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For Problem E

Based on the wear data shown in (Figure x), the wear in the 39-48 range is primarily influenced by environmental factors, exhibiting a mild weathering effect, and it is assumed that no significant wear is caused to the stairs.

On this basis, using the principles of probability theory, we can assume that the wear amount of the stairs caused each year follows a Gaussian distribution.

The total wear rate of the stairs is determined by both the average annual wear rate and the time (i.e., the number of years since construction), and their relationship is expressed as:

R\_{\text{total}} = R\_{\text{year}} \cdot T

Where R\_{\text{total}}​ represents the total wear amount, R\_{\text{daily}}​ is the annual wear amount, and T refers to the time span, which is the total number of years since the stairs were constructed.

Since division operations do not alter the distribution pattern of a Gaussian distribution, the estimated value of the construction year of the stairs also follows a Gaussian distribution. Therefore, we can further calculate the probability of the time period provided by the archaeologist within this estimated Gaussian distribution. The higher the probability, the more reliable the estimated value.

For Problem F

The primary method for repairing building materials involves reinforcing cracks by filling them with polymer materials\cite{ YTLX200401034}. In addition, the steel plate bonding method is commonly used for strengthening flexural or tensile members (such as stair platform beams, platform slabs, and stair slabs) under normal working conditions and static loads. Closed protection by wrapping the original staircase components in steel plates is also an option.

For repairs or renovations that are visually apparent, their presence can typically be determined through direct observation. However, for polymer reinforcement methods that are not easily discernible to the naked eye, we have designed the following detection method:

We first calculate the difference between the measured original depth data of surface depressions and the optimized depth data from ....., identifying points with significant positive differences as abnormal depression points. These points may indicate abnormal depressions caused by repairs or renovations.

Next, we conduct secondary sampling on the treads where the abnormal depression points are located, significantly increasing the sampling density to 300 samples per square meter. The same abnormal point analysis is repeated on the newly collected data. If the newly identified abnormal points exhibit circular or strip-shaped clustered distributions (as shown in reference figures), we preliminarily conclude that the area may have undergone repairs using polymer reinforcement methods.

To further verify the use of polymer materials for reinforcement, we employ the Torrent Permeability Test Method to measure the air permeability of the building material\cite{sena2015non}. Polymer materials typically exhibit strong adhesion, no powder residues, water resistance, and polishability. However, their air permeability differs significantly from that of the base stone material. By comparing air permeability values, we can further confirm the presence and extent of repaired regions.

When further analyzing areas with abnormal air permeability, we perform water absorption tests on the associated staircase treads. The specific method is as follows:

To further study areas with abnormal air permeability, we performed water absorption tests on the associated staircase treads. The specific steps are as follows:

**Testing Procedure**

1. Attach cobalt chloride test paper, fully saturated with water and turned pink, to both the clustered abnormal points and the normal areas of the staircase treads.
2. Cover the test paper with appropriately sized covers to prevent water evaporation.
3. Use a D345 camera to continuously monitor and record the color change of the test paper.

**Analysis of Test Results**

Cobalt chloride test paper turns blue upon water absorption. By monitoring the rate at which the paper turns blue, we can assess the water absorption capacity of different areas of the stone material.

If the test paper in the clustered abnormal points turns blue at a delayed rate, it indicates poor water absorption capacity in that area, further supporting the conclusion that polymer reinforcement has been applied.

Finally, by combining the test results with the surface water permeability characteristics of the material and comparing the water absorption differences

between the clustered abnormal points and normal areas, the presence and spatial distribution of polymer material repairs can be further confirmed.

Through the above multi-level detection method, not only can the presence of repairs or renovations be scientifically determined, but the repaired areas can also be accurately identified.

For Problem G

To determine whether the source of the target material aligns with the archaeologist's hypothesis, we recommend sampling from what the archaeologist believes to be the original source and conducting destructive mechanical experiments to obtain precise material parameters. These experimentally obtained parameters are then used to replace the tabulated values and are re-integrated into the Step-pit model to derive the optimal fitting result. Finally, this result is compared to the fitting result obtained using the tabulated parameters, with the coefficient of determination R^2 used as the evaluation criterion.

**Goodness-of-fit calculation formula:**

R^2 = \frac{\text{SSR}}{\text{SST}} = 1 - \frac{\text{SSE}}{\text{SST}}, \quad 0 \leq R^2 \leq 1

* **The sum of Squares for Error (SSE):** The sum of squared differences between the actual values and predicted values, reflecting the magnitude of errors.
* **The sum of Squares for Regression (SSR):** The sum of squared differences between the predicted values and the mean of the actual values, representing the variation explained by the model.
* **The total Sum of Squares (SST):** The sum of squared differences between the actual values and their mean, equivalent to the total variation. The relationship satisfies:

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\end{itemize}

SST=SSE+SSR\text{SST} = \text{SSE} + \text{SSR}

R^2 represents the proportion of the total variation that can be explained by the model. The closer the R^2 value is to 1, the higher the consistency of the model. The R^2 value can be used to evaluate the match between the archaeologist's hypothesized material source and the actual results. When R^2 exceeds 0.8, it indicates that the source of the target material is consistent with the archaeologist's hypothesis.

