Assignment 14, Authomata Theory

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Contents

1 Problems

1.1 Problem 1

For each language below over the alphabet $\Sigma = \{a, b\}$ specify the number of equivalence classes and the distinguishing extension that distinguishes between the said classes:

1. $L = \{b, bb, bbb, bba\}$.
2. All words which begin and end in ba.
3. $L = \{c^i b^i a^i \mid i \geq 0\}$.

1.1.1 Answer 1

We can distinguish between words having one character by appending ϵ, since $b.\epsilon$ is in the language and $a.\epsilon$ isnt. There are four words of length 2, namely aa, ab, ba and bb, and we can distinguish between bb and the rest of the words by appending ϵ, since, again, only bb is in the language. Finally, there are 8 words of length 3, of which bba and bbb can be distinguished from the rest by appending ϵ. Words of length 4 and greater are indistinguishable since no word in L is that long. Hence, there must be in total 4 equivalence classes.

1.1.2 Answer 2

At first, I will define δ for this language, thus simultaneously showing it is regular:

	a	b
q_0	q_f	q_1
q_1	q_2	q_f
$*q_2$	q_3	q_4
q_3	q_3	q_4
q_4	q_5	q_3
$*q_5$	q_3	q_4

Thus establishing the upper bound on number of classes: 6. Now, I will use a "representative" of every class to find the distinguishing extension.

	b	ba	baa	bab
baba	ba		ϵ	a
bab	ba	a	a	
baa	ba	ϵ		
ba	ϵ			

Apparently, *baba* and *ba* are indistinguishable, i.e. q_2 and q_5 could be unified. Hence, the minimal automata has 5 states. Hence, according to Myhill-Nerode there are 5 equivalence classes.

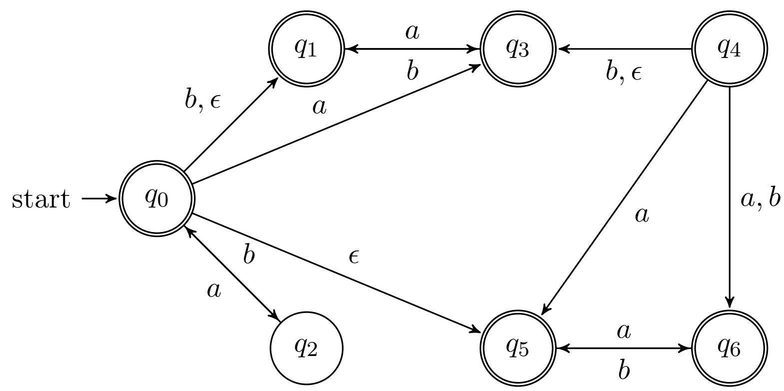
1.1.3 Answer 3

Not only this language doesn't answer the requirement of having only two letters in its alphabet, it is also not a regular language. Once I prove the language isn't regular, it immediately follows that it has infinitely many equivalence classes.

Suppose, for contradiction, L is regular. If this is the case, its automaton must have a finite number of classes, say n. Now consider all words in L starting with c^0, c^1 and so on trhough to c^n. In total, there would be $n+1$ such words, which, by pigeonhole principle implies that at least two of them belong in the same class, i.e. must end in the same suffix, but clearly for any two c^k and c^r, where $k \neq r$ it is also the case that $a^k b^k \neq a^r b^r$, hence the initial assumption must be false. Hence L is not regular and, consequently, has infinitely many equivalence classes.o

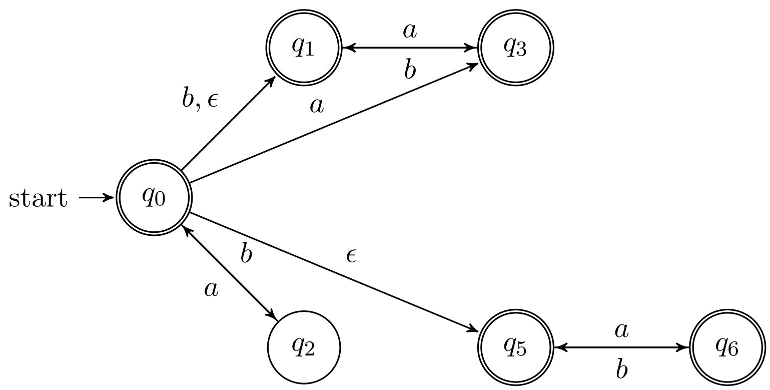
1.2 Problem 2

Build the reduced DFA of the given NFA:

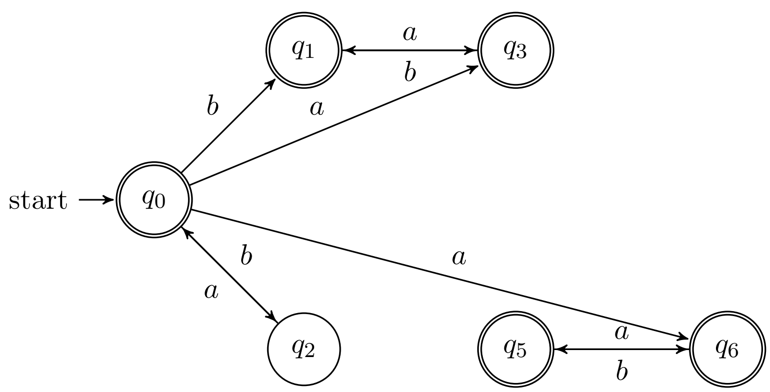


1.2.1 Answer 4

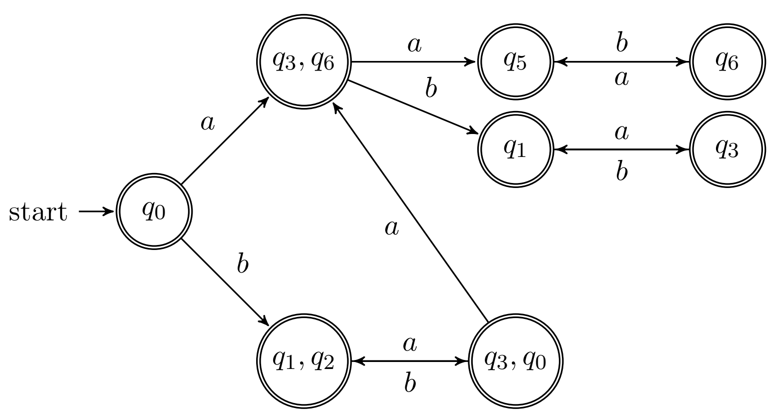
In the first step, notice that there are no arrows leading to q_4, this means we can safely remove it and all the arrows leading from it:



In the next step, we can replace all ϵ-transitions by the arcs leading directly to the nodes inside ϵ-closure of the source node.



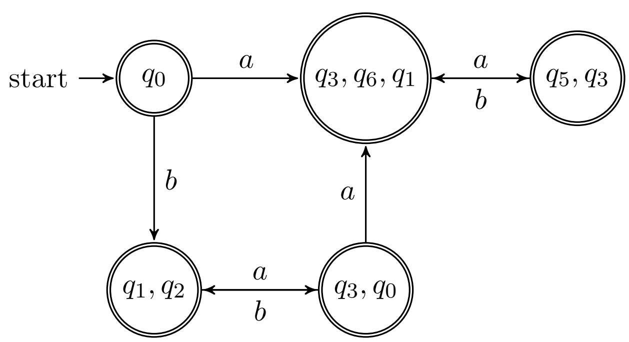
Now we can create product automation to obtain a DFA:



Now we are ready to build the table of "representatives" of the classes:

	a	b	aa	ab	ba	aab
aba	a	a		a	a	a
aab	b	aa	b		b	
ba	aa	b	a	b		
ab		aa	a			
aa	a	a				
b	b					

Which means that q_5 and q_3 can be unified, similarly q_1 and q_6 and $\{q_3, q_6\}$ with q_1. This gives us the following reduced DFA:



1.3 Problem 3

Prove that the language $L = \{0^r 1^s 2^t 0^{t+3} \mid r, s, t \geq 1\}$ is not regular using Myhill-Nerode theorem.

