

```
In[401]:= SimpSymbol[symbol_] :=  $\lambda^{\text{StringCount}[\text{SymbolName}[\text{symbol}], "x"]}$ 
```

```
In[402]:= SimpSymbol[n134x25]
```

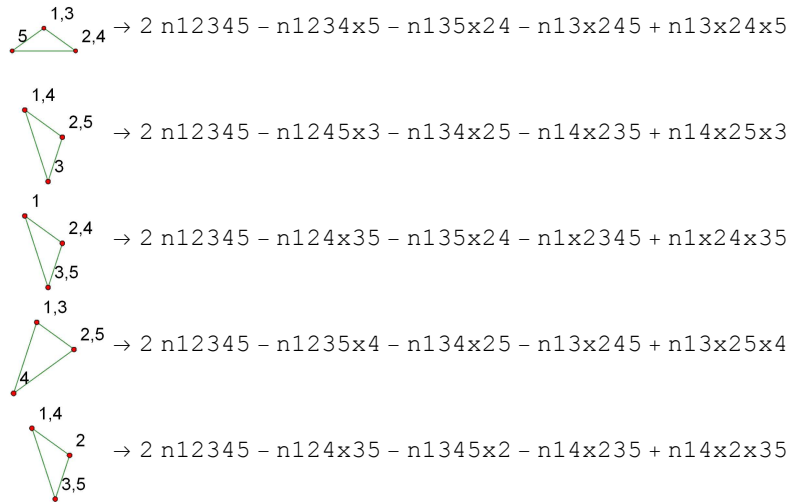
```
Out[402]=  $\lambda$ 
```

```
In[403]:= simpRep = Table[k → SimpSymbol[k], {k, realyNullAtomVars}]; Take[simpRep, 3]
```

```
Out[403]= {n1x2x3x4x5 →  $\lambda^4$ , n1x2x3x45 →  $\lambda^3$ , n1x2x35x4 →  $\lambda^3$ }
```

```
In[414]:= Table[allGraphs[k, "graph"] -> allGraphs[k, "colofourrealnull"],  
               {k, {alfaKey, betaKey, gammaKey, deltaKey, epsilon1Key}}] // TableForm
```

```
Out[414]//TableForm=
```



```
In[420]:= Intersection[{n13x24x5, n12345, n1234x5, n135x24, n13x245},  
                       {n13x25x4, n12345, n1235x4, n134x25, n13x245}, {n14x25x3, n12345, n1245x3, n134x25, n14x235},  
                       {n14x2x35, n12345, n124x35, n1345x2, n14x235}, {n1x24x35, n12345, n124x35, n135x24, n1x2345}]
```

```
Out[420]= {n12345}
```

```
In[419]:= Intersection[
  Table[ListofVars[allGraphs[k, "colofourrealnull"]], {k, {alfa1Key, beta1Key, gamma1Key, delta1Key, epsilon1Key}}]]
```

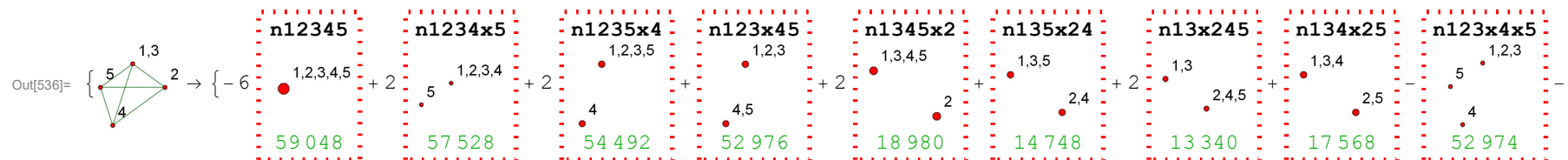
```
Out[419]= {{n13x24x5, n12345, n1234x5, n135x24, n13x245},
  {n13x25x4, n12345, n1235x4, n134x25, n13x245}, {n14x25x3, n12345, n1245x3, n134x25, n14x235},
  {n14x2x35, n12345, n124x35, n1345x2, n14x235}, {n1x24x35, n12345, n124x35, n135x24, n1x2345}}
```

```
Intersection[{n123x45, n134x25, n135x24, n13x2x4x5, n12345, n1234x5,
  n1235x4, n123x4x5, n1345x2, n134x2x5, n135x2x4, n13x245, n13x24x5, n13x25x4, n13x2x45},
{n124x35, n134x25, n145x23, n14x2x3x5, n12345, n1234x5, n1245x3, n124x3x5, n1345x2, n134x2x5,
  n145x2x3, n14x235, n14x23x5, n14x25x3, n14x2x35}, {n124x35, n13x245, n15x234, n1x24x3x5, n12345,
  n1234x5, n1245x3, n124x3x5, n135x24, n13x24x5, n15x24x3, n1x2345, n1x234x5, n1x245x3, n1x24x35},
{n125x34, n13x245, n14x235, n1x25x3x4, n12345, n1235x4, n1245x3, n125x3x4, n134x25, n13x25x4,
  n14x25x3, n1x2345, n1x235x4, n1x245x3, n1x25x34}, {n12x345, n135x24, n14x235, n1x2x35x4, n12345,
  n1235x4, n124x35, n12x35x4, n1345x2, n135x2x4, n14x2x35, n1x2345, n1x235x4, n1x24x35, n1x2x345}]
```

```
Intersection[
  Table[ListofVars[allGraphs[k, "colofourrealnull"]], {k, {quad1Key, quad2Key, quad3Key, quad4Key, quad5Key}}]]
```

```
Table[allGraphs[k, "graph"] →
  {allGraphs[k, "colofourrealnull"] /. repcolofourrealnullgraph2, allGraphs[k, "colofourrealnull"]},
  {k, {quad1Key, quad2Key, quad3Key, quad4Key, quad5Key}}]
```

