알고리즘

01. Divide & Conquer

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미디어소프트웨어학과
민경하

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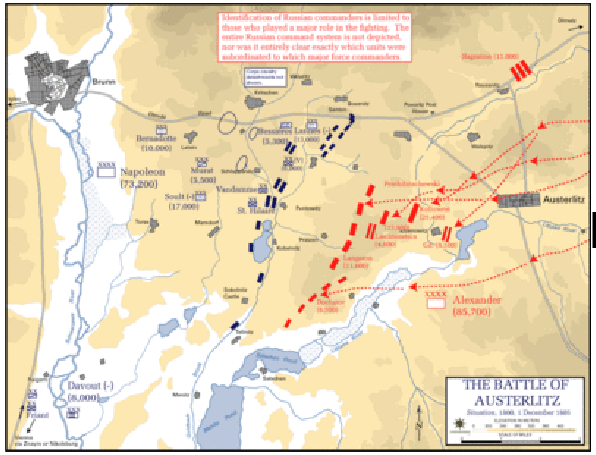
1.0 Introduction

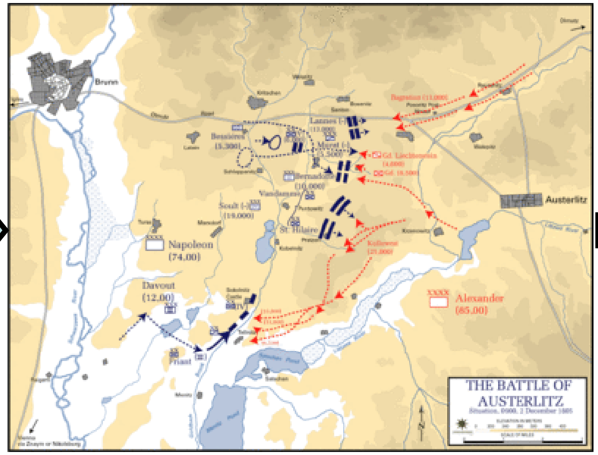
(1) Battle of Austerlitz

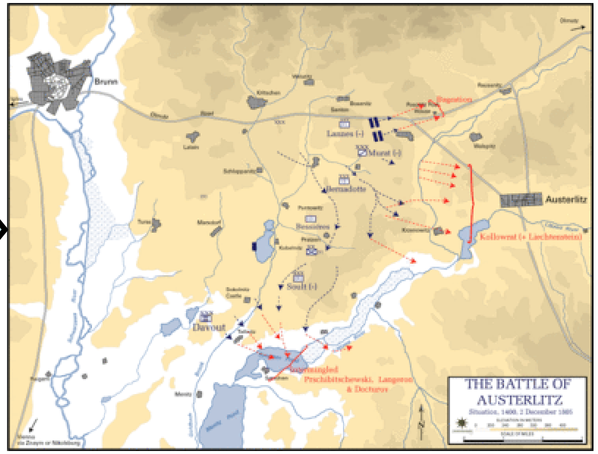
1.0 Introduction

(1) Battle of Austerlitz (1805)

- Battle of three emperors
- France (65,000) VS Austria + Russia (80,000)
- The end of the 3rd anti-France aliance







