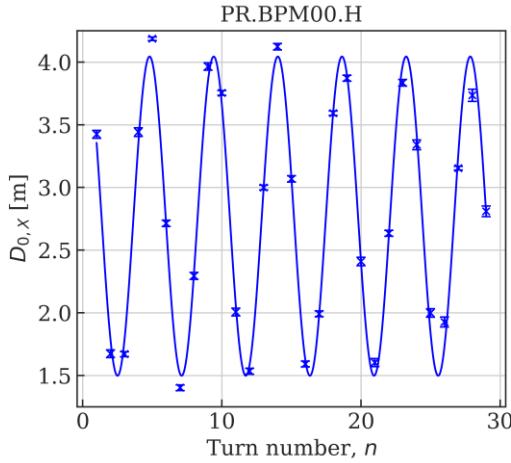
D in first turns (BPM) (1)



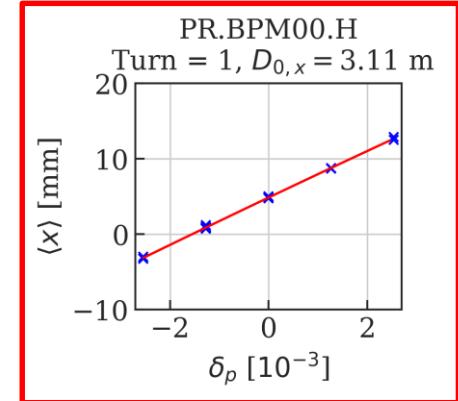
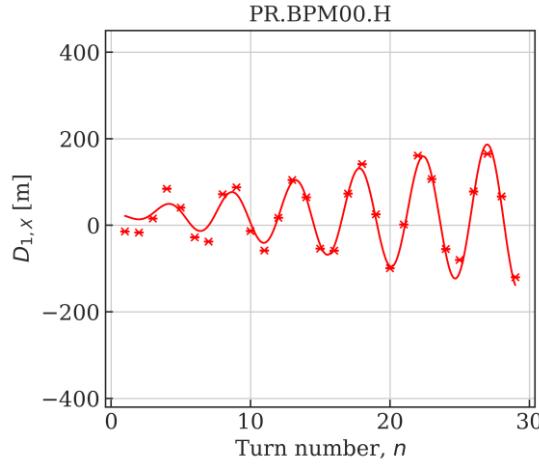
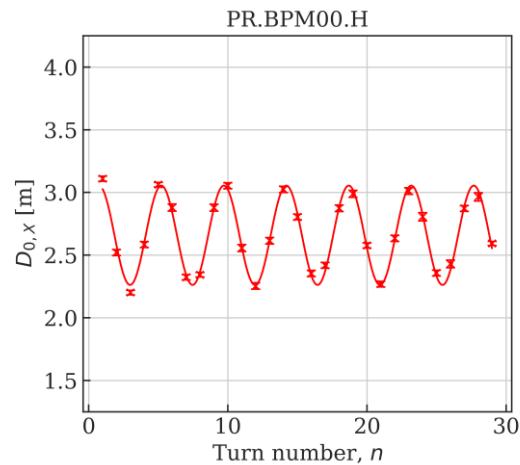
D vs. model in first turns (BPM) (2)



D measurements (BPM)



OP transfer line optics

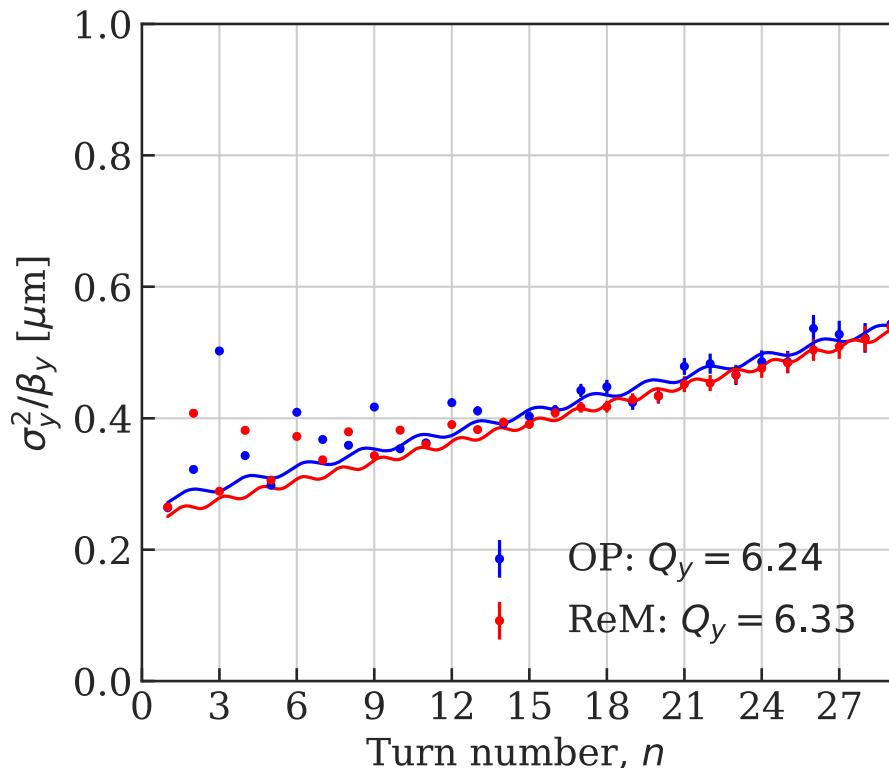


ReM transfer line optics

Beam profile measurements - V

- Fitting with a 5-parameter Gaussian with linear baseline:

$$\frac{\sigma_y^2(n)}{\beta_y} = \frac{\varepsilon_y}{2} \left(\left(M_g + \frac{1}{M_g} \right) + \left(M_g - \frac{1}{M_g} \right) \cos(\phi + 4\pi(n-1)q_x) \right) + \frac{\sigma_{BSG}^2(n)}{\beta_y}$$



- Care to be taken with fit parameters:
 - Only a few useful turns
 - Fit only made on later turns!

Free parameters	OP optics	ReM optics
q_x	0.23	0.204
ε [mm mrad]	0.62	0.58
ϕ	4.65	-0.48
M_g	1.02	1.00
$\beta_y \Delta y'^2_{rms}$ [μm]	0.019	0.020

Dispersion beating after injection

- **If D-mismatch is small, $M_D/D_0 \ll 1$:**

- Envelope will oscillate with q_x :

$$\overline{D_0}^2 \left(1 + \frac{M_D}{\overline{D_0}} \cos(\theta + 2\pi(n-1)q_x) \right)^2 \approx \overline{D_0}^2 + 2\overline{D_0} M_D \underbrace{\cos(\theta + 2\pi(n-1)q_x)}_{\text{envelope oscillating at } q_x} + O(M_D^2)$$

