Simulating Quantum Algorithms for Indoor Localization

Quantum Computing Workshop

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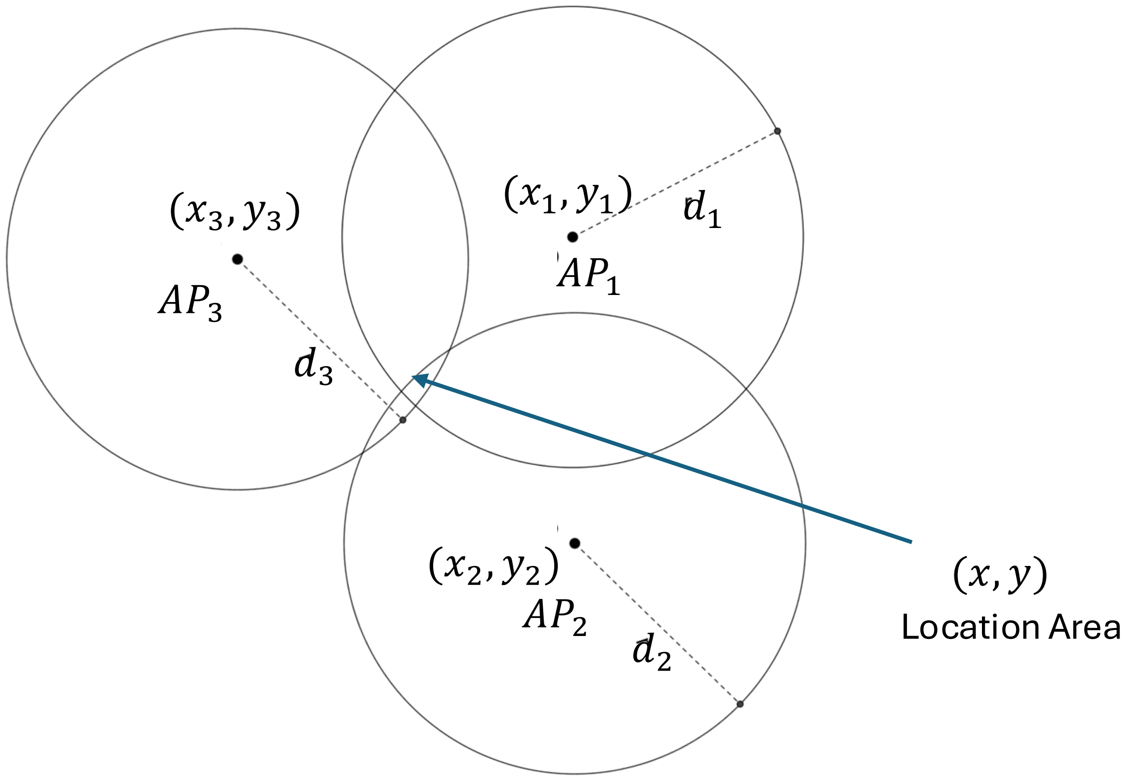
Introduction

Indoor Localization

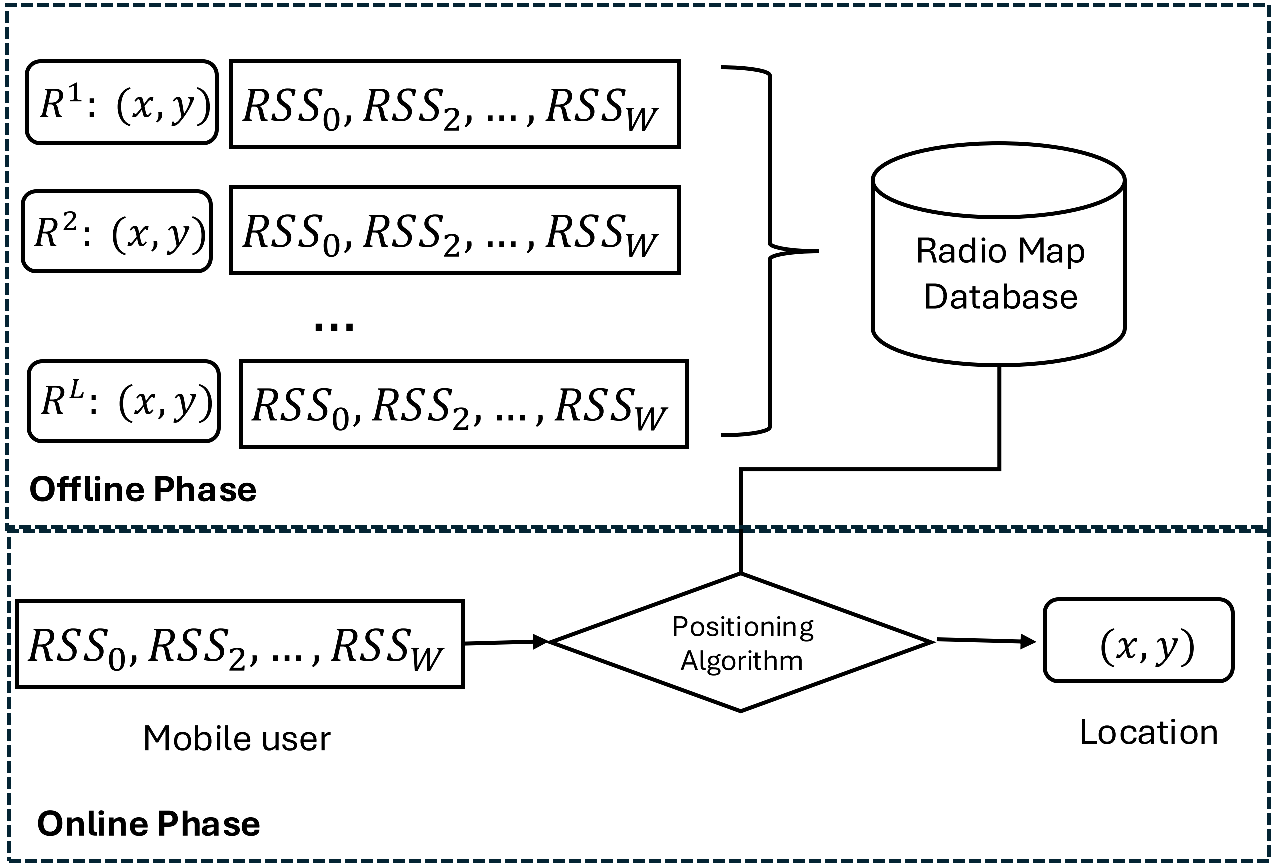
- Identifying and tracking the position of objects inside buildings where Global Positioning System (GPS) signals are weak.
  - The position of object is estimated based on the received signal strength indicator (RSSI) across different Access Points (APs e.g., WiFi).
  - Navigation and Wayfinding
  - Asset Tracking
  - Emergency Response
  - Retail Insights
  - Smart Building Management

