hw3-2-a

February 4, 2025

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1 Question 2

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[84]:
```python
# Python matrix for distances
distances_matrix_w_HTX = [
    [0, 343, 163, 367, 526, 443, 589, 735,    614,  269],  # Baton Rouge, LA
    [343, 0, 262, 465, 528, 681, 349, 741,    339,  434],  # Little Rock, AR
    [163, 262, 0, 247, 381, 431, 414, 594,    606,  441],  # Jackson, MS
    [367, 465, 247, 0, 161, 211, 280, 369,    791,  633],  # Montgomery, AL
    [526, 528, 381, 161, 0, 260, 250, 214,    848,  793],  # Atlanta, GA
    [443, 681, 431, 211, 260, 0, 489, 355,    999,  710],  # Tallahassee, FL
    [589, 349, 414, 280, 250, 489, 0, 462,    678,  779],  # Nashville, TN
    [734, 741, 594, 369, 214, 355, 462, 0,    1061, 1002], # Columbia, SC
    [614, 339, 606, 791, 848, 999, 678, 1061, 0,    445],  # Oklahoma City, OK
    [269, 434, 441, 633, 793, 710, 779, 1002, 445,  0]     # Houston, TX
]

# City names corresponding to the rows/columns
cities = {
    0: "Baton Rouge, LA",
    1: "Little Rock, AR",
    2: "Jackson, MS",
    3: "Montgomery, AL",
    4: "Atlanta, GA",
    5: "Tallahassee, FL",
    6: "Nashville, TN",
    7: "Columbia, SC",
    8: "Oklahoma City, OK",
    #9: "Houston, TX"
}
```

I grabbed the houston data but I'm pretty sure I won't use it.

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[85]:
```
import pandas as pd

# Remove the last row
distances_matrix = distances_matrix_w_HTX[:-1]

# Remove the last element of each row
distances_matrix = [row[:-1] for row in distances_matrix]

# Create a DataFrame from the distances matrix
df_distances = pd.DataFrame(distances_matrix)

# Rename the columns and index to match the city names
df_distances.columns = [cities[i] for i in range(len(df_distances.columns))]
df_distances.index = [cities[i] for i in range(len(df_distances.index))]

display(df_distances)
```

	Baton Rouge, LA	Little Rock, AR	Jackson, MS	\
Baton Rouge, LA	0	343	163	
Little Rock, AR	343	0	262	
Jackson, MS	163	262	0	
Montgomery, AL	367	465	247	
Atlanta, GA	526	528	381	
Tallahassee, FL	443	681	431	
Nashville, TN	589	349	414	
Columbia, SC	734	741	594	
Oklahoma City, OK	614	339	606	

	Montgomery, AL	Atlanta, GA	Tallahassee, FL	\
Baton Rouge, LA	367	526	443	
Little Rock, AR	465	528	681	
Jackson, MS	247	381	431	
Montgomery, AL	0	161	211	
Atlanta, GA	161	0	260	
Tallahassee, FL	211	260	0	
Nashville, TN	280	250	489	
Columbia, SC	369	214	355	
Oklahoma City, OK	791	848	999	

	Nashville, TN	Columbia, SC	Oklahoma City, OK
Baton Rouge, LA	589	735	614
Little Rock, AR	349	741	339
Jackson, MS	414	594	606
Montgomery, AL	280	369	791
Atlanta, GA	250	214	848
Tallahassee, FL	489	355	999
Nashville, TN	0	462	678
Columbia, SC	462	0	1061

```
Oklahoma City, OK            678          1061                  0
```

[86]:
```
demand = [4590,
          3055,
          6205,
          5080,
          10912,
          22244,
          7051,
          5282,
          8023]

total_demand = sum(demand)

print(total_demand)
```

```
72442
```