- **If D-mismatch is large, $M_D/D_0 \sim 1$:**

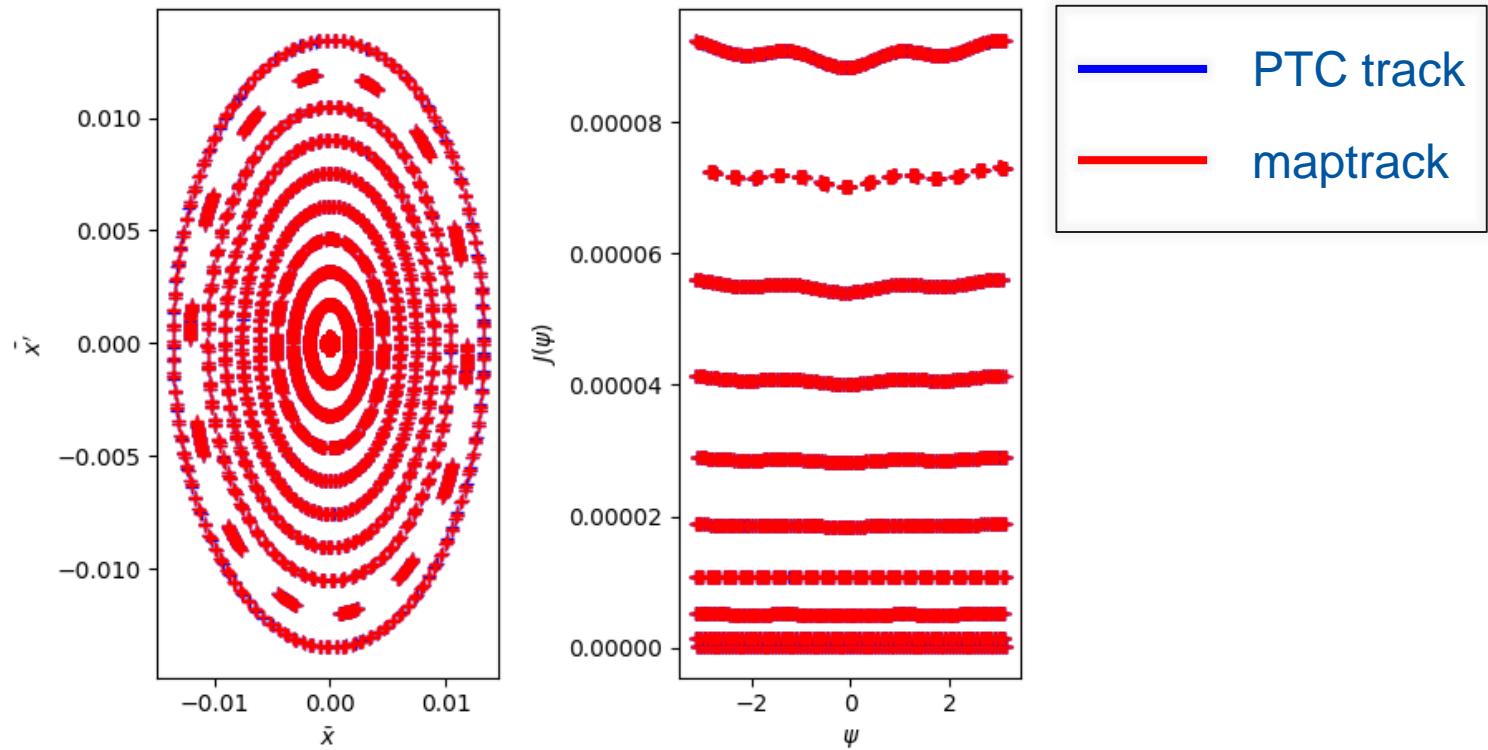
- Envelope will oscillate with a component at $2q_x$:

$$O(M_D^2) = M_D^2 \cos^2(\theta + 2\pi(n-1)q_x) = \frac{M_D^2}{2} (1 + \cos 2(\theta + 2\pi(n-1)q_x))$$

$\underbrace{\phantom{\frac{M_D^2}{2}}}_{\text{envelope oscillating at } 2q_x}$

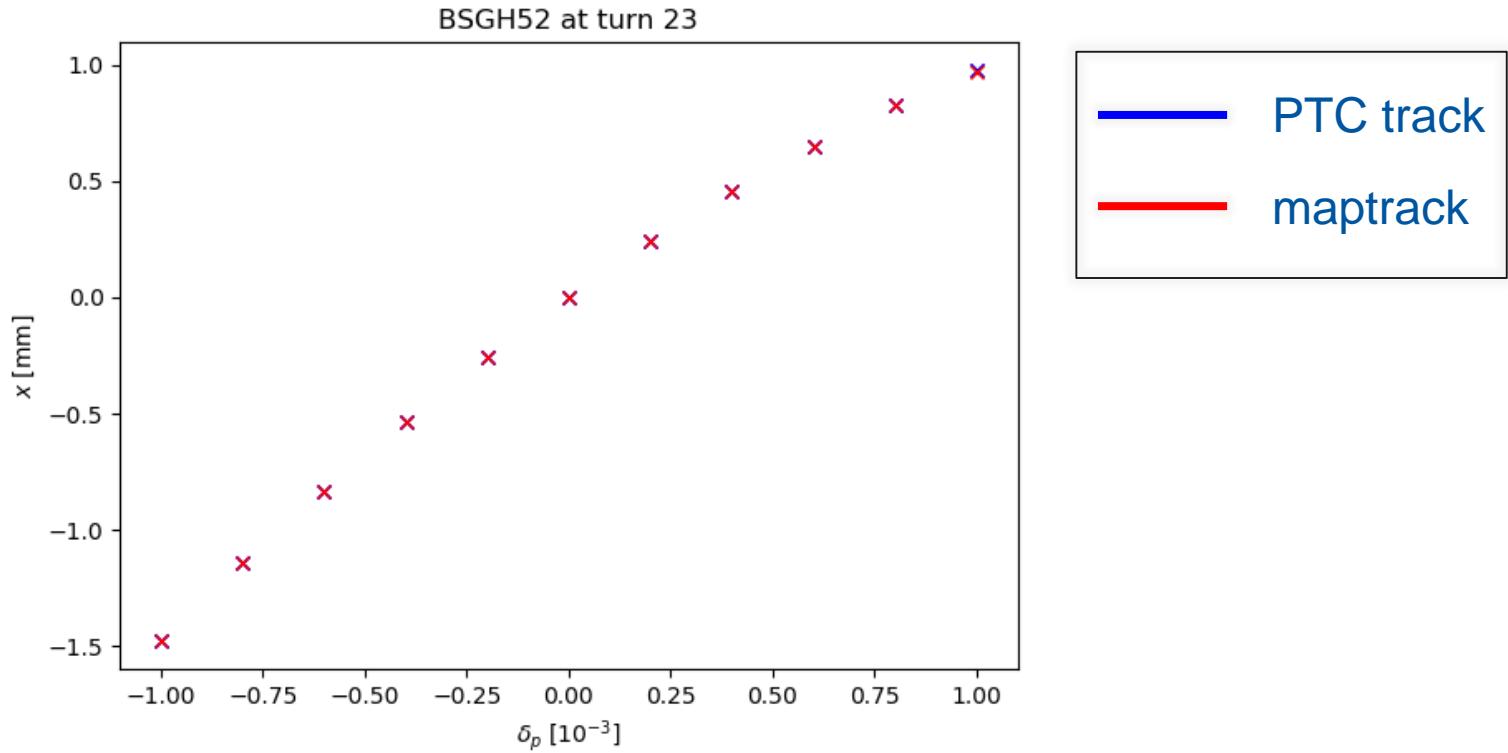
Benchmarking maptrack

- Maps exported from PTC at **5th order** and a few particles tracked for 300 turns:



Benchmarking maptrack

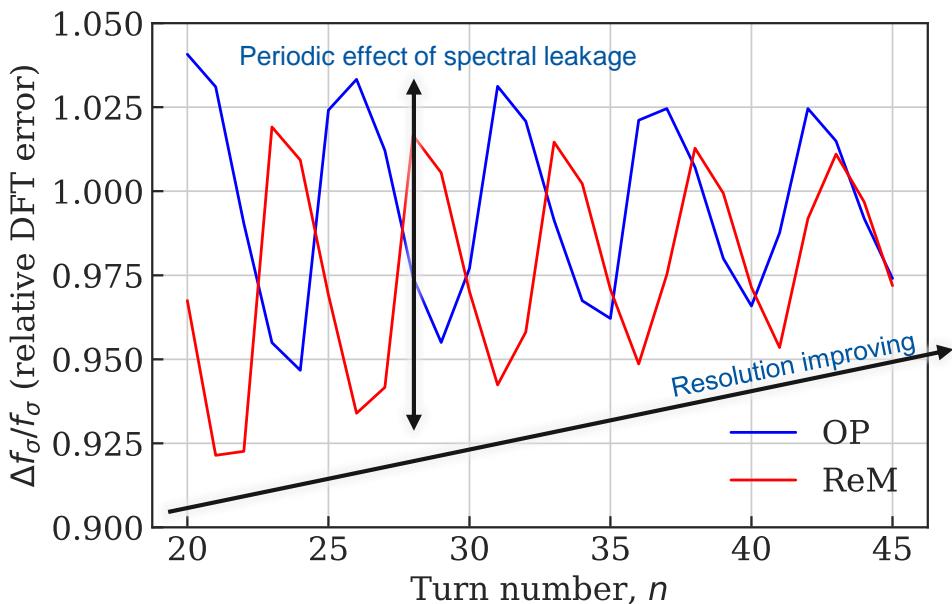
- Turn-by-turn dispersion to 5th order:



See extra slides for convergence

Resolution of DFT – no window

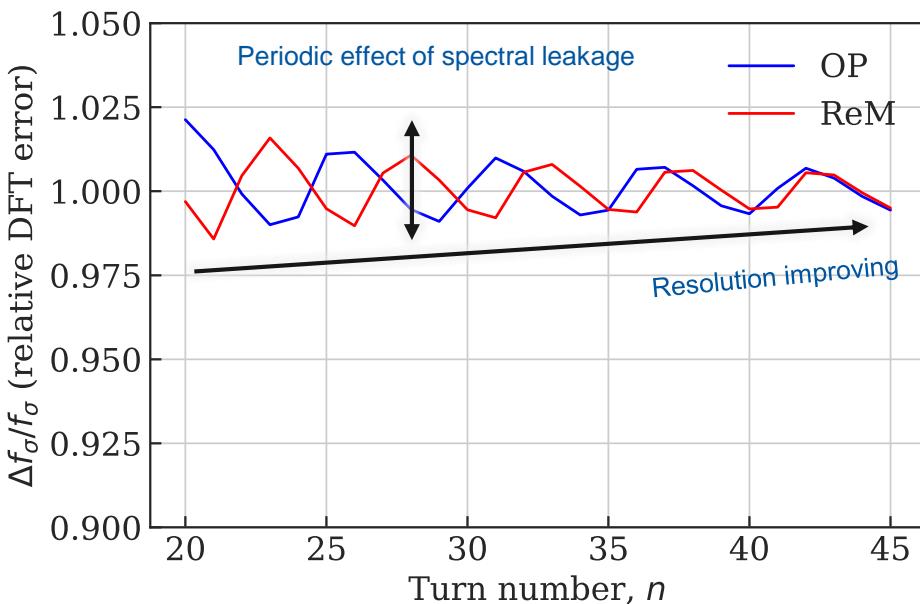
- Analytic expression for T-by-T beam size to check DFT:
 - Sample the analytic function every turn, for different numbers of turns, perform DFT without windowing
 - python numpy implementation using `fft.rfft()` and parabolic interpolation of the spectrum



- f_σ is accurate to better than $\pm 5\%$ for 29 turns
- Spectral leakage seen:
 - Non-periodic sampling
- Resolution improves with number of samples
 - Little benefit in resolution above 30 turns

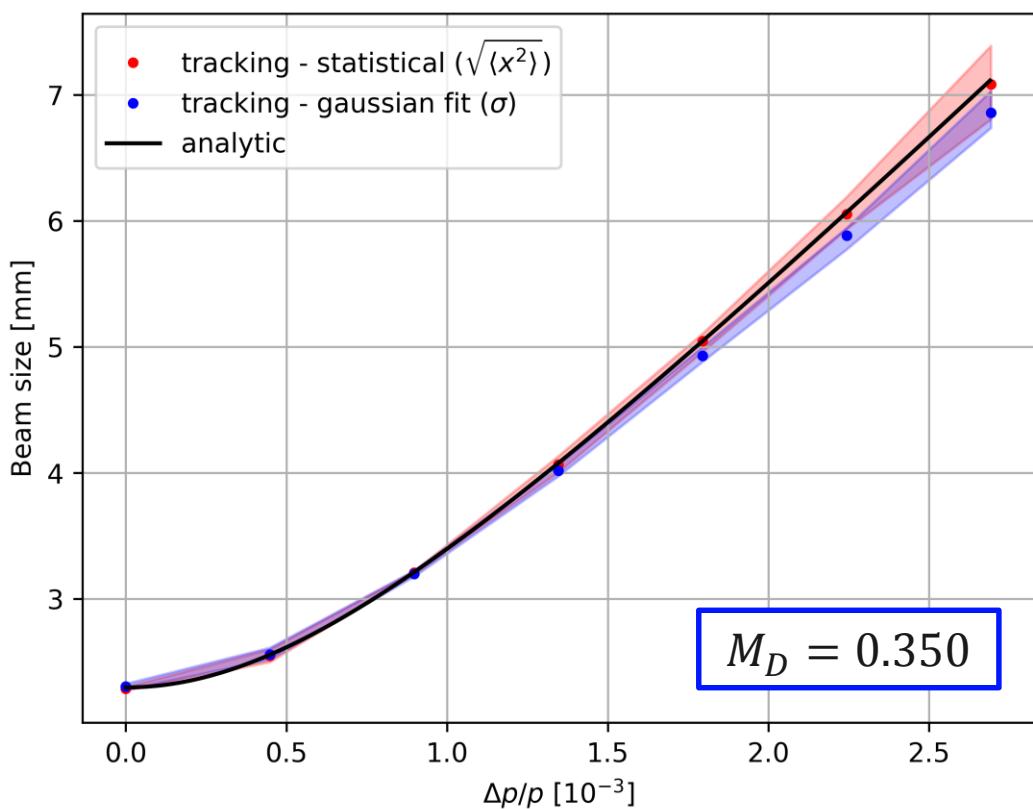
Resolution of FFT - Hanning

- Analytic expression for T-by-T beam size to check DFT:
 - Sample the analytic function every turn, for different numbers of turns, perform DFT without windowing
 - python numpy implementation using `fft.rfft()` and parabolic interpolation of the spectrum
 - With **Hanning window** f_σ is accurate to better than $\pm 1.5\%$ for 29 turns
 - Spectral leakage seen:
 - Non-periodic sampling
 - Resolution improves with number of samples
 - Little benefit in resolution above 30 turns

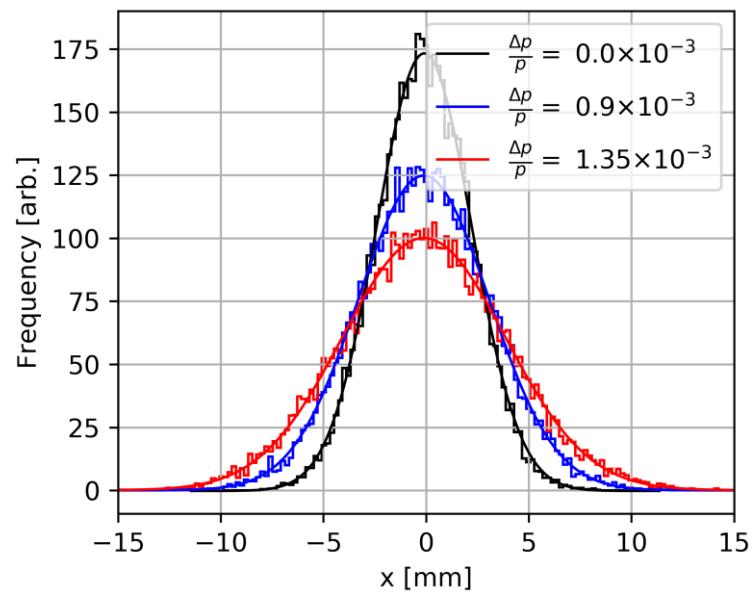


Comparison to analytic results (1)

- Dispersion mismatch (OP transfer line optics):

