## Fuzzy Moving Average Trading System With Genetic Algorithm Optimization

EN.525.770.81 - Intelligent Algorithms Liam Ventura Original Authors: Tan Chee Wei, Laxman Singh, Ramasamy Muthuraman, and Isaac Varun Kumar

#### Motivation

- The Moving Average Strategy (MAS) is one of the most popular technical indicators that informs traders of existing trends and helps identify upcoming reversals
  - Generates buy or sell signals by seeing if the difference between the long and short-term MA is positive or negative [1][2]
- Fuzzy Logic integration allows to describe the strength of the signal to generate controller actions (i.e. buy or sell)
- Genetic Algorithm implementation will optimized the trading strategy parameters in order to maximize returns

### Simple Moving Average

#### Simple Moving Average (SMA)

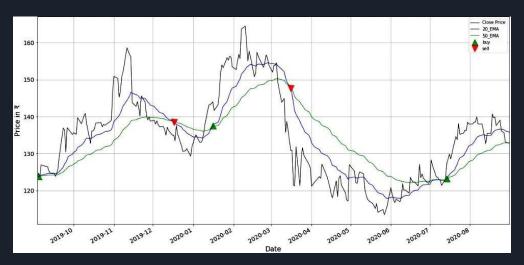
A = Average in period n

n = Number of time periods (window)

$$SMA = \frac{A_1 + A_2 + \dots + A_n}{n}$$

## Simple Moving Average Strategy

- When the Short-term (Fast) Moving Average crosses <u>above</u> the Long-term (Slow) Moving Average -> Indicates a *Buy* Signal
- When the Short-term (Fast) Moving Average crosses <u>below</u> the Long-term (Slow) Moving Average -> Indicates a *Sell Signal*



Source: Linkedin - Patrick Nabriya

## Simple Moving Average Strategy

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- When the Short-term (Fast) Moving Average crosses <u>below</u> the Long-term (Slow) Moving Average -> Indicates a *Sell Signal*

```
maBuy = lambda day: (
   fastMA[day-2] < slowMA[day-2] and
   fastMA[day-1] > slowMA[day-1]
)
maSell = lambda day: (
   fastMA[day-2] > slowMA[day-2] and
   fastMA[day-1] < slowMA[day-1]
)</pre>
```

#### Fuzzy System - Concept

- Moving Average provides buy and sell signals, if using the simple MA strategy then must immediate take action when signals cross over. This can cause extra costs or losses.
- Using Fuzzy Logic we can wait to make decisions until a certain threshold is exceeded
- Difference Between two Moving Averages is divided into 7 extents [1][2]:
  - Extremely Low (EL), Very Low (VL), Low (L), Middle (M), High (H), Very High (VH), Extremely High (EH)
  - Moving Average Differences acts as centroids for Pi Shaped Input Membership Function
  - Fuzzy Input Memberships then get mapped to Fuzzy Controller Based on Rules
- Output Membership Functions have a range of [-1,1] mapped to Negative, Medium, Positive
  - Y > (0.5) indicates a strong buy (bullish trend) -> Results in Buy Action
  - Y < (-0.5) indicate a strong sell (bearish trend) -> Results in Sell Action
- Rule 1: If x is EL or x is VL or x is L then y is Negative
- Rule 2: If x is M then y is Medium
- Rule 3: If x is EL or x is VH or x is H then y is Positive

#### Fuzzy System - Implementation

- Difference Between two Moving Averages is divided into 7 extents array P:
  - Extremely Low (EL), Very Low (VL), Low (L), Middle (M), High (H), Very High (VH), Extremely High (EH) [1][2]
  - These values serve as the parameters for the input membership function as defined below:

$$f(x; a, b, c, d) = \begin{cases} 0, & x \le a \\ 2\left(\frac{x-a}{b-a}\right)^2, & a \le x \le \frac{a+b}{2} \\ 1 - 2\left(\frac{x-b}{b-a}\right)^2, & \frac{a+b}{2} \le x \le b \\ 1, & b \le x \le c \\ 1 - 2\left(\frac{x-c}{d-c}\right)^2, & c \le x \le \frac{c+d}{2} \\ 2\left(\frac{x-d}{d-c}\right)^2, & \frac{c+d}{2} \le x \le d \\ 0, & x \ge d \end{cases}$$

