Assignment 13, Introduction to Statistics

Oleg Sivokon

<2015-03-27 Fri>

Contents

1 Problems

1.1 Problem 1

Given the probabilities of two basketball teams A and B of

winning $P(W|A) = 0.6$, $P(W|B) = 0.3$.

drawing $P(D) = 0.1$.

1. What is the chance the teams will play only two games to establish the winner?
2. The third game was a draw. What is the chance A wins in the first round?
3. Provided A wins the tournament, what is the chance that at least three games had been played?

1.1.1 Answer 1

The way to look at this problem is equivalent to counting all game pairs where either A wins twice or B wins twice or either one of the games is a draw, and dividing it into all possible outcomes (the Ω) of two consequent games. Since the probabilities of winning are independent, using the product law, A will win with probability 0.6×0.6 and B will win with the probability 0.3×0.3. Whereas the probability of two games finishing with any result is trivially 1. Since the probabilities of both teams winning subsequent games are disjoint, we sum them, up, and this gives us $0.1 + 0.36 + 0.09 - 0.01 = 0.54$ chance of tournament ending after playing just two games. (0.01 was added to compensate for the case when teams draw two times).

1.1.2 Answer 2

Since we already given that there was a third game, we will only concern ourselves with three different possibilities this could've happen:

1. Two draws.
2. A wins B wins.
3. B wins A wins.

It is easy to see (from commutativity of multiplication) that options (2) and (3) are equally likely, thus the chance of A being the first to win is $(1 - 0.1 \times 0.1)/2 = 0.495$.

1.1.3 Answer 3

The chance of A not winning in just two games is the chance of A winning two subsequent games, or winning a game and drawing. Since the probability of A is a given, substracting from it the chance of A winning in under three games will give us the chance of A winning in three games or more: $0.6 - 0.6 \times 0.6 + 2 \times 0.1 \times 0.6 = 0.36$.

Below is the model testing the answers:

```
import random

def random_game():
    game = random.random()
    if game < 0.1: return 'draw'
    elif game < 0.4: return 'b'
    else: return 'a'

def tournament():
    winner = None
```

```
    previous = None
    turns = 0
    game = []
    while not winner:
        candidate = random_game()
        turns += 1
        if candidate == 'a' and previous in ['a', 'draw']:
            winner = 'a'
        elif candidate == 'b' and previous in ['b', 'draw']:
            winner = 'b'
        previous = candidate
        game.append(candidate)
    return winner, turns, game

def game_ends_in_two(tries):
    two_rounds_games = 0
    for i in xrange(tries):
        winner, turns, game = tournament()
        if turns == 2:
            two_rounds_games += 1
    return float(two_rounds_games) / tries

def a_wins_three_rounds_game(tries):
    games = 0
    a_wins = 0
    for i in xrange(tries):
        winner, turns, game = tournament()
        if len(game) > 2 and game[2] == 'draw':
            games += 1
            if game[0] == 'a': a_wins += 1
    return float(a_wins) / games

def chance_player_wins(player, times):
    wins = 0
    for i in xrange(times):
        winner, turns, game = tournament()
        if winner == player:
            wins += 1
    return float(wins) / times
```

```
def chance_game_won_by(player, times):
    wins = 0
    for i in xrange(times):
        if player == random_game():
            wins += 1
    return float(wins) / times

def at_least_three_games_played(times):
    good = 0
    games = 0
    for i in xrange(times):
        winner, turns, game = tournament()
        if winner == 'a':
            games += 1
            if len(game) > 2:
                good += 1
    return float(good) / games
```

1.2 Problem 2

Some of the plates produced in a factory can be defective in two different ways: with a chance of 0.15 there can be cracks in a plate and with a chance of 0.25 the coloring of the plate may not be uniform. The chance the plate will be defective is 0.35.

1. One plate was found to be defective, what is the chance of this plate to have cracks?
2. One plate was found to have cracks, what is the chance it will also have uneven coating?
3. A plate was found to have no cracks, what is the chance of the plate to be painted unevenly?