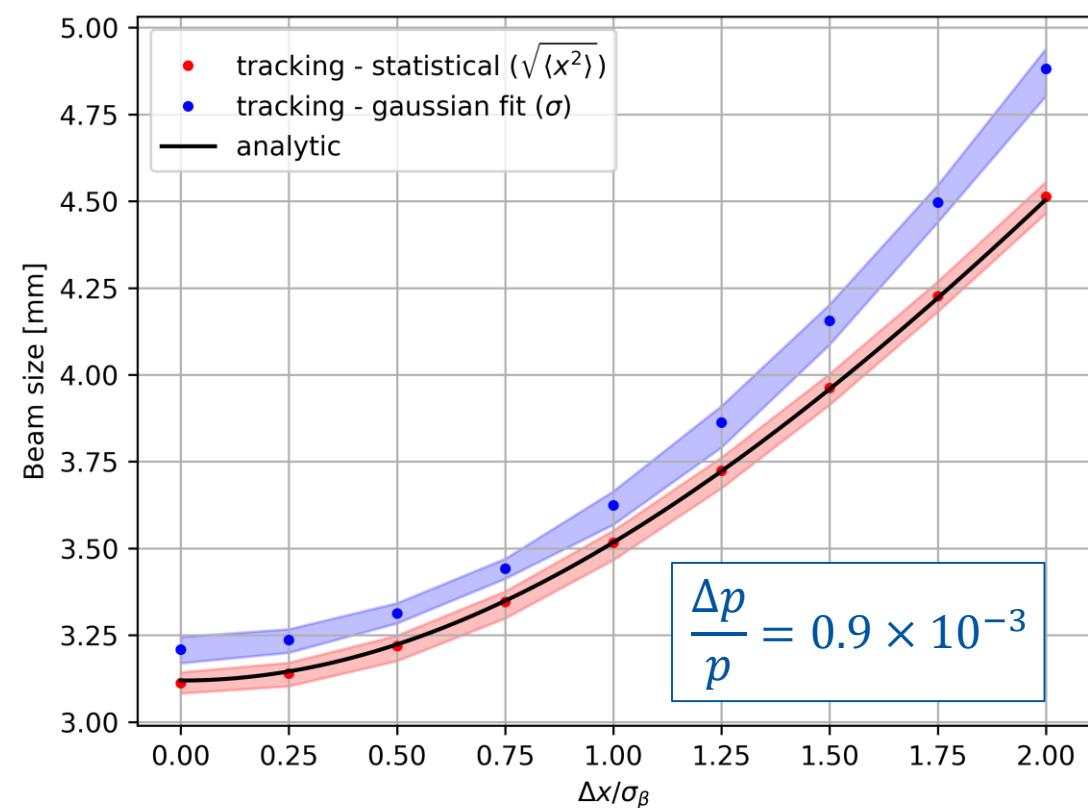


$$\Delta\varepsilon = \frac{M_D^2}{2} \left(\frac{\Delta p}{p} \right)^2$$

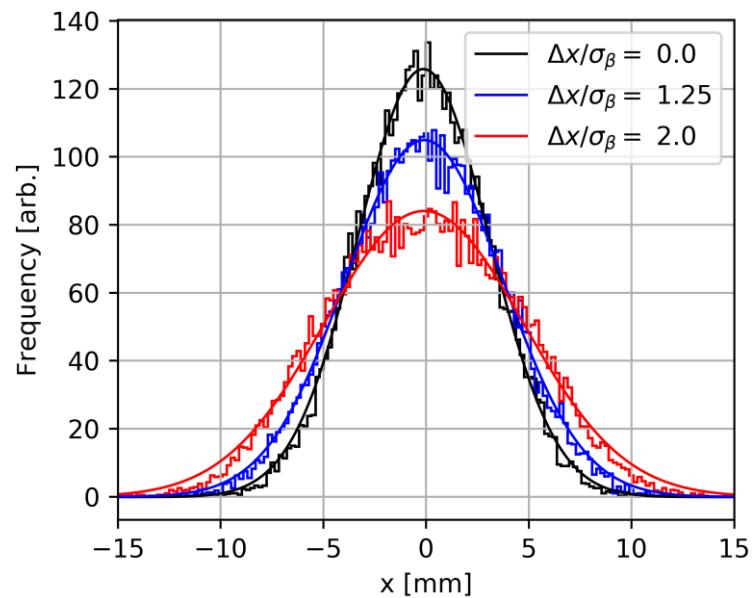


Comparison to analytic results (2)

- Injection mis-steering (optics matched):



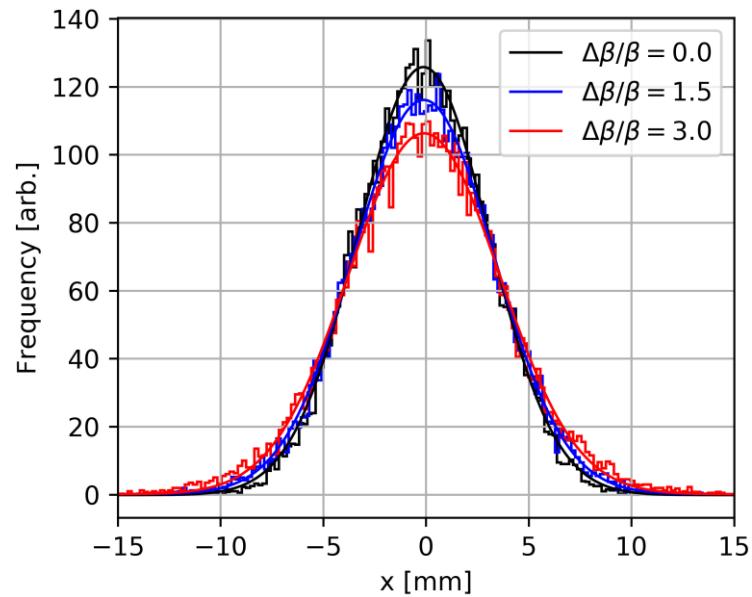
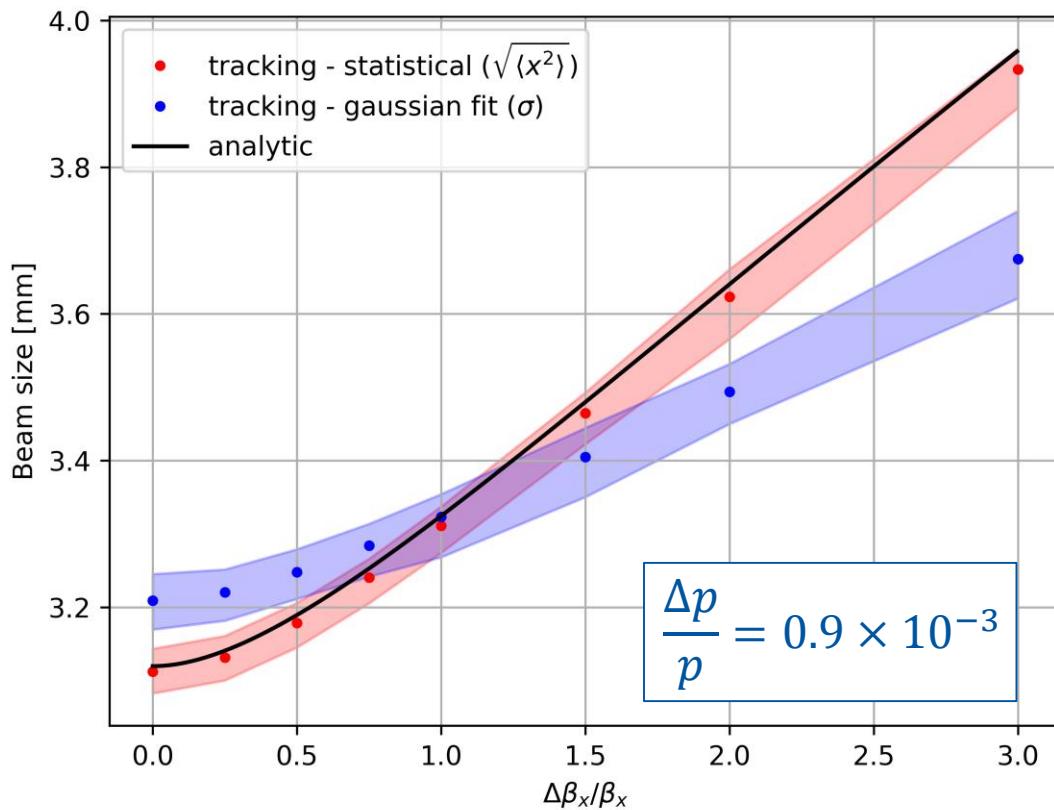
$$\Delta \varepsilon = \frac{\Delta x^2 + (\alpha \Delta x)^2}{2\beta}$$



