Assignment 11, Authomata Theory

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Contents

1 Problems

1.1 Problem 1

Given following languages over the alphabet $\{a, b\}$

- $L_1 = \emptyset$.
- $L_2 = \{\epsilon, aa\}$.
- $L_3 = \{\epsilon, a, aa, ab, abb\}$.
- $L_4 = \{aabb, aabbb, aa, aaa\}$.
- $L_5 = \{\epsilon, b, bbb, abab, abba, aabb\}$.
- $L_6 = \{\epsilon, bbbaa, baba, aaab, aabba, aa\}$.

1. What are the following languages:
   - $L_4 L_4$.
   - $(L_1 \cup L_2 \cup L_3)^R$.
   - $L_3 L_1 L_6$.
2. Define exponentiation as follows: $L^K = \{x \in L \mid \exists y \in K.(|y| = |x|)\}$. What are the languages $L_4^{L_5}$ and $L_6^{L_1}$.

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:- use_module(library(lists)).

concatentated_member(L1, L2, L3) :-
    member(M1, L1), member(M2, L2),
    string_concat(M1, M2, L3).

concatentated(L1, L2, L3) :-
    findall(X, concatentated_member(L1, L2, X), X),
    list_to_set(X, L3).

assignment_11a :-
    X = ["aabb", "aabbb", "aa", "aaa"],
    concatentated(X, X, Y),
    [First | Rest] = Y,
    write("$$\\{"),
    write(First),
    maplist(format(',\\allowbreak ~s'), Rest),
    write("\\}$$").
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$\{aabbaabb, aabbaabbb, aabbaa, aabbaaa, aabbbaabb, aabbbaabbb, aabbbaa, aabbbaaa, aaaabb, aaaabbb, aaaa, aaaaa, aaaaabb, aaaaabbb, aaaaaa\}$

1.1.1 Answer 1

1. Concatenation of L_4 with itself gives: $L_4L_4 = \{aabbaabb, aabbaabbb, aabbaa, aabbaaa, aabbbaabb, aabbbaa, aabbbaaa, aaaabbb, aaaa, aaaaa, aaaaabb, aaaaabbb\}$
2. $(L_1 \cup L_2 \cup L_3)^R = \{\epsilon, a, aa, ba, bba\}$.
3. $L_3L_1L_6 = \emptyset$. This is so because there are no words in language L_1 to concatenate with.

1.1.2 Answer 2

1. $L_4^{L_5} = \{aaa, aabb\}$.
2. $L_6^{L_1} = \emptyset$.

1.2 Problem 2

Let L_1, L_2 and L_3 be languages over some alphabet Σ. Prove or disprove:

1. $(L_1 \cup L_2)L_3 = L_1L_3 \cup L_2L_3$.
2. $(L_1 \cap L_2)L_3 = L_1L_3 \cap L_2L_3$.

1.2.1 Answer 3

First, I will prove $(L_1 \cup L_2)L_3 \subset L_1L_3 \cup L_2L_3$. Assume to the contrary that there is $w \in (L_1 \cup L_2)L_3$ which is not in $L_1L_3 \cup L_2L_3$. Put $w = xy$ where $x \in (L_1 \cup L_2)$ and $y \in L_3$ (this implies $L_3 \neq \emptyset$ and at least one of $(L_1 \cup L_2) \neq$

$\emptyset$. Suppose x comes from L_1, then it has to be in $L_1L_3 \cup L_2L_3$ because it is in L$_1$ L$_3$\$, similartly if it originates in L_2. Suppose now $L_3 = \emptyset$, then there is an empty set on both sides of equation (by definition of concatenation). Suppose both L_1 and L_2 are empty, then, again, we have emtpy set on both sides of the equation. Thus we showed that it is impossible for w not to be in the $L_1L_3 \cup L_2L_3$, hence the original argument must be true.

Similarly, to prove $L_1L_3 \cup L_2L_3 \subset (L_1 \cup L_2)L_3$, assume there exists $w \in L_1L_3 \cup L_2L_3$, not a amember of $(L_1 \cup L_2)L_3$. Again, $w = xy$ where $y \in L_3$ and x may be a member of L_1, L_2 or both. Suppose, again, the sets aren't empty. If w came from L_1L_3, then x came from L_1, but it is a member of $(L_1 \cup L_2)$ and similarly if it came from L_2. Since $y \in L_3$ and L_3 is present on both sides, it is not possible for w to not be a member of $(L_1 \cup L_2)L_3$. As in previous case, whenever L_3 or $(L_1 \cup L_2)$ are empty, both sides of equation contain an empty set. Hence we proved both directions, hence the conjecture is true.

1.2.2 Anser 4

This conjecture isn't generally true. Suppose $L_1 = L_3 = \{a\}$ and $L_3 = \{\epsilon, aa\}$. Then:

1. $(L_1 \cap L_2)L_3 = \emptyset$.
2. $L_1L_3 \cap L_2L_3 = \{aa\}$.

I.e. both sides of equation are not equal. This completes the proof.