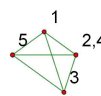
```
In[536]:= Table[allGraphs[k, "graph"] →
  {allGraphs[k, "colofourrealnull"] /. repcolofourrealnullgraph2, allGraphs[k, "colofourrealnull"]},
  {k, {quad1Key, quad2Key, quad3Key, quad4Key, quad5Key}}]
```



$$\begin{array}{c}
\boxed{n134 \times 2 \times 5} \quad \boxed{n13 \times 24 \times 5} \quad \boxed{n135 \times 2 \times 4} \quad \boxed{n13 \times 25 \times 4} \quad \boxed{n13 \times 2 \times 45} \quad \boxed{n13 \times 2 \times 4 \times 5} \\
\begin{array}{c} 1,3,4 \\ 5 \quad 2 \end{array} \quad \begin{array}{c} 1,3 \\ 5 \quad 2,4 \end{array} \quad \begin{array}{c} 1,3,5 \\ 4 \quad 2 \end{array} \quad \begin{array}{c} 1,3 \\ 4 \quad 2,5 \end{array} \quad \begin{array}{c} 1,3 \\ 4,5 \quad 2 \end{array} \quad \begin{array}{c} 1,3 \\ 5 \quad 2 \end{array} \\
17\,514 \quad 13\,284 \quad 14\,586 \quad 13\,176 \quad 13\,124 \quad 13\,122
\end{array}$$

$$-6\,n12345 + 2\,n1234 \times 5 + 2\,n1235 \times 4 + n123 \times 45 -$$

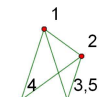
$$n123 \times 4 \times 5 + 2\,n1345 \times 2 + n134 \times 25 - n134 \times 2 \times 5 + n135 \times 24 - n135 \times 2 \times 4 + 2\,n13 \times 245 - n13 \times 24 \times 5 - n13 \times 25 \times 4 - n13 \times 2 \times 45 + n13 \times 2 \times 4 \times 5 \},$$



$$\begin{array}{c}
\boxed{n12345} \quad \boxed{n1234 \times 5} \quad \boxed{n1245 \times 3} \quad \boxed{n124 \times 35} \quad \boxed{n15 \times 234} \quad \boxed{n1 \times 2345} \quad \boxed{n135 \times 24} \quad \boxed{n13 \times 245} \quad \boxed{n124 \times 3 \times 5} \\
\begin{array}{c} 1,2,3,4,5 \\ 5 \end{array} \quad \begin{array}{c} 1,2,3,4 \\ 5 \end{array} \quad \begin{array}{c} 1,2,4,5 \\ 3 \end{array} \quad \begin{array}{c} 1,2,4 \\ 3,5 \end{array} \quad \begin{array}{c} 1,5 \\ 2,3,4 \end{array} \quad \begin{array}{c} 1 \\ 2,3,4,5 \end{array} \quad \begin{array}{c} 1,3,5 \\ 2,4 \end{array} \quad \begin{array}{c} 1,3 \\ 2,4,5 \end{array} \quad \begin{array}{c} 1,2,4 \\ 5 \quad 3 \end{array} \\
59\,048 \quad 57\,528 \quad 45\,416 \quad 43\,908 \quad 21\,24 \quad 728 \quad 14\,748 \quad 13\,340 \quad 43\,902
\end{array}$$

