

# Assignment 11, Infinitesimal Calculus

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# 1 Problems

## 1.1 Problem 1

1. Prove that for any natural number  $n$  it holds that:

$$4^n \geq \binom{2n}{n}.$$

2. Prove by induction, or in any other way, that for any natural number  $n$  it holds that:

$$\binom{2n}{n} \geq \frac{4^n}{2n+1}.$$

### 1.1.1 Answer 1

### 1.1.2 Answer 2

## 1.2 Problem 2

1. Given  $k, l \in \mathbb{N}$ , prove that  $a = k + l\sqrt{2}$  is irrational.
2. Prove that for every natural number  $n$  it holds that:

$$\sum_{i=0}^n \sqrt{2}^i$$

is irrational.

### 1.2.1 Answer 3

### 1.2.2 Answer 4

## 1.3 Problem 3

1. Given real numbers  $a$  and  $b$  prove that if

$$\frac{|a|}{2} > \left| b - \frac{a}{2} \right|,$$

then

$$|b - a| < |a|.$$

### 1.3.1 Answer 5

## 1.4 Problem 4

Given  $a, b, c \in \mathbb{R}$ ,

1. Prove that if  $a > 0$  and  $a + b > a + c$ , then  $b > c$ .
2. Prove that if  $a > 0$  and  $ab > ac$ , then  $b > c$ .
3. Prove that if  $|a| > |b|$  iff  $a^2 > b^2$ .
4. Prove that if  $b > c$  and  $|a - b| > |a - c|$ , then  $b > a$ .
5. Show (my means of example) that from  $b > c$  and  $b > a$  it doesn't follow that  $|a - b| > |a - c|$ .

1.4.1 Answer 6

1.4.2 Answer 7

1.4.3 Answer 8

1.4.4 Answer 9

1.4.5 Answer 10

1.5 Problem 5

Solve the equation:

$$\lfloor |x+1| - |x-1| \rfloor = x.$$

1.5.1 Answer 11

1.6 Problem 6

**Definition:** set  $A$  of real numbers is called **dense in interval**  $I$  if for every  $x, y \in I$  s.t.  $x < y$  there exists  $a \in A$  such that  $x < a < y$ .

1. Let  $A$  be dense in interval  $[0, 1]$ , prove that set  $B = \{na | a \in A, n \in \mathbb{N}\}$  is dense in interval  $[0, \infty)$ .
2. Let  $A = \mathbb{R}$ , prove that  $A$  isn't dense in  $I$  iff exists an open interval  $(x, y)$  in  $I$ , such that  $A \cap (x, y) = \emptyset$ .
3. Let  $A$  be the real numbers in interval  $[0, 1]$ , prove that the set  $C = \{\frac{a+1}{n^2} | a \in A, n \in \mathbb{N}\}$  isn't dense in  $[0, 1]$ .

**1.6.1 Answer 12**

**1.6.2 Answer 13**

**1.6.3 Answer 14**