$$f(x;a,b,c,d) = \begin{cases} 0, & x \leq a \\ 2\left(\frac{x-a}{b-a}\right)^2, & a \leq x \leq \frac{a+b}{2} \\ 1-2\left(\frac{x-b}{b-a}\right)^2, & \frac{a+b}{2} \leq x \leq b \\ 1, & b \leq x \leq c \\ 1-2\left(\frac{x-c}{d-c}\right)^2, & c \leq x \leq \frac{c+d}{2} \\ 2\left(\frac{x-d}{d-c}\right)^2, & \frac{c+d}{2} \leq x \leq d \\ 0, & x \geq d \end{cases}$$

$$\mu_{pi}(x;a,b,c,d) = \begin{cases} EL, & a = P(0), b = P(0), c = P(0), d = P(1) \\ VL, & a = P(0), b = P(1), c = P(1), d = P(2) \\ L, & a = P(1), b = P(2), c = P(2), d = P(3) \\ M, & a = P(2), b = P(3), c = P(3), d = P(4) \\ H, & a = P(3), b = P(4), c = P(4), d = P(5) \\ VH, & a = P(4), b = P(5), c = P(5), d = P(6) \\ EH, & a = P(5), b = P(6), c = P(6), d = P(6) \end{cases}$$

Source: Matlab - Pi MF

#### Fuzzy System - Implementation

- The Output Membership Function is defined as such
  - Negative (Neg), Medium (Med), Positive (Pos)

$$f(x;a,b,c,d) = \begin{cases} 0, & x \leq a \\ 2\left(\frac{x-a}{b-a}\right)^2, & a \leq x \leq \frac{a+b}{2} \\ 1-2\left(\frac{x-b}{b-a}\right)^2, & \frac{a+b}{2} \leq x \leq b \\ 1, & b \leq x \leq c \\ 1-2\left(\frac{x-c}{d-c}\right)^2, & c \leq x \leq \frac{c+d}{2} \\ 2\left(\frac{x-d}{d-c}\right)^2, & \frac{c+d}{2} \leq x \leq d \\ 0, & x \geq d \end{cases} \qquad \mu_{pi}(x;a,b,c,d) = \begin{cases} Neg, & a = -1, b = -1, c = -1, d = 0 \\ Med, & a = -1, b = 0, c = 0, d = 1 \\ Pos, & a = 0, b = 1, c = 1, d = 1 \end{cases}$$

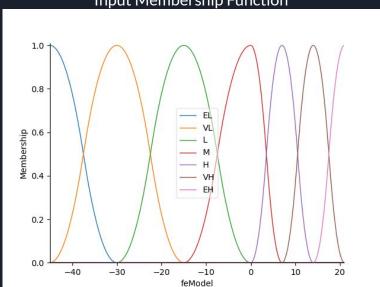
$$\mu_{pi}(x;a,b,c,d) = egin{cases} Neg, & a=-1,b=-1,c=-1,d=0 \ Med, & a=-1,b=0,c=0,d=1 \ Pos, & a=0,b=1,c=1,d=1 \end{cases}$$

Source: Matlab - Pi MF

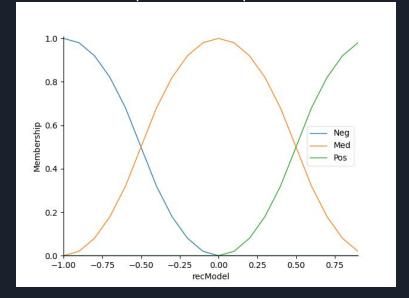
#### Fuzzy System - Implementation

- Difference Between two Moving Averages is divided into 7 extents:
  - Extremely Low (EL), Very Low (VL), Low (L), Middle (M), High (H), Very High (VH), Extremely High (EH) [1][2]
  - Fuzzy Input Memberships then get mapped to Fuzzy Controller Based on Rules

Input Membership Function



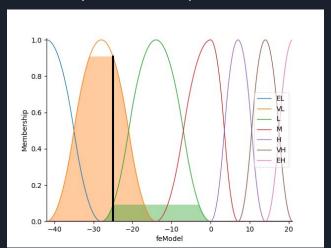
**Output Membership Function** 



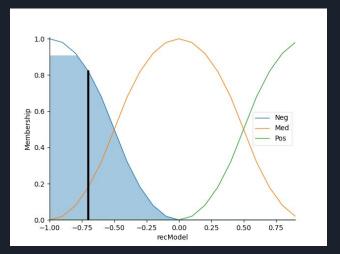
## Fuzzy System - Implementation Testing Sell Signal Fast Moving Average