Let's create our P-Median Model

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[87]:
```
from gurobipy import Model, GRB, quicksum
import pandas as pd

H = demand

C = distances_matrix

I = range(9) # Demand

J = range(9) # Supply

f = None # we don't need it for this formulation

# DataFrame to hold solutions for each value of p
df_solutions = pd.DataFrame(columns=["p", "Facility", "Assigned Cities", "Total␣
 ↪Cost"])

def solve_p_median(p):
    # Create the model
    model = Model("P-Median")

    # Decision variables
    y = model.addVars(I, J, vtype=GRB.BINARY, name="y")    # Assignment variables
    x = model.addVars(J, vtype=GRB.BINARY, name="x")       # Facility location␣
 ↪variables
```

```
    # Objective function
    model.setObjective(
        quicksum(H[i] * C[i][j] * y[i, j] for i in I for j in J),
        GRB.MINIMIZE
    )

    # Constraints
    # Each city must be assigned to one facility
    model.addConstrs((quicksum(y[i, j] for j in J) == 1 for i in J),␣
↪name="Assignment")

    # Assignment is only possible to open facilities
    model.addConstrs((y[i, j] <= x[j] for i in I for j in J),␣
↪name="OpenFacility")

    # Exactly p facilities must be opened
    model.addConstr(quicksum(x[j] for j in J) == p, name="FacilityCount")

    # Solve the model
    model.optimize()

    # Output results
    if model.status == GRB.OPTIMAL:
        print("Optimal solution found:")
        supply_route = []
        total_cost = model.objVal
        for i in I:
            for j in J:
                if y[i, j].x > 0.5:
                    supply_route.append([cities[i], cities[j]])
                    print(f"City {cities[i]} is assigned to facility␣
↪{cities[j]}.")
        print("Facilities opened at:")
        facilities = []
        for j in J:
            if x[j].x > 0.5:
                facilities.append(cities[j])
                print(f"Facility at {cities[j]}")

        # Save results to CSV
        df_facilities = pd.DataFrame(facilities, columns=["City"])
        df_facilities.to_csv(f"hw3-2-a-p{p}.csv", index=False)

        # Append results to solutions DataFrame
        df_solutions.loc[len(df_solutions)] = [p, facilities, supply_route,␣
↪total_cost]
```

```
    else:
        print("No optimal solution found.")

    return model

for i in range(1, 5):
    model = solve_p_median(i)
```

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 23.6.0
23G93)

CPU model: Apple M2 Max
Thread count: 12 physical cores, 12 logical processors, using up to 12 threads

Optimize a model with 91 rows, 90 columns and 252 nonzeros
Model fingerprint: 0x260866df
Variable types: 0 continuous, 90 integer (90 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [7e+05, 2e+07]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 1e+00]
Found heuristic solution: objective 3.556654e+07
Presolve removed 91 rows and 90 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.00 seconds (0.00 work units)
Thread count was 1 (of 12 available processors)

Solution count 2: 2.13576e+07 3.55665e+07

Optimal solution found (tolerance 1.00e-04)
Best objective 2.135758700000e+07, best bound 2.135758700000e+07, gap 0.0000%
Optimal solution found:
City Baton Rouge, LA is assigned to facility Montgomery, AL.
City Little Rock, AR is assigned to facility Montgomery, AL.
City Jackson, MS is assigned to facility Montgomery, AL.
City Montgomery, AL is assigned to facility Montgomery, AL.
City Atlanta, GA is assigned to facility Montgomery, AL.
City Tallahassee, FL is assigned to facility Montgomery, AL.
City Nashville, TN is assigned to facility Montgomery, AL.
City Columbia, SC is assigned to facility Montgomery, AL.
City Oklahoma City, OK is assigned to facility Montgomery, AL.
Facilities opened at:
Facility at Montgomery, AL
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 23.6.0
23G93)
```

```
CPU model: Apple M2 Max
Thread count: 12 physical cores, 12 logical processors, using up to 12 threads

Optimize a model with 91 rows, 90 columns and 252 nonzeros
Model fingerprint: 0x7cae7d15
Variable types: 0 continuous, 90 integer (90 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [7e+05, 2e+07]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+00]
Found heuristic solution: objective 2.766092e+07
Presolve time: 0.00s
Presolved: 91 rows, 90 columns, 252 nonzeros
Variable types: 0 continuous, 90 integer (90 binary)

Root relaxation: objective 1.416479e+07, 47 iterations, 0.00 seconds (0.00 work
units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0     0               0    1.416479e+07 1.4165e+07  0.00%     -    0s