Introduction

RSS Ranging w/ Trilateration and RSS Fingerprinting



$$d_1^2 = (x_1 - x)^2 + (y_1 - y)^2$$
$$d_2^2 = (x_2 - x)^2 + (y_2 - y)^2$$
$$d_3^2 = (x_3 - x)^2 + (y_3 - y)^2$$



Introduction

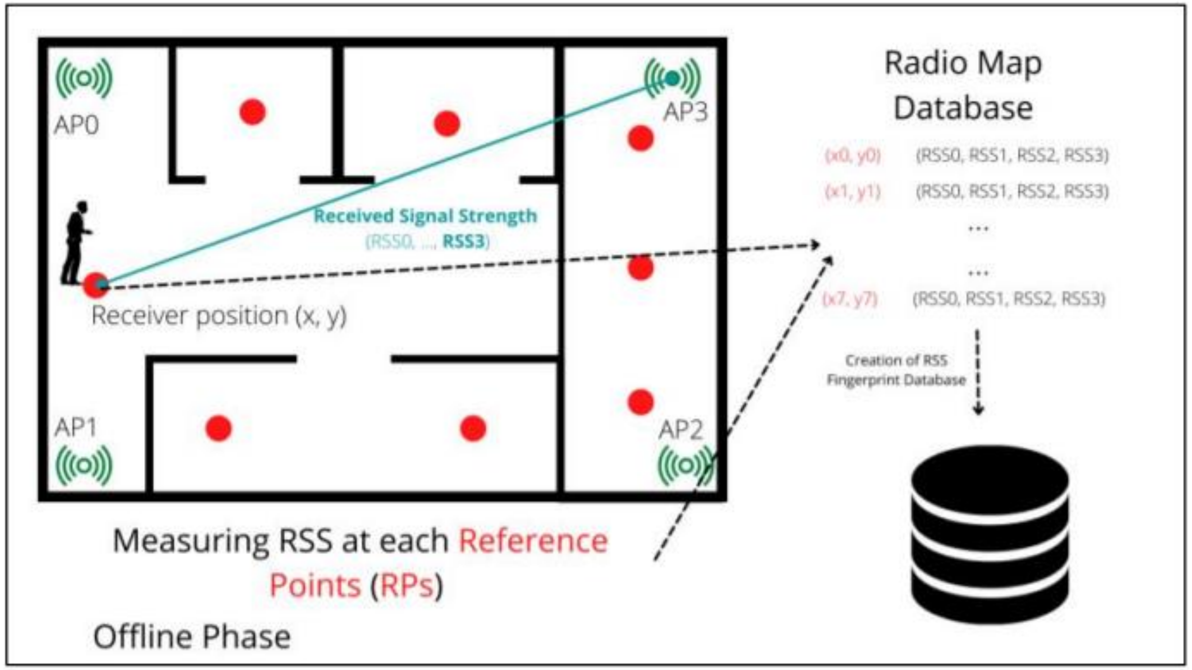
Indoor Localization

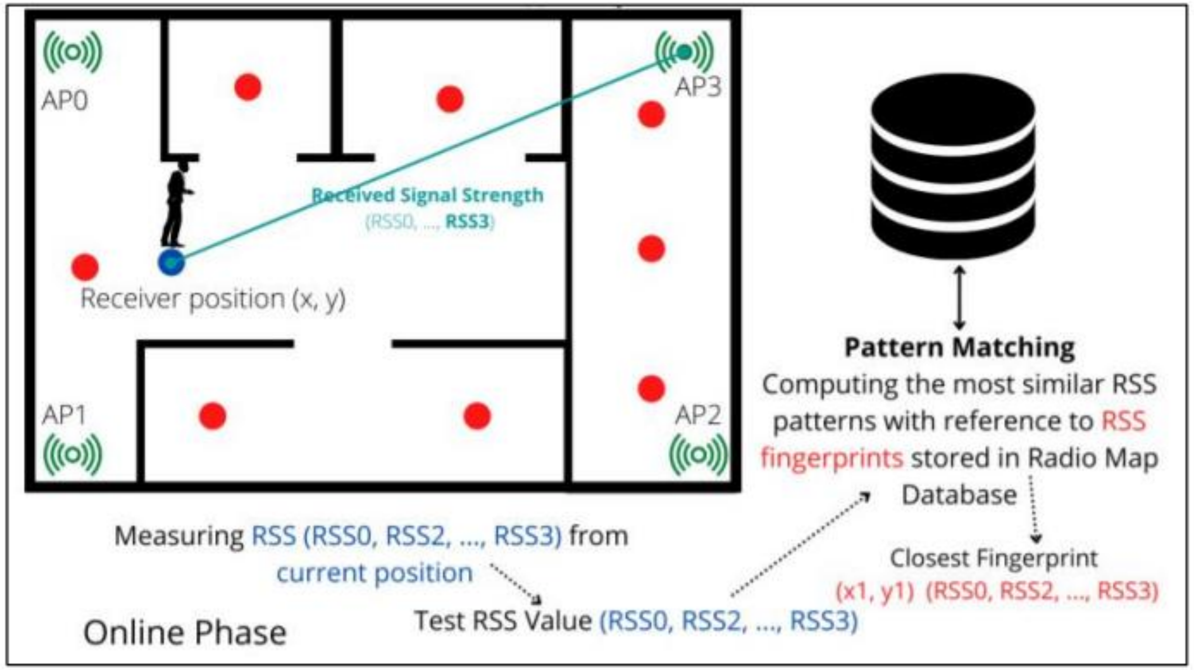
- RSS fingerprinting is more favored than trilateration because trilateration is sensitive to environmental factors like fluctuations in signals, obstacles which affect the accuracy of distance estimation
- RSS trilateration requires careful calibration of path loss model