1.2.1 Answer 4

Total probability of having cracks is given to be 0.15, the probability of being defective is 0.35, thus the chance of a plate having cracks, provided it is defective is 0.15 in 0.35, i.e. $4.285\,714\,285\,71 \times 10^{-1}$.

1.2.2 Answer 5

The chance of a plate having both cracks and uneven coating is one in three. This is easy to see using the formula $P(A \cap B) = P(A) + P(B) - P(A \cup B)$. Substituting gives: $P(A \cap B) = 0.15 + 0.25 - 0.35 = 0.05$.

1.2.3 Answer 6

Of all plates 0.65 aren't defective, of the rest 0.1 have cracks, but are painted properly (recall the result obtained in 1.2.2.), thus the chance of a plate to have been painted unevenly is $1 - 0.65 - 0.1 = 0.25$.

1.3 Problem 3

Three coffee grinding machines produce all the coffee packed at a factory. Machine A grinds 0.55 of all the coffee, machine B grinds 0.3 and machine C grinds the remaining 0.15 of coffee. The coffee can be of fine or of a coarce grind. With a chance of 0.4, the machine A produces fine grinds of coffee. The machine B produces fine grinds with the 0.5 chance. It is also known that the chance of producing fine grind of coffee overall is 0.4.

1. A chosen pack of coffee was produced by machine C. What is the chance the coffee was ground finely?
2. A chosen pack of coffee was found to be of a fine grind. What is the chance it was produced by machine B?
3. Are events "the coffee is finely ground" and "the coffee was ground by the machine A" are independent?

1.3.1 Answer 7

The chance of a pack of coffee to be ground finely, given it came from machine C is the total chance of coffee being ground finely sans the chance it was ground finely and came from the machine A or B. Thus: $x = \frac{0.4-0.55\times0.4-0.3\times0.5}{0.15} = 0.2$.

1.3.2 Answer 8

The chance of a pack of coffee originating from machine B is the chance it was a finely ground coffe produced by machine B divided by the total chance it was finely ground: $x = \frac{0.3\times0.5}{0.4} = 0.375$.

1.3.3 Answer 9

These events are not independent. Independent events are such that their intersection is an empty set, but there are clearly packs of coffee produced by machine B, which are also finely ground *(exactly half of them)*.

1.4 Problem 4

Given a choice of three loaded coins, A with a chance of tails being $\frac{1}{3}$, B with the chance of tails being $\frac{1}{2}$ and C with the chance of tails being $\frac{2}{3}$. A random coin is selected.

1. What is the chance of tossing tails?
2. Same coin is tossed one more time, what is the chance it lends tails twice?
3. Given the coin landed tails twice, what is the chance the coin tossed is the fair one?
4. Given the coin landed tails twice, what is the chance it will lend tails again?

1.4.1 Answer 10

Since there is no preference towards any one of three coins, we will treat the chance of choosing one as being equally likely. Thus the chance of tossing tails is simply the average of the three: $x = \left(\frac{1}{3} + \frac{1}{2} + \frac{2}{3}\right) \times \frac{1}{3} = \frac{1}{2}$.

1.4.2 Answer 11

Using the product law and the previous answer gives $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

1.4.3 Answer 12

We can calculate how each one of the coins contributes towards the total chance of tossing tails (obtained in the previous answer). This is given by $P(A) = \frac{1}{9}$, $P(C) = \frac{4}{9}$ and $P(B) = \frac{1}{4}$. The probability of choosing the fair coin given it landed tails twice is the $\frac{P(B)}{P(A)+P(B)+P(C)} = \frac{1}{4} \times \frac{36}{29} = \frac{9}{29}$.

1.4.4 Answer 13

The chance of coin landing whichever way is independent of how it landed before, so it will be the same as the chance of a coin landing tails, already given in the 1.4.1, viz. $\frac{1}{2}$.