Explored 1 nodes (47 simplex iterations) in 0.00 seconds (0.00 work units)
Thread count was 12 (of 12 available processors)

Solution count 2: 1.41648e+07 2.76609e+07

Optimal solution found (tolerance 1.00e-04)
Best objective 1.416478600000e+07, best bound 1.416478600000e+07, gap 0.0000%
Optimal solution found:
City Baton Rouge, LA is assigned to facility Little Rock, AR.
City Little Rock, AR is assigned to facility Little Rock, AR.
City Jackson, MS is assigned to facility Little Rock, AR.
City Montgomery, AL is assigned to facility Tallahassee, FL.
City Atlanta, GA is assigned to facility Tallahassee, FL.
City Tallahassee, FL is assigned to facility Tallahassee, FL.
City Nashville, TN is assigned to facility Little Rock, AR.
City Columbia, SC is assigned to facility Tallahassee, FL.
City Oklahoma City, OK is assigned to facility Little Rock, AR.
Facilities opened at:
Facility at Little Rock, AR
Facility at Tallahassee, FL
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 23.6.0
23G93)
```

```
CPU model: Apple M2 Max
Thread count: 12 physical cores, 12 logical processors, using up to 12 threads

Optimize a model with 91 rows, 90 columns and 252 nonzeros
Model fingerprint: 0x03bcf16c
Variable types: 0 continuous, 90 integer (90 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [7e+05, 2e+07]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 3e+00]
Found heuristic solution: objective 2.990353e+07
Presolve time: 0.00s
Presolved: 91 rows, 90 columns, 252 nonzeros
Variable types: 0 continuous, 90 integer (90 binary)

Root relaxation: objective 9.144098e+06, 39 iterations, 0.00 seconds (0.00 work
units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0     0               0    9144098.0000 9144098.00  0.00%     -    0s

Explored 1 nodes (39 simplex iterations) in 0.01 seconds (0.00 work units)
Thread count was 12 (of 12 available processors)

Solution count 2: 9.1441e+06 2.99035e+07

Optimal solution found (tolerance 1.00e-04)
Best objective 9.144098000000e+06, best bound 9.144098000000e+06, gap 0.0000%
Optimal solution found:
City Baton Rouge, LA is assigned to facility Tallahassee, FL.
City Little Rock, AR is assigned to facility Oklahoma City, OK.
City Jackson, MS is assigned to facility Atlanta, GA.
City Montgomery, AL is assigned to facility Atlanta, GA.
City Atlanta, GA is assigned to facility Atlanta, GA.
City Tallahassee, FL is assigned to facility Tallahassee, FL.
City Nashville, TN is assigned to facility Atlanta, GA.
City Columbia, SC is assigned to facility Atlanta, GA.
City Oklahoma City, OK is assigned to facility Oklahoma City, OK.
Facilities opened at:
Facility at Atlanta, GA
Facility at Tallahassee, FL
Facility at Oklahoma City, OK
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 23.6.0
23G93)
```

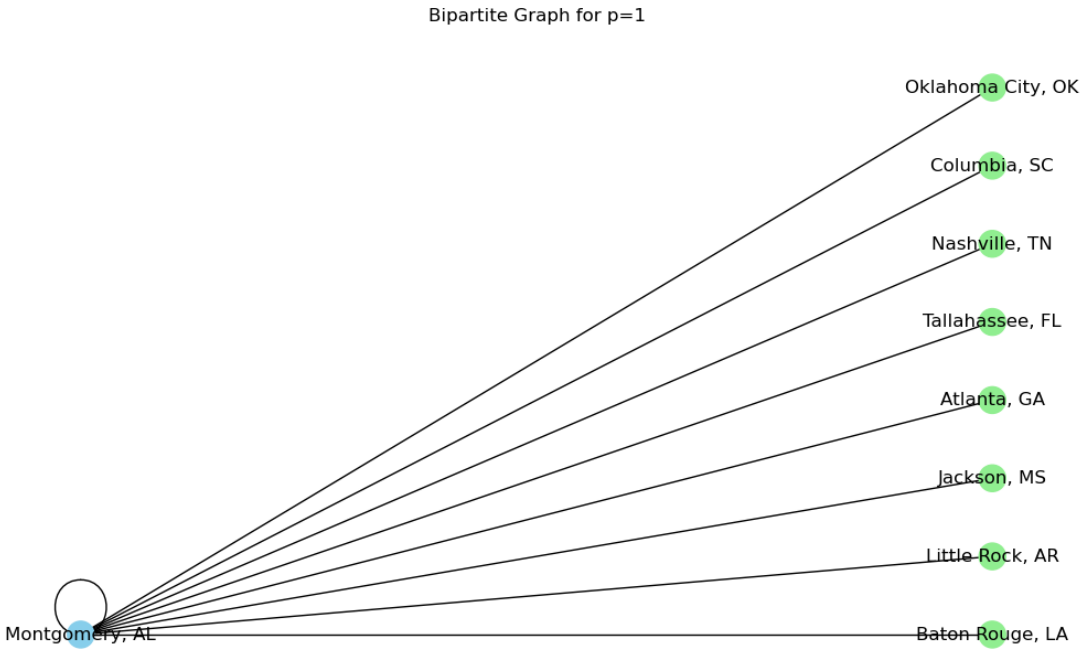
```
CPU model: Apple M2 Max
Thread count: 12 physical cores, 12 logical processors, using up to 12 threads