$$\begin{array}{c}
\boxed{n1 \times 234 \times 5} \quad \boxed{n13 \times 24 \times 5} \quad \boxed{n15 \times 24 \times 3} \quad \boxed{n1 \times 245 \times 3} \quad \boxed{n1 \times 24 \times 35} \quad \boxed{n1 \times 24 \times 3 \times 5} \\
\begin{array}{c} 1 \\ 5 \quad 2,3,4 \end{array} \quad \begin{array}{c} 1,3 \\ 5 \quad 2,4 \end{array} \quad \begin{array}{c} 1,5 \\ 3 \quad 2,4 \end{array} \quad \begin{array}{c} 1 \\ 3 \quad 2,4,5 \end{array} \quad \begin{array}{c} 1 \\ 3,5 \quad 2,4 \end{array} \quad \begin{array}{c} 1 \\ 5 \quad 2,4 \end{array} \\
666 \quad 13\,284 \quad 1620 \quad 218 \quad 168 \quad 162
\end{array}$$

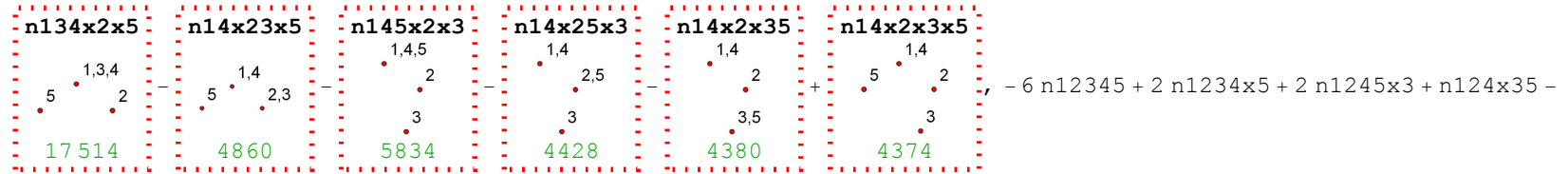
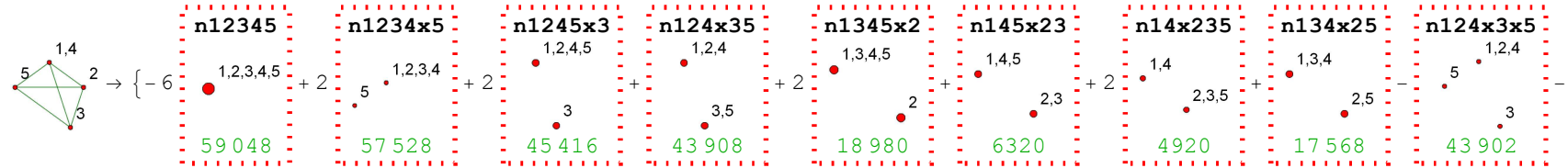
$$n124 \times 3 \times 5 + 2\,n135 \times 24 + n13 \times 245 - n13 \times 24 \times 5 + n15 \times 234 - n15 \times 24 \times 3 + 2\,n1 \times 2345 - n1 \times 234 \times 5 - n1 \times 245 \times 3 - n1 \times 24 \times 35 + n1 \times 24 \times 3 \times 5 \},$$



$$\begin{array}{c}
\boxed{n12345} \quad \boxed{n1235 \times 4} \quad \boxed{n12 \times 345} \quad \boxed{n124 \times 35} \quad \boxed{n1345 \times 2} \quad \boxed{n1 \times 2345} \quad \boxed{n14 \times 235} \quad \boxed{n135 \times 24} \quad \boxed{n12 \times 35 \times 4} \\
\begin{array}{c} 1,2,3,4,5 \\ 5 \end{array} \quad \begin{array}{c} 1,2,3,5 \\ 4 \end{array} \quad \begin{array}{c} 1,2 \\ 3,4,5 \end{array} \quad \begin{array}{c} 1,2,4 \\ 3,5 \end{array} \quad \begin{array}{c} 1,3,4,5 \\ 2 \end{array} \quad \begin{array}{c} 1 \\ 2,3,4,5 \end{array} \quad \begin{array}{c} 1,4 \\ 2,3,5 \end{array} \quad \begin{array}{c} 1,3,5 \\ 2,4 \end{array} \quad \begin{array}{c} 1,2 \\ 4 \quad 3,5 \end{array} \\
59\,048 \quad 54\,492 \quad 39\,392 \quad 43\,908 \quad 18\,980 \quad 728 \quad 4920 \quad 14\,748 \quad 39\,372
\end{array}$$

