CS5401 FS2018 Assignment 1d

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Contents

Introduction

Assignment 1d involved implementing a Multi-Objective Evolutionary Algorithm (MOEA) to more effectively solve Light Up puzzles by balancing the fulfillment of three objectives:

1. maximize the number of cells lit up (represented in this implementation as a ratio of lit cells to the total number of white cells)

2. minimize the number of bulbs shining on each other

3. minimize the number of black cell adjacency constraint violations

For BONUS #1, a fourth objective was added, namely minimizing the number of bulbs placed on the board.

This report outlines this solution's particular implementation of a MOEA, the impact of initialization strategies on the MOEA's performance, a comparison between parent selection, survival strategy, and survival selection strategies on MOEA performance, as well as the impact of increasing the number of objectives on non-domination and MOEA performance (BONUS #1).

MOEA Overview

The MOEA implemented in this assignment is based on the NSGA-II algorithm. It begins, similar to a standard evolutionary algorithm, by creating an initial population using either uniform random or validity enforced plus uniform random initialization, the settings for which are specified in the algorithm configuration file. That population is evaluated and the subfitnesses are determined and assigned to each individual in the population.

The population is then evaluated on the basis of non-domination. A list of Pareto fronts is created from the initial population where all genotypes in a given front are not dominated by any other genotypes in that front while genotypes in higher level fronts are dominated by genotypes in lower level fronts. The 'best' genotypes, those in the best level of non-domination, are assigned to level number one. Subsequent levels increase in increments of one for other levels of non-domination.

The fitness of each genotype is then set to its level in the list of Pareto fronts, with individuals exhibiting a smaller fitness (level number) are more fit. A binary tournament selection is performed to choose breeding parents. Then offspring are created using an n-point crossover recombination (with n determined in the configuration file). Following that, mutation is performed, completing the child population.

For the standard NSGA-II configuration (exhibited in the deliverables configuration folder), the plus survival strategy is exhibited, combining the children and parent populations into one large population from which to choose the new population. Individuals are then selected for survival using a binary tournament selection and the process is repeated using the new population until the end of the experiment.

Impact of Initialization on MOEA Performance

The effect of Validity Enforced plus Uniform Random versus Uniform Random initialization was examined in this experiment for both the provided puzzle and randomly generated puzzles. One would assume that an initialization method utilizing Validity Enforced initialization would outperform a solely Uniform Random initialization. After performing statistical analysis on the experiment data, it was concluded that there is in fact no tangible difference between the initialization methods for the tested puzzles. Table 6 and Table 7 each display statistical analysis supporting this finding.

This statistical analysis consisted of performing an f-test, which determined if variances could be treated as equal. In both cases, the f-test yielded that unequal variances should be assumed. Following the f-test, the two-tailed t-test was performed assuming unequal variances. This test yielded (in both cases) that neither initialization method was statistically better for the set of Light Up puzzles tested.

To visually interpret the data, plots of evaluations versus average local subfitness and evaluations versus local best subfitness were graphed for each of the subfitnesses collected in this experiment (not including the bonus): ratio of lit cells to total number of white cells, number of bulbs shining on each other, and number of black cell constraints not met. For concision, figures pertaining to only the provided puzzle are discussed in this section as the randomly generated puzzle results are quite similar. Figures 1, 2, 3, 7, 8, and 9 depict experiment plots for the provided puzzle while figures 4, 5, 6, 10, 11, and 12 depict experiment plots for the randomly generated puzzle.

Figure 1 and Figure 7 depict the lit cell ratio subfitness plots for the Validity Enforced

plus Uniform Random and Uniform Random initialized experiments, respectively. In the uniform random case (Figure 7), both the average local list cell ratio and the local best lit cell ratio started lower than those of the validity enforced experiment. However, as the experiments progressed, both the average and best subfitnesses of each experiment reached appreciatively the same point without the best subfitness plateauing. This implies that letting the experiment run for longer would produce a more fit solution, with respect to the lit cell ratio. Note that the lit cell ratio subfitness, the metric was maximized.

