Assignment 11, Discrete Mathematics

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Contents

1 Problems

1.1 Problem 1

In all questions that follow, establish whether $x \in y$ or $x \subseteq y$.

1. $x = \{1, 2\}, y = \{1, 2, 3\}$.
2. $x = \{3\}, y = \{\{1\}, \{2\}, \{3\}\}$.
3. $x = \{1, 2\}, y = \{\{1, 2\}, 3\}$.
4. $x = \{1, 3\}, y = \{\{1, 2\}, 3\}$.
5. $x = \emptyset, y = \emptyset$.
6. $x = \{\emptyset\}, y = \{\emptyset\}$.
7. $x = \{1\}, y = \{1, 2\}$.
8. $x = \emptyset, y = P(\{1, 2, 3\})$.

1.1.1 Answer 1

1. $x \subseteq y$.
2. $x \in y$.
3. $x \in y$.
4. $x \notin y \wedge x \nsubseteq y$.
5. $x \subseteq y$.
6. $x \subseteq y$.
7. $x \subseteq y$.
8. $x \subseteq y$.

1.2 Problem 2

Prove or disprove:

1. $(A \setminus B) \setminus B = A \setminus B$.
2. $A \setminus (B \setminus A) = A$.
3. $A \subseteq P(A)$.

1.2.1 Answer 2

The statement is true. Suppose for contradiction that there is a $b \in A \setminus B$ s.t. $b \notin (A \setminus B) \setminus B$. Then, we know that $b \in A$ and $b \notin B$. But we assumed $b \notin A \vee b \in B$. This is a contradiction, hence $(A \setminus B) \setminus B = A \setminus B$. Additionally, it is easy to see that set subtraction is idempotent (i.e. if applied repeatedly, will yield the same result after first application).

1.2.2 Answer 3

The statement is true. Suppose for contradiction there was an $a \in A$, which is not in $A \setminus (B \setminus A)$. Then such a would have to be in A, but not in $(B \setminus A)$. I will now show that no elements of $(B \setminus A)$ is in A, thus subtracting this group would not remove any elements of A, i.e. $A \cap (B \setminus A) = \emptyset$. Suppose b was in A and in $(B \setminus A)$ at the same time, then it would have to be in A, but A is excluded from $(B \setminus A)$. This is in contradiction to the previous assumtion. Therefore $A \setminus (B \setminus A) = A$.

1.2.3 Answer 4

This statement isn't true in general. An example which disproves it would be an assignment: $A = \{1, 2\}$. $P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. $A \not\subseteq P(A)$.

1.3 Problem 3

Prove the following identity using $A \setminus B = A \cap B'$: $(A_1 \cup A_2) \setminus (B_1 \cap B_2) = (A_1 \cup B_1) \cup (A_1 \cup B_2) \cup (A_2 \cup B_1) \cup (A_2 \cup B_2)$.

1.3.1 Answer 5

The proof doesn't require discussion. It is accomplished by simple set-theoretical manipulations.

$$
\begin{aligned}
(A_1 \cup A_2) \setminus (B_1 \cap B_2) &\iff && \text{Given} \\
A_1 \setminus (B_1 \cap B_2) \cup A_2 \setminus (B_1 \cap B_2) &\iff && \text{Union over subtraction} \\
A_1 \cap (B_1 \cap B_2)' \cup A_2 \cap (B_1 \cap B_2)' &\iff && \text{Required identity} \\
A_1 \cap (B_1' \cup B_2') \cup A_2 \cap (B_1' \cup B_2') &\iff && \text{De Morgan's law} \\
(A_1 \cap B_1') \cup (A_1 \cap B_2') \cup (A_2 \cap B_1') \cup (A_2 \cap B_2') &\iff && \text{Union over subtraction} \\
(A_1 \setminus B_1) \cup (A_1 \setminus B_2) \cup (A_2 \setminus B_1) \cup (A_2 \setminus B_2) &\iff && \text{Required identity}
\end{aligned}
$$