1.0 Introduction

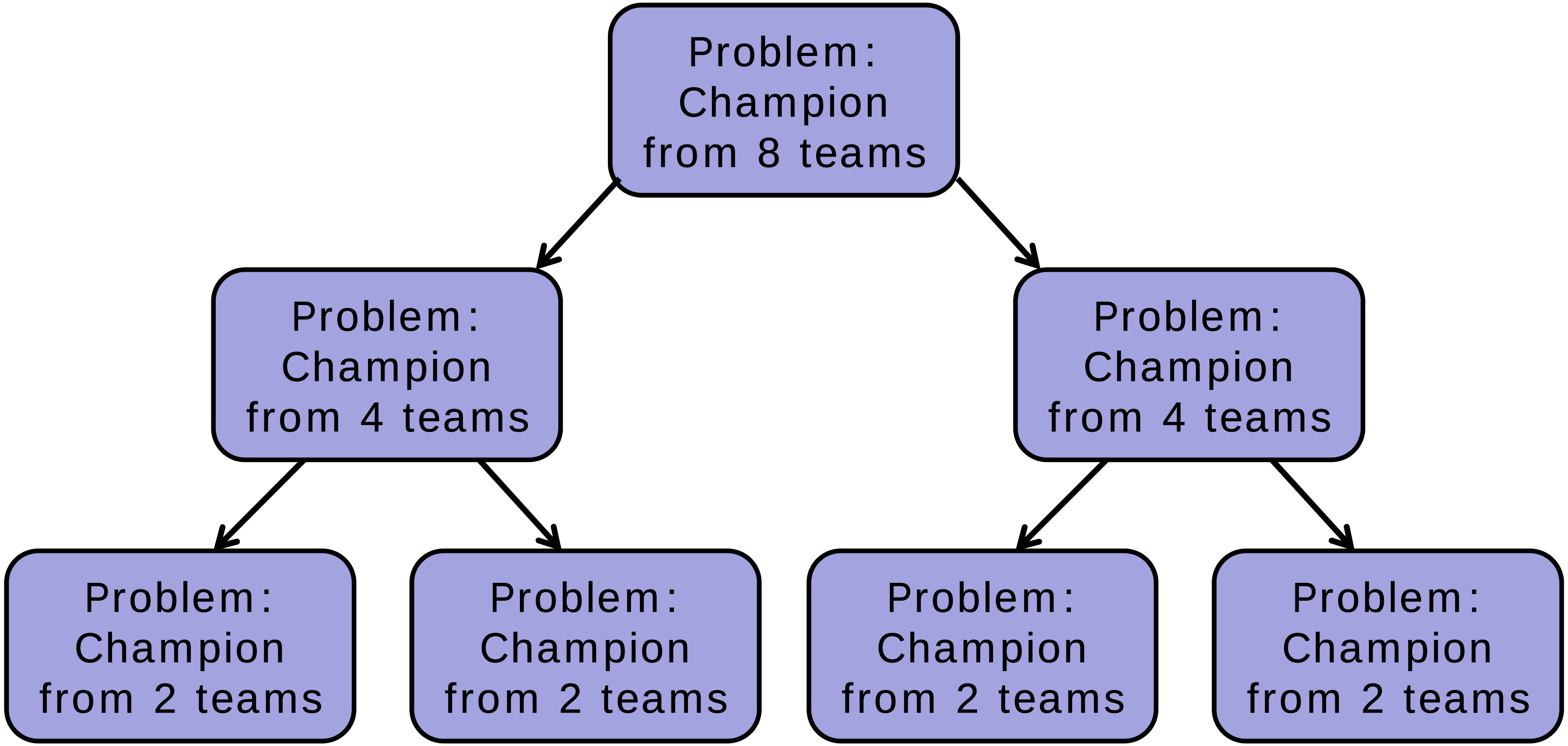
(2) League VS Tournament

- Ex) Elite-8 of Worldcup 2014
  - BRA, COL, FRA, GER, NED, CRC, ARG, BEL
- Champion by league
  - How many games do they play?
- Champion by tournament
  - How many games do they play?

1.0 Introduction

(2) League VS Tournament

– Champion by tournament



1.0 Introduction

(2) League VS Tournament

– Champion by tournament

```
int Champion8 ( {BRA, COL, FRA, GER, NED, CRC, ARG, BEL} )
{
    Lwinner = Champion4 ( {BRA, COL, FRA, GER} );
    Rwinner = Champion4 ( {NED, CRC, ARG, BEL} );

    return Winner ( Lwinner, Rwinner );
}
```

1.0 Introduction

(2) League VS Tournament

– Champion by tournament

```
int Champion4 ( {BRA, COL, FRA, GER} )
{
    Lwinner = Champion2 ( {BRA, COL} );
    Rwinner = Champion2 ( {FRA, GER} );

    return Winner ( Lwinner, Rwinner );
}
```

```
int Champion4 ( {NED, CRC, ARG, BEL} )
{
    Lwinner = Champion2 ( {NED, CRC} );
    Rwinner = Champion2 ( {ARG, BEL} );

    return Winner ( Lwinner, Rwinner );
}
```

1.0 Introduction

(2) League VS Tournament

– Champion by tournament

```
int Champion2 ( {BRA, COL} )
{
    Lwinner = Champion1 ( {BRA} );      //     Unnecessary
    Rwinner = Champion1 ( {COL} );      //     Unnecessary

    return Winner ( BRA, COL);
}
```

```
int Champion2 ( {BRA, COL} )
{
    Lwinner = Champion1 ( {BRA} );      //     Unnecessary
    Rwinner = Champion1 ( {COL} );      //     Unnecessary

    return Winner ( BRA, COL);
}
```