Figure 2 and Figure 8 depict the evaluations versus subfitness plots for the bulb shine constraint violations. These plots behaved quite similarly to the lit cell ratio plots in that the Validity Enforced plus Uniform Random plot started with a higher average number of bulb shine constraints violated while the plain Uniform Random plot had a lower number of initial constraint violations. This is logical as enforcing validity has the potential to place more bulbs, which creates opportunity for more bulbs to shine on each other, further adding to the number of bulb shine violations. The local best for both plots stayed right at zero bulb constraint violations, implying that most, if not all, experiments always had at least one member of the population with no bulb shine constraints. Because this subfitness is to be minimized as part of the MOEA, it is peculiar that the average number of bulb constraints increases as the experiments continue. This implies that as more bulbs are placed on the board, the multi-objective nature of the algorithm allows for more and more bulbs to shine on each other, keeping those individuals with more bulb-shine in the population so long as the levels of non-domination dictate it.

The third subfitness, black cell constraint violations, was examined for both initialization schemes in Figure 3 and Figure 9. Before examining these plots, it was hypothesized that the Validity Enforced plus Uniform Random initialization method would be superior with respect to minimizing the black cell constraint violations when compared to the Uniform Random only initialization. While the statistical analysis was not granular enough to definitively prove this, the graphs provide anecdotal evidence that the Validity Enforced plus Uniform Random initialization method is in fact superior when it comes to the black cell constraint violations. This conclusion was drawn because the Validity Enforced plus Uniform Random method had lower average and local best black cell constraints violated at each point on the graph, across all experiments.

Comparison of Parent Selection, Survival Strategy, and Survival Selection Strategies

The following MOEA configurations were compared to determine which combination was optimal.

- Parent Selection
  - Fitness Proportional
  - Binary Tournament

- Survival Selection
  - Fitness Proportional
  - Binary Tournament

- Survival Strategy

  - Plus
  - Comma

Note that for these comparisons, all MOEA configurations were tested against randomly generated puzzles for concision.

Tables 1, 2, 3, 4, and 5 outline the type of configurations compared, the figures associated with each MOEA, and the table containing statistical analysis results for each comparison, where applicable. Note: not all MOEA configuration subfitness graphs are included in this report for clarity.

TODO: ADD FIGURES? REMOVE FIGURE LABELS FROM OUTLINE TABLES?

Table 1 outlines the comparison made between an MOEA configured with fitness proportional parent and survival selection versus tournament parent and survival selection, both using the plus survival strategy. Table 8 illustrates an f-test being applied resulting in equal variance assumption followed by a t-test assuming equal variances. The result of the t-test was that the MOEA configuration employing tournament for both parent and survival selection was superior, for the given test cases, to the MOEA configured to use fitness proportional selection for both the parent and survival selection. The fact that the binary tournament configuration produced the better performing algorithm means that a lower selection pressure drives up algorithm performance, as a binary tournament applies less selection pressure than fitness proportional selection.

The next comparison, outlined in Table 2 and statistically analyzed in Table 9, compared a fitness proportional parent and survival selection configuration with a tournament parent and survival selection configuration, with both configurations using a comma survival strategy. This comparison again involved an f-test which yielded that equal variances could be assumed, followed by a t-test assuming equal variances. The t-test yielded that neither configuration was statistically better for the given test cases. Unlike for the plus survival strategy, the comma survival strategy does not seem to be as sensitive to selection pressure brought about by varying survival and parent selection methods. This may be due to the fact that the comma survival strategy automatically removes all parent genotypes from the population after each subsequent generation, only allowing children genotypes to comprise each new generation, ensuring that stale, yet optimally fit, genotypes that have existed for generations are removed for new genotypes to take their place.

The third comparison is outlined in Table 3 and is statistically analyzed in Table 10. This experiment yielded that the fitness proportional parent selection and tournament survival selection configuration outperformed the tournament parent selection and fitness proportional survival selection configuration. With the statistical analysis of this comparison, unequal variances were assumed by the f-test and the t-test yielded the results mentioned above. This finding showed that higher selection pressure for parent selection (fitness proportional selection) was more effective than higher selection pressure for survival selection.

