Stock Portfolio Optimization

An Algorithmic Improvement in Python's View

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Background: Stock Portfolio

- Stock portfolio investment is the art and science of making decisions about investing mix of stocks.
- Allocate portfolio investment across stocks

Stock Name	Weights	Return	Variance
Google	50%	3%	0.01
Apple	30%	2%	0.02
Facebook	20%	1%	0.03

- Mean Return $= 50\% \times 3\% + 30\% \times 2\% + 20\% \times 1\% = 2.3\%$
- Risk $= 50\% \times 0.01 + 30\% \times 0.02 + 20\% \times 0.03 = 0.017$
- Our goal: Choose weights to minimize the risk of a portfolio given an objective return.

Mean-variance portfolio selection

- Equivalent formula:

$$\min_\alpha \frac{1}{2}\alpha' V\alpha$$
$$\text{s.t.}\hat{\mu}'\alpha = \hat{r}$$

where

$\alpha \in \mathbb{R}^n$ is a weight vector
$V \in \mathbb{R}^{n\times n}$ is a covariance matrix of a portfolio
$\hat{\mu} \in \mathbb{R}^n$ is a mean return vector
$\hat{r} \in \mathbb{R}$ is the objective return

Analytical Solution

- Lagrangian function: $\mathcal{L} = \frac{1}{2}\alpha' V\alpha + \nu(\hat{\mu}'\alpha - \hat{r})$
- Set graident to 0: $\nabla_\alpha \mathcal{L} = V\alpha + \nu\hat{\mu} = 0 \Rightarrow \alpha = -\nu V^{-1}\hat{\mu}$
- Substitute into constraint: $\hat{r} = \hat{\mu}'\alpha = -\nu\hat{\mu}' V^{-1}\hat{\mu} \Rightarrow \nu = -\frac{\hat{r}}{\hat{\mu}' V^{-1}\hat{\mu}}$
- Thus, $\alpha^* = \hat{r} \cdot \frac{V^{-1}\hat{\mu}}{\hat{\mu}' V^{-1}\hat{\mu}} = \hat{r}\gamma^*$

Potential Problems

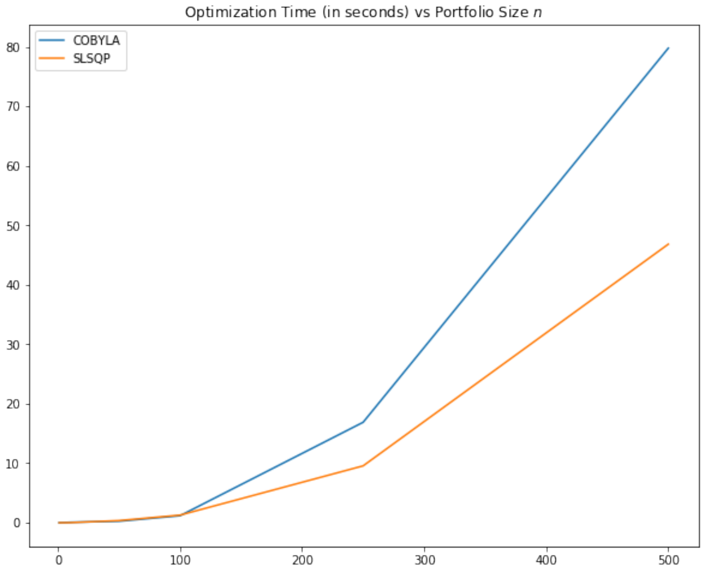
However, it is not ideal in real market situations.

- The constraints are too basic for actual transactions.
  e.g. No short-sells constraints, Diversitification constraints
- There are tens of thousands stocks in the world, not to mention other forms of assets.
- ......

General Optimizer

As a typical constrained quadratic optimization problem, we can also apply built-in functions (e.g. `scipy.optimize.minimize.SLSQP()`) from `SciPy` to solve it.

Result


Optimization Time (in seconds) vs Portfolio Size n
COBYLA
SLSQP
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500

Potential Problems

- As shown in the previous result, the portfolio optimization is slow.
- The result may not converge.

Project Plan

- Optimization in `SciPy` remains in the quadratic camp and many cannot incorporate constrains.
- Karmarka *et al.* (1950) proposes combining duality theory with Newton method, which can help us to reformulate the problem into a linear one.
- Our plan is to survey the active researches in this area, develop/implement a better optimization algorithm in python.
- Can be further improved by using CUDA, cython, *etc.*