Assignment 13, Linear Algebra 1

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Contents

1 Problems

1.1 Problem 1

1. Find all solutions of $z^3 + 3i\overline{z} = 0$.
2. Let z_1, z_2 be complex numbers. Prove that unless $z_1 z_2 = 1$ and $|z_1| = |z_2| = 1$, then $\frac{z_1+z_2}{1+z_1 z_2}$ is a real number.

1.2 Problem 2

Let $\mathbb{Q}$ denote the field of rational numbers. And K defined as follows:

$$K = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Q} \right\}.$$

Is K a field under matrix addition and multiplication?

1.3 Problem 3

Verify that V is a vectors space over field $\mathbf{F}$:

1. $\mathbf{F} = \mathbb{C}, V = \mathbb{M}^{\mathbb{C}}_{n\times n}$ and addition defined to be the rational addition, while multiplication is defined to be $\lambda \times A = |\lambda| \times A$.
2. $\mathbf{F} = \mathbb{R}, V = \{(x, y) \in \mathbb{R}^2 \mid y = 2x\}$, with addition being the addition of $\mathbb{R}^2$ and multiplication $\lambda \times (x, y) = (\lambda x, 0)$.

1.4 Problem 4

Given sets:

1. $K = \{(x, y, z, t) \in \mathbb{R}^4 \mid x + y - z + t = 0 \wedge 2x + y + z - 3t = 0\}$.
2. $L = \{f \in V \mid f\left(\frac{1}{2}\right) > f(2)\}$, V is the vector space of all real-valued functions.
3. $M = \{p(x) \in \mathbb{R}^4[x] \mid p(-3) = 0\}$.

4. $R = \{(x, y) \in \mathbb{R}^3 \mid x^2 + y^2 = 0\}$.
5. $R = \{(x, y) \in \mathbb{R}^3 \mid x^2 - y^2 = 0\}$.

Fore each of sets given, assert them being vector spaces. In case they are, prove this in two different ways.

1.5 Problem 5

Given vector space V and $\vec{v}_1, \vec{v}_2, \vec{v}_3$ distinct vectors prove or disprove:

1. If $\mathrm{Sp}\{\vec{v}_1, \vec{v}_2\} = \mathrm{Sp}\{\vec{v}_1, \vec{v}_3\}$, then $\vec{v}_2$ is a multiple of $\vec{v}_3$.
2. If $\vec{v}_1 - 2\vec{v}_2 + \vec{v}_3 = \vec{0}$, then $\mathrm{Sp}\{\vec{v}_1, \vec{v}_2\} = \mathrm{Sp}\{\vec{v}_1, \vec{v}_3\}$.
3. If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent, then $\mathrm{Sp}\{\vec{v}_1, \vec{v}_2\} = \mathrm{Sp}\{\vec{v}_1 + \vec{v}_3, \vec{v}_3 + \vec{v}_3\}$.

1.6 Problem 6

Given following subspaces of $\mathbb{R}^4$: $U = \mathrm{Sp}\{(1, 1, 2), (2, 2, 1)\}$ and $W = \mathrm{Sp}\{(1, 3, 4), (2, 5, 1)\}$, find spanning set of $U \cap W$.