BONUS #1: Impact of Increasing Number of Objectives on Number of Non-Domination and MOEA Performance

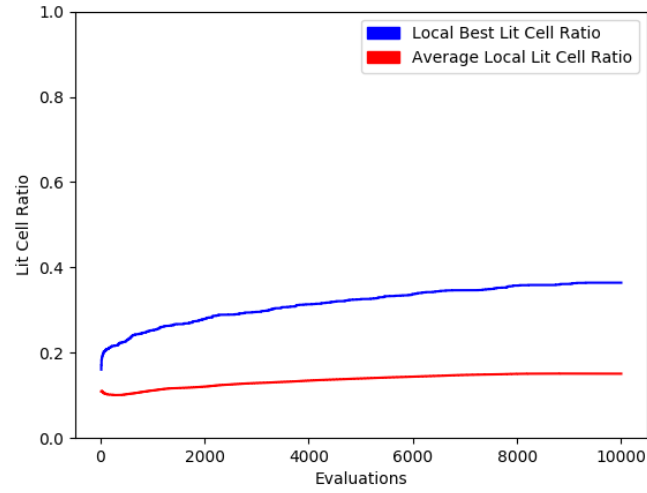TODO: need to create config files and run test for this

Figure 1: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Lit Cell Ratio Subfitness, Validity Enforced plus Uniform Random Initialized, Provided Puzzle**, Averaged Over All Runs

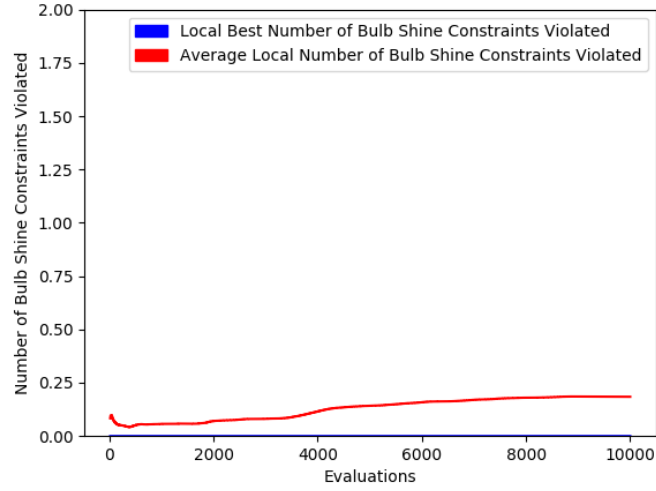# Appendix: Figures and Tables

Figure 2: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Bulb Shine Constraint Subfitness, Validity Enforced plus Uniform Random Initialized, Provided Puzzle**, Averaged Over All Runs
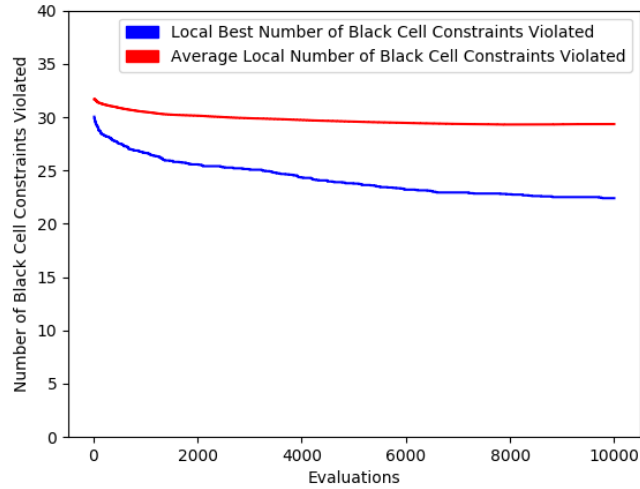

