# Assignment 15, Linear Algebra 1

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## Contents

# 1 Problems

## 1.1 Problem 1

For each of the given transformations check if it is linear:

1. $T : \mathbb{R}_2[x] \to \mathbb{R}_4[x]$ defined as $T(f(x)) = (x^3 - x)f(x^2)$.
2. $T : \mathbb{M}^{\mathbb{R}}_{n\times n} \to \mathbb{M}^{\mathbb{R}}_{n\times n}$ defined as $T(X) = AXA$ for some $A \in \mathbb{M}^{\mathbb{R}}_{n\times n}$.

### 1.1.1 Answer 1

### 1.1.2 Answer 2

## 1.2 Problem 2

1. Does there exist an isomorphism $T : \mathbb{R}_3[x] \to \mathbb{R}^3$ for which $T(x^2 + 2x) = (1, 2, 1)$, $T(x + 1) = (0, 1, 1)$, $T(x^2 - 2) = (1, 0, -1)$?
2. Given linear space $V$ and linear transformations $S, T : V \to V$, prove that, if $V$ is finite-dimensional and $\ker S = \{0\}$, then $\operatorname{im} TS = \operatorname{im} S$.

### 1.2.1 Answer 3

### 1.2.2 Answer 4

## 1.3 Problem 3

Let $T : \mathbb{R}^5 \to \mathbb{R}^5$ be a linear transformation s.t. $T^2 = 0$.

1. Prove that $\operatorname{im} T \subseteq \ker T$.
2. What are the possible values for the dimension of $\ker T$?
3. Let $U$ be a subspace of $\mathbb{R}^5$ s.t. $\dim U = 3$, prove that $U \cap \ker T \neq \{0\}$.

#### 1.3.1 Answer 5

#### 1.3.2 Answer 6

#### 1.3.3 Answer 7

### 1.4 Problem 4

Let $a \in \mathbb{R}$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Let $B = ((1,0,0),(1,1,0),(1,1,1))$ be a basis in $\mathbb{R}^3$. Then $T$ with respect to the basis $B$ is given by

$$[T]_B = \begin{bmatrix} a & 1-1 & 0 \\ a & 2a & 2a+2 \\ a+1 & a+1 & 2a+2 \end{bmatrix}.$$

Also, $(2,2,2) \in \ker T$.

1. Find $a$ and compute $T(x,y,z)$ for any $(x,y,z) \in \mathbb{R}^3$.
2. Find the matrix representing $T$ with respect to standard basis.
3. Find basis for $\operatorname{im} T$ and $\ker T$.

#### 1.4.1 Answer 8

#### 1.4.2 Answer 9

#### 1.4.3 Answer 10

### 1.5 Problem 5

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $T(x,y) = (x+2y, y)$

1. Find the basis $B$ of $\mathbb{R}^2$ s.t.
$$[T]_B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$
2. Prove that
$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

### 1.5.1 Answer 11

### 1.5.2 Answer 12

## 1.6 Problem 6

Let $a, b, c \in \mathbb{R}$, prove that $A \sim B \sim C$.

$$A = \begin{bmatrix} b & c & a \\ c & a & b \\ a & b & c \end{bmatrix}, B = \begin{bmatrix} c & a & b \\ a & b & c \\ b & c & a \end{bmatrix}, C = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}.$$

### 1.6.1 Answer 13