Assignment 15, Authomata Theory

Oleg Sivokon

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Contents

1 Problems

1.1 Problem 1

Given context-free grammar $V = \{S, M, N, W, X, Y, Z\}$ s.t. $T = \{1, 0\}$

$$\begin{aligned}
&S \to M \mid XN \mid W \mid 0N \mid 1Z1 \\
&M \to 0M0 \mid N \\
&N \to N0 \mid 0 \\
&W \to 0W \mid 00W0 \\
&X \to 0X1 \mid 0 \mid 0Y0 \\
&Z \to W \ .
\end{aligned}$$

1. Is V ambiguous?
2. Give a normalized grammar equivalent to V.

1.1.1 Answer 2

It is easier to normalize the grammar first and then to look for ambiguities, thus the answers are in reverse order.

1. Any derivation containing W cannot terminate, and so does Z.
2. Further, we can eliminate the rule $M \to N$.
3. Y has no derivation rules, thus we can also remove it.

Thus obtaining:

$$\begin{aligned}
&S \to M \mid XM \mid 0M \\
&M \to 0M0 \mid 0 \mid 0M \\
&X \to 0X1 \mid 0 \ .
\end{aligned}$$

1. It is easy to see that M derives number of zeros greater than one, thus $M \to 0M0$ is redundant. Subsequently, $S \to 0M$ is already covered by $S \to M$.

What remains is:

$$
\begin{aligned}
&S \to M \mid XM \\
&M \to 0 \mid 0M \\
&X \to 0 \mid 0X1 \ .
\end{aligned}
$$

1.1.2 Answer 1

Now it is easy to see that the string 00 can be derived in two different ways:

- $S \to M,\ M \to 0M,\ M \to 0.$
- $S \to XM,\ X \to 0,\ M \to 0.$

Hence V is ambiguous.

1.2 Problem 2

Given context-free grammar $V = \{S, M, N, W, X, Y, Z\}$ s.t. $T = \{1, 0\}$

$$
\begin{aligned}
&S \to 0W11 \mid 0X1 \mid 0Y \\
&W \to S \mid Z \\
&X \to S \mid W \\
&Y \to 1 \\
&Z \to X \ .
\end{aligned}
$$

1. Bring V to Chomsky's normal form.
2. What is the language of V?

1.2.1 Answer 3

1. We can easily eliminate Y variable, thus removing $Y \to 1$ rule, and adding $S \to 01$ rule.
2. We can eliminate Z variable by removing $Z \to X$ and $W \to Z$ rules and adding $W \to X$ rule.
3. We can eliminate X variable by removing $X \to S \mid W$ and $S \to 0X1$ rules, and adding: $S \to 0S1$ rule.
4. Finally, we can eliminate W variable by removing $W \to S$ and $S \to 0W11$ rules and adding $S \to 0S11$ rule.

The resulting grammar will be:

$$S \to 0S11 \mid 0S1 \mid 01 \ .$$

Since this is still not CNF, I introduce an extra variable: X and derivation rules $X \to 0$, $Y \to 1 \mid 11$ and $Z \to SY$ thus obtaining:

$$\begin{aligned} &S \to XZ \mid 01 \\ &X \to 0 \\ &Y \to 1 \mid 11 \\ &Z \to SY \ . \end{aligned}$$

Which is in CNF.

1.2.2 Answer 4

Using the results from the previous answer it is easy to see that the language $L(V) = \{0^n 1^k \mid n \leq k \wedge n, k > 0\}$.