Figure 3: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Black Cell Constraint Subfitness, Validity Enforced plus Uniform Random Initialized, Provided Puzzle**, Averaged Over All Runs
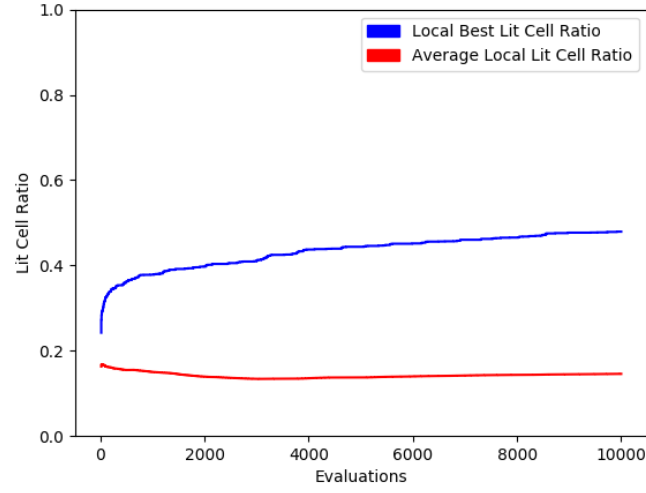
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Figure 4: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Lit Cell Ratio Subfitness, Validity Enforced plus Uniform Random Initialized, Randomly Generated Puzzle**, Averaged Over All Runs
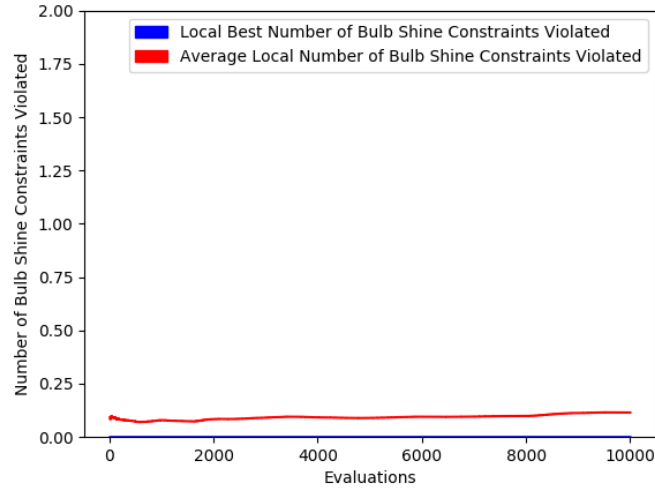

Figure 5: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Bulb Shine Constraint Subfitness, Validity Enforced plus Uniform Random Initialized, Randomly Generated Puzzle**, Averaged Over All Runs

Figure 6: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Black Cell Constraint Subfitness, Validity Enforced plus Uniform Random Initialized, Randomly Generated Puzzle**, Averaged Over All Runs

Figure 7: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Lit Cell Ratio Subfitness, Uniform Random Initialized, Provided Puzzle**, Averaged Over All Runs

Figure 8: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Bulb Shine Constraint Subfitness, Uniform Random Initialized, Provided Puzzle**, Averaged Over All Runs
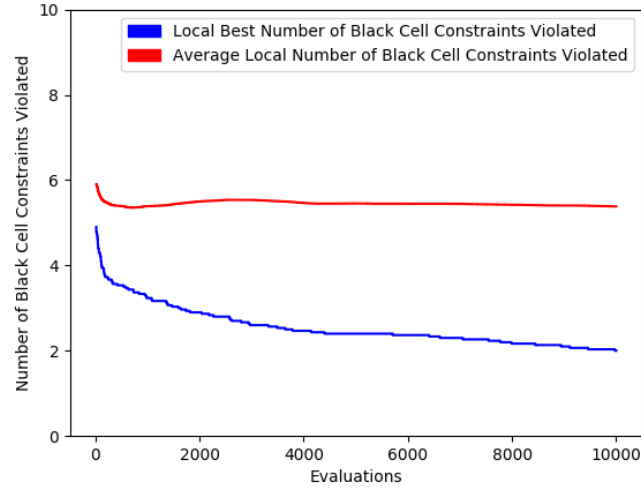