[30 30 10 1]
Slow Moving Average
[10 20 50 20]
Difference of Moving Averages
[ 20 10 -40 -19]
Adjusted to 7 extents
[-42.0, -28.0, -14.0, 0, 7.0, 14.0, 21.0]
Exampel Difference Value:-25
Membership:-0.7012028573764877, Extent Value:feModel : -25

#### Input Membership Function



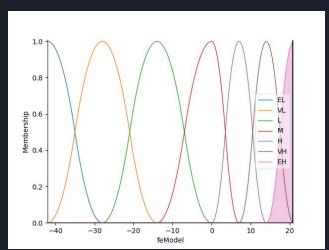
#### **Output Membership Function**



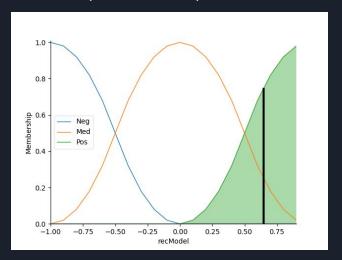
## Fuzzy System - Implementation Testing Buy Signal Fast Moving Average

Fast Moving Average
[30 30 10 1]
Slow Moving Average
[10 20 50 20]
Difference of Moving Averages
[ 20 10 -40 -19]
Adjusted to 7 extents
[-42.0, -28.0, -14.0, 0, 7.0, 14.0, 21.0]
Exampel Difference Value:25
Membership:0.646550290939318, Extent Value:feModel : 20.90000000000894

#### Input Membership Function



#### **Output Membership Function**



### Genetic Algorithm - Implementation

- 1. Initialize Population of n Strategies (Individuals)
  - Calculate Fitness of for each Strategy
  - b. Generation = 0
- 2. For generation in Generations(size):
  - a. Generation = Generation + 1
    - i. Keep Best 30% of Strategies (Elitism)
  - b. Selection: (Tournament Style)
  - c. Generate offsprings of Best Strategies
  - d. Mutate offsprings
  - e. Introduce a subset of new random strategies
  - f. Evaluate Fitness of Population
  - g. End
- 3. Output the best strategy

```
@nb.jit(nopython=True)
def getCompounded(tradeRes: np.array):
    ''' Get compounded return '''
    invest = 1
    for perc in tradeRes:
        invest = (1+perc)*invest
    return invest
```

Maximization of fitness function: Compounded Profit Returns from trades executed

## Implementation - Changes from Authors

- Authors used Adaptive Moving Average, Typical Moving Average, and Triangular
   Moving Average in addition to Simple Moving Average
  - Using Exponential Moving Average and Weighted Moving Average
- Authors had a fixed long (m) and short (n) period lengths [1][2]
  - $om = \{10, 20, 50, 100, 150, 200\}$ ;  $n = \{1, 3, 5, 10, 15, 20\}$  and generated 32 combinations
  - Changed to let the GA iterate through a maximum and minimum window range
- Authors used *Backtrader* to implement backtest [2]
  - Wrote own method of backtesting
- Authors examined Crude Oil Futures [1][2]
  - This implementation will take a look at Blue Chip Tech Stocks

#### Genetic Algorithm - Details of Changes

- Optimizable Parameters -
  - Moving Average Type: Simple Exponential, Weighted
  - Moving Average Price Field: (Open, Low, High, Close, Adj Close)
  - Moving Average Length: range [3 300]
- What can be optimized:
  - Mean: Mean Percentage of gain/loss over all trades made
  - Median: Median Percentage of gain/loss over all trades made
  - Compounded: Multiples of Initial Investment from applying strategy
- Since Optimizing Strategy Across Multiple Tickers will end up with fitness values for each ticker, to combine fitness value across all tickers can implement
  - Min: the minimum of all Fitness Values
  - Mean: the mean of all Fitness Values
  - Median: the median of all Fitness Values
  - Max: the max of all Fitness Values

### Implementation - Technical Indicators

#### Simple Moving Average (SMA)

A = Average in period n

n = Number of time periods (window)

#### Exponential Moving Average (EMA)

P = Price ; K = 2 / (N+1)

N = Number of time period (window)

