To investigate the wear of stairs further, we first need to clarify the mechanical mechanism of human action on stairs during walking. To this end, this article developed the Step-pit model

to quantify the wear and tear produced on the stairs by each step during walking.

**1.Model preparation**

Based on the SLIM, we have simplified the force exerted by a person walking on a stair. By analyzing the IR pressure distribution map using the grid method, the length and area of the foot as a whole and the three main force regions can be obtained as follows:

Where Li represents the length of region i and Si refers to the area of region i.

Then, coordinate axes are established on the profile of each step of the stairs, taking the i-th step as an example (as shown in Fig. n). According to relevant studies, the average step width of stairs is about 32.5 cm and the height is about 14.5 cm \cite{WOS:000783553300001}

Using half the length of the grid edge in Figure n as a unit of measurement, therefore, the width of the stair treads is about 48 unit lengths and the height is about 21 unit lengths.

To ensure that the landing point always remains within the countertop of the stairs, their range of values must satisfy the following constraints (due to space limitations, only the coordinate range of point M is presented here. The coordinate ranges for points U and C will be derived and explained in detail later based on the relationships among the three points):

\begin{table}[h!]

\centering

\setlength{\tabcolsep}{8pt} % 调整列间距

\renewcommand{\arraystretch}{1.5} % 增加行距

\caption{Landing Coordinate Range for Point M}

\label{tab:M\_coordinates}

\begin{tabular}{@{}>{\centering}p{3cm}@{\hskip 10pt}c@{\hskip 10pt}c@{}}

\toprule

\textbf{Walking Direction} & \textbf{Front-Foot Landing} &

\textbf{Full-Foot Landing} \\ \midrule

Ascending & [0, 14] & [18, 30] \\

Descending & \multicolumn{2}{c}{\centering [4, 14]} \\ \bottomrule

\end{tabular}

\end{table}

U: The pressure centers of **Hallux**

M:The pressure centers of **Metatarsals Two and Three Metatarsals Four and Five**

C: The pressure centers of **Medial Calcaneus Lateral Calcaneus**

This section sets the geometric foundation for the subsequent analysis and provides a rational coordinate frame.

**2.Walking patterns and Landing point analysis**

In this model, it is assumed that a person walks up and down stairs in two ways and only two ways:

·  **Full-foot Walking**: The entire foot applies force to the stairs.

·  **Forefoot Walking**: Only the forefoot region applies force, while the rear foot remains suspended in the air.

In addition, each individual used only one walking style during a complete flight of stairs. To standardize the analysis, M is used as a benchmark.

Calculation of standard coordinates

The standard coordinates on the i+1st step can be obtained recursively from the standard coordinates of the i-th step:

Although walking styles vary, regardless of the style, the human stride length tends to keep the landing point of the next step within a “comfort zone”. When the expected landing point is outside the comfort zone, the walker adjusts the stride length to bring it back within a reasonable range. The comfort zone can be expressed as:

On this basis, the standard coordinates of the i+1-th stair need to be adjusted according to the constraints of the comfort zone. The specific calculation process is shown in Figure N.

After multiple iterations, the model generates a probability distribution curve for the pressure in the second main stress area on the upper foot of the stair table surface (with respect to the variable x, see Fig. N for details).

 Combining the coordinate relationships of the three main force regions, the probability distributions of the first and third force regions can be further plotted in Fig. N.

**3.Wear depth analysis of a single step**.

Literature \cite{ WOS:000678697003082}provides proportional relationships between the forces exerted by the three main force regions of the foot when a person walks on stairs. These ratios reflect the magnitude of the forces exerted on the stairs by the different regions. The results were plotted as a trilinear diagram below:

Where FGi is the vertical pressure exerted by region i on the stairs and G is the total vertical pressure.

Assume that the wear volume can be approximated as a regular quadrangular pyramid, then based on its geometric characteristics, the wear depth in region m is calculated as:

Where Wm and Sm represent the wear volume and area of the m-th region, respectively.

The wear analysis of forefoot walking is similar to that of full-foot walking, but the proportional distribution of forces is different from that of full-foot walking due to the fact that there are only two main force regions applying forces. In Fig. N, a trilinear diagram was plotted to show the relationship between the force distribution in the two regions. The depth of wear was calculated in the same way as for full-footed walking, with only the parameters adjusted.

**4. conclusion**