Figure 9: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Black Cell Constraint Subfitness, Uniform Random Initialized, Provided Puzzle**, Averaged Over All Runs
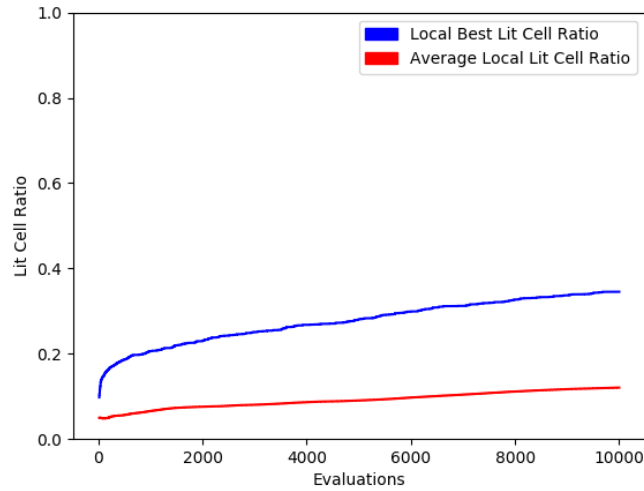
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Figure 10: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Lit Cell Ratio Subfitness, Uniform Random Initialized, Randomly Generated Puzzle**, Averaged Over All Runs
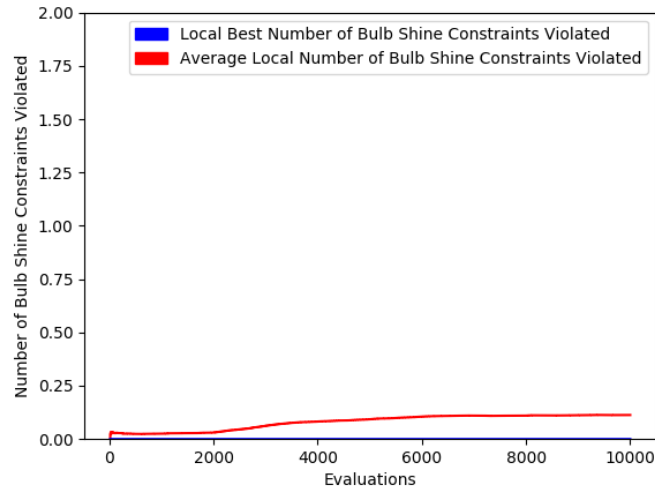
Figure 11: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Bulb Shine Constraint Subfitness, Uniform Random Initialized, Randomly Generated Puzzle**, Averaged Over All Runs
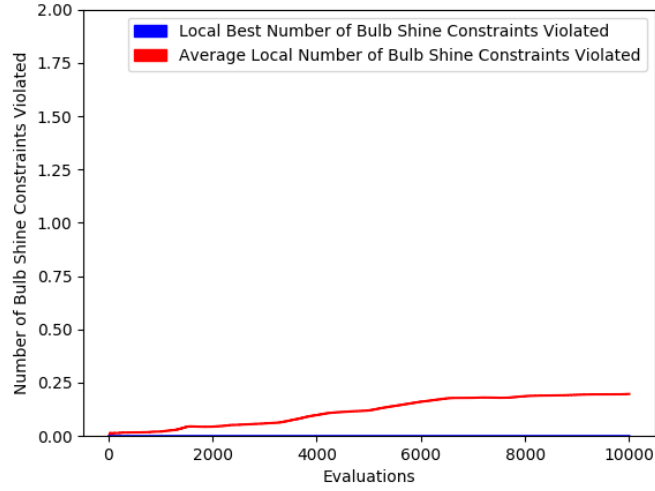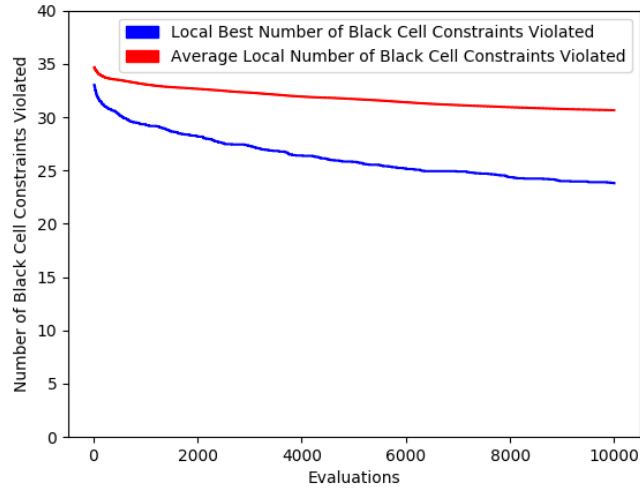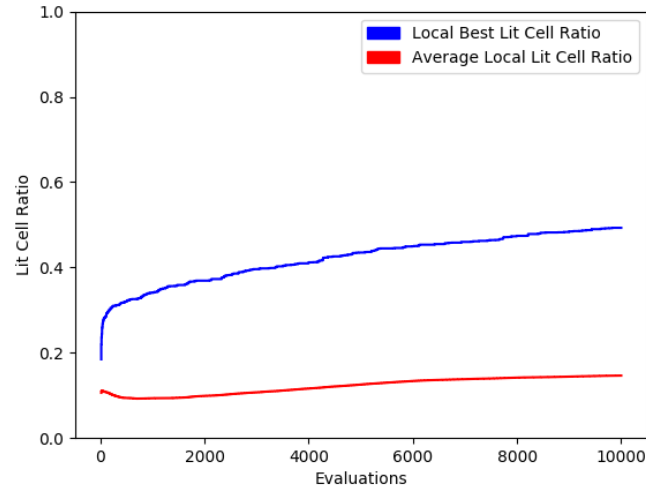
10
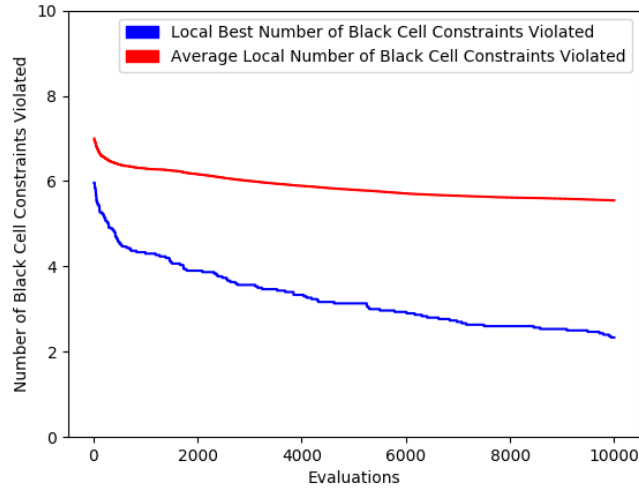
Figure 12: Evaluations versus Average Local Subfitness and Evaluations versus Local Best Subfitness for the **Black Cell Constraint Subfitness, Uniform Random Initialized, Randomly Generated Puzzle**, Averaged Over All Runs

