Assignment 14, Introduction to Statistics

Oleg Sivokon

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1 Problems

1.1 Problem 1

An ant travels 4 meters to the left with a chance 0.4 and 3 meters to the right with a chance 0.6 every day.

1. What is the expected value and the variance of ant's position to the right of starting point?
2. What is the probability of ant moving to the right for the first time on the fourth day?
3. What is the chance the ant will walk 9 meters to the right by the end of day ten?
4. What is the expected value and variance after ten days?

1.1.1 Answer 1

$E(x) = 0.4 \cdot (-4) + 0.6 \cdot 3 = 1.8 - 1.6 = 0.2$. In other words: the expected value is just a weighted sum of all possible outcomes.

1.1.2 Answer 2

In order to move right for the first time on the fourth day the ant needs to move left on three previous days, hence, using the product law: $P(x) = 0.4^3 = 0.064$.

1.1.3 Answer 3

The chance of moving 9 meters to the right is the chance of moving x days to the left and y days to the right s.t. $x \cdot (-4) + y \cdot 3 = 9$. Since we also know that the ant needs to spend exactly 10 days to get to its destination, we obtain a system of linear equations:

$$\begin{cases} -4x + 3y = 9 \\ \quad x + y = 10 \end{cases}$$

$$\begin{aligned} x &= 10 - y \\ 3y - 4(10 - y) &= 9 \\ 3y - 40 + 4y &= 9 \\ 7y &= 49 \\ y &= 7 \\ x &= 3\,. \end{aligned}$$

From this, the chance of getting 9 meters to the right by the end of the tenth day is $P(x) = 0.6^7 \cdot 0.4^3 = 1.791\,590\,4 \times 10^{-3}$.

1.1.4 Answer 4

The expected value by the day ten is just the sum of ten expected values of the movement carried out in one day, thus $E(10x) = 10E(x) = 10 \cdot 0.2 = 2$.

1.2 Problem 2

Yarden rolls a four-sided fair die until he rolls 1.

1. What is the chance that he will roll the die exactly four times?
2. What is the expected value of times he will roll the die?

Sharon, too, rolls the same die, but she will cease after rolling it just six times. What is the chance Sharon will not roll 1?

Alon rolls the same die. He wants to roll 1 three times, not necessarily in succession. What is the chance he will roll the dice eight times?

1.2.1 Answer 5

It's either Yarden rolls 1 or he rolls 2, 3, or 4, since the die is fair, the chance of rolling 1 is $\frac{1}{4}$, the chance of not rolling 1 is thus $\frac{3}{4}$. The chance of not rolling 1 for the first three times and rolling it afterwards is $\left(\frac{3}{4}\right)^3 \cdot \frac{1}{4} = \frac{27}{64 \cdot 4} = \frac{27}{256}$.

1.2.2 Answer 6

Since Yarden expects to roll 1 with a chance $\frac{1}{4}$, he expects to finish after $\frac{x}{4} = 1$ tries, i.e. on the fourth roll.

1.2.3 Answer 7

As discussed in 1.2.1, the chance of not rolling 1 is $\frac{3}{4}$, hence the chance of not rolling 1 even after eight trials is $\left(\frac{3}{4}\right)^8 = \frac{6561}{65536}$.

1.2.4 Answer 8

This is simply the chance of rolling 1 three times and rolling "not 1" five times, hence $\left(\frac{3}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^3 = \frac{234}{1024 \cdot 64} = \frac{243}{65536}$.

1.3 Problem 3

Yoel throws a ball at ten bottles standing next to each other. With probability of 0.1 he can scatter all of the bottles. He will miss entirely with a probability 0.2. The bottles are rearranged after each throw.

1. What is the chance Yoel will hit all the bottles in exactly 2 throws?

2. What is the chance that in two throws Yoel will hit all the bottles, in three throws he will miss entirely, and the rest he will neither miss, nor hit all the bottles?

3. What is the expected value and the variance of the number of time Yoel hitting all the bottles?

1.3.1 Answer 9

The chance of hitting all the bottles in two tries is just the sum of the chance of hitting all bottles on the first and on the second tries. This is so because the events are independent. Thus the total probability is $2 \cdot 0.1 = 0.2$.

