Assignment 11, Infinitesimal Calculus

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Contents

1 Problems

1.1 Problem 1

1. Prove that for any natural number n it holds that:

$$4^n \geq \binom{2n}{n}.$$

2. Prove by induction, or in any other way, that for any natural number n it holds that:

$$\binom{2n}{n} \geq \frac{4^n}{2n+1}.$$

1.1.1 Answer 1

1.1.2 Answer 2

1.2 Problem 2

1. Given $k, l \in \mathbb{N}$, prove that $a = k + l\sqrt{2}$ is irrational.

2. Prove that for every natural number n it holds that:

$$\sum_{i=0}^{n} \sqrt{2}^i$$

is irrational.

1.2.1 Answer 3

1.2.2 Answer 4

1.3 Problem 3

1. Given real numbers a and b prove that if
$$\frac{|a|}{2} > \left| b - \frac{a}{2} \right|,$$
then
$$|b - a| < |a| .$$

1.3.1 Answer 5

1.4 Problem 4

Given $a, b, c \in \mathbb{R}$,

1. Prove that if $a > 0$ and $a + b > a + c$, then $b > c$.
2. Prove that if $a > 0$ and $ab > ac$, then $b > c$.
3. Prove that if $|a| > |b|$ iff $a^2 > b^2$.
4. Prove that if $b > c$ and $|a - b| > |a - c|$, then $b > a$.
5. Show (my means of example) that from $b > c$ and $b > a$ it doesn't follow that $|a - b| > |a - c|$.

1.4.1 Answer 6

1.4.2 Answer 7

1.4.3 Answer 8

1.4.4 Answer 9

1.4.5 Answer 10

1.5 Problem 5

Solve the equation:

$$\lfloor |x+1| - |x-1| \rfloor = x.$$

1.5.1 Answer 11

1.6 Problem 6

Definition: set A of real numbers is called **dense in interval** I if for every $x, y \in I$ s.t. $x < y$ there exists $a \in A$ such that $x < a < y$.

1. Let A be dense in interval $[0, 1]$, prove that set $B = \{na | a \in A, n \in \mathbb{N}\}$ is dense in interval $[0, \infty)$.
2. Let $A = \mathbb{R}$, prove that A isn't dense in I iff exists an open interval (x, y) in I, such that $A \cap (x, y) = \emptyset$.
3. Let A be the real numbers in interval $[0, 1]$, prove that the set $C = \{\frac{a+1}{n^2} | a \in A, n \in \mathbb{N}\}$ isn't dense in $[0, 1]$.

1.6.1 Answer 12

1.6.2 Answer 13

1.6.3 Answer 14