Comparison to analytic results (3)

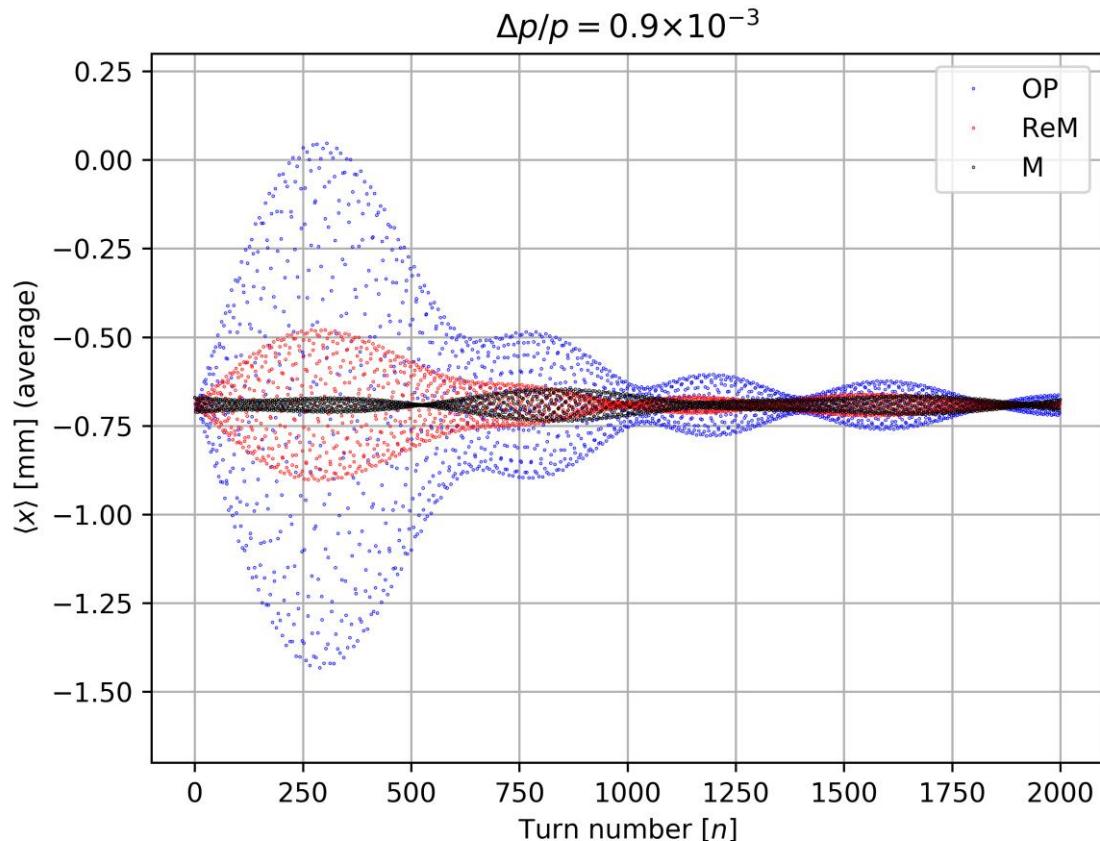
- Betatronic mismatch:

$$\varepsilon = \varepsilon_0 \frac{1}{2} \left[\frac{\beta}{\beta_0} + \frac{\beta_0}{\beta} + \beta \beta_0 \alpha_0^2 \left(\frac{1}{\beta} - \frac{1}{\beta_0} \right)^2 \right]$$



Tracking studies: $E_i = 0.9$ eVs

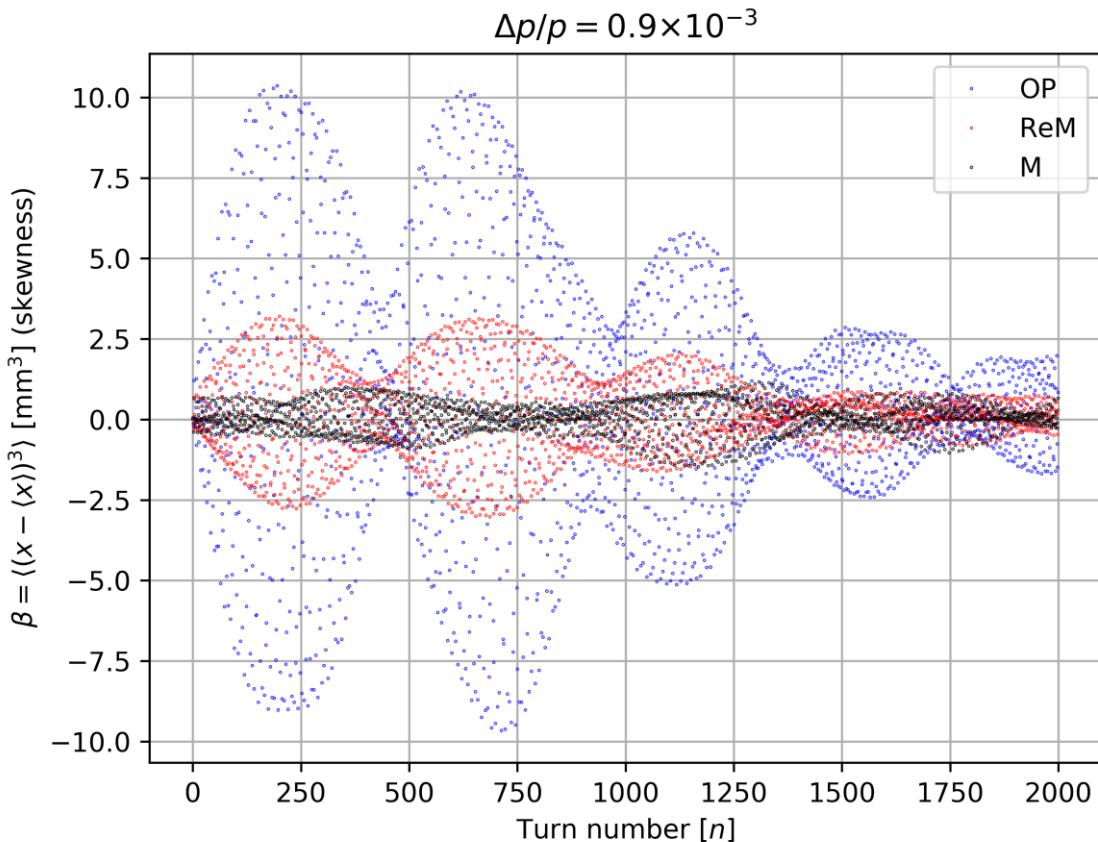
Average beam position:



- Comments:
 - Average beam position oscillates as filamentation ensues, (see skewness)
 - Even matched optics is perturbed: asymmetry of non-linear dispersion

