信陽師範學院
数学与统计学院
SCHOOL OF MATHEMATICS AND STATISTICS

第7章 MATLAB符号运算

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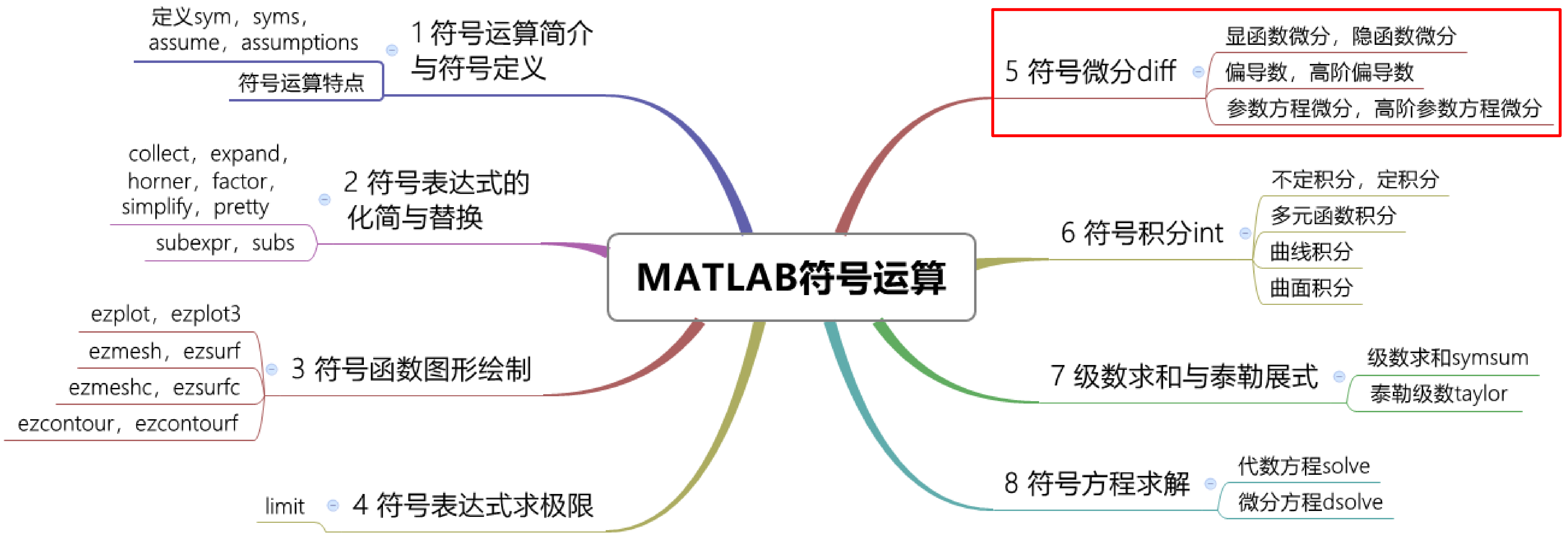
第7章 MATLAB符号运算思维导图



符号运算与数值运算的区别：数值计算的表达式、矩阵变量中不允许有未定义的自由变量，而符号计算可以含有未定义的符号变量。符号计算存放的是精确数据，耗存储空间，运行速度慢，但结果精度高；数值计算则是以一定精度来计算的，计算结果有误差，但是运行速度快。

1. diff函数简介

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函数diff：实现函数求导和求微分，可以实现一元函数求导和多元函数求偏导。

- diff(S)，实现表达式S的求导，自变量由函数symvar确定。
- diff(S,'v')，实现表达式对指定变量v的求导，该语句还可以写为diff(S,sym('v'))。
- diff(S,n)，求S的n阶导。
- diff(S,'v',n)，求S对v的n阶导，该表达式还可以写为diff(S,n,'v')。

例1：求如下函数的微分

$$(1)\ y_1=\sqrt{1+e^x}\qquad (2)\ y_2=x\cos x\qquad (3)\begin{cases}x=a\cos t\\ y=b\sin t\end{cases}\qquad (4)\ z=\frac{xe^y}{y^2}\qquad (5)\ x^2+y^2+z^2=a^2$$

(1) 缺省，(2) 2阶和3阶导数，(3) 参数方程1阶、2阶导数，(4) 偏导数，(5) 隐函数导数

1. diff函数简介

- 偏导数：已知二元函数$f(x,y)$，若求$\dfrac{\partial^{m+n} f}{\partial x^m \partial y^n}$，则可以用函数 f = diff(diff(f,x,m),y,n) 或 f = diff(diff(f,y,n),x,m)。
- 隐函数的偏导数，公式如下：

$$\frac{\partial x_i}{\partial x_j} = -\frac{\dfrac{\partial}{\partial x_j} f(x_1, x_2, \cdots, x_n)}{\dfrac{\partial}{\partial x_i} f(x_1, x_2, \cdots, x_n)}$$