Optimize a model with 91 rows, 90 columns and 252 nonzeros
Model fingerprint: 0x98239b45
Variable types: 0 continuous, 90 integer (90 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [7e+05, 2e+07]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 4e+00]
Found heuristic solution: objective 3.297366e+07
Presolve time: 0.00s
Presolved: 91 rows, 90 columns, 252 nonzeros
Variable types: 0 continuous, 90 integer (90 binary)

Root relaxation: objective 5.259558e+06, 25 iterations, 0.00 seconds (0.00 work
units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0     0               0    5259558.0000 5259558.00  0.00%     -    0s

Explored 1 nodes (25 simplex iterations) in 0.01 seconds (0.00 work units)
Thread count was 12 (of 12 available processors)

Solution count 2: 5.25956e+06 3.29737e+07

Optimal solution found (tolerance 1.00e-04)
Best objective 5.259558000000e+06, best bound 5.259558000000e+06, gap 0.0000%
Optimal solution found:
City Baton Rouge, LA is assigned to facility Jackson, MS.
City Little Rock, AR is assigned to facility Jackson, MS.
City Jackson, MS is assigned to facility Jackson, MS.
City Montgomery, AL is assigned to facility Atlanta, GA.
City Atlanta, GA is assigned to facility Atlanta, GA.
City Tallahassee, FL is assigned to facility Tallahassee, FL.
City Nashville, TN is assigned to facility Atlanta, GA.
City Columbia, SC is assigned to facility Atlanta, GA.
City Oklahoma City, OK is assigned to facility Oklahoma City, OK.
Facilities opened at:
Facility at Jackson, MS
Facility at Atlanta, GA
Facility at Tallahassee, FL
Facility at Oklahoma City, OK
```

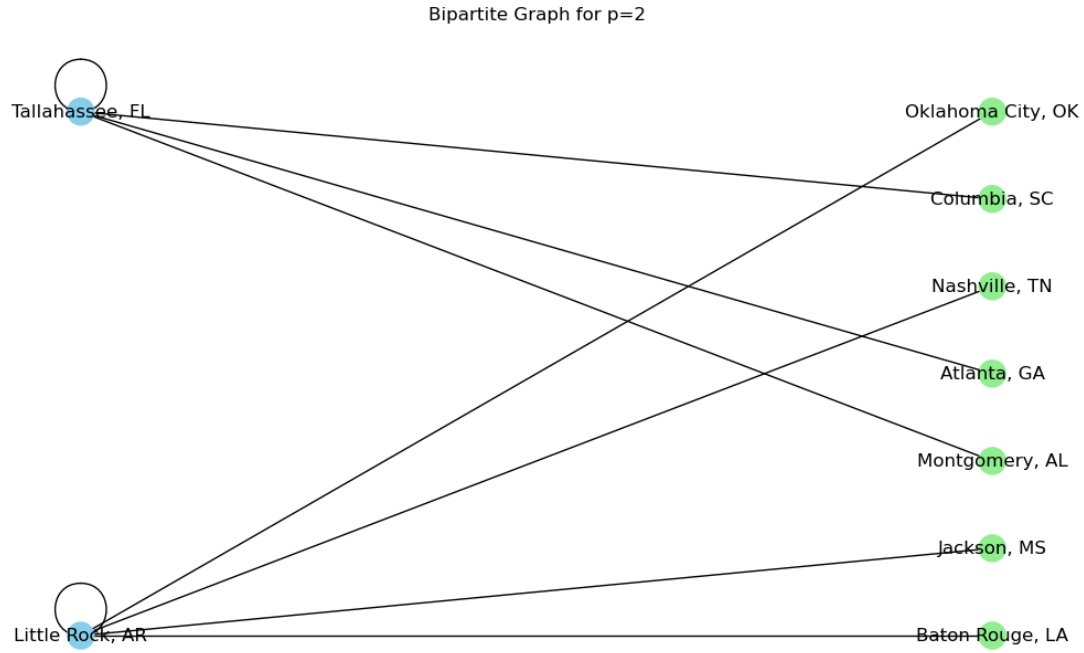
[88]:
```
import locale