1.0 Introduction

(2) League VS Tournament

– Champion by tournament

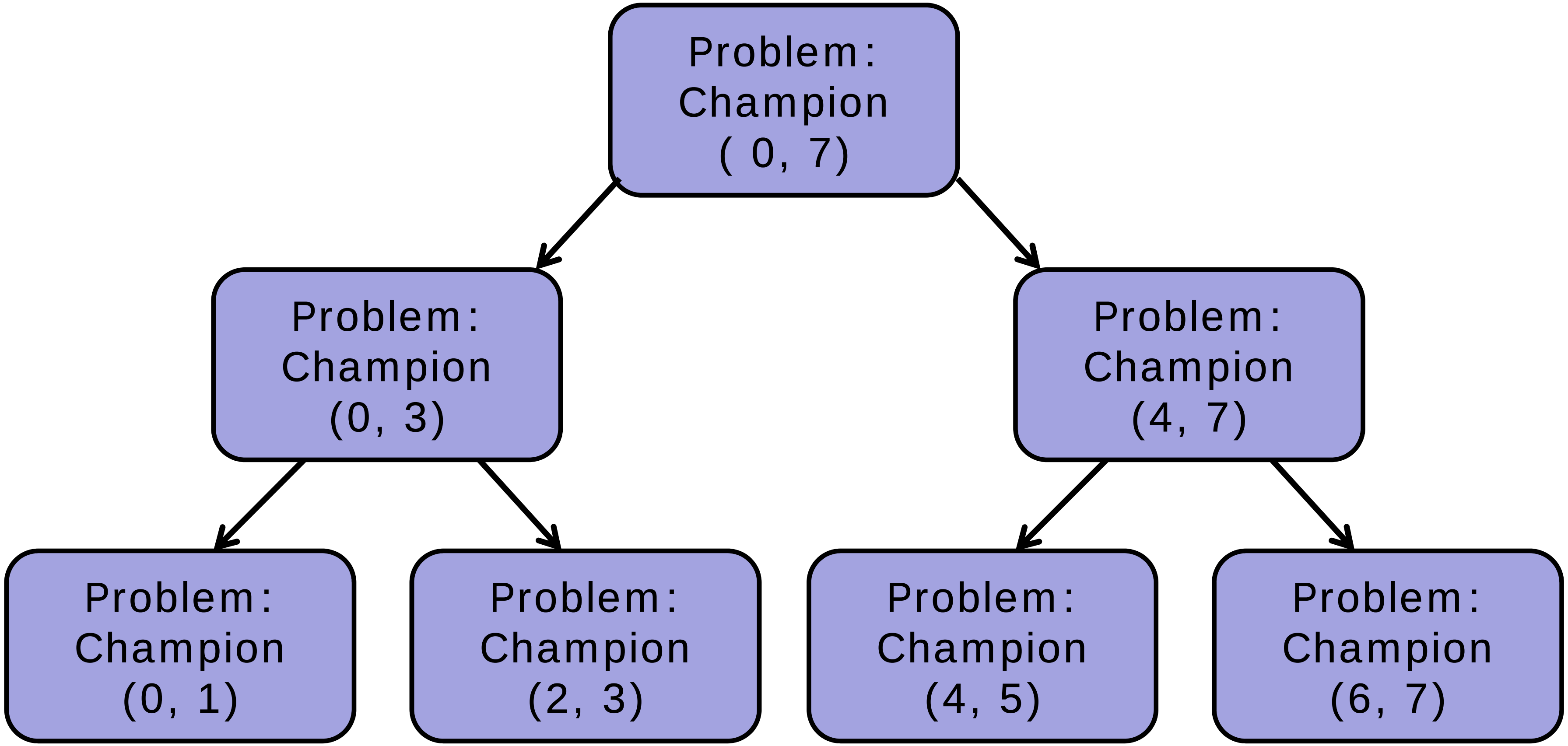
```
char sweet8[8] = {BRA, COL, FRA, GER, NED, CRC, ARG, BEL};
int Champion ( int s, int e )    //initially, s = 0, e = 7
{
    int m = (s + e)/2;
    Lwinner = Champion ( s, m );  //     s = 0, m = 3
    Rwinner = Champion ( m+1, e );//     m+1 = 4, e = 7

    return Winner ( Lwinner, Rwinner );
}
```

1.0 Introduction

(2) League VS Tournament

– Champion by tournament



1.0 Introduction

(2) League VS Tournament

– Champion by tournament

```
char sweet8[8] = {BRA, COL, FRA, GER, NED, CRC, ARG, BEL};
int Champion ( int s, int e )     //initially, s = 0, e = 7
{
    int m = (s + e)/2;
    Lwinner = Champion ( s, m ); //     s = 0, m = 3
    Rwinner = Champion ( m+1, e );//    m+1 = 4, e = 7

    return Winner ( Lwinner, Rwinner );
}
```

– Do we miss something?