#### Weighted Moving Average (WMA)

P = Price; W = Weight; n= number of periods in weight group

• More weight assigned to earlier prices

$$SMA = \frac{A_1 + A_2 + \dots + A_n}{n}$$

$$EMA_n = P_n * k + EMA_{n-1} * (1 - k)$$

$$WMA = \frac{\sum_{t=1}^{n} W_t * P_t}{\sum_{t=1}^{n} W_t}$$

#### Implementation - Backtesting Strategy

- Looks the original price dataframe and the dataframe that contains trades executed
- Starts with initial investment and based on strategy evaluation (in this case compounded)
   returns the equity curve for the optimized strategy and buy and hold strategy

#### Backtesting Algorithm

```
def getEquityCurve(df: pd.DataFrame, df_backtest:pd.DataFrame, init_invest:float) -> Tuple[list, list]:
    invest_val = init_invest
    equity_curve=[]
    dates= []

for trade in range(len(df_backtest)):
    df_trade = df[
        (df['Date'] >= df_backtest.loc[trade, 'Bought_On']) &
        (df['Date'] <= df_backtest.loc[trade, 'Sold_On'])]
    equity = invest_val*df_trade['Adj Close'].values/df_trade['Adj Close'].values[0]

equity_curve += list(equity)
    dates += list(df_trade['Date'])
    invest_val = equity[-1]
    return dates, equity_curve</pre>
```

#### **Example of Trades Executed for Google**

```
Bought_On,Sold_On,Profit

2 2017-03-27 00:00:00-04:00,2017-05-31 00:00:00-04:00,0.0606629446651159

3 2017-09-08 00:00:00-04:00,2018-01-04 00:00:00-05:00,0.03886269358640271

4 2018-02-06 00:00:00-05:00,2018-02-20 00:00:00-05:00,-0.05220708781818317

5 2018-03-19 00:00:00-04:00,2018-08-06 00:00:00-04:00,0.1211857755633996

6 2018-10-31 00:00:00-04:00,2018-11-19 00:00:00-05:00,-0.08843273685112818

7 2018-12-06 00:00:00-05:00,2019-04-01 00:00:00-04:00,0.0777672428525138

8 2019-06-11 00:00:00-04:00,2019-09-04 00:00:00-04:00,0.04493714943021021
```

#### Model - Parameters

Training Ticker = [ NVDA, AMZN, GOOG, TSLA, TSM]

Testing Tickers = [ AAPL, GOOGL, META]

Number of strategies = 40

Keep Best (Elitism) = 30%

Number of Generations = 20

Moving Average Types = [Simple, Exponential, Weighted]

Moving Average Fields = [Open, Low, High, Close, Adj Close]

Minimum Moving Average Window (Days) = 3

Maximum Moving Average Window (Days) = 200

Max Window Frame Mutation Increment = +/- Range(-10,10)

Mutation of Strategy = 20 %

Percentage of Crossover = 33%

Strategy Evaluation = Compounded

#### Model - Data

- Data is retrieved from Yahoo Finance Using *yfinance* module
- Starting Date = 1/1/2017
- End Date = 1/1/2020
  - Selected this Data Frame to not take into account of volatility during covid years

```
import yfinance as yf
import os

TRAINING_TICKERS = ['NVDA', 'AMZN', 'GOOG', 'TSLA', 'AAPL']

TESTING_TICKERS = ['META', 'GOOGL', 'TSM']

LOWER_DATE = '2017-01-01'

UPPER_DATE = '2020-01-01'

for ticker in TRAINING_TICKERS+TESTING_TICKERS:
    if not os.path.exists('Data/' + ticker + '.csv'):

data = yf.download(ticker, start=LOWER_DATE, end=UPPER_DATE)
    data.to_csv('Data/' + ticker + '.csv')
```