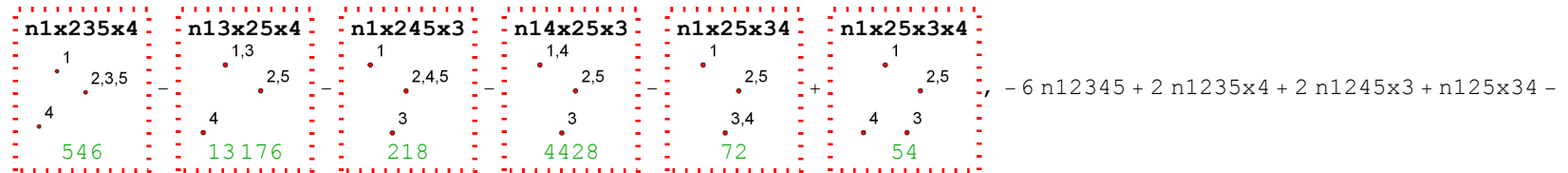
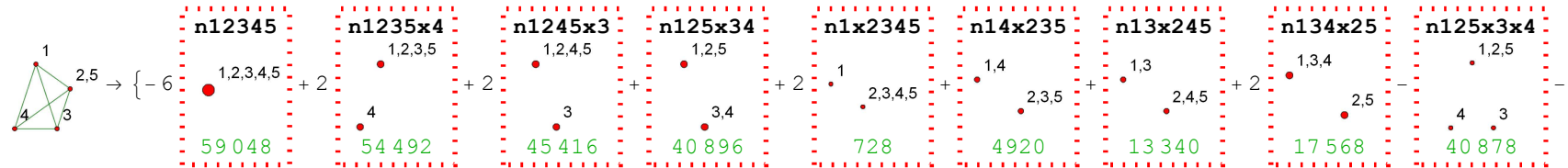
$$\begin{array}{c}
\boxed{n135 \times 2 \times 4} \quad \boxed{n1 \times 235 \times 4} \quad \boxed{n1 \times 2 \times 345} \quad \boxed{n14 \times 2 \times 35} \quad \boxed{n1 \times 24 \times 35} \quad \boxed{n1 \times 2 \times 35 \times 4} \\
\begin{array}{c} 1,3,5 \\ 4 \quad 2 \end{array} \quad \begin{array}{c} 1 \\ 4 \quad 2,3,5 \end{array} \quad \begin{array}{c} 1 \\ 2 \quad 3,4,5 \end{array} \quad \begin{array}{c} 1,4 \\ 2 \quad 3,5 \end{array} \quad \begin{array}{c} 1 \\ 2,4 \quad 3,5 \end{array} \quad \begin{array}{c} 1 \\ 4 \quad 2 \end{array} \\
14\,586 \quad 546 \quad 26 \quad 4380 \quad 168 \quad 6
\end{array}$$

$$n12 \times 35 \times 4 + 2\,n1345 \times 2 + n135 \times 24 - n135 \times 2 \times 4 + n14 \times 235 - n14 \times 2 \times 35 + 2\,n1 \times 2345 - n1 \times 235 \times 4 - n1 \times 24 \times 35 - n1 \times 2 \times 345 + n1 \times 2 \times 35 \times 4 \},$$



$-6 \text{n12345} + 2 \text{n1234x5} + 2 \text{n1245x3} + \text{n124x35} -$

$\text{n124x3x5} + 2 \text{n1345x2} + \text{n134x25} - \text{n134x2x5} + \text{n145x23} - \text{n145x2x3} + 2 \text{n14x235} - \text{n14x23x5} - \text{n14x25x3} - \text{n14x2x35} + \text{n14x2x3x5}\}$,



$-6 \text{n12345} + 2 \text{n1235x4} + 2 \text{n1245x3} + \text{n125x34} -$

$\text{n125x3x4} + 2 \text{n134x25} + \text{n13x245} - \text{n13x25x4} + \text{n14x235} - \text{n14x25x3} + 2 \text{n1x2345} - \text{n1x235x4} - \text{n1x245x3} - \text{n1x25x34} + \text{n1x25x3x4}\}$

```
In[408]:= ChromaticPolynomial[allGraphs[alfalKey, "graph"], x]
```

```
Out[408]= 2 x - 3 x^2 + x^3
```

```
In[421]:= Select[Keys[allGraphs], allGraphs[#, "colofourrealnull"] == n12345 &]
```

```
Out[421]= {59 048}
```

```
In[424]:= allGraphs[k5Key, "graph"]
```

