hw4_1

November 20, 2024

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1 Question 1

Solving for flow rate

[85]:
```
import numpy as np

# Define the coefficient matrix A and the constants vector b
A = np.array([
    [-1,    .1,     0,      0],
    [1,     -1,     0,      0.2],
    [0,     .2,     -1,     0],
    [0,     .7,     0,      -1]
])

# Right-hand side constants vector (all zero in this case)
b = np.array([-1, 0, 0, 0])

# Solve the system using np.linalg.solve if it's square and has a unique␣
↪solution
# If it does not have a unique solution, we can use np.linalg.lstsq to get a␣
↪least-squares solution
try:
    lam = np.linalg.solve(A, b)
except np.linalg.LinAlgError:
    # If matrix A is singular, use least-squares solution
    lam, residuals, rank, s = np.linalg.lstsq(A, b, rcond=None)

# The flow rates are the following
lam
```

[85]:
```
array([1.13157895, 1.31578947, 0.26315789, 0.92105263])
```

The solution for the outflow values is:

$$\lambda_1 = 1.13157895$$
$$\lambda_2 = 1.31578947$$
$$\lambda_3 = 0.26315789$$
$$\lambda_4 = 0.92105263$$

Now let's get the utilization

[86]:
```
# Define the vector te
te = np.array([0.8, 0.7, 3, 1])

# Calculate the vector u as the product of lam and te
u = lam * te
u
```

[86]:
```
array([0.90526316, 0.92105263, 0.78947368, 0.92105263])
```

$$u_1 = 0.90526316$$
$$u_2 = 0.92105263$$
$$u_3 = 0.78947368$$
$$u_4 = 0.92105263$$

Let's now find the SCV for arrivals

[87]:
```
from sympy import symbols, Eq, solve

# Define symbols for the variables
scv_a_1, scv_a_2, scv_a_3, scv_a_4 = symbols('scv_a_1, scv_a_2, scv_a_3,␣
↪scv_a_4')
```

$$C_a^2(1)$$

[88]:
```
p21=0.1
scv_e_2=0.75

eq1 = Eq(scv_a_1,1/lam[0]+(lam[1]*p21/
↪lam[0])*(p21*((1-(u[1])**2)*scv_a_2+(u[1])**2*scv_e_2)+1-p21))
```

$$C_a^2(2)$$

[89]:
```
p42=0.2
scv_e_4=0.5

p12=1
scv_e_1=1.5

eq2 = Eq(scv_a_2,
         lam[3]*p42/
 ↪lam[1]*(p42*((1-(u[3])**2)*scv_a_4+(u[3])**2*scv_e_4)+1-p42)+
            lam[0]*p12/
 ↪lam[1]*(p12*((1-(u[0])**2)*scv_a_1+(u[0])**2*scv_e_1)+1-p12))
```

$$C_a^2(3)$$

[90]:
```
p23 = 0.2

eq3 = Eq(scv_a_3,
         lam[1]*p23/
 ↪lam[2]*(p23*((1-(u[1])**2)*scv_a_2+(u[1])**2*scv_e_2)+1-p23))
```

$$C_a^2(4)$$

[91]:
```
p24 = 0.7
scv_e_2 = 0.75
eq4 = Eq(scv_a_4,
         lam[1]*p24/
 ↪lam[3]*(p24*((1-(u[1])**2)*scv_a_2+(u[1])**2*scv_e_2)+1-p24))
```

[92]:
```
# Solve the system of equations
solution = solve((eq1, eq2, eq3, eq4), (scv_a_1, scv_a_2, scv_a_3, scv_a_4))

print("Solution:", solution)
scv_a_1 = solution[scv_a_1]
scv_a_2 = solution[scv_a_2]
scv_a_3 = solution[scv_a_3]
scv_a_4 = solution[scv_a_4]
```

```
Solution: {scv_a_1: 0.998133040094418, scv_a_2: 1.33974178577721, scv_a_3:
0.967888289623990, scv_a_4: 0.887609013683965}
```

$$C_a^2(1) = 0.998133040094418$$
$$C_a^2(2) = 1.33974178577721$$
$$C_a^2(3) = 0.967888289623990$$
$$C_a^2(4) = 0.887609013683965$$

Let's now work on the WIP of each station

Now let's solve for Cycle Time

$$CT_s(k) = \frac{C_a^2(k) + C_e^2(k)}{2} \cdot \frac{u_k}{1 - u_k} \cdot t_e(k) + t_e(k)$$

[93]:
```
CT_1=(scv_a_1+scv_e_1)/2 * u[0]/(1-u[0]) * te[0]+te[0]
CT_1
```

[93]: 10.3484196199164

[94]:
```
CT_2=(scv_a_2+scv_e_2)/2 * u[1]/(1-u[1]) * te[1]+te[1]
CT_2
```

[94]: 9.23311229192359

[95]:
```
CT_3=(scv_a_3+scv_e_2)/2 * u[2]/(1-u[2]) * te[2]+te[2]
CT_3
```

[95]: 12.6631216291349

[96]:
```
CT_4=(scv_a_4+scv_e_4)/2 * u[3]/(1-u[3]) * te[3]+te[3]
CT_4
```

[96]: 9.09438591315646

[97]:
```
print("CT_1:", CT_1, "CT_2:", CT_2, "CT_3:", CT_3, "CT_4:", CT_4)
```

```
CT_1: 10.3484196199164 CT_2: 9.23311229192359 CT_3: 12.6631216291349 CT_4:
9.09438591315646
```

Now let's get the WIP of each workstation

$$\text{WIP}_s(k) = \lambda_k \cdot CT_s(k)$$

[98]:
```
WIP_1=CT_1*lam[0]
WIP_1
```

[98]:

11.7100537804318

[99]:
```
WIP_2=CT_2*lam[1]
WIP_2
```

[99]:

12.1488319630573

[100]:
```
WIP_3=CT_3*lam[2]
WIP_3
```

[100]:

3.33240042871972

[101]:
```
WIP_4=CT_4*lam[3]
WIP_4
```

[101]:

8.37640807790726

[102]:
```
print("WIP_1:", WIP_1, "WIP_2:", WIP_2, "WIP_3:", WIP_3, "WIP_4:", WIP_4)
```

```
WIP_1: 11.7100537804318 WIP_2: 12.1488319630573 WIP_3: 3.33240042871972 WIP_4:
8.37640807790726
```

[103]:
```
WIP = WIP_1 + WIP_2 + WIP_3 + WIP_4
WIP
```

[103]:

35.5676942501161

Cycle Time of a Job in the System

$$CT_s = \frac{WIP_s}{TH_s}$$

[105]:
```
TH=1
CT=WIP/1
CT
```

[105]:

35.5676942501161

Profit of the System

[109]:
```
lam_3_exit=lam[2]
lam_3_exit
```

[109]:
```
0.26315789473684215
```

[110]:
```
lam_4_exit=lam[3]*.8
lam_4_exit
```

[110]: 0.736842105263158

[111]:
```
lam_exit=lam_3_exit+lam_4_exit
lam_exit
```

[111]: 1.0