Table 1: Fitness Proportional Parent & Survival Selection Configuration vs Tournament Parent & Survival Selection Configuration, Randomly Generated Puzzle, Comma Survival Strategy, Comparison Info

	Configuration A	Configuration B
Parent Selection	Fitness Proportional	Tournament Selection
Survival Selection	Fitness Proportional	Tournament Selection
Survival Strategy	Plus	Plus
Subfitness 1 Graph (lit cell ratio)	Figure N/A	Figure N/A
Subfitness 2 Graph (bulb shine constraint violations)	Figure N/A	Figure N/A
Subfitness 3 Graph (black cell constraint violations)	Figure N/A	Figure N/A
Statistical Analysis	**Table 8**	

11

Table 2: Fitness Proportional Parent & Survival Selection Configuration vs Tournament Parent & Survival Selection Configuration, Randomly Generated Puzzle, Comma Survival Strategy, Comparison Info

	Configuration A	Configuration B
Parent Selection	Fitness Proportional	Tournament Selection
Survival Selection	Fitness Proportional	Tournament Selection
Survival Strategy	Comma	Comma
Subfitness 1 Graph (lit cell ratio)	Figure N/A	Figure N/A
Subfitness 2 Graph (bulb shine constraint violations)	Figure N/A	Figure N/A
Subfitness 3 Graph (black cell constraint violations)	Figure N/A	Figure N/A
Statistical Analysis	**Table 9**	