- 参数方程的导数：$y = f(t)$，$x = g(t)$，则

$$\frac{dy}{dx} = \frac{f'(t)}{g'(t)} \qquad \frac{d^2 y}{dx^2} = \frac{d}{dt}\left(\frac{f'(t)}{g'(t)}\right)\frac{1}{g'(t)} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{1}{g'(t)}$$

1. diff函数简介

```
>> syms a b t x y z;
>> fh1=sqrt(1+exp(x));
%求(1)，未指定求导变量和阶数，按缺省规则处理
>> df1 = diff(fh1)
df1 = exp(x)/(2*(exp(x) + 1)^(1/2))
>> fh2=x*cos(x);
>>  df2 = diff(fh2,x,2)  %求(2)，求f对x的二阶导数
df2 = - 2*sin(x) - x*cos(x)
>> df23 = diff(fh2,x,3) %求(2)，求f对x的三阶导数
df23 = x*sin(x) - 3*cos(x)
>>  ft1=a*cos(t);  ft2=b*sin(t);
%求(3)。按参数方程求导公式求y对x的导数
>> df3 = diff(ft2)/diff(ft1)
df3 = -(b*cos(t))/(a*sin(t))
>> df32 = (diff(ft1)*diff(ft2,2)-diff(ft1,2)*diff(ft2))/(diff(ft1))^3
%求(3)。求y对x的二阶导数
df32 = -(a*b*cos(t)^2 + a*b*sin(t)^2)/(a^3*sin(t)^3)
>> fh4=x*exp(y)/y^2;
>> df4x = diff(fh4,x)   %求(4)。z对x的偏导数
df4x = exp(y)/y^2
>> df4y = diff(fh4,y)   %求(4)。z对y的偏导数
df4y = (x*exp(y))/y^2 - (2*x*exp(y))/y^3
>> fh5=x^2+y^2+z^2-a^2;
%求(5)。按隐函数求导公式求z对x的偏导数
>> zx = -diff(fh5,x)/diff(fh5,z)
zx = -x/z
>> zy = -diff(fh5,y)/diff(fh5,z) %按隐函数求导公式求z对y的偏导数
zy = -y/z
```

2. 函数微分

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例2：计算微分(1) $\dfrac{\partial^2 (y^2 \sin x^2)}{\partial x^2}$；　(2) $\dfrac{\partial}{\partial y}\left(\dfrac{\partial^2}{\partial x^2} y^2 \sin x^2\right)$；(3) $f(x)=\dfrac{\sin x}{x^2+4x+3}$， $\dfrac{d^4 f(x)}{dx^4}$

```
>> syms x y t
>> D1 = diff(sin(x^2)*y^2,x,2)
D1 =
  2*y^2*cos(x^2) - 4*x^2*y^2*sin(x^2)
>> D2 = diff(D1,y)
D2 =
  4*y*cos(x^2) - 8*x^2*y*sin(x^2)
>> fh = sin(x)/(x^2 + 4*x + 3);
>> fh1 = diff(fh)
fh1 =
    cos(x)/(x^2 + 4*x + 3) - (sin(x)*(2*x + 4))/(x^2 + 4*x + 3)^2
```

```
fh4 = diff(fh,4)
%式子非常长，想办法化简
>> fh4s = collect(simplify(fh4),[sin(x),cos(x)])
fh4s =
((x^8 + 16*x^7 + 72*x^6 - 32*x^5 - 1094*x^4 -
3120*x^3 - 3120*x^2 + 192*x + 1581)/(x^2 + 4*x +
3)^5)*sin(x) + (-(- 8*x^7 - 112*x^6 - 552*x^5 -
1040*x^4 + 296*x^3 + 4080*x^2 + 5640*x +
2448)/(x^2 + 4*x + 3)^5)*cos(x)
>> subexpr(fh4s)
```