Introduction

Signal (Wi-Fi) Fingerprinting for Indoor Localization

- Offline or Calibration Phase
- Online or Tracking Phase





Introduction

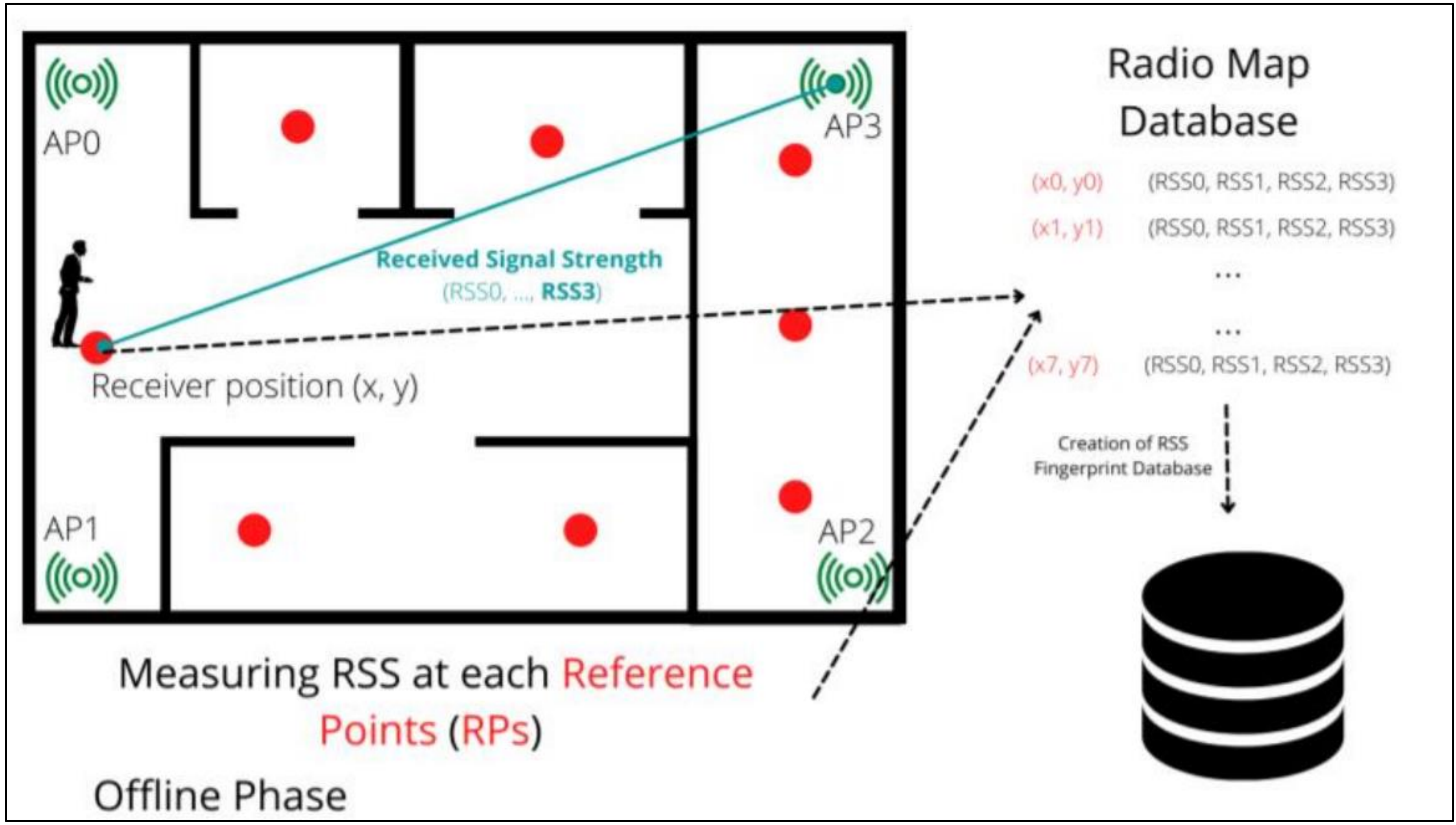
Offline Phase

- 4 access points (Green)
- 7 reference locations (Red)
- For each position (x, y)

