Assignment 14, Linear Algebra 1

Oleg Sivokon

<2016-05-20 Fri>

Contents

1 Problems

1.1 Problem 1

Given f, g, h are functions from $\mathbb{R}$ to $\mathbb{R}$, check that all of them are linearly independent when:

1. $f(x) = \sin x,\ g(x) = \cos x,\ h(x) = x\cos x$.
2. $f(x) = x(x-1),\ g(x) = x(x-2),\ h(x) = (x-1)(x-2)$.
3. $f(x) = \sin^2 x,\ g(x) = \cos^2 x,\ h(x) = 3$.

1.1.1 Answer 1

1.1.2 Answer 2

1.1.3 Answer 3

1.2 Problem 2

Given the following subsets of $\mathbb{R}^4$:

$$U = \{(x, y, z, t) \in \mathbb{R}^4 \mid x - y + z = 0 \land x - y - 2t = 0\}$$
$$W = \mathrm{Sp}\{(1, 0, 1, 1), (0, 1, 0, -1), (1, 0, 1, 0)\}$$

1. Prove that U and W are subspaces of $\mathbb{R}^4$.
2. Find basis for U, W and $U + W$.
3. Find basis for $U \cap W$.
4. Find subspace T of $\mathbb{R}^4$ s.t. $U \oplus T = \mathbb{R}^4$.

1.2.1 Answer 4

1.2.2 Answer 5

1.2.3 Answer 6

1.2.4 Answer 7

1.3 Problem 3

Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ and $\vec{w}$ be vectors in linear space V. Given $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is linearly independent and that $\vec{w} \notin \mathrm{Sp}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$, prove that $\vec{v_1} \notin \mathrm{Sp}\{\vec{v}_1 + \vec{w}, \vec{v}_2 + \vec{w}, \ldots, \vec{v}_k + \vec{w}\}$.

1.3.1 Answer 8

1.4 Problem 4

Let U and W be distinct linear subspaces of $\mathbb{R}^5$ of dimension 3. Suppose $(2, 1, 0, 1), (1, 0, 1, 1) \in U \cap W$, what is the dimension of $U + W$?

1.4.1 Answer 9

1.5 Problem 5

Let A and B be square matrices of size n, $n \geq 2$. Suppose A and B are of the rank 1,

1. what are the possible ranks of $A + B$?
2. What is the possible rank of $A + B$ when they both are of rank 2?
3. Prove that it is possible to write any matrix of rank 2 as a sum of matrices of rank 1.

1.5.1 Answer 10

1.6 Problem 6

Given bases $B = (\vec{u}_1, \vec{u}_2, \vec{u}_3)$ and $C = (\vec{v}_1, \vec{v}_2, \vec{v}_3)$ both in $\mathbb{R}^3$ s.t.

$$\begin{aligned}
\vec{u}_1 &= (2, 1, 1) \\
\vec{u}_2 &= (2, -1, 1) \\
\vec{u}_3 &= (1, 2, 1) \\
\vec{v}_1 &= (3, 1, -5) \\
\vec{v}_2 &= (1, 1, -3) \\
\vec{v}_3 &= (-1, 0, 2)
\end{aligned}$$

1. Write the matrix of change of basis from B to C and its inverse.
2. Compute the coordinate vector $[w]_B$ where $\vec{w} = (-5, 8, -5)$.
3. Similarly, compute $[w]_C$.

1.6.1 Answer 11

1.6.2 Answer 12

1.6.3 Answer 13