# Set the locale to 'en_US.UTF-8' for currency formatting
locale.setlocale(locale.LC_ALL, 'en_US.UTF-8')

# Display the df_solutions DataFrame with formatted Total Cost
df_solutions['Total Cost'] = df_solutions['Total Cost'].apply(lambda x: locale.
 ↪currency(x, grouping=True))
display(df_solutions)
```

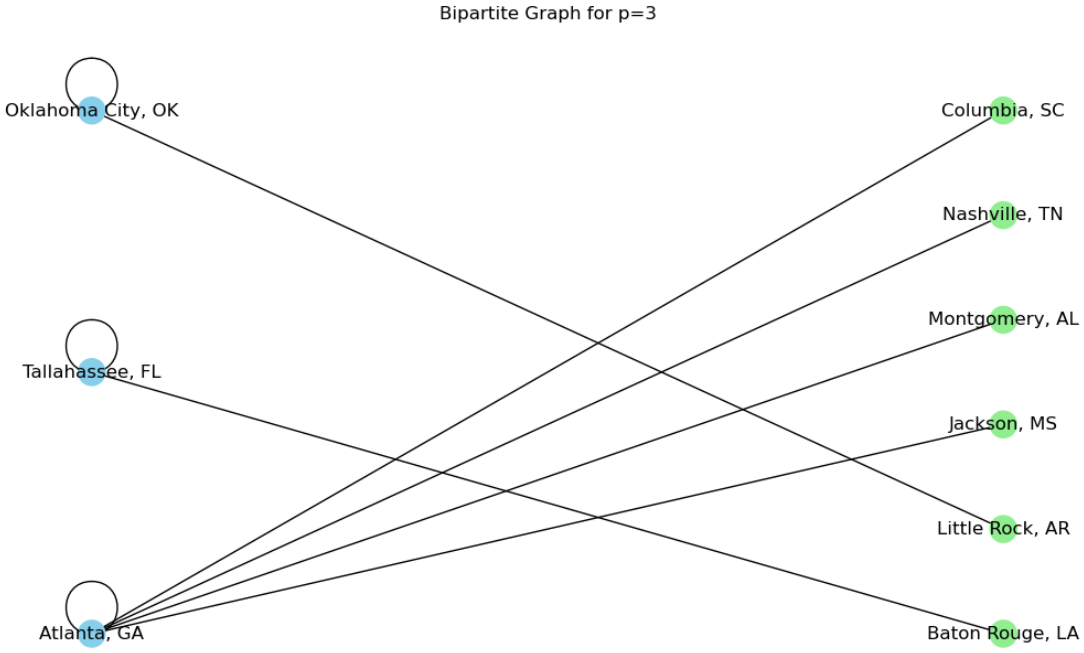
```
   p                                          Facility  \
0  1                                  [Montgomery, AL]
1  2                [Little Rock, AR, Tallahassee, FL]
2  3  [Atlanta, GA, Tallahassee, FL, Oklahoma City, OK]
3  4  [Jackson, MS, Atlanta, GA, Tallahassee, FL, Ok…