Table 3: Fitness Proportional Parent Selection & Tournament Survival Selection Configuration vs Tournament Parent Selection & Fitness Proportional Survival Selection Configuration, Randomly Generated Puzzle, Plus Survival Strategy, Comparison Info

	Configuration A	Configuration B
Parent Selection	Fitness Proportional	Tournament Selection
Survival Selection	Tournament Selection	Fitness Proportional Selection
Survival Strategy	Plus	Plus
Subfitness 1 Graph (lit cell ratio)	Figure N/A	Figure N/A
Subfitness 2 Graph (bulb shine constraint violations)	Figure N/A	Figure N/A
Subfitness 3 Graph (black cell constraint violations)	Figure N/A	Figure N/A
Statistical Analysis	**Table 10**	

Table 4: Fitness Proportional Parent Selection & Tournament Survival Selection Configuration vs Tournament Parent Selection & Fitness Proportional Survival Selection, Randomly Generated Puzzle, Comma Survival Strategy, Comparison Info

	Configuration A	Configuration B
Parent Selection	Fitness Proportional	Tournament Selection
Survival Selection	Tournament Selection	Fitness Proportional
Survival Strategy	Comma	Comma
Subfitness 1 Graph (lit cell ratio)	Figure N/A	Figure N/A
Subfitness 2 Graph (bulb shine constraint violations)	Figure N/A	Figure N/A
Subfitness 3 Graph (black cell constraint violations)	Figure N/A	Figure N/A
Statistical Analysis	**Table 11**	

Table 5: Tournament Parent & Survival Selection (Comma Survival Strategy) vs Tournament Parent & Survival Selection (Plus Selection Strategy), Randomly Generated Puzzle, Comparison Info

	Configuration A	Configuration B
Parent Selection	Tournament Selection	Tournament Selection
Survival Selection	Tournament Selection	Tournament Selection
Survival Strategy	Comma	Plus
Subfitness 1 Graph (lit cell ratio)	Figure N/A	Figure N/A
Subfitness 2 Graph (bulb shine constraint violations)	Figure N/A	Figure N/A
Subfitness 3 Graph (black cell constraint violations)	Figure N/A	Figure N/A
Statistical Analysis	**Table 12**	

Table 6: Statistical Analysis performed on the Uniform Random and Validity Enforced Uniform Random Initialized, Randomly Generated Puzzle, EA configurations

	random_gen	random_gen_uniform_random_init
mean	1.10325284795678	1.0764116503308314
variance	0.04970344216568983	0.04355857958196582
standard deviation	0.22294268807406498	0.20870692269775296
observations	30	30
df	29	29
F	1.141071234248146	
F critical	0.5373999648406917	
Unequal variances assumed		
observations	30	
df	31	
t Stat	0.47331304656977363	
P two-tail	0.6377741699825987	
t Critical two-tail	2.0395	
Nether random_gen_uniform_random_init nor random_gen is statistically better		

Table 7: Statistical Analysis performed on the Uniform Random and Validity Enforced Uniform Random Initialized, Provided Puzzle, EA configurations

	website_puzzle	website_puzzle_uniform_random_init
mean	0.8031737242867948	0.7959489651519909
variance	0.0006036666878279215	0.00058947613524421
standard deviation	0.024569633254878164	0.02427912962287178
observations	30	30
df	29	29
F	1.02407348183676	
F critical	0.5373999648406917	
Unequal variances assumed		
observations	30	
df	31	
t Stat	1.1263571505364551	
P two-tail	0.26465388827990055	
t Critical two-tail	2.0395	
Nether website_puzzle_uniform_random_init nor website_puzzle is statistically better		

Table 8: Statistical Analysis performed on the Fitness Proportional Parent & Survival Selection Configuration vs Tournament Parent & Survival Selection Configuration, Randomly Generated Puzzle, Plus Survival Strategy

	random_gen_fitness_proportional_parent_fitness_proportional_survival_plus	random_gen_tournament_parent_tournament_survival_plus
mean	0.9712902683331008	1.1621500278718004
variance	0.05379675879756281	0.0613398685104960
standard deviation	0.23194128308165152	0.2476688686744784
observations	30	30
df	29	29
F	0.877027618478956	
F critical	0.5373999648406917	
Equal variances assumed		
observations	30	
df	58	
t Stat	-3.02905123657769584	
P two-tail	0.0036590951523001046	
t Critical two-tail	2.0017	
random_gen_tournament_parent_tournament_survival_plus is statistically better than random_gen_fitness_proportional_parent_fitness_proportional_survival_plus		