Date	me     000
0   2017-01-03 00:00:00-05:00   26.1   26.5925   24.845   25.5025   25.143   150199   1   2017-01-04 00:00:00-05:00   25.85   26.375   25.3825   26.0975   25.7296   119922   2   2017-01-05 00:00:00-05:00   26.1325   26.455   25.2625   25.435   25.0764   98429   3   2017-01-06 00:00:00-05:00   25.7125   26.0625   25.3   25.775   25.4116   82285   4   2017-01-09 00:00:00-05:00   25.875   27   25.875   26.82   26.4419   91624	900   900
1   2017-01-04 00:00:00-05:00   25.85   26.375   25.3825   26.0975       25.7296   119922         2   2017-01-05 00:00:00-05:00   26.1325   26.455   25.2625   25.435       25.0764   98429         3   2017-01-06 00:00:00-05:00   25.7125   26.0625   25.3   25.775       25.4116   82285         4   2017-01-09 00:00:00-05:00   25.875   27   25.875   26.82       26.4419   91624	000
2   2017-01-05 00:00:00-05:00   26.1325   26.455   25.2625   25.435   25.0764   98429   3   2017-01-06 00:00:00-05:00   25.7125   26.0625   25.3   25.775   25.4116   82285   4   2017-01-09 00:00:00-05:00   25.875   27   25.875   26.82   26.4419   91624	
3   2017-01-06 00:00:00-05:00   25.7125   26.0625   25.3   25.775   25.4116   82285   4   2017-01-09 00:00:00-05:00   25.875   27   25.875   26.82   26.4419   91624	00
4   2017-01-09 00:00:00-05:00   25.875   27   25.875   26.82   26.4419   91624	
	00 I
	00 I
5   2017-01-10 00:00:00-05:00   26.9525   27.2975   26.4075   26.6175   26.2422   88092	00
6   2017-01-11 00:00:00-05:00   26.5   26.55   26.0375   26.29   25.9193   52566	00 I
7   2017-01-12 00:00:00-05:00   26.0575   26.175   25.405   25.86   25.4954   62561	00 I
8   2017-01-13 00:00:00-05:00   25.9   26.25   25.765   25.8575   25.4929   45782	00 I
9   2017-01-17 00:00:00-05:00   25.75   25.8   25.1425   25.2775   24.9211   58061	คค เ
	00

### Model - Initial Baseline Strategy

```
STARTING_STRAT = {
   'fast_ma_type': 'simple',
   'slow_ma_type': 'simple',
   'fast_ma_field': 'Adj Close',
   'slow_ma_field': 'Adj Close',
   'fast_ma_period': 5,
   'slow_ma_period': 20}
```

#### Simple MA Model - Optimized Strategies

Optimization time: 196.72148323059082

```
Optimal Mean Strategy Parameters: {'fast ma type': 'weighted', 'slow ma type': 'exponential', 'fast ma field': 'Adj Close', 'slow_ma_field': 'Adj Close', 'fast_ma_period': 13, 'slow_ma_period': 18}

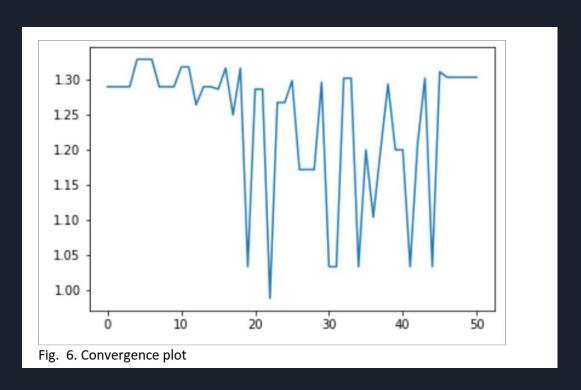
Optimal Max Strategy Parameters: {'fast ma type': 'weighted', 'slow_ma_type': 'exponential', 'fast ma field': 'Close', 'slow_ma_field': 'Low', 'fast_ma_period': 14, 'slow_ma_period': 16}

Optimal Median Strategy Parameters: {'fast ma type': 'weighted', 'slow_ma_type': 'exponential', 'fast ma field': 'Close', 'slow_ma_field': 'Close', 'fast_ma_period': 13, 'slow_ma_period': 16}

Optimal Min Strategy Parameters: {'fast ma type': 'weighted', 'slow ma type': 'simple', 'fast ma field': 'Open', 'slow_ma_field': 'Low', 'fast_ma_period': 5, 'slow_ma_period': 6}
```

# Results - Author's Result for Max Fitness Function Optimization

Max Fitness = 1.36



# Results - Author's Backtesting Results (Crude Oil)

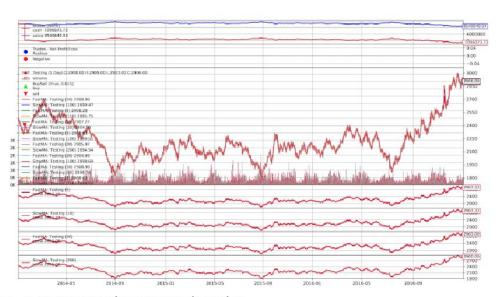


Fig.	8.	<b>Training</b>	and	testing	trading	pla	ot
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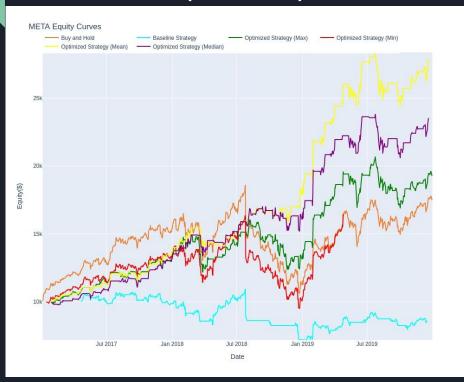
Initial Investment	\$10,000,000
ROI = (Net Profit)/Cost * 100	%
Author's Max Strategy	-17.05983

## Results - Simple MA Parameters Training Fitness



Strategy	Fitness
Mean	2.013362
Max	3.152996
Median	2.423178
Min	1.629903

# Results - Simple MA Backtesting Results (META)



Initial Investment	\$10,000
ROI = (Net Profit)/Cost * 100	Percent (%)
Buy and Hold	75.63
Baseline	-15.01
Optimized (Max)	92.7
Optimized (Min)	62.84
Optimized (Mean)	178.57
Optimized (Median)	135.54

## Results - Simple MA Backtesting Results (GOOGL)



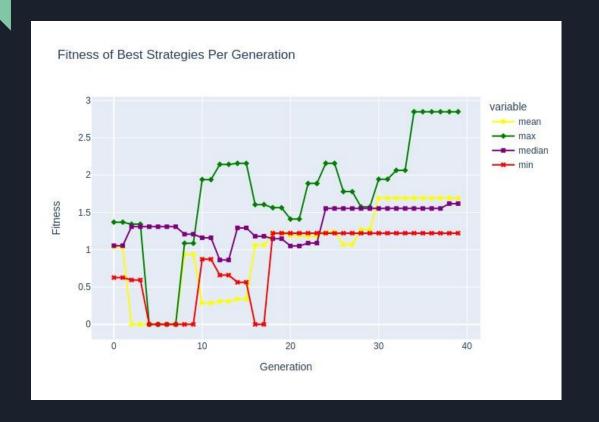
Initial Investment	\$10,000
ROI = (Net Profit)/Cost * 100	Percent (%)
Buy and Hold	65.80
Baseline	11.52
Optimized (Max)	95.26
Optimized (Min)	99.14
Optimized (Mean)	129.39
Optimized (Median)	141.98

# Results - Simple MA Backtesting Results (AAPL)



Initial Investment	\$10,000
ROI = (Net Profit)/Cost * 100	Percent (%)
Buy and Hold	164.66
Baseline	89.86
Optimized (Max)	91.78
Optimized (Min)	131.13
Optimized (Mean)	49.22
Optimized (Median)	51.40

### Results - Fuzzy Parameters Training Fitness

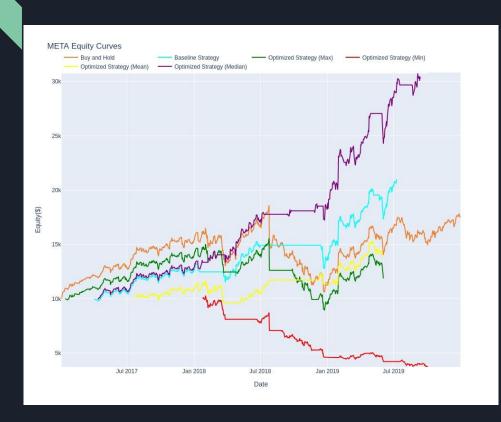


Strategy	Fitness
Mean	1.692051
Max	2.848038
Median	1.617651
Min	1.221721

### Fuzzy Model - Optimized Strategies

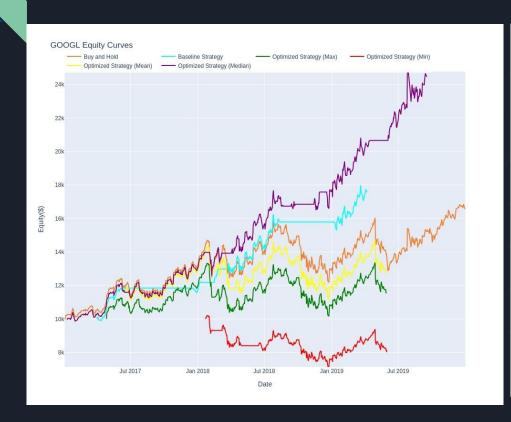
```
Optimization time: 2446.594869852066
Optimal Mean Strategy Parameters: { 'fast ma type': 'weighted', 'slow ma type':
'exponential', 'fast ma field': 'Open', 'slow ma field': 'Low',
'fast ma period': 11, 'slow ma period': 21}
Optimal Max Strategy Parameters: {'fast ma type': 'weighted', 'slow ma type':
'exponential', 'fast ma field': 'High', 'slow ma field': 'Low',
'fast ma period': 11, 'slow ma period': 28}
Optimal Median Strategy Parameters: { 'fast ma type': 'exponential',
'slow ma type': 'simple', 'fast ma field': 'Close', 'slow ma field': 'Open',
'fast ma period': 3, 'slow ma period': 12}
Optimal Min Strategy Parameters: { 'fast ma type': 'weighted', 'slow ma type':
'exponential', 'fast ma field': 'High', 'slow ma field': 'Open',
'fast ma period': 12, 'slow ma period': 17}
```

## Results - Fuzzy Backtesting (META)



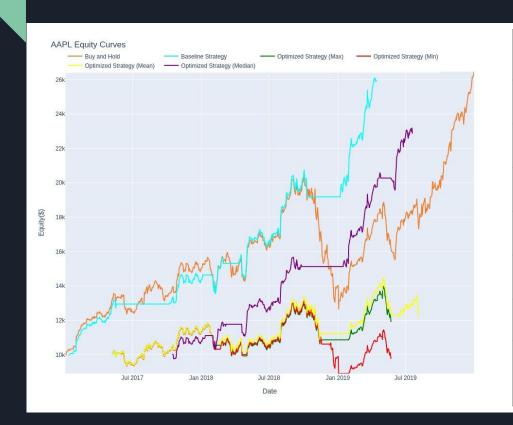
Initial Investment	\$10,000
ROI = (Net Profit)/Cost * 100	Percent (%)
Buy and Hold	75.6375
Baseline	110.1757
Optimized (Max)	18.8014
Optimized (Min)	-62.74246
Optimized (Mean)	38.9113
Optimized (Median)	202.7797

## Results - Fuzzy Backtesting (GOOGL)



Initial Investment	\$10,000
ROI = (Net Profit)/Cost * 100	Percent (%)
Buy and Hold	65.8036
Baseline	75.8266
Optimized (Max)	15.5649
Optimized (Min)	-19.81175
Optimized (Mean)	27.2372
Optimized (Median)	144.706

## Results - Fuzzy Backtesting (AAPL)



Initial Investment	\$10,000
ROI = (Net Profit)/Cost * 100	Percent (%)
Buy and Hold	164.6595
Baseline	159.523
Optimized (Max)	19.087
Optimized (Min)	-2.41
Optimized (Mean)	22.3
Optimized (Median)	132.08

#### Conclusion

- Combination of Fuzzy Logic Rule and Moving Average Strategy can form a Fuzzy Moving Average System (FMAS) to generate trade signals
- Genetic algorithm methods can be used for better strategy optimization
- Time Complexity using Python is Exponential when comparing the time to Optimize a
   Simple MA Strategy vs. Fuzzy Moving Average System
  - o 196.72148323059082 vs 2446.594869852066
- Author concluded that trend over training years show a downward or sideways price movement
  - Explains why author's model sells most of the time in test period even though market is bullish
  - Training data not being a combination of upward and downward trends leads to model biased towards downward trends
- Adjusted implementation of model across Blue Chip Tech Stocks showed that the model
  was able to optimize strategy parameters and increased performance for majority of
  testing Tech Stocks as opposed to Buy and Hold Strategy

#### References

- 1. Liu, X., An, H., Wang, L., & Guan, Q. (2016). Quantified moving average strategy of crude oil futures market based on fuzzy logic rules and genetic algorithms. *Physica A-statistical Mechanics and Its Applications*, 482, 444-457.
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