                                    Assigned Cities      Total Cost
0  [[Baton Rouge, LA, Montgomery, AL], [Little Ro…  $21,357,587.00
1  [[Baton Rouge, LA, Little Rock, AR], [Little R…  $14,164,786.00
2  [[Baton Rouge, LA, Tallahassee, FL], [Little R…   $9,144,098.00
3  [[Baton Rouge, LA, Jackson, MS], [Little Rock,…   $5,259,558.00
```

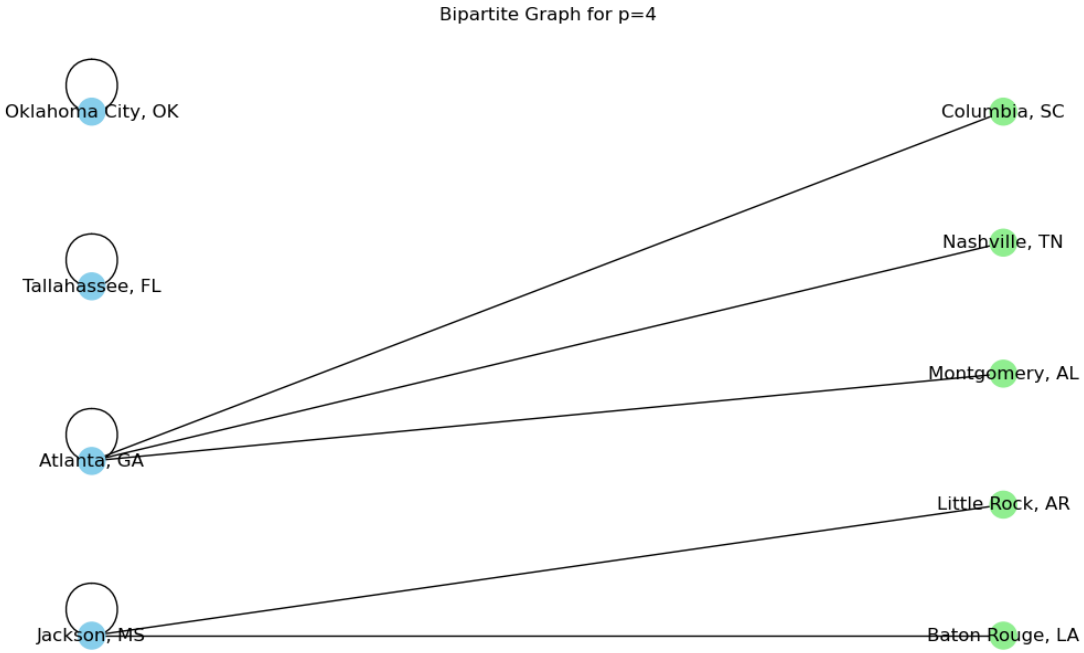
[89]:
```
import networkx as nx

import matplotlib.pyplot as plt

def plot_bipartite_graph(df_solutions):
    for index, row in df_solutions.iterrows():
        p = row['p']
        facilities = row['Facility']
        assigned_cities = row['Assigned Cities']

        B = nx.Graph()

        # Add nodes with the node attribute "bipartite"
        B.add_nodes_from(facilities, bipartite=0)
        B.add_nodes_from([city for city, _ in assigned_cities], bipartite=1)

        # Add edges
        for city, facility in assigned_cities:
            B.add_edge(city, facility)

        # Draw the bipartite graph
        plt.figure(figsize=(10, 6))
        pos = nx.drawing.layout.bipartite_layout(B, facilities)
        nx.draw(B, pos, with_labels=True, node_color=['skyblue' if node in␣
 ↪facilities else 'lightgreen' for node in B.nodes()])
```