Table 9: Statistical Analysis performed on the Fitness Proportional Parent & Survival Selection Configuration vs Tournament Parent & Survival Selection Configuration, Randomly Generated Puzzle, Comma Survival Strategy

	random_gen_fitness_proportional_parent_fitness_proportional_survival_comma	random_gen_tournament_parent_tournament_survival_comma
mean	1.0292821828716079	1.053450949371725
variance	0.036008620889199365	0.026268301489784093
standard deviation	0.18975937628796993	0.16207498724289351
observations	30	30
df	29	29
F	1.3708012641473353	
F critical	0.5373999648406917	
Equal variances assumed		
observations	30	
df	58	
t Stat	-0.52154275129917932	
P two-tail	0.6039746747540546	
t Critical two-tail	2.0017	
Nether random_gen_tournament_parent_tournament_survival_comma nor random_gen_fitness_proportional_parent_fitness_proportional_survival_comma is statistically better		

Table 10: Statistical Analysis performed on the Fitness Proportional Parent Selection & Tournament Survival Selection Configuration vs Tournament Parent Selection & Fitness Proportional Survival Selection Configuration, Randomly Generated Puzzle, Plus Survival Strategy

	random_gen_fitness_proportional_parent_tournament_survival_plus	random_gen_tournament_parent_fitness_proportional_survival_plus
mean	1.1900153275106078	0.9222749663618831
variance	0.06049925303163906	0.02800642701535593
standard deviation	0.2459659590911699	0.16735125545252294
observations	30	30
df	29	29
F	2.160190556022379	
F critical	0.5373999648406917	
Unequal variances assumed		
observations	30	
df	31	
t Stat	4.846489025268326	
P two-tail	1.2117965929260821e-05	
t Critical two-tail	2.0395	
random_gen_fitness_proportional_parent_tournament_survival_plus is statistically better than random_gen_tournament_parent_fitness_proportional_survival_plus		

Table 11: Statistical Analysis performed on the Fitness Proportional Parent Selection & Tournament Survival Selection Configuration vs Tournament Parent Selection & Fitness Proportional Survival Selection, Randomly Generated Puzzle, Comma Survival Strategy

	random_gen_fitness_proportional_parent_tournament_survival_comma	random_gen_tournament_parent_fitness_proportional_survival_comma
mean	1.1186544987668476	1.0557988630231476
variance	0.04925392618449863	0.027554956293447713
standard deviation	0.22193225584510834	0.16599685627579733
observations	30	30
df	29	29
F	1.787479742662873	
F critical	0.5373999648406917	
Unequal variances assumed		
observations	30	
df	31	
t Stat	1.221342582229864	
P two-tail	0.22729007385576178	
t Critical two-tail	2.0395	
Nether random_gen_tournament_parent_fitness_proportional_survival_comma nor random_gen_fitness_proportional_parent_tournament_survival_comma is statistically better		

Table 12: Statistical Analysis performed on the Tournament Parent & Survival Selection (Comma Survival Strategy) vs Tournament Parent & Survival Selection (Plus Selection Strategy), Randomly Generated Puzzle

	random_gen_tournament_parent_tournament_survival_comma	random_gen_tournament_parent_tournament_survival_plus
mean	1.053450949371725	1.1621500278718004
variance	0.026268301489784093	0.06133986851049603
standard deviation	0.16207498724289351	0.2476688686744784
observations	30	30
df	29	29
F	0.4282418943446097	
F critical	0.5373999648406917	
Unequal variances assumed		
observations	30	
df	31	
t Stat	-1.9776642228780876	
P two-tail	0.05349296004844127	
t Critical two-tail	2.0395	
Nether random_gen_tournament_parent_tournament_survival_plus nor random_gen_tournament_parent_tournament_survival_comma is statistically better		