1.3.1 Answer 5

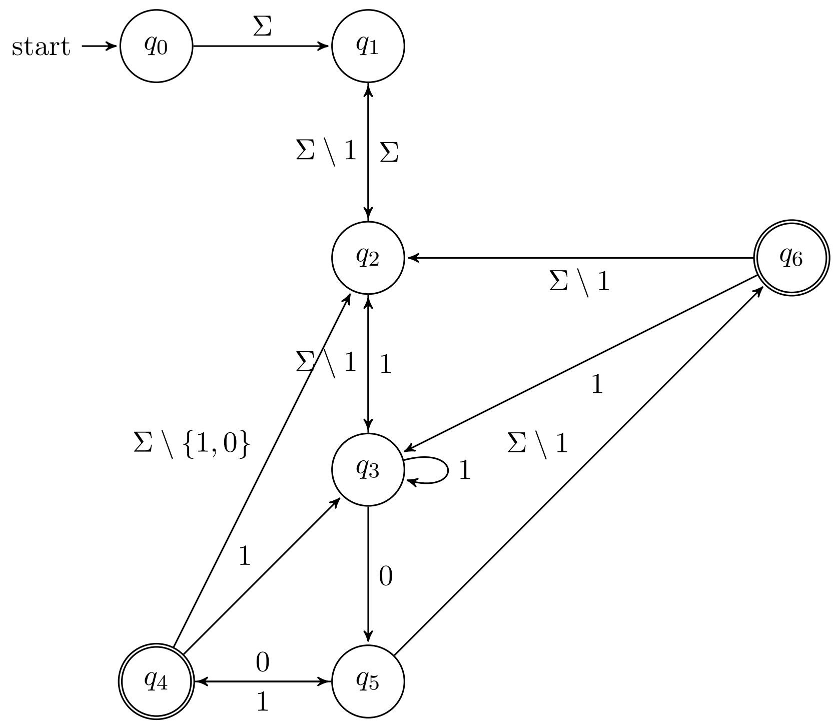
Assume, for contradiction, L is regular. Then according to Myhill-Nerode theorem the free monoid Σ^* where $\Sigma = \{0, 1, 2\}$ is partitioned into finite number of equivalence classes by the relation R_L defined on L, where words in Σ^* are related iff they don't have a distinguishing extension. Let the number of equivalence classes created by R_L be n. Consider the set of words where $r = 1$, $s = 1$ and $t = 1 \dots n + 1$. By pigeonhole principle there must be two words in this set in the same equivalence class. Suppose without loss of generality $w_1 = 0^1 1^1 2^i$ and $w_2 = 0^1 1^1 2^j$ where $i \neq j$ are these words. Then it follows that there does not exist a distinguishing extension for them, but if we choose 0^{i+3} to be the distinguishing extension, then w_1 is in the language, but w_2 isn't since $0^{i+3} \neq 0^{j+3}$. Hence the initial claim is false, hence the language is not regular.

1.4 Problem 4

Given alphabet Σ s.t. $|\Sigma| \geq 2$ and $1, 0 \in \Sigma$, in how many equivalence classes does the relation R_L partition its free monoid, if the language for this relation is given by regular expression $\Sigma\Sigma^+ 10\Sigma$?

1.4.1 Answer 6

In the first step I'll design a DFA to accept this language:



Let's verify whether every two "representatives" of this automaton have a distinguishing extension:

For the ease of notation $\Omega = \Sigma \setminus 1$.

	Σ	$\Sigma\Sigma$	$\Sigma\Sigma 1$	$\Sigma\Sigma 10$
$\Sigma\Sigma 10\Omega$	ϵ	ϵ	ϵ	ϵ
$\Sigma\Sigma 10$	1	1	1	
$\Sigma\Sigma 1$	0	0		
$\Sigma\Sigma$	101			

Since we could find a distinguishing extension for each pair of the states in the automaton, it follows that the number of equivalence classes is the same as the number of the states, i.e. 7.