we record the RSS vector

- We collect several

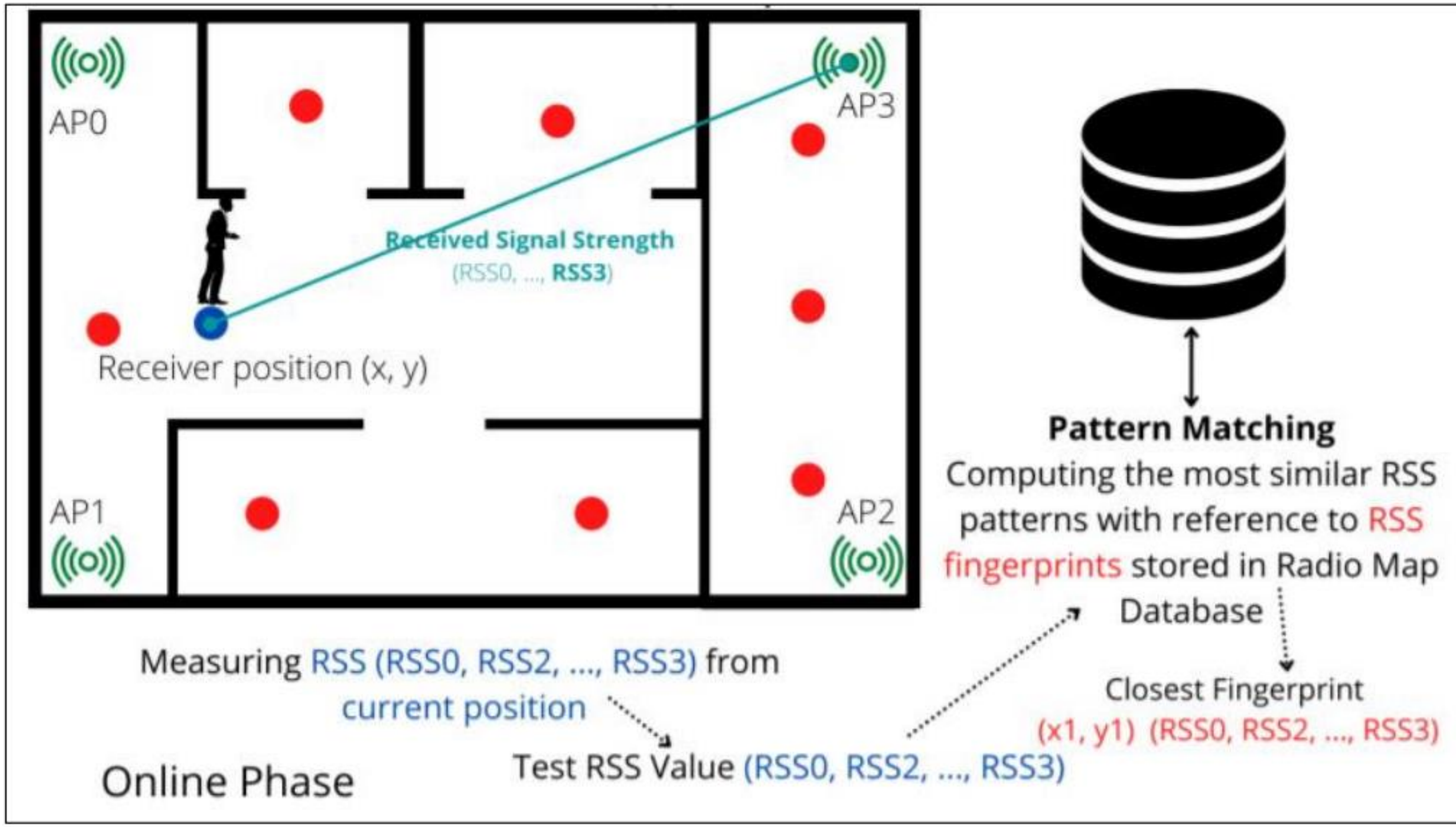
fingerprints at different reference locations to build the fingerprint database or the radio map.



Introduction

Online Phase

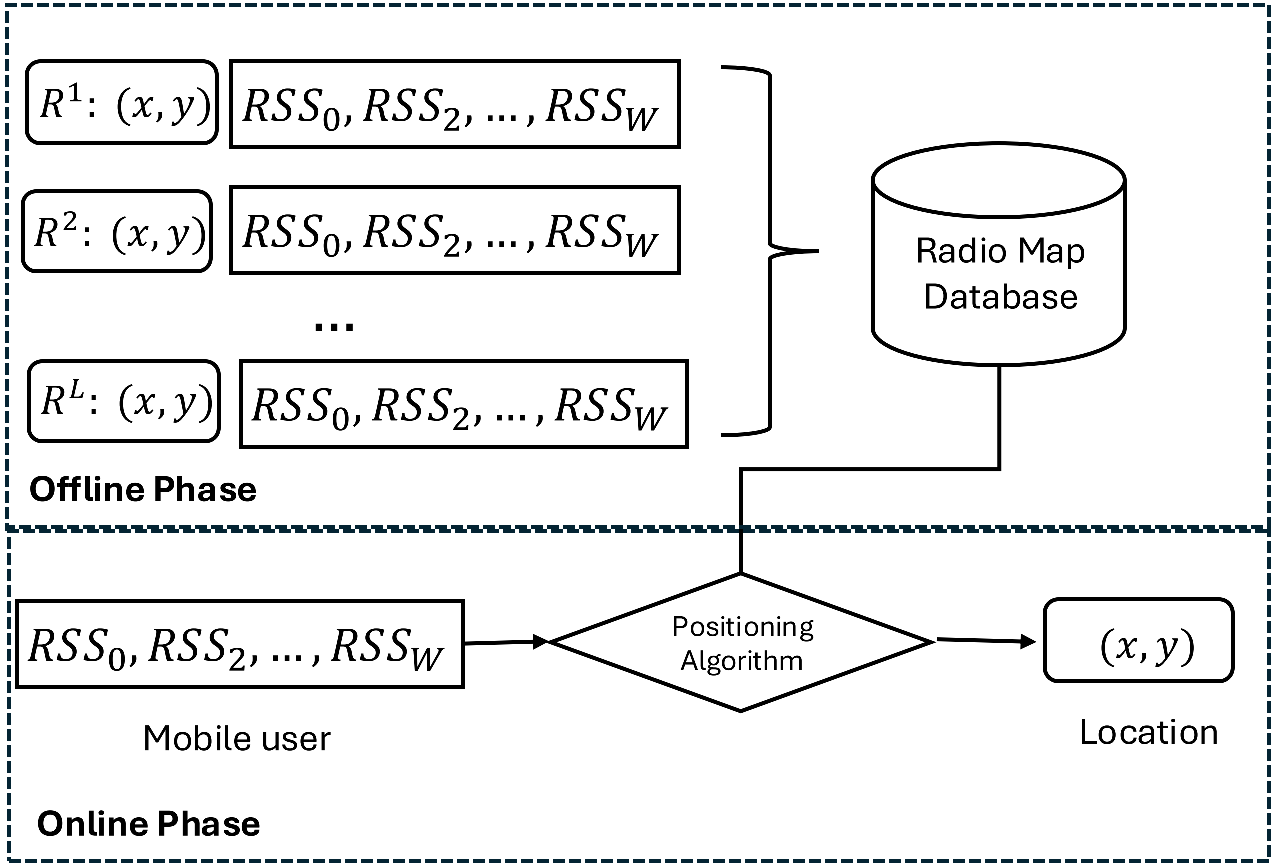
- Given receiver's RSS vector, we compare it to all fingerprint vectors and we get the closest Fingerprint based on a vector similarity measure.
- The label of the most similar vector is the estimated position.



Introduction

Indoor Localization: How to Identify Closest Match

- Compare the mobile user's RSS vector to each of fingerprint vectors in the database
- Compute vector similarity of $RSSI_u$ and $RSSI_l$
  - Distance-based
  - Cosine similarity
  - Other vector-based similarity measures
- Finding the closest match takes $O(LW)$, where
  - L: number of fingerprints of reference locations in the radio map database
  - W: Number of Access Points (WiFi) sending signals to the mobile user
- It is possible to find the closest match by preparing the quantum states of the vectors and apply swap test circuit, reducing the problem to $O(L \log_2 W)$



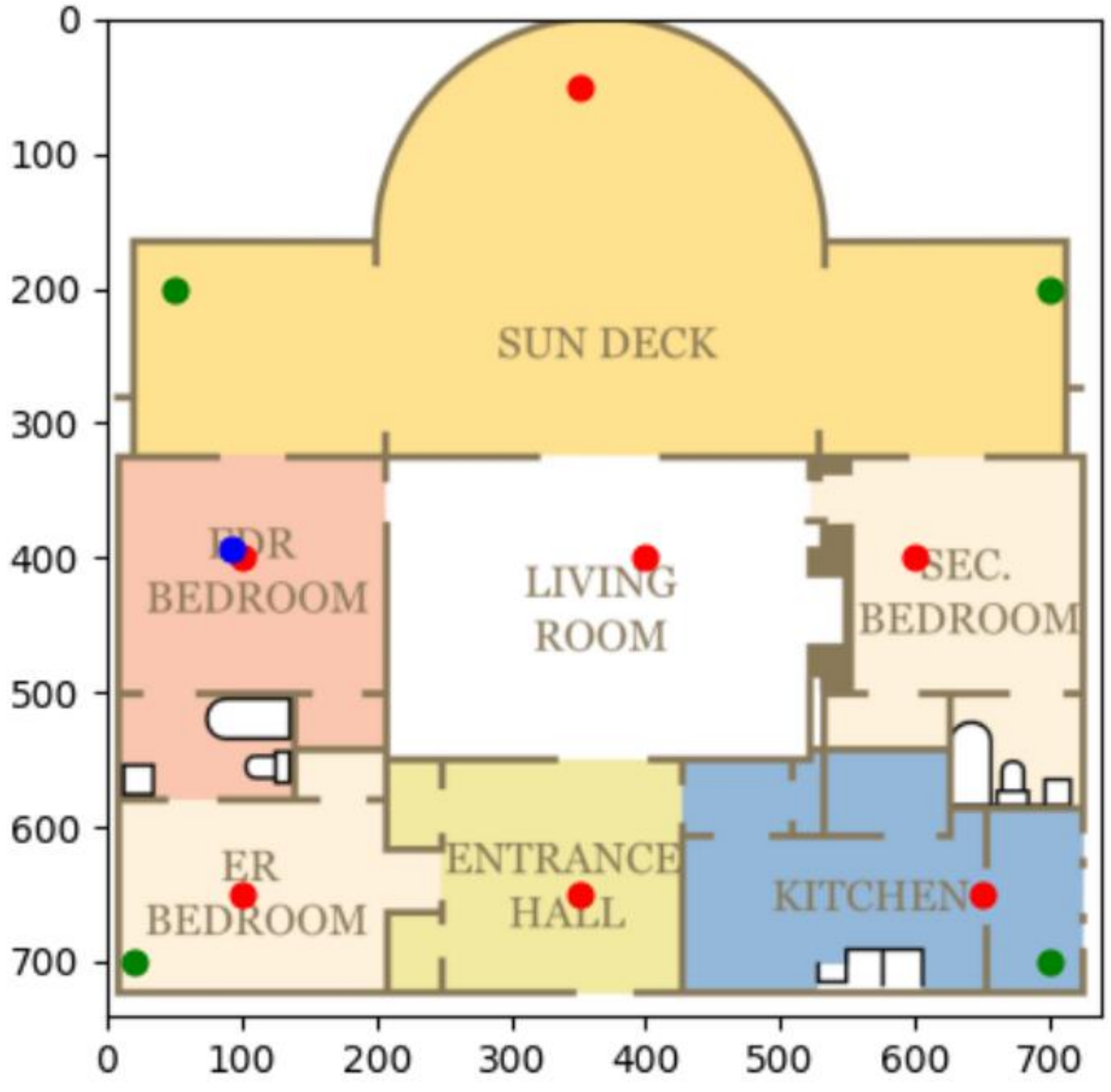
Preliminaries

Simulating a Radio Map

- RSSI vectors from four APs was constructed based on an obstacle-free distance estimation model

$$RSS(d) = -10nlog_{10}(d) - C$$

- Log-Normal Shadowing Model
- n: path exponent (5)
- C: Environment constant (-20)



A. Prabakar. (2015) RSSI-based accurate indoor localization scheme for wireless sensor networks. [Online]. Available: https://www.youtube.com/watch?v=CWvRJdF7oVEt=199s

Sample App

[Link](#)

Quantum Algorithms

Qubit

- A **qubit** is the basic unit of information that can be in the state $|0\rangle$ or $|1\rangle$.
- **Superposition** states that if a quantum system can exist in state $|0\rangle$ or $|1\rangle$, then it can also exist in state $\alpha_0|0\rangle + \alpha_1|1\rangle$ where $\alpha_0, \alpha_1 \in \mathbb{C}$, which we refer to as the complex amplitudes of the state.
- The **act of measuring** the state collapses the given state into either of the states 0 or 1 with a probability $|\alpha_0|^2$ of being in state 0 and $|\alpha_1|^2$ of being in state 1.

Quantum Algorithms

Quantum Registers

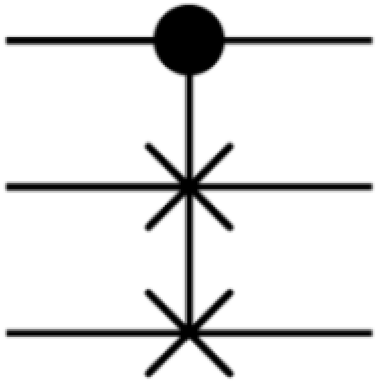
- A **quantum register** may compose of one or more qubits.
- The **state** of **quantum register** is a tensor product of the state of its component qubits, denoted $q_1 \otimes q_2 \otimes \ldots \otimes q_n$ for an n-qubit register.
- A **quantum register** can also be in a **superposition state**, for example in a 2-qubit quantum register:

$$|\psi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

Quantum Algorithms

Quantum Operations

- The state of a qubit is transformed through the application of quantum operation.
  - Basic quantum operations act on one or two qubits at a time



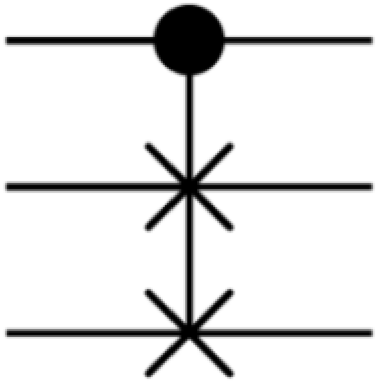
$$\mathbf{CSWAP}(|0\rangle|\psi_1\rangle|\psi_2\rangle = |0\rangle|\psi_1\rangle|\psi_2\rangle$$

$$\mathbf{CSWAP}(|1\rangle|\psi_1\rangle|\psi_2\rangle = |1\rangle|\psi_2\rangle|\psi_1\rangle$$

Quantum Algorithms

Quantum Operations

- The state of a qubit is transformed through the application of quantum operation.
  - Basic quantum operations act on one or two qubits at a time



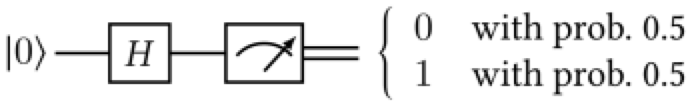
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_4 & 0_4 \\ 0_4 & SWAP \end{pmatrix}$$

Quantum Algorithms

Quantum Operations

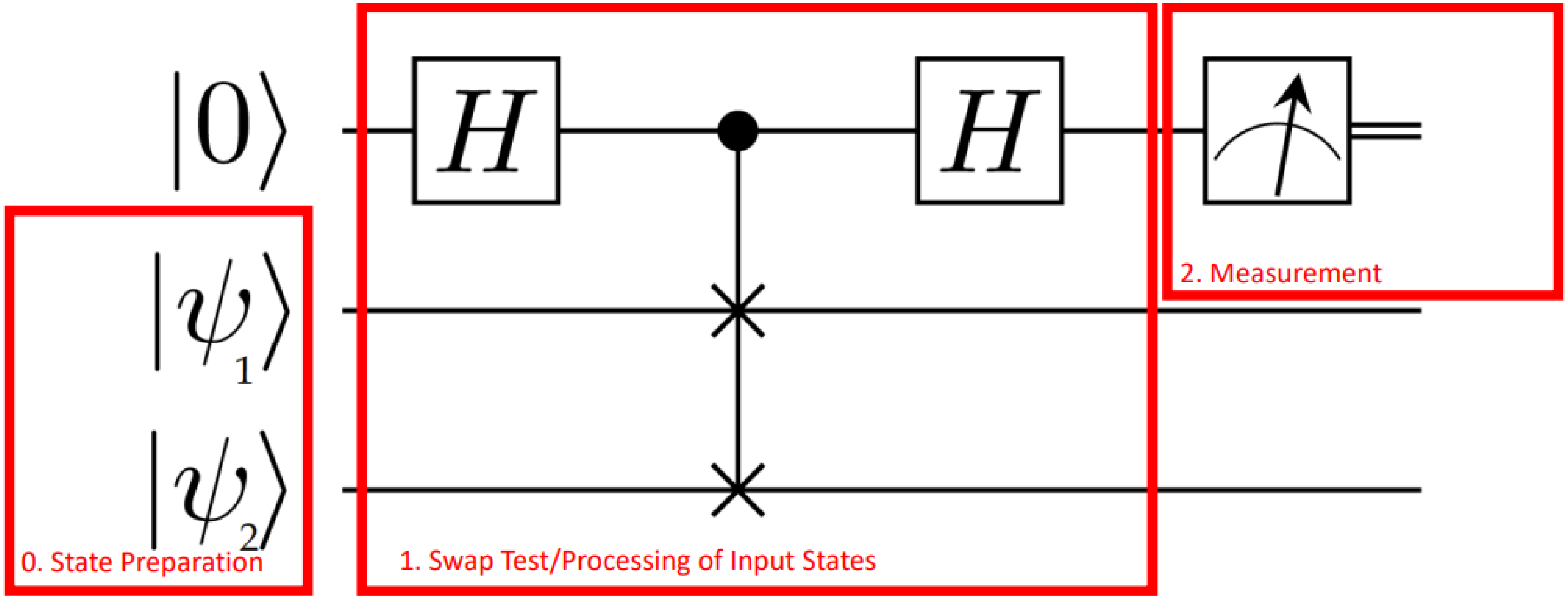
- The state of a qubit is transformed through the application of quantum operation.
  - Basic quantum operations act on one or two qubits at a time
  - Measurement is an irreversible operation that collapses a given state $|\psi\rangle$ into one of its basis states

For example, measuring a qubit whose quantum state is represented by $\psi = a|0\rangle + b|1\rangle$ will collapse its state in $|0\rangle$ with probability a^2 and into $|1\rangle$ with probability b^2 .



Quantum Swap Test

Quantum Swap Test Circuit: Stages



Quantum State Preparation

Quantum State Preparation using Amplitude Encoding

- Also known as arbitrary state preparation
- Given a numerical input vector $v = (x_0, x_1, \ldots, x_{n-1})^T$ as input
- We use amplitudes to encode the data
- The input vector must be normalized to length 1
- The input vector is encoded in the amplitudes of the quantum state as follows

$$|\psi\rangle = \sum_{i=0}^{n-1} \widehat{x_i}|i\rangle \qquad where\ \widehat{x_k} = \frac{x_k}{||v||}$$

M. Weigold, J. Barzen, F. Leymann, and M. Salm. 2020. Data Encoding Patterns for Quantum Algorithms. HILLSIDE Proc. of Conf. on Pattern Lang. of Prog. 22 (October 2020), 10 pages

Quantum State Preparation

Quantum State Preparation using Amplitude Encoding

- Example: $v = [1, 0, 2, 3]$
- $||v|| = \sqrt{1^2 + 0^2 + 2^2 + 3^2} = \sqrt{14}$

- Prepared quantum state:
- $|\psi\rangle = \frac{1}{\sqrt{14}}(1|00\rangle + 0|01\rangle + 2|10\rangle + 3|11\rangle)$

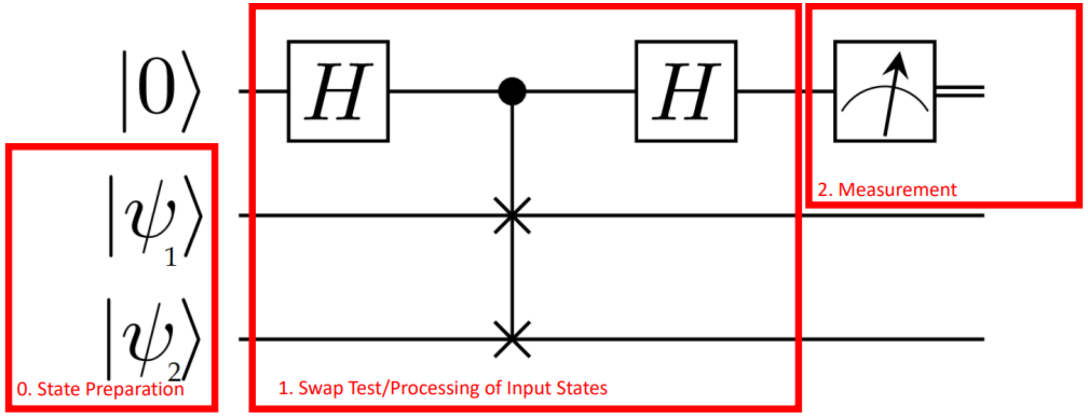
$$\boxed{|\psi\rangle = \sum_{i=0}^{n-1} \widehat{x_i}|i\rangle \qquad where\ \widehat{x_k} = \frac{x_k}{||v||}}$$

Apply this for each of the fingerprint $RSSI_l$ and $RSSI_u$ (measured by mobile object)

M. Weigold, J. Barzen, F. Leymann, and M. Salm. 2020. Data Encoding Patterns for Quantum Algorithms. HILLSIDE Proc. of Conf. on Pattern Lang. of Prog. 22 (October 2020), 10 pages

Quantum Swap Test

Quantum Swap Test: Algorithm



Input: Two quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ stored in separate qubit registers, each containing $N = \log_2(n)$ qubits; An ancillary qubit a, initialized $|0\rangle$; Number of times the circuit runs P.

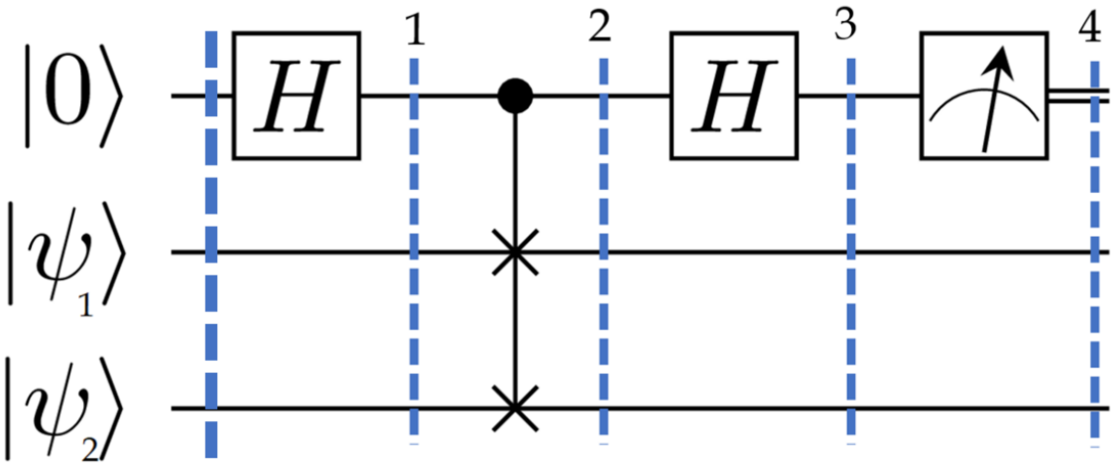
Output: $|\langle\psi_1|\psi_2\rangle|^2$

```
for j from 0 to P − 1 do
    Apply Hadamard gate to a.
    for i from 0 to N − 1 do
        Apply CSWAP(a, ψ_1i, ψ_2i)
    end for
    Apply Hadamard gate to a.
    Measure a in the Z-basis and record
each of their measurement M_k as either
0 or 1.
    Compute s = 1 − (2/P) Σ_{i=1}^{P} M_i
end for
```

Quantum Swap Test

Quantum Swap Test Circuit

- Buhrman et al., 2001 showed that quantum swap test reduces the cost of computing similarity from W to $\log_2 W$, where W is the size of the bit string.



