# Assignment 14, Discrete Mathematics

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## Contents

# 1 Problems

## 1.1 Problem 1

1. Develop the identity $(3-2)^n = 1$ using binom of Newton formula:

$$\sum_{i=0}^{n} \binom{?}{?} 3^? \cdot (??)^? = 1$$

   And verify the identity for the case $n = 4$.

2. Let number of ways to distribute $k$ identical balls between 10 boxes is $D(10, k)$. Paint three boxes green and the remaining seven—red. Derive:

$$D(10, k) = \sum_{i=0}^{k} ???,$$

   and verify for the case $k = 3$.

### 1.1.1 Answer 1

$$\sum_{i=0}^{n} \binom{i}{n} 3^i \cdot (-2)^{n-i} = 1.$$

**Solution:** *(using Maxima)*

```
n: 4;
tex(sum(binomial(n, i) * 3^i * (-2)^(n - i), i, 0, n));
```

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*(hand-made)*

$$\sum_{i=0}^{4} \binom{i}{4} 3^i \cdot (-2)^{4-i}$$
$$= 1 \cdot 3^0 \cdot (-2)^4 + 4 \cdot 3^1 \cdot (-2)^3 + 6 \cdot 3^2 \cdot (-2)^2 + 4 \cdot 3^3 \cdot (-2)^1 + 1 \cdot 3^4 \cdot (-2)^0$$
$$= 16 - 96 + 216 - 216 + 81$$
$$= 1$$

#### 1.1.2 Answer 2

### 1.2 Prolbem 2

How many permutations of a string $AAABBCCDD$ can you form s.t. they don't contain subsequences $AAA$, $BB$, $CC$ or $DD$?

#### 1.2.1 Answer 3

The total number of ways in which we can arrange the sequence $AAABBCCDD$ is $n(\Omega) = \frac{9!}{3!2!2!2!} = 7560$. Then we find all permutations which contain sequences of consequtive letters, $AAA$, $BB$ and so on.

$$\begin{aligned}
\frac{9!}{3!2!^3} &- \left(\frac{7!}{2!^3} + 3 \cdot \frac{8!}{3!2!^2}\right) + \left(3 \cdot \frac{6!}{2!^2} + 3 \cdot \frac{7!}{3!2!}\right) - \left(3 \cdot \frac{5!}{2!} + \frac{6!}{3!}\right) + 4! \\
&= 7560 - (630 + 5040) + (540 + 1260) - (180 + 120) + 24 \\
&= 3414 \, .
\end{aligned}$$

We count in stages: first we find all permutations of the string containing $AAA$ or duplicated characters, divided by the internal orderings of the remaining duplicates. Some of these permuations will also intersect with each other, thus, we want to subtract duplicates such as $AAA \cup BB$, but now we subtracted some of the duplicates twice, so we need to add them back. Those which we counted twice are those containing three subsequences, and so on. Finally:

```prolog
is_prefix([], _).
is_prefix(_, []) :- fail.
is_prefix([X | Xs], [X | Ys]) :- is_prefix(Xs, Ys).

not_allowed([a, a, a]).
not_allowed([b, b]).
not_allowed([c, c]).
not_allowed([d, d]).

prefix_allowed(Sofar) :-
    not_allowed(Bad), is_prefix(Bad, Sofar).

valid_seqence(X, [], X).
valid_seqence(Sofar, Pool, Result) :-
    select(E, Pool, Rem), Next = [E | Sofar],
    \+prefix_allowed(Next),
    valid_seqence(Next, Rem, Result).
valid_seqence(X) :-
    valid_seqence([], [a, a, a, b, b, c, c, d, d], X).

sans_repetitions :-
    findall(X, valid_seqence(X), X),
    list_to_set(X, Y),
    length(Y, Result),
    format('$$~p$$', [Result]).
```

$$3414$$

### 1.3 Problem 3

Four families (all distinct) went out to barbecue. They took 8 steaks and 10 kebabs. In how many ways is it possible to distribute the food to the families, while every family has to have at least one meal?

#### 1.3.1 Answer 4

We can distribute all meals in the following way:

| i(S) | i(K) | i(O) | r(S) | r(K) | r(O) |
|---|---|---|---|---|---|
| 4 | 0 | $\binom{0}{4} = 1$ | 4 | 10 | $\binom{4}{14} = 1001$ |
| 3 | 1 | $\binom{1}{4} = 4$ | 5 | 9 | $\binom{5}{14} = 2002$ |
| 2 | 2 | $\binom{2}{4} = 6$ | 6 | 8 | $\binom{6}{14} = 3003$ |
| 1 | 3 | $\binom{1}{4} = 4$ | 7 | 9 | $\binom{7}{14} = 3432$ |
| 0 | 4 | $\binom{0}{4} = 1$ | 8 | 8 | $\binom{8}{14} = 3003$ |
| | | $2^4 = 16$ | | | 43758 |

$i(X)$ stands for how many meals were distributed *initially*. $r(X)$ stands for *remaining* meals. $S$ stands for *steaks*, $K$ stands for *kebabs*, $O$ stands for *orderings*.

```
:- use_module(library(clpfd)).

feed_families([(S, K), (S1, K1), (S2, K2), (S3, K3)]) :-
    Steaks = [S, S1, S2, S3],
    Kebabs = [K, K1, K2, K3],
    Steaks ins 0..8, sum(Steaks, #=, 8),
    Kebabs ins 0..10, sum(Kebabs, #=, 10),
    append([Steaks, Kebabs], Meals),
    maplist(indomain, Meals),
    Total is (S + K) * (S1 + K1) * (S2 + K2) * (S3 + K3),
    Total > 0.

barbecue :-
    findall(X, feed_families(X), X),
    length(X, Result),
    format('$$~p$$', [Result]).
```

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