Assignment 12, Discrete Mathematics

Oleg Sivokon

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1 Problems

1.1 Problem 1

Let M be the set of all relations over $A = \{1, 2, 3\}$.

1. How many members does M have?
2. Let S be a set of relations over M, defined as follows:

$$S = \{R_1 R_2 | R_1, R_2 \in M \wedge R_1 R_2 = R_2 R_1\}.$$

   Show that S is not an equivalence relation.

1.1.1 Answer 1

The number of elements of M is $|M|^2 = 3^2 = 9$. This can be proved from a partial sum of a recurence: $a_{n+1} = a_n + 2n - 1$, but I will omit the proof. Intuitively, the number of relations of a set of cardinality $n+1$, is as big as the number of relations of a set with cardinality n plus a number of relations we can create from the element added to the elements already counted $(n-1)$, plus the number of relations to the element added from elements already counted $(n-1)$, plus the relation from the element added to itself.

1.1.2 Answer 3

I will prove that S is not an equivalence using the definition of equivalence which states that a relation is an equivalence if it is **relfexive**, **symmetric** and **transitive**. It is easy to see that the definition of reflexivity requires that *all* members of M be present in the relation, but, for example, $(1, 2)$ is absent from S. Suppose, for contradiction, it was present in S, then it would imply that there exists a pair $(a, b) \in M$ s.t. $((1, 2), (a, b)) \in S$ and $((a, b), (1, 2)) \in S$. By looking at the first and the last members of the two members of S we know that $(1, 2) \circ (a, b) = (1, b)$ and $(a, b) \circ (1, 2) =$

$(a, 2)$, in other words it is necessary that: $(1, 2) \circ (a, b) = (1, b) = (1, 2) = (a, 2) = (a, b) \circ (1, 2)$, which in turn, means that $b = 2$ and $a = 1$. Plugging these values back into original equilaty gives $(1, 2) \circ (1, 2) = (1, 2)$, which is obviously false. Hence, by contradiction, the proof is complete.