1.3.2 Answer 10

Using the polynomial distribution, the chance of hitting all the bottles twice, missing three times and neither missing nor hitting all the bottles is $\frac{10!}{2!3!5!} \cdot 0.2^3 \cdot (1 - 0.1 - 0.2)^{10-2-3} = 0.01 \cdot 0.008 \cdot 0.16807 = 3.388\,291\,2 \times 10^{-1}$.

1.3.3 Answer 11

Using binomial distribution formula: $E(x) = pn$, expected value is $10 \cdot 0.1 = 1$. The variance is $V(x) = np(1 - p) = 1 \cdot 0.9 = 0.9$.

1.4 Problem 4

Yaron receives SMSs with expected value given by Poisson distribution of 5 messages per hour.

1. What is the chance that Yaron will receive 8 messages in two hours?

2. Yaron received 8 messages in two hours, what is the chance he received only one message in the first hour?
3. Yaron defines an hour as "bad" if during that hour he received at most one message. He randomly selects hours and verifies the record of number of messages received during that hour until he encounter the "bad" hour for the first time.
   - What is the chance he will have to look into four records?
   - What is the expected value of "not bad" hours?

1.4.1 Answer 12

Since this is Poisson distribution we assume message arrivals to be uniformly distributed along the time-line, hence $\lambda = 5 \cdot 2 = 10$. Thus, the cance is calculated using $p(X = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$. Hence $p(X) = e^{-8} \cdot \frac{10^8}{8!} = 8.320\,005\,652\,34 \times 10^{-1}$.

1.4.2 Answer 13

Similar to the question above, new $\lambda = 8/2 = 4$, the probability thus is $p(X) = e^{-4} \cdot \frac{4^1}{1!} = 7.326\,255\,555\,48 \times 10^{-2}$.

1.4.3 Answer 14

Since the expected value for Poisson distribution is equal to λ (5 in our case), Yaron should expect to receive one message per hour with the chance $p(X) = e^{-5} \cdot \frac{5^1}{1!} = 3.368\,973\,499\,54 \times 10^{-2}$. Since he looked in four records, the chance will be four times that high, i.e. $1.347\,589\,399\,82 \times 10^{-1}$.

1.4.4 Answer 15

In the previous answer we've calculated the expected value for "bad" hour, its complement is the expectation for the "not bad" hour, viz.

$9.663\,102\,650\,05 \times 10^{-1}$.

1.5 Problem 5

Omri has a 0.5 chance of having a meeting in development department. He also has a 0.2 chance of managerial meeting. While at the same time, there will be no meeting at all on that day with a chance of 0.4. Whether Omri has a managerial meeting is independent of whether he has to meet the developers.

1. Let X be the number of meetings Omri has to attend during the day, find probability function $p(X)$.
2. What is the chance that in 2 days out of 5 Omri will have to attend no meeting whatsoever?
3. Suppose Omri had to attend at least one meeting on each day of the week, what is the chance Omri had to attend two meetings on two days during the same week?

1.5.1 Answer 16

It is expedient that we first find the probability of both meetings happening on the same day. Let the even of having a managerial meeting be M, and the even of having a development meeting be D. The even of having no meeting whatsoever will be denoted by N, then $p(M \cap D) = p(N) + p(M) + p(D) - p(\Omega) = 0.4 + 0.2 + 0.5 - 1 = 0.1$. Since all probabilities must sum up to one, the probability of having exactly one meeting is $1 - 0.4 - 0.1$. Hence, the probability function is given by:

x	0	1	2
$p(X)$	0.4	0.5	0.1

1.5.2 Answer 17

The answer can be given using binomial distribution formula: $X \sim B(n, p)$, where n is the number of trials (5), and k is the number of successes (2):

$$\begin{aligned} \binom{n}{k} p^k (1-p)^{n-k} &= \binom{5}{2} 0.4^2 \cdot 0.6^3 \\ &= 10 \cdot 0.16 \cdot 0.216 \\ &= 0.3456 \, . \end{aligned}$$

1.5.3 Answer 18

A way to look at this problem could be as follows: a chance of having some meetings (either one or two) is $p(M \cup D) = 0.6$. The chance of having two meetings is $p(M \cap D) = 0.1$ (as we already calculated in 1.5.1. That is a chance of having two meetings, provided we know some meeting took place is one in six. If we try it five times, then our chances grow five-fold, viz. $\frac{1}{6} \cdot 5 = \frac{5}{6}$ for one meeting, and half of that for two meetings: $\frac{5}{12}$.