1.0 Introduction

(2) League VS Tournament

– Champion by tournament

```
char sweet8[8] = {BRA, COL, FRA, GER, NED, CRC, ARG, BEL};
int Champion ( int s, int e )     //initially, s = 0, e = 7
{
    if ( s == e )
        return s;

    int m = (s + e)/2;
    Lwinner = Champion ( s, m );  //     s = 0, m = 3
    Rwinner = Champion ( m+1, e );//     m+1 = 4, e = 7

    return Winner ( Lwinner, Rwinner );
}
```

– Do we miss something? → **degenerate case**

(2) League VS Tournament

– Champion by tournament

```
char sweet8[8] = {BRA, COL, FRA, GER, NED, CRC, ARG, BEL};
int Champion ( int s, int e )    //initially, s = 0, e = 7
{
    if ( s == e )
        return s;                                         → Degenerate case

    int m = (s + e)/2;                                    → Divide

    Lwinner = Champion ( s, m );
    Rwinner = Champion ( m+1, e );                        → Conquer

    return Winner ( Lwinner, Rwinner );                   → Combine
}
```

1.0 Introduction

(3) Key idea of divide & conquer

- Solve a problem of *n* inputs by splitting the input into *k* subsets

– Three steps of divide & conquer

- Divide
  - – Breaking a problem into subproblems
- Conquer
  - – Recursively solving these subproblems
- Combine (optional)
  - – Appropriately combining their answers

1.0 Introduction

(4) Abstract algorithm for DnC (recursive)

```
global n, A(1:n);
DnC ( int p, int q )
{
    int m;
    if ( SMALL (p, q) )
        return G (p, q);
    else
        m ← DIVIDE (p, q);
        return COMBINE ( DnC (p, m), DnC (m+1, q) );
}
```

– Most divide & conquer algorithms are implemented using recursive call

1.0 Introduction

(5) Three check points

- Same format
- Reduced problem size
- Degenerate case

1.0 Introduction

- Same format

```
global n, A(1:n);
DnC ( int p, int q )
{
    int m;
    if ( SMALL (p, q) )
        return G (p, q);
    else
        m ← DIVIDE (p, q);
        return COMBINE ( DnC (p, m), DnC (m+1, q) );
}
```

1.0 Introduction

- Reduced problem size

```
global n, A(1:n);
DnC ( int p, int q )
{
        int m;
        if ( SMALL (p, q) )
                return G (p, q);
        else
                m ← DIVIDE (p, q);
                return COMBINE ( DnC (p, m), DnC (m+1, q) );
}
```

1.0 Introduction

- Degenerate case

```
global n, A(1:n);
DnC ( int p, int q )
{
    int m;
    if ( SMALL (p, q) )
        return G (p, q);
    else
        m ← DIVIDE (p, q);
        return COMBINE ( DnC (p, m), DnC (m+1, q) );
}
```

1.0 Introduction

(6) Performance analysis for DnC

- T (n): time complexity of DnC ( ) for n inputs
- g (n): for small input
- f_1 (n): for DIVIDE ( )
- f_2 (n): for COMBINE ( )

$$T(n) = \begin{cases} g(n), & \text{for small } n \\ 2T(n/2) + f_1(n) + f_2(n), & \text{otherwise} \end{cases}$$

$$T(n) = \begin{cases} g(n), & \text{for small } n \\ aT(n/b) + O(n^d), & \text{otherwise} \end{cases}$$

1.0 Introduction

(7) The simplest divide & conquer algorithm

- Binary search:
  - Let $A = \{a_1, \dots a_n\}$ be a list of elements which are sorted in nondecreasing order.
  - Determine whether x is in A or not. If x is in A, then find i such that $a_i = x$.

1.0 Introduction

- Straightforward search

```
int bsearch_straightforward ( int s, int e, int A[], int x )
{
    for ( i = s; i <= e; i++ ) {
        if ( A[i] == x )
            return i;
    }

    return NONE;
}
```

  - Performance analysis?

1.0 Introduction

- Binary search (Divide & Conquer)

```
int bsearch ( int s, int e, int A[], int x )
{
    if ( s == e )
        if ( A[s] == x )
            return s;
        else
            return NONE;
    m ← (s+e)/2;
    if ( A[m] == x )
        return m;
    else if ( A[m] > x )
        return bsearch ( s, m, A, x );
    else
        return bsearch ( m + 1, e, A, x );
}
```

1.0 Introduction

- Binary search → check three key points

```
int bsearch ( int s, int e, int A[], int x )
{
    if ( s == e )
        if ( A[s] == x )
            return s;
        else
            return NONE;
    m ← (s+e)/2;
    if ( A[m] == x )
        return m;
    else if ( A[m] > x )
        return bsearch ( s, m, A, x );
    else
        return bsearch ( m + 1, e, A, x );
}
```

1.0 Introduction

- Binary search (Divide & Conquer)
  - Performance analysis

$$T(n) = \begin{cases} 1, & \text{for } n = 1 \\ T(n/2) + 1, & \text{otherwise} \end{cases}$$

$$T(n) = \log n$$