Tracking studies: $E_i = 0.9$ eVs

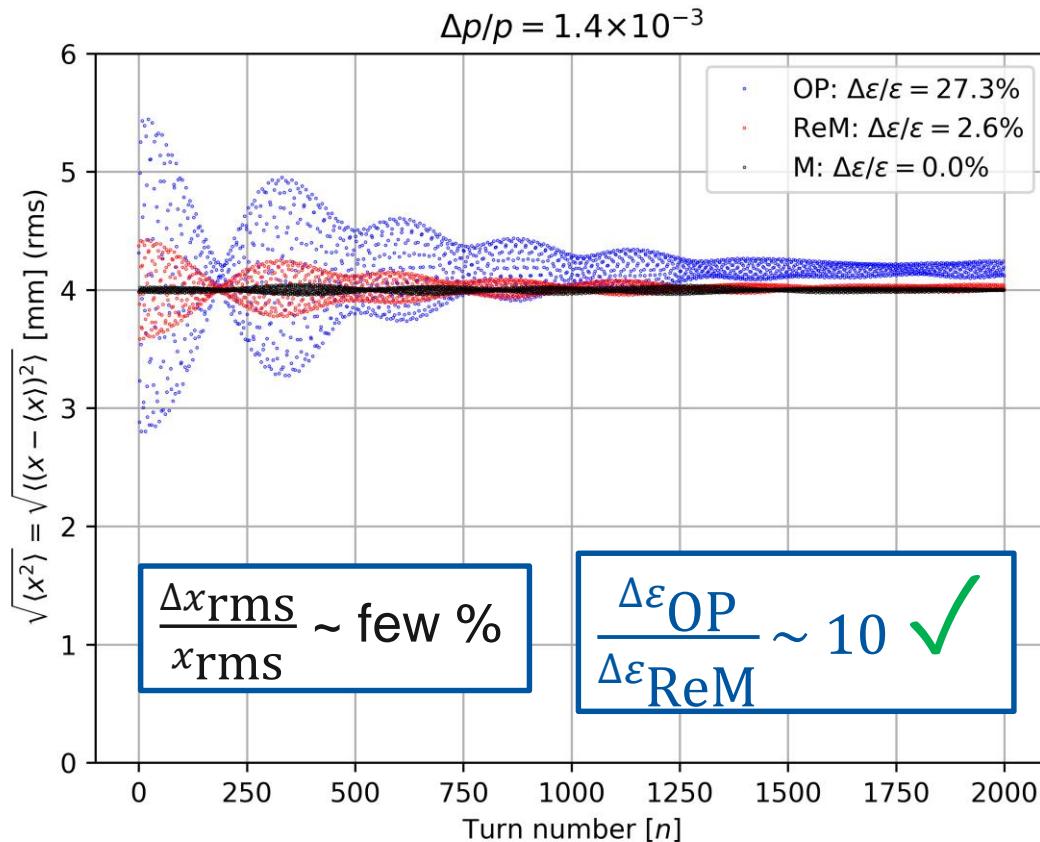
Skewness:



- Comments:
 - Skewness indicator of filamentation
 - Asymmetric non-linear dispersion induces asymmetry across beam
 - Driving oscillation of average position

Tracking studies: $E_i = 1.5$ eVs

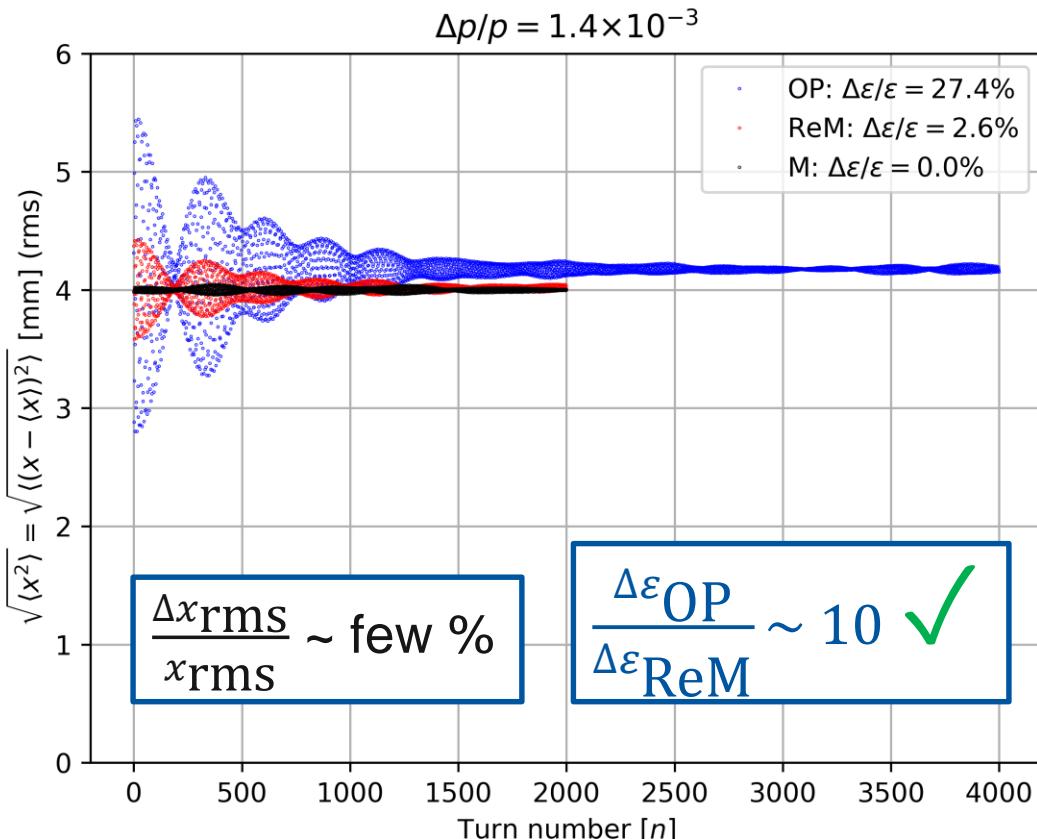
RMS horizontal beam size:



- **Comments:**
 - RMS beam size is still shaking at 2000 turns
 - Analytic results verified

Tracking studies: $E_I = 1.5$ eVs

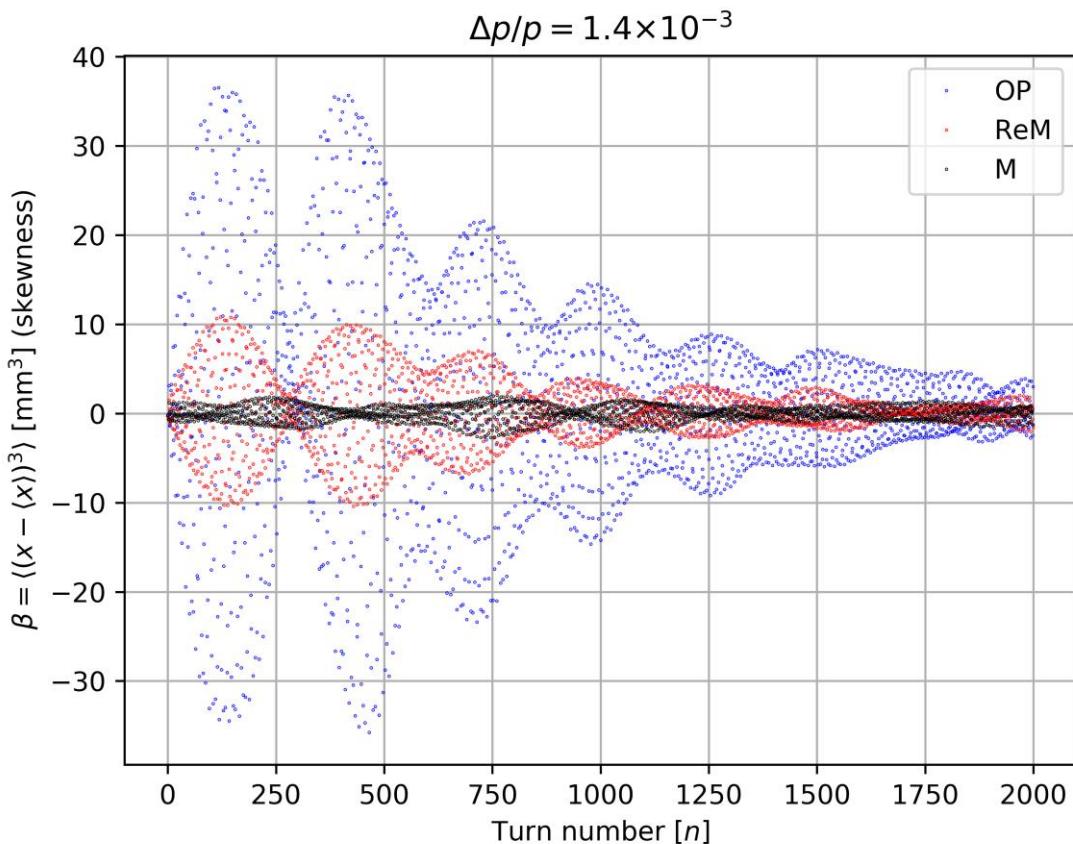
RMS horizontal beam size:



- Comments:
 - Check made to 4000 turns...
 - Jitter of ~ 80 μm on the beam size remains
 - Due to non-linear dispersion

Tracking studies: $E_i = 1.5$ eVs

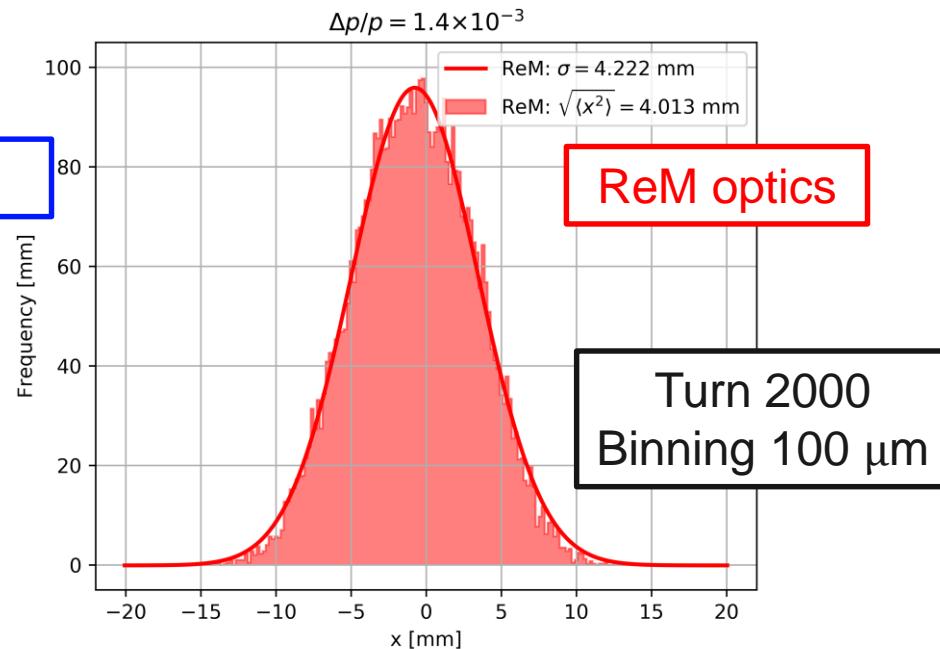
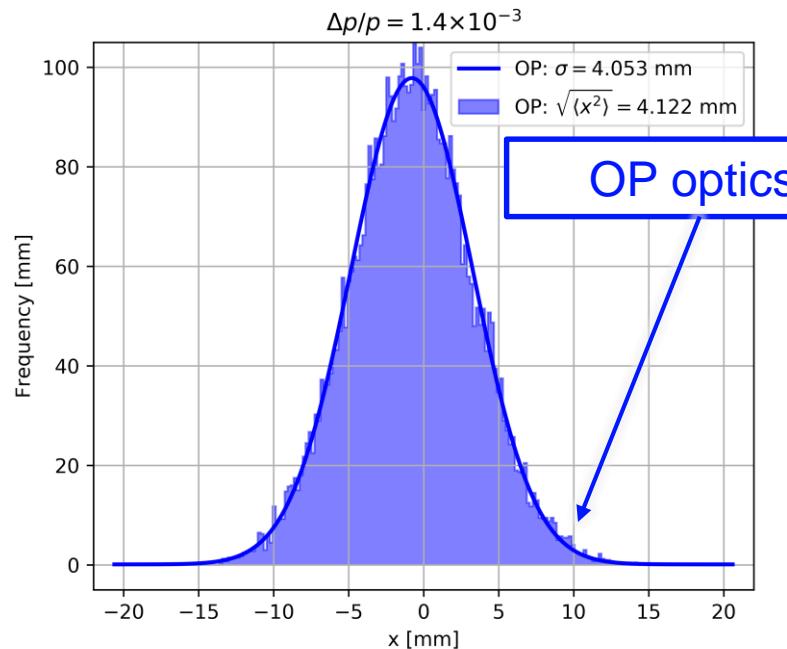
Skewness:



- Comments:
 - Skewness indicator of filamentation
 - Asymmetric non-linear dispersion induces asymmetry across beam
 - Driving oscillation of average position

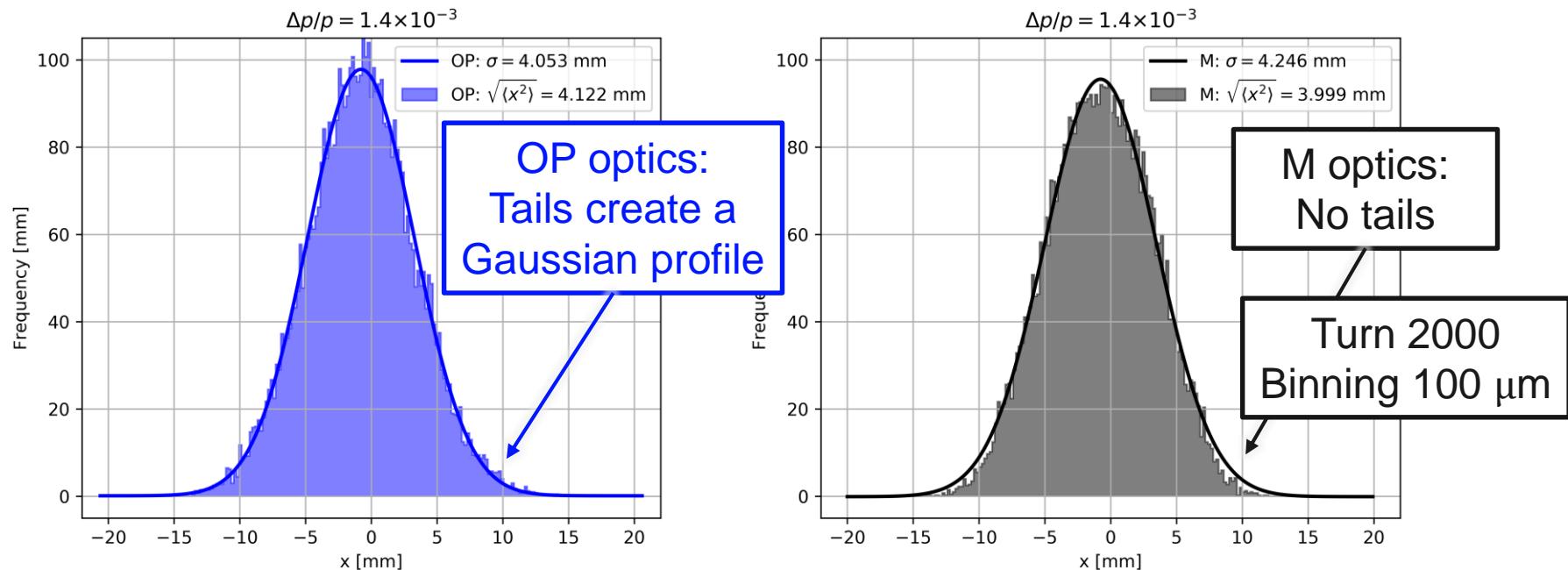
Tracking studies: $E_i = 1.5$ eVs

- No discernable difference between the two transfer line optics:
 - The effect of the tails generated during filamentation has a significant effect when fitted with a Gaussian function (σ)
 - **No surprise that beam size measurements were inconclusive**



