Assignment 15, Authomata Theory

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Contents

1 Problems

1.1 Problem 1

Given context-free grammar $V = \{S, M, N, W, X, Y, Z\}$ s.t. $T = \{1, 0\}$

$$\begin{aligned}
&S \to M \mid XN \mid W \mid 0N \mid 1Z1 \\
&M \to 0M0 \mid N \\
&N \to N0 \mid 0 \\
&W \to 0W \mid 00W0 \\
&X \to 0X1 \mid 0 \mid 0Y0 \\
&Z \to W\ .
\end{aligned}$$

1. Is V ambiguous?
2. Give a normalized grammar equivalent to V.

1.1.1 Answer 2

It is easier to normalize the grammar first and then to look for ambiguities, thus the answers are in reverse order.

1. Any derivation containing W cannot terminate, and so does Z.
2. Further, we can eliminate the rule $M \to N$.
3. Y has no derivation rules, thus we can also remove it.

Thus obtaining:

$$\begin{aligned}
&S \to M \mid XM \mid 0M \\
&M \to 0M0 \mid 0 \mid 0M \\
&X \to 0X1 \mid 0\ .
\end{aligned}$$

1. It is easy to see that M derives number of zeros greater than one, thus $M \to 0M0$ is redundant. Subsequently, $S \to 0M$ is already covered by $S \to M$.

What remains is:

$$
\begin{aligned}
&S \to M \mid XM \\
&M \to 0 \mid 0M \\
&X \to 0 \mid 0X1 \ .
\end{aligned}
$$

1.1.2 Answer 1

Now it is easy to see that the string 00 can be derived in two different ways:

- $S \to M$, $M \to 0M$, $M \to 0$.
- $S \to XM$, $X \to 0$, $M \to 0$.

Hence V is ambiguous.