Assignment 16, Linear Algebra 1

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1 Problems

1.1 Problem 1

Given the matrix:

$$A = \begin{pmatrix} 0 & a & 1 \\ a & 0 & -1 \\ 0 & 0 & a \end{pmatrix}$$

a being a real number.

1. What are the values of a that make this matrix diagonalisable?
2. Put $a = 1$, find diagonal matrix D and invertible matrix P such that $D = A^{-1}PA$.

1.1.1 Answers 1

1.2 Problem 2

1. Prove that there doesn't exist a 3×3 matrix with a characteristic polynomial $p(x) = x^7 - x^5 + x^3$.
2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with characteristic polynomial $p(x) = x^2 + 2x - 3$.
   (a) Porve that linear mapping $2T + I$ is an isomorphism.
   (b) What is the characteristic polynomial of T^3.
3. Let A be a 5×5 matrix such that $\mathrm{tr}(A) = 0$ and $\rho(A) = 1$. Find all eigenvalues of A.

1.2.1 Answer 4

1.3 Problem 3

Prove or disprove (by providing an example) each one of the following:

1. If two matrices A and B share characteristic polynomial, then their rank is the same.
2. Put $m > 1$, m being a real number. If $A^m = 0$, then 0 is the **only** eigenvalue of A.
3. Matrices: $A = \begin{pmatrix} 2 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are similar.
4. Matrices: $C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ are similar.

1.3.1 Answer 5

1.4 Problem 4

Let $\vec{u}, \vec{v}$ be vectors distinct from $\vec{0}$ in $\mathbb{R}^n$. Given that $\vec{u}$ is orthogonal to $\vec{v}$ and $\|u\| = \|v\|$. Find all values of the real number a such that $\vec{u}a + \vec{v}$ is orthogonal to $\vec{v}a + \vec{u}$.

1.5 Problem 5

Let U_1 and U_2 be subspaces of $\mathbb{R}^n$.

1. Let $\mathbb{R}^n = U_1 \oplus U_2$, prove that $U_1^\perp \cap U_2^\perp = \{\vec{0}\}$ and $\mathbb{R}^n = U_1^\perp \oplus U_2^\perp$.
2. Let $U_1 + U_2 = \mathbb{R}^n$, does it follow that $\mathbb{R}^n = U_1^\perp + U_2^\perp$?