Assignment 13, Introduction to Statistics

Oleg Sivokon

<2015-03-27 Fri>

Contents

1 Problems

1.1 Problem 1

Given the probabilities of two basketball teams A and B of

winning $P(W|A) = 0.6$, $P(W|B) = 0.3$.

drawing $P(D) = 0.1$.

1. What is the chance the teams will play only two games to establish the winner?
2. The third game was a draw. What is the chance A wins in the first round?
3. Provided A wins the tournament, what is the chance that three games had been played?

1.1.1 Answer 1

The game can end in two turns if:

1. Teams draw the first time and then either one of them wins, the probability of this happening is $0.1 * (0.6 + 0.3) = 0.09$.
2. Alternatively, if A wins, either draw or repeated victory will do, the probability of this happening is $0.6 * (0.6 + 0.1) = 0.42$.
3. Finally, if B wins, the probability of ending early is: $0.3 * (0.3 + 0.1) = 0.12$

Summing up gives the total chances of the game ending in two rounds: $0.09 + 0.42 + 0.12 = 0.63$.

1.1.2 Answer 2

Since we already given that there was a third game, we will only concern ourselves with three different possibilities this could've happen:

1. Two draws.
2. A wins B wins.
3. B wins A wins.

It is easy to see (from commutativity of multiplication) that options (2) and (3) are equally likely, thus the chance of A being the first to win is $(1 - 0.1 \times 0.1)/2 = 0.495$.

1.1.3 Answer 3

Similarly to the first answer, A wins in two games if it either:

1. A wins, A wins, $p = 0.6 * 0.6 = 0.36$.
2. A wins, draw, $p = 0.6 * 0.1 = 0.06$.
3. draw, A wins, $p = 0.1 * 0.6 = 0.06$.

All together: 0.48. The chance A wins in three games is:

1. A wins, B wins A wins, $p = 0.6 * 0.3 * 0.6 = 0.108$.
2. B wins, A wins, A wins, $p = 0.3 * 0.6 * 0.6 = 0.108$.
3. draw, draw, A wins, $p = 0.1 * 0.1 * 0.6 = 0.006$.

All together: 0.222. Which makes the total chances of A to win $0.48 + 0.222 = 0.702$, thus the fraction of the tournaments won by A that lasted for three rounds is $\frac{0.222}{0.702} = 3.162\,393\,2 \times 10^{-1}$.

Below is the model testing the answers:

```
import random

def random_game():
    game = random.random()
    if game < 0.1: return 'draw'
    elif game < 0.4: return 'b'
    else: return 'a'

def finite_tournament():
    outcomes = []
    wins = ['drawa', 'drawb', 'aa', 'bb', 'adraw', 'bdraw']
    for g in range(3):
        outcomes.append(random_game())
        if ''.join(outcomes) in wins: break
    return outcomes

def game_ends_in_two(tries):
    two_rounds_games = 0
    for i in xrange(tries):
        game = finite_tournament()
        if len(game) == 2:
            two_rounds_games += 1
    return float(two_rounds_games) / tries

def a_wins_three_rounds_game(tries):
    games, a_wins = 0, 0
    for i in xrange(tries):
        game = finite_tournament()
        if len(game) > 2 and game[2] == 'draw':
            games += 1
            if game[0] == 'a': a_wins += 1
    return float(a_wins) / games

def at_least_three_games_played(times):
    good, games = 0, 0
    for i in xrange(times):
        game = finite_tournament()
        if game[-1] == 'a' or ''.join(game) == 'adraw':
            games += 1
            if len(game) > 2: good += 1
    return float(good) / games

print '''
+ Game ends in two turns: $\\num{%f}$
+ A wins first round: $\\num{%f}$
+ Three games were played: $\\num{%f}$
''' % (game_ends_in_two(100000),
       a_wins_three_rounds_game(100000),
       at_least_three_games_played(100000))
```

- Game ends in two turns: $6.313\,70 \times 10^{-1}$
- A wins first round: $4.864\,79 \times 10^{-1}$
- Three games were played: $3.181\,14 \times 10^{-1}$

1.2 Problem 2

Some of the plates produced in a factory can be defective in two different ways: with a chance of 0.15 there can be cracks in a plate and with a chance of 0.25 the coloring of the plate may not be uniform. The chance the plate will be defective is 0.35.

1. One plate was found to be defective, what is the chance of this plate to have cracks?
2. One plate was found to have cracks, what is the chance it will also have uneven coating?
3. A plate was found to have no cracks, what is the chance of the plate to be painted unevenly?

1.2.1 Answer 4

Total probability of having cracks is given to be 0.15, the probability of being defective is 0.35, thus the chance of a plate having cracks, provided it is defective is 0.15 in 0.35, i.e. $4.285\,714\,285\,71 \times 10^{-1}$.

1.2.2 Answer 5

The chance of a plate having both cracks and uneven coating is one in three. This is easy to see using the formula $P(A \cap B) = P(A) + P(B) - P(A \cup B)$. Substituting gives: $P(A \cap B) = 0.15 + 0.25 - 0.35 = 0.05$.

1.2.3 Answer 6

Of all plates 0.65 aren't defective, of the rest 0.1 have cracks, but are painted properly (recall the result obtained in 1.2.2.), thus the chance of a plate to have been painted unevenly is $1 - 0.65 - 0.1 = 0.25$.