```
        plt.title(f'Bipartite Graph for p={p}')
        plt.show()

plot_bipartite_graph(df_solutions)
```

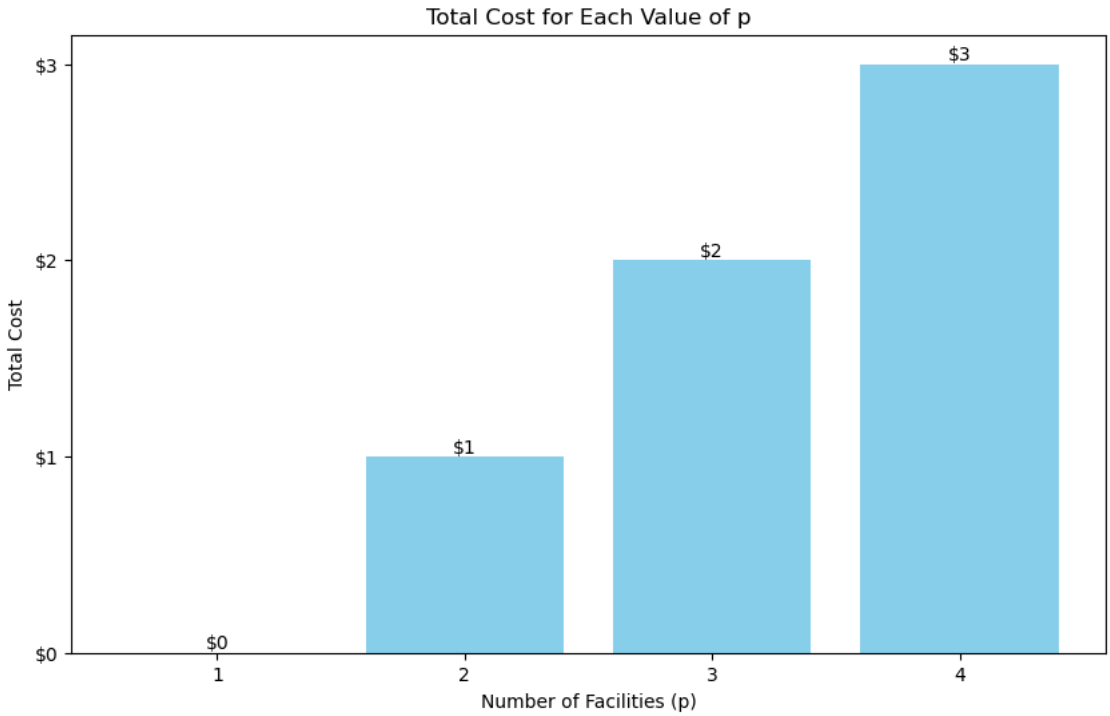





Bipartite Graph for p=3
Oklahoma City, OK
Tallahassee, FL
Atlanta, GA
Columbia, SC
Nashville, TN
Montgomery, AL
Jackson, MS
Little Rock, AR
Baton Rouge, LA


Bipartite Graph for p=4
Oklahoma City, OK
Tallahassee, FL
Atlanta, GA
Jackson, MS
Columbia, SC
Nashville, TN
Montgomery, AL
Little Rock, AR
Baton Rouge, LA

[90]:
```
import locale
import matplotlib.pyplot as plt
from matplotlib.ticker import FuncFormatter

# Set the locale to 'en_US.UTF-8' for currency formatting
locale.setlocale(locale.LC_ALL, 'en_US.UTF-8')

# Extract the values of p and the corresponding total costs
p_values = df_solutions['p']
total_costs = df_solutions['Total Cost']

# Create a bar plot
plt.figure(figsize=(10, 6))
bars = plt.bar(p_values, total_costs, color='skyblue')
plt.xlabel('Number of Facilities (p)')
plt.ylabel('Total Cost')
plt.title('Total Cost for Each Value of p')
plt.xticks(p_values)  # Ensure all p values are shown on the x-axis

# Add labels on top of the bars
for bar in bars:
    yval = bar.get_height()
    plt.text(bar.get_x() + bar.get_width()/2, yval, locale.currency(yval,␣
 ↪grouping=True).split('.')[0], ha='center', va='bottom')

# Format y-axis as currency without decimals
formatter = FuncFormatter(lambda x, pos: locale.currency(x, grouping=True).
 ↪split('.')[0])
plt.gca().yaxis.set_major_formatter(formatter)

plt.show()
```



Tekin thought this looked correct.