For Problem H

**Number of Users in a Typical Day for Staircase**

In ... (referenced analysis), we have already obtained the total number of users during the operational period of the staircase. Assuming that the number of people using the stairs per day remains constant during this period, the number of people using the stairs in a typical day can be expressed as:

\text{Number of Users in a Typical Day} = \frac{\text{Total Number of Users}}{\text{Total Days of Usage}}

**Short-Time High Traffic or Long-Time Low Traffic**

Fatigue strength is significantly influenced by loading frequency, defined as the frequency at which force is applied to the

material\cite{Yokobori1976}.\cite{Takezono1980}.\cite{HeimbachHeimbach+1970+377+380}. Higher loading frequency accelerates the formation of micro-cracks within the material, which can rapidly propagate in a short

period\cite{ SJESAC88A958454EEE1CD4ED092FB8A0E8F8}.\cite{ SJESF46245B4D88236414B6977C781CEC048}, ultimately leading to fracture. This indicates that higher loading frequencies directly increase the probability of fracture by expediting crack propagation. Consequently, when a large number of people use the staircase within a short period, the increased loading frequency makes the formation and propagation of cracks within the staircase more likely, thereby increasing the risk of fracture.

Additionally, due to abrupt geometric transitions, the outer edges of staircases often exhibit stress concentration, which further accelerates fatigue damage. This stress concentration makes cracks more likely to propagate, ultimately resulting in material fracture. Observations of real structures corroborate this phenomenon, as cracks are predominantly concentrated along the outer edges of staircases, confirming the role of stress concentration.

In summary, when a large number of people use the staircase within a short period, the increased loading frequency and stress concentration at the outer edges are primary contributors to crack formation and propagation, ultimately leading to material damage and even fracture.

To quantify this effect, we evaluate the depression depth X\_6 in the 0–3 step region of the staircase using the following equation:

\text{LFP} = X\_6 - k\_A A\_{0-3} - k\_B B\_{0-3} - k\_C C\_{0-3} - k\_D D\_{0-3}

In cases of significant normal wear, the fracture risk in this region increases notably.

To assess the relationship between X\_6 and the loading frequency parameter \text{LFP}, a Mann-Whitney U test is used. Since X\_6 and X\_6 + \text{LFP} share similar data distributions and do not require identical distributions, the test is applied to determine whether X\_6 + \text{LFP} is significantly greater than X\_6. The hypotheses are as follows:

* **Null Hypothesis (H\_0)**: X\_6 + \text{LFP} is not significantly greater than X\_6.
* **Alternative Hypothesis (H\_1)**: X\_6 + \text{LFP} is significantly greater than X\_6.

\begin{itemize}

\item Null Hypothesis (H\(\_0\)): \(X\_6 + \text{LFP}\) is not significantly greater than \(X\_6\).

\item Alternative Hypothesis (H\(\_1\)): \(X\_6 + \text{LFP}\) is significantly greater than \(X\_6\).

\end{itemize}

The U statistic is calculated as follows:

$$U = n^2 + \frac{n}{2}(n + 1) - R(X\_6)$$

At a significance level of \alpha = 0.05, the rejection region is U < 8. The decision criteria are:

1. If U < 8, the alternative hypothesis H\_1 is accepted, indicating material loss at the staircase edges caused by fractures.
2. If U \geq 8, the null hypothesis H\_0 is accepted, suggesting that material loss at the staircase edges is primarily due to normal wear and bending, with no additional fractures.

Based on the Mann-Whitney U test results, the following conclusions can be drawn:

* If a significant difference exists between X\_6 and LFP\text{LFP}, it indicates that material loss at the staircase edges is not solely caused by normal wear and bending but is also due to a large number of people using the staircase in a short time.
* If no significant difference exists between X\_6 and \text{LFP}, it indicates that a small number of people use the stairs over an extended time.

\begin{itemize}

\item If a significant difference exists between \( X\_6 \) and \( \text{LFP} \), it indicates that material loss at the staircase edges is not solely caused by normal wear and bending but is also due to a large number of people using the staircase in a short time.

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\end{itemize}

By integrating the Mann-Whitney U test results with the theoretical analysis, this method enables a reasonable determination of whether the staircase was used by a large number of people over a short time or by a small number of people over a long time.