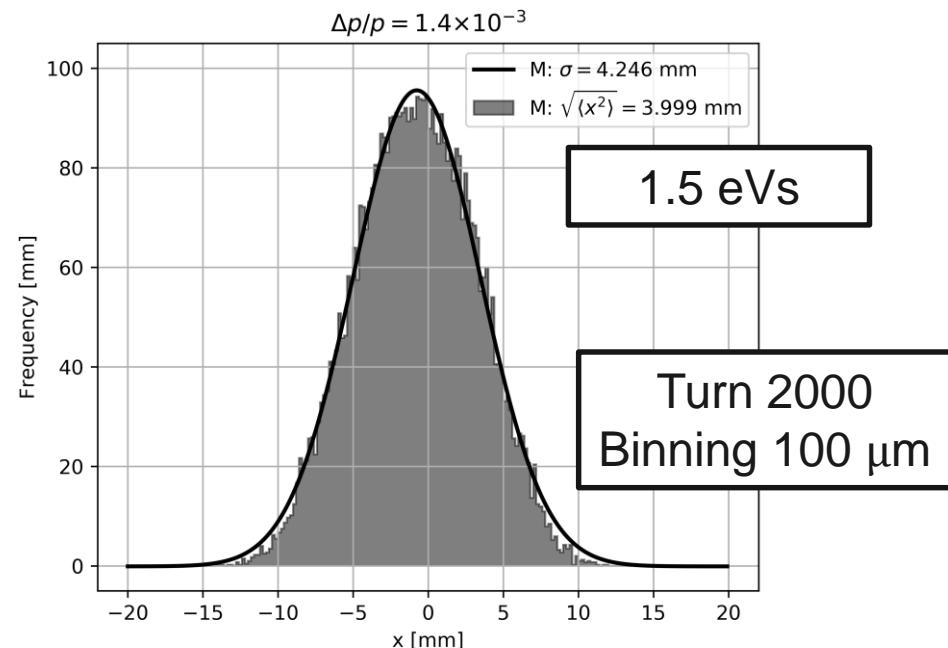
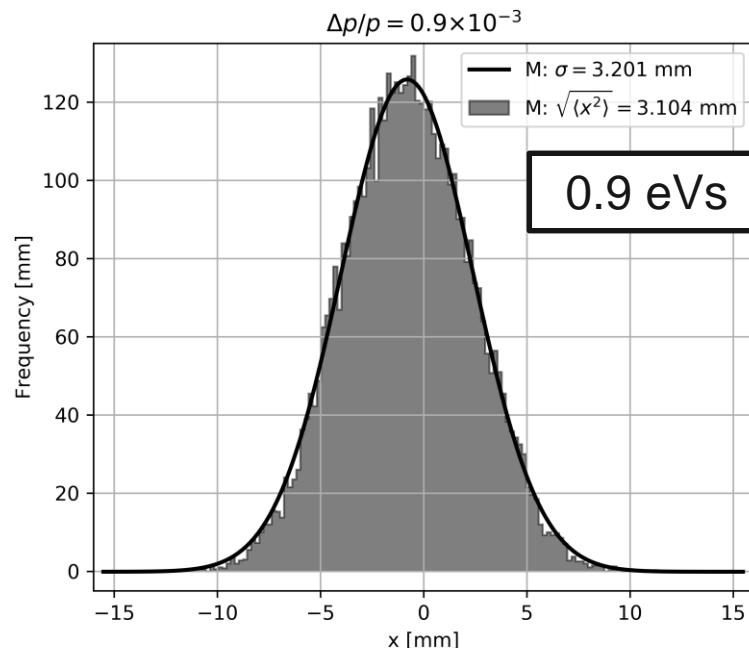
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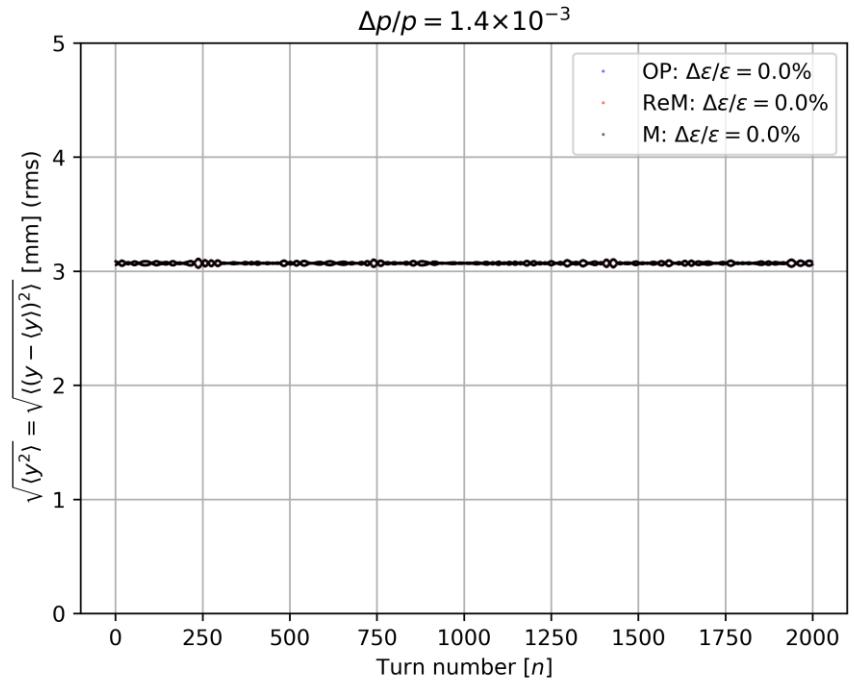
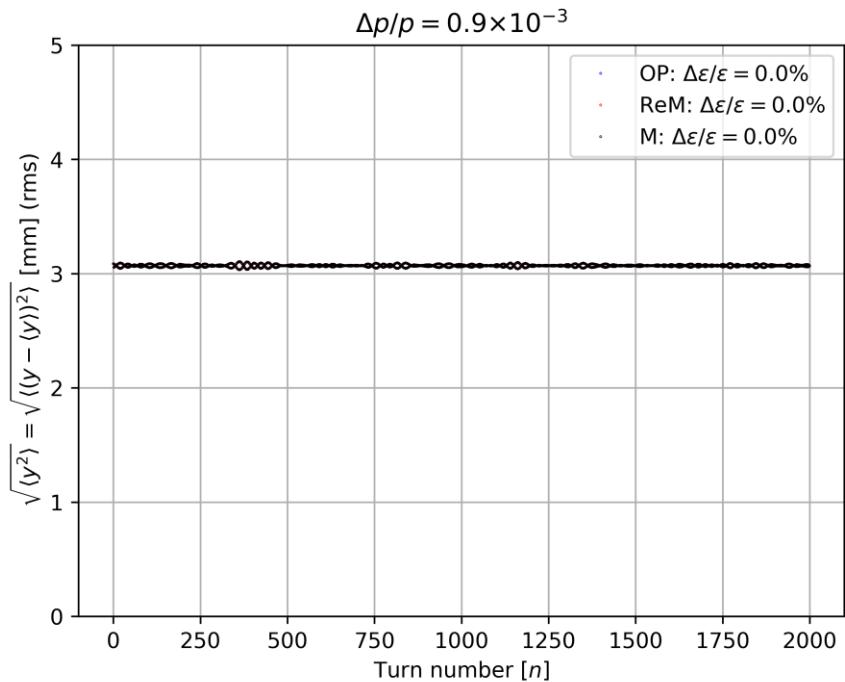
Tracking studies: matched case

- Fitting a Gaussian to the beam profile over-estimates the RMS beam size:
 - Exaggerated by the larger momentum spread:



Tracking studies: vertical plane

RMS vertical beam size:



A quick aside... emittance

- Emittance was computed in two ways as the beam size is still beating after 2000 turns (at 1 – 2 %):

1. Average RMS beam size taken in the last 10 turns

- Emittance computed using the closed dispersion and betatron functions removed in quadrature:

$$\varepsilon = \sqrt{\langle x^2 \rangle_{10 \text{ turns}} - (D \frac{\Delta p}{p})^2 / \beta}$$

- *Mathematically correct: no assumptions made on distribution*

2. Last turn statistical emittance:

$$\varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

- *Dispersion computed from the particle distribution itself (statistical) and removed particle-by-particle*



A quick aside... convergence

- Both approaches are suitable and analytic results verified:

