MONOPEDAL JUMPER ON A TRAMPOLINE

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ORIGINAL PROPOSAL AND CHANGES:

Original: A sideways view of a leg of a monopedal jumper on a trampoline from hip to foot which would bounce on a trampoline represented by a spring system.

Configuration variables were written as [q1, q2, q3, q4, q5], where, q1 was the x coordinate of the centre of mass of the body (somewhere near lower abdomen).

Changes: Configuration variables were written as [q2, q3, q4, q5], where, **q1 was not considered**. *(Because I could not handle very well, the random motion and could not make sense of the perturbations riding on the desired trajectory of each angle, that came with this DOF)*.

Thus setting q1 to zero permanently.

MODELLING ANALOGY

Here, only one leg of the person is simulated. The simulation is in side view. The working is the following images:

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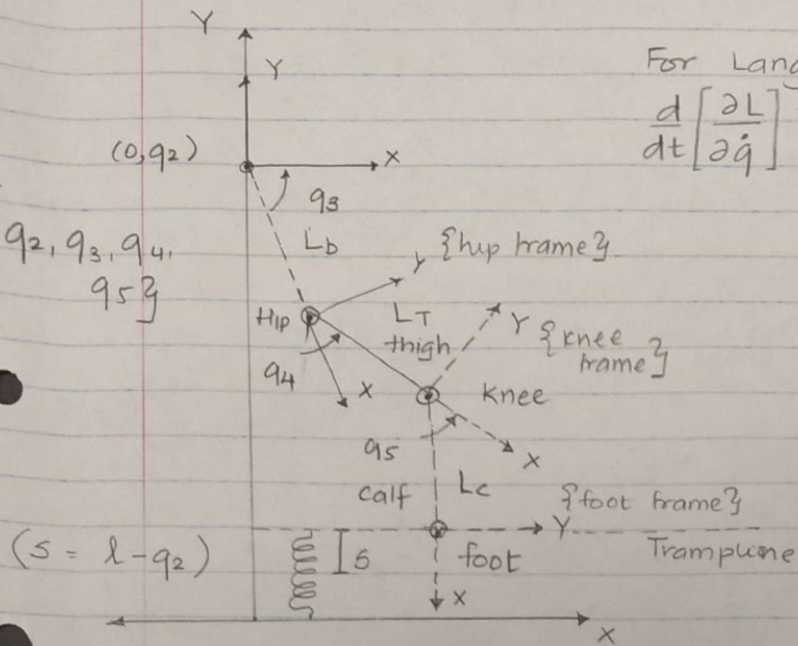
$q = \{q_2, q_3, q_4, q_5\}$



For Langrangian, L

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = \lambda_1 \nabla \phi_1 + \lambda_2 \nabla \phi_2$$

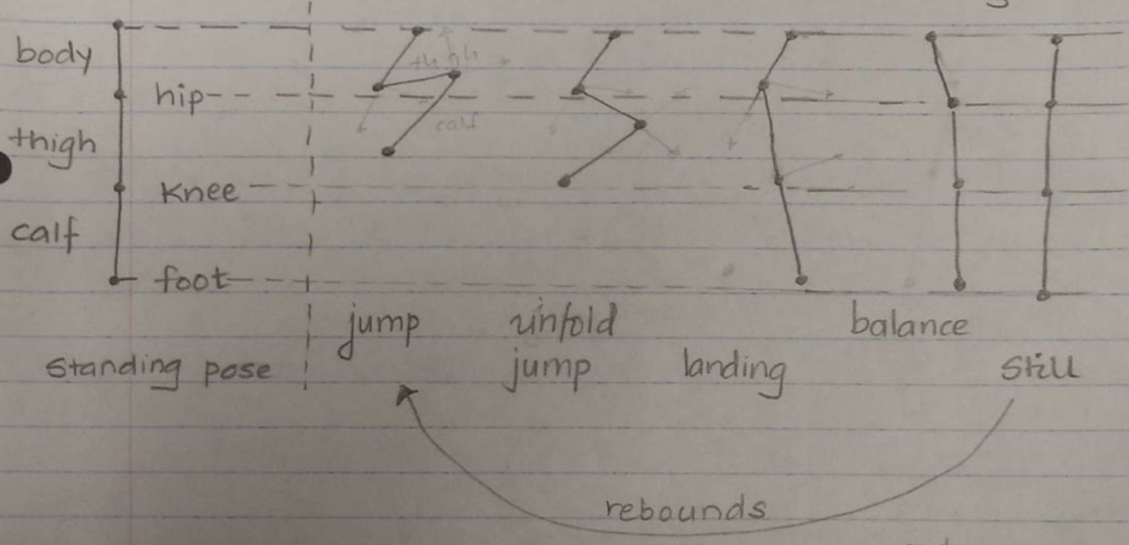
$$\phi_1 = Y_{foot} + 4 X_{foot}^2$$

$$\phi_2 = X_{hip}^2 + Y_{hip}^2 = (L_b + L_T + L_c - q_2[t])^2$$

$$K.E = \sum \tfrac{1}{2} V^T . I . V$$

$$I = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & \frac{1}{3} m l^2 \end{bmatrix}$$

CONSIDER A MAN AS FOLLOWS :



FOR IMPACTS

$$\Phi_{IMP} = Y_{foot} - H_{trampoline}$$

$$\frac{\partial L}{\partial \dot{q}}\Big|_{\tau^-}^{\tau^+} = \lambda \nabla \phi$$

$$\text{Hamiltonian}\Big|_{\tau^-}^{\tau^+} = 1/2 \times K_{trampoline} \times (L_B + L_T + L_c - q_2)^2$$

USE OF MATERIAL TAUGHT IN CLASS

1. **Change in coordinates**
   a. The ground frame is situated at the actual fixed ground.
   b. It is translated to the **CG of the body, at (0, q2)**, using configuration variable q2.
   c. From there on, frames from **"joint i" is rotated by "angle qi" and translated by length L_{i+1} to reach "joint i+1"**.
2. **Euler Lagrange equations with constraints**
   a. **Each joint was considered as a torsional spring**, thus instead of using body forces as "external", these were now considered as potential energy components:

$$U = \sum \tfrac{1}{2} K \Theta^2$$

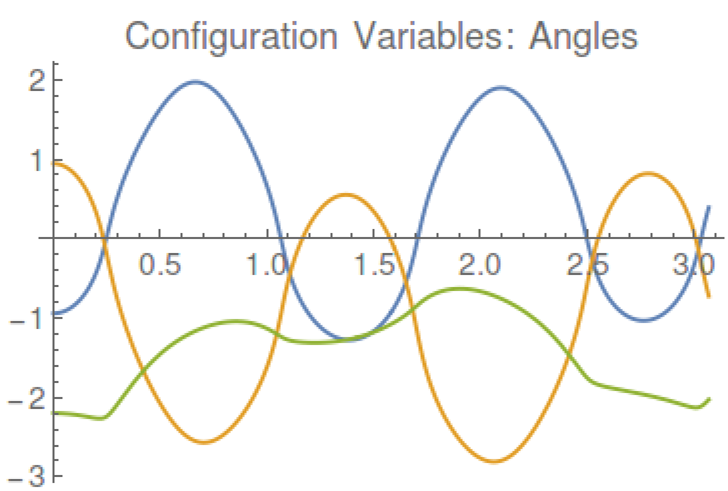
   b. Constraints were put to avoid joint collisions or overlapping of links or prevent their breaking up - it was essential for the paths to have a eccentricity greater than zero.
   c. Also, this ensures that there is a jump axis and every point of the **foot / hip** trajectory maintain their position from a point on this axis so a stable jump can be executed. Each **constraint was the simplest form of parabola facing downward:**

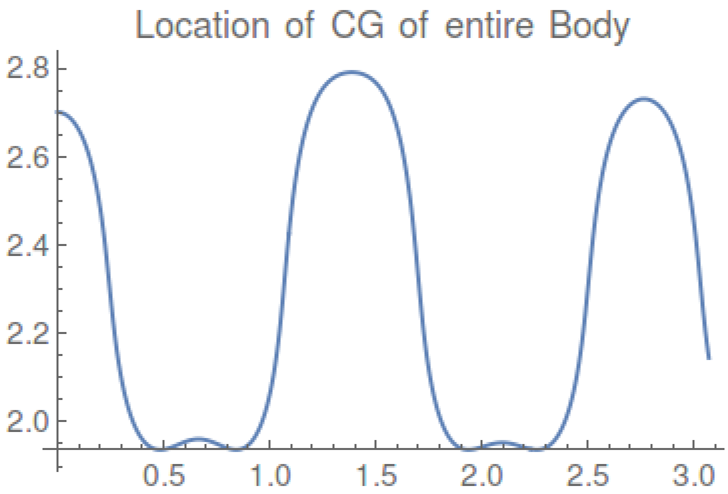
$$\Phi 1 : Y\_foot = -4 \ (X\_foot)^2$$

The 4 factor on the RHS just makes the trajectory wider.

$$\Phi 2: \ (X\_hip)^2 + (Y\_hip)^2 - (Lb+Lt+Lc - q2)^2 = 0$$

Here are the joint trajectories due the constraints:





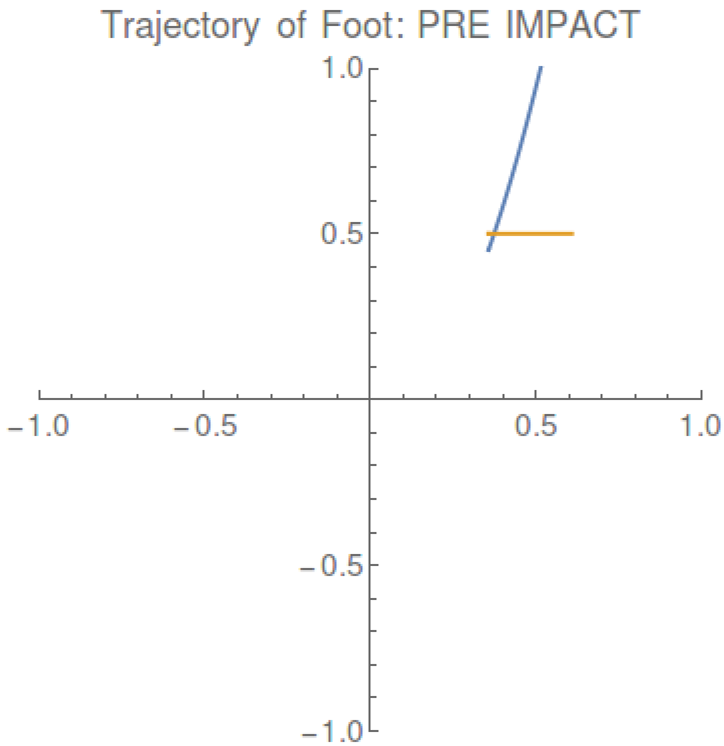
3. **Rotational Inertia:**
   a. Each leg link (thigh, calf, foot) was given a fixed mass in as per rough estimate of proportions.
   b. However **uniform mass distribution** was considered and inertia was calculated as a **rod with axis at end**.
   c. Thus, **inertia matrix** for a mass M was:

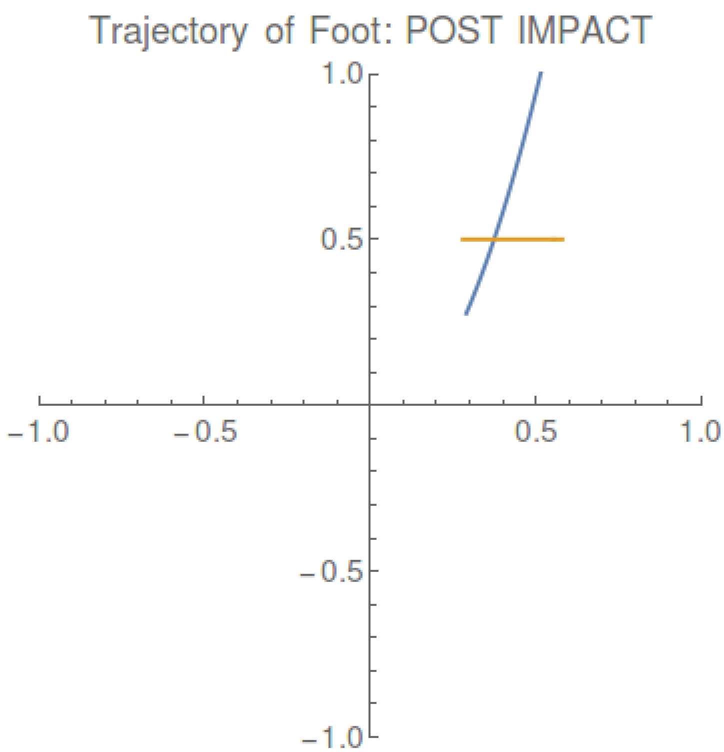
$$\begin{matrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & \frac{1}{3} M L^2 \end{matrix}$$

4. **Impacts:**
   a. The impact was considered when the heel first touches the trampoline.
   b. The Hamiltonian difference at the $^{-}T_{impact}$ and $+T_{impact}$ would be energy of the trampoline due to compression = $\frac{1}{2}\ K_{trampoline}\ (L\text{-}q2[t])^2$.
   c. Here Hamiltonian can be approximated as the total energy and hence equation can be formulated as above.

RESULTS

1. The leg starts from a mid-air position - as if jumping onto the trampoline
2. It then unfolds and then jumps up in rebound .
3. The lower bound on the oscillation on the Centre of Gravity of body is interpreted as Lt+ Lc + Lb/2 which fits the thought process of designing the constraints. (please refer above graph "Location of CG of Entire Body")
4. Here are the trajectory the foot follows wrt to the trampoline surface:


Trajectory of Foot: PRE IMPACT
1.0
0.5
−1.0
−0.5
0.5
1.0
−0.5
−1.0


Trajectory of Foot: POST IMPACT
1.0
0.5
−1.0
−0.5
0.5
1.0
−0.5
−1.0

CONCLUSION

Based on the above graphs of the foot, the jumper performs well.

The Animation closely follows a “forward stationery jump” as drawn on the hand written sheet attached in MODELLING section.

However here, the motion could be controlled a bit more. The torsional spring joints sort of worked around designing trajectory control forces.

The system could be made more robust with control forces and exact trajectory derivation not an approximation to a quadratic curve - which I have done. (I would love to explore this though!)

REFERENCES

1. Leyendecker, Koch, Ringkamp, Ober-Blobaum, 2014, “A DISCRETE VARIATIONAL APPROACH TO NON-SMOOTH DYNAMICS AND OPTIMAL CONTROL”
2. Blajer, Czaplicki, 2003, “CONTACT MODELING AND IDENTIFICATION OF PLANAR SOMERSAULTS ON THE TRAMPOLINE”
3. Koch, Leyendecker, “OPTIMAL CONTROL OF STANDING JUMP MOVEMENTS”