Assignment 17, Authomata Theory

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Contents

1 Problems

1.1 Problem 1

Prove that language $L = \{a^i b^{i+j} c^j \mid 1 \leq i \leq j\}$ is not context-free.

1.1.1 Answer 2

Suppose, for contradiction, L is context-free, then, according to pumping lemma, the following applies:

1. p is the "pumping length".
2. For every word $z \in L$, $z = uvwxy$, s.t.
3. $|vwx| \leq p$.
4. $|vx| \geq 1$.
5. $uv^n wx^n y \in L$.

Consider $p = i$, then there are five distinct ways to decompose w into $uvwxy$. Of them three will decompose in a way such that both v and x are the same symbol, i.e. both v and x are either a, b or c.

It is easy to see none of the above can be pumped: if $v = a^r$ and $x = a^s$ then by pumping a, eventually there will be more as in z than cs, which contradicts $i \leq j$. Similarly, if we pump bs, eventually there will be more bs than as and cs together. Similarly for cs.

Another two possible decompositions are:

1. $v = a^r$ and $x = b^s$. However, again, if we pump as, i.e. $r \neq 0$, then eventually there will be more as than cs. And similarly for bs. When we pump as and bs together, eventually there will be more as than cs, again, contradicting $i \leq j$.

2. Thus the only case worth considering is where $v = b^r$ and $x = c^s$. Consider the word $z = a^p b^{2p} c^p \in L$ with this decomposition. If either $r = 0$ or $s = 0$, we proceed as above, however, if $r = s \neq 0$, then it must be the case that for all words $z' = a^p b^{p+p-r+r*i} c^{p-r+r*i}$, $z' \in L$. but it is not the case for $i = 0$. Since $|a^p| > |c^{p-r}|$ contrary to the required $i \leq j$.

These are all the possible decompositions of z, since neither can be pumped, it must be the case that L is not context-free.

1.2 Problem 2

Prove that context-free languages are not closed under *max* operation.

1.2.1 Answer 2

Recall the definition of *max*:

$$max(L) = \{u \in L \mid \forall v \in \Sigma^* : uv \in L \implies v = \epsilon\} .$$

Let's take $L = \{a^n b^m c^k \mid n \leq k \vee m \leq k\}$. L is context-free, since we can give a grammar $L(G) = L$ as follows:

$$\begin{aligned}
&S \to X \mid Y \\
&X \to aXC \mid C \\
&C \to bCc \mid Cc \mid bBCc \mid c \\
&B \to bB \mid b \\
&Y \to AZ \\
&A \to aA \mid \epsilon \\
&Z \to bZc \mid Q \\
&Q \to cQ \mid c .
\end{aligned}$$

However, the $max(L) = \{a^n b^n c^n\}$, which is known to be non-context-free.