Series of Tensor Multiplications:

$$\underbrace{(H\otimes I\otimes I)}_{\text{2 to 3}}\underbrace{(\mathbf{CSWAP})}_{\text{1 to 2}}\underbrace{(H\otimes I\otimes I)}_{\text{init to 1}}\underbrace{|0\rangle|\psi_1\rangle|\psi_2\rangle}_{\text{initial state}}$$

Harry Buhrman, Richard Cleve, John Watrous, and Ronald de Wolf Phys. Rev. Lett. 87, 167902 – Published 26 September 2001

Quantum Swap Test

Quantum Swap Test Circuit

- Compare states using ancillary and controlled-swap gates
- Let X be the outcome of ancillary qubit after measuring.
- An observer measures that the ancillary qubit collapses into 0 with probability:

$$P(X = 0) = \frac{1}{2} + \frac{1}{2} |\langle \psi_1 | \psi_2 \rangle|^2$$

Harry Buhrman, Richard Cleve, John Watrous, and Ronald de Wolf Phys. Rev. Lett. 87, 167902 – Published 26 September 2001

Quantum Swap Test: Processing & Outcomes

Quantum Swap Test: $\boldsymbol{P(X=0)=\frac{1}{2}+\frac{1}{2}|\langle\psi_1|\psi_2\rangle|^2}$

- Suppose that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal
- $P(X=0)=\frac{1}{2}+\frac{1}{2}|\langle\psi_1|\psi_2\rangle|^2$
- Taking the inner product: $\langle\psi_1|\psi_2\rangle=0$
- $P(X=0)=\frac{1}{2}$
- This means that when two states are orthogonal, we expect that the probability of observing the ancillary qubit equal to 0 is $\frac{1}{2}$
- $P(X=0)=\frac{1}{2}$

Quantum Swap Test: Processing & Outcomes

Quantum Swap Test: $\boldsymbol{P(X=0)=\frac{1}{2}+\frac{1}{2}|\langle\psi_1|\psi_2\rangle|^2}$

- Suppose that $|\psi_1\rangle = |\psi_2\rangle$
- $P(X=0)=\frac{1}{2}+\frac{1}{2}|\langle\psi_1|\psi_1\rangle|^2$
- Taking the inner product: $\langle\psi_1|\psi_1\rangle = 1$
- $P(X=0)=1$
- This means that when two states are equal, we expect that the probability of observing the ancillary qubit equal to 0 is 1
- $P(X=0)=1$

Cosine Similarity

Connection bet. Quantum Swap Test Circuit and Cosine Similarity

- To demonstrate that preparing the quantum states of the given vectors and running the circuit P times is the same as computing the cosine similarity, we can take the result of the circuit as Bernoulli Random Variable.
- Given that the probability of success $p = \frac{1}{2} + |\langle\psi_1|\psi_2\rangle|^2$ and the success event is observing the ancillary qubit collapse into 0 state. Then the expected value is:

$$\langle X \rangle = p \times 0 + (1 - p) \times 1$$

Cosine Similarity

Connection bet. Quantum Swap Test Circuit and Cosine Similarity

- Expected Value of Bernoulli Random Variable

$$\langle X\rangle = p \times 0 + (1 - p) \times 1$$

$$\langle X \rangle = 1 - p$$

$$\langle X \rangle = 1 - \frac{1}{2} - |\langle\psi_1|\psi_2\rangle|^2$$

$$\langle X \rangle = \frac{1}{2} - |\langle\psi_1|\psi_2\rangle|^2$$

Cosine Similarity

Connection bet. Quantum Swap Test Circuit and Cosine Similarity

- If we run the circuit P times and store each result $\mathrm{M_k} \in \{0, 1\}$, then the average value of r_i's is the expected value $\langle X \rangle$, i.e.,

$$\langle X \rangle = \frac{1}{P}\sum_{i=1}^{P} r_i = \frac{1}{2} - |\langle\psi_1|\psi_2\rangle|^2$$

- Rearranging, the square of inner product is:

$$|\langle\psi_1|\psi_2\rangle|^2 = 1 - \frac{2}{P}\sum_{i=1}^{P} M_i$$

Swap Test Circuit

Applicability

- The circuit can be used for problems comparing the similarity between vectors.
  - Not limited to localization: semantic search, fetching relevant documents for retrieval augmented generation in large language models.
  - Demonstrates the concept of superposition & entanglement.
  - Introduces quantum state preparation early on.
- Can be introduced early on, since the concepts used are easy to follow (e.g., emphasis for superposition, what measurement operation does, how its probabilistic nature can be used to solve problems). Suitable for introductory quantum algorithm classes.

Conclusion

- A circuit for measuring vector similarity is discussed in depth. Simulations are done using Qiskit SDK.
- Sample app that includes radio map generation.
- Code Repository: link
- Circuit for Quantum State Preparation and Swap Test

Future Work

- Better generation of radio map/way of generating received signal strength indicators per room/section can be explored.
- Expand the scenario it can simulate, open field vs. crowded and how it affects radio signals.
- Creating another applicative scenario: finding a moving object with estimated location based on its RSS values.
- Computing the similarity value using real quantum hardware like IBM Q.

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