1.3 Problem 3

Three coffee grinding machines produce all the coffee packed at a factory. Machine A grinds 0.55 of all the coffee, machine B grinds 0.3 and machine C grinds the remaining 0.15 of coffee. The coffee can be of fine or of a coarce grind. With a chance of 0.4, the machine A produces fine grinds of coffee. The machine B produces fine grinds with the 0.5 chance. It is also known that the chance of producing fine grind of coffee overall is 0.4.

1. A chosen pack of coffee was produced by machine C. What is the chance the coffee was ground finely?
2. A chosen pack of coffee was found to be of a fine grind. What is the chance it was produced by machine B?
3. Are events "the coffee is finely ground" and "the coffee was ground by the machine A" are independent?

1.3.1 Answer 7

The chance of a pack of coffee to be ground finely, given it came from machine C is the total chance of coffee being ground finely sans the chance it was ground finely and came from the machine A or B. Thus: $x = \frac{0.4-0.55\times0.4-0.3\times0.5}{0.15} = 0.2$.

1.3.2 Answer 8

The chance of a pack of coffee originating from machine B is the chance it was a finely ground coffe produced by machine B divided by the total chance it was finely ground: $x = \frac{0.3 \times 0.5}{0.4} = 0.375$.

1.3.3 Answer 9

These events are not independent. Independent events are such that their intersection is an empty set, but there are clearly packs of coffee produced by machine B, which are also finely ground *(exactly half of them)*.

1.4 Problem 4

Given a choice of three loaded coins, A with a chance of tails being $\frac{1}{3}$, B with the chance of tails being $\frac{1}{2}$ and C with the chance of tails being $\frac{2}{3}$. A random coin is selected.

1. What is the chance of tossing tails?
2. Same coin is tossed one more time, what is the chance it lends tails twice?
3. Given the coin landed tails twice, what is the chance the coin tossed is the fair one?
4. Given the coin landed tails twice, what is the chance it will lend tails again?

1.4.1 Answer 10

Since there is no preference towards any one of three coins, we will treat the chance of choosing one as being equally likely. Thus the chance of tossing tails is simply the average of the three: $x = \left(\frac{1}{3} + \frac{1}{2} + \frac{2}{3}\right) \times \frac{1}{3} = \frac{1}{2}$.

1.4.2 Answer 11

There is an equal chance to select all coins, therefore we will need to divide the total in three. For each coin the chance of throwing tails subsequently is given by the probability of throwing tails squared, this gives: $(\frac{2}{3})^2 \times (\frac{1}{2})^2 \times (\frac{1}{2})^2) \times \frac{1}{3} = \frac{29}{108}$.

1.4.3 Answer 12

We can calculate how each one of the coins contributes towards the total chance of tossing tails (obtained in the previous answer). This is given by $P(A) = \frac{1}{9}$, $P(C) = \frac{4}{9}$ and $P(B) = \frac{1}{4}$. The probability of choosing the fair coin given it landed tails twice is the $\frac{P(B)}{P(A)+P(B)+P(C)} = \frac{1}{4} \times \frac{36}{29} = \frac{9}{29}$.

1.4.4 Answer 13

The fact that the coin landed tails twice shifts the probabilities of that coin towards being the C coin. We already calculated the total probability of a coin landing tails after two tosses (in 1.4.2), now we need to find how much every coin contributes to that probability and multiply that with the probability of each coin landing on tails. To get single contributions we will multiply each contribution with the total's inverse: $\frac{108}{29}$. $\frac{2*108}{3*29} + \frac{108}{2*29} * \frac{108}{3*29} = \frac{162}{29}$.

Below is the code that models the coin tosses:

```
import random

def select_coin():
    tails = 0
    chance = random.random() * 3
    if chance <= 1: tails = 1.0 / 3
    elif chance <= 2: tails = 0.5
    else: tails = 2.0 / 3
    return tails

def chance_of_tails(times):
    return sum(select_coin() for x in xrange(times)) / times
```

```
def chance_two_tails(times):
    return sum(select_coin() ** 2
               for x in xrange(times)) / times

def chance_of_fair_two_tails(times):
    fair, total = 0.0, 0.0
    for x in xrange(times):
        coin = select_coin() ** 2
        if coin == 0.25: fair += coin
        total += coin
    return fair / total

def chance_third_tails(times):
    two_thirds, half, third, total = 0.0, 0.0, 0.0, 0.0
    for x in xrange(times):
        coin = select_coin() ** 2
        if coin == 0.25: half += coin
        elif coin < 0.25: third += coin
        else: two_thirds += coin
        total += coin
    two_thirds /= total
    half /= total
    third /= total
    return two_thirds * 2 / 3 + half / 2 + third / 3

print '''
+ Probability of tossing tails: $\\num{%s}$
+ Probability of tossing two tails: $\\num{%s}$
+ Chance of fair coin given two tails: $\\num{%s}$
+ Chance third toss will be tails: $\\num{%s}$
''' % tuple(f(100000)
            for f in [chance_of_tails,
                      chance_two_tails,
                      chance_of_fair_two_tails,
                      chance_third_tails])
```

- Probability of tossing tails: $4.992\,05 \times 10^{-1}$
- Probability of tossing two tails: $2.689\,519\,444\,44 \times 10^{-1}$
- Chance of fair coin given two tails: $3.131\,517\,642\,1 \times 10^{-1}$
- Chance third toss will be tails: $5.684\,453\,294\,13 \times 10^{-1}$