2. 函数微分

例3：试推导函数$F(t) = t^2 f(t) sint$的3阶导函数公式，并得出$f(t) = e^{-t}$时的3阶导数，将这样得出的结果与直接求导的结果相比较。

```
>> syms t f(t)
>> Ft = t^2*f(t)*sin(t);
>> DFt3 = diff(Ft,t,3)   %对t求三阶导数
>> res1 = subs(DFt3,f(t),exp(-t))
res1 = 6*exp(-t)*cos(t) - 6*exp(-t)*sin(t) + 2*t^2*exp(-t)*cos(t) + 2*t^2*exp(-t)*sin(t) - 12*t*exp(-t)*cos(t)
>> res1 = simplify(res1)
res1 = 2*exp(-t)*(3*cos(t) - 3*sin(t) + t^2*cos(t) + t^2*sin(t) - 6*t*cos(t))
>> Ftsubs = subs(Ft,f(t),exp(-t));   %先把exp(-t)带入，再求三阶导数
>> res2 = simplify(diff(Ftsubs,t,3))
res2 = 2*exp(-t)*(3*cos(t) - 3*sin(t) + t^2*cos(t) + t^2*sin(t) - 6*t*cos(t))
```

2. 函数微分

例4：试求三阶导数矩阵 $H(x)=\begin{bmatrix} 4\sin 5x & e^{-4x^2} \\ 3x^2+4x+1 & \sqrt{4x^2+2} \end{bmatrix}$

```
syms x;
Hx = [4*sin(5*x),exp(-4*x^2); 3*x^2+4*x+1,sqrt(4*x^2+2)];
H3 = diff(Hx,x,3)
H3 =
[ -500*cos(5*x),                    192*x*exp(-4*x^2) - 512*x^3*exp(-4*x^2)]
[             0, (24*2^(1/2)*x^3)/(2*x^2 + 1)^(5/2) - (12*2^(1/2)*x)/(2*x^2 + 1)^(3/2)]
Hxs = simplify(H3)  %查看简化结果
Hxs =
[ -500*cos(5*x),    -64*x*exp(-4*x^2)*(8*x^2 - 3)]
[             0, -(12*2^(1/2)*x)/(2*x^2 + 1)^(5/2)]
```

3. 隐函数的偏导数

隐函数的高阶偏导数算法：

$$\frac{\partial x_i}{\partial x_j} = -\frac{\dfrac{\partial}{\partial x_j} f(x_1, x_2, \cdots, x_n)}{\dfrac{\partial}{\partial x_i} f(x_1, x_2, \cdots, x_n)}$$

$$F_n(x, y) = \frac{\partial^n y}{\partial x^n}$$

$$= \frac{\partial F_{n-1}(x, y)}{\partial x} + \frac{\partial F_{n-1}(x, y)}{\partial y} F_1(x, y)$$

```
function dy = impldiff(f,x,y,n)
  if mod(n,1) ~= 0  % n非整数
     error('n should positive integer, please correct.')
  else
     F1 = -simplify(diff(f,x)/diff(f,y)); % 一阶偏导
     dy = F1;
     for i = 2:n
        % 按递推公式编写
        dy = simplify(diff(dy,x) + diff(dy,y)*F1);
     end
  end
end
```

3. 隐函数的偏导数

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例5：已知隐函数 $f(x,y)=(x^2-2x)e^{-x^2-y^2-xy}=0$，试求 $\dfrac{\partial^3 y}{\partial x^3}$

```
>> syms x y;
>> f = (x^2-2*x)*exp(-x^2-y^2-x*y);
>> F1 = impldiff(f,x,y,1)   %调用高阶偏导数算法
F1 =
    (2*x + 2*x*y - x^2*y + 4*x^2 - 2*x^3 - 2)/(x*(x + 2*y)*(x - 2))
>> F3 = impldiff(f,x,y,3)   %调用高阶偏导数算法
F3 =
    (3*(- 3*x^3 + 6*x^2 + 4*x - 4)^3)/(2*x^3*(x + 2*y)^5*(x - 2)^3) + (4*(x^3 - 3*x^2 + 6*x -
4))/(x^3*(x + 2*y)*(x - 2)^3) + (3*(- 9*x^7 + 54*x^6 - 108*x^5 + 60*x^4 + 40*x^3 - 48*x^2 + 64*x -
32))/(2*x^3*(x + 2*y)^3*(x - 2)^3)
```

4. 参数方程的导数

- 参数方程的导数：$y=f(t),\quad x=g(t)$，则 $\dfrac{d^n y}{dx^n}$
- 递推公式如下：

$$\frac{dy}{dx}=\frac{f'(t)}{g'(t)}$$

$$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{f'(t)}{g'(t)}\right)\frac{1}{g'(t)}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{1}{g'(t)}$$

……

$$\frac{d^n y}{dx^n}=\frac{d}{dt}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)\frac{1}{g'(t)}$$

```
function result = paradiff(y,x,t,n)
  if mod(n,1) ~=0
     error('n should positive integer, please correct');
  elseif n==1
     result = diff(y,t)/diff(x,t);
  else
     %递归调用
     result = diff(paradiff(y,x,t,n-1),t)/diff(x,t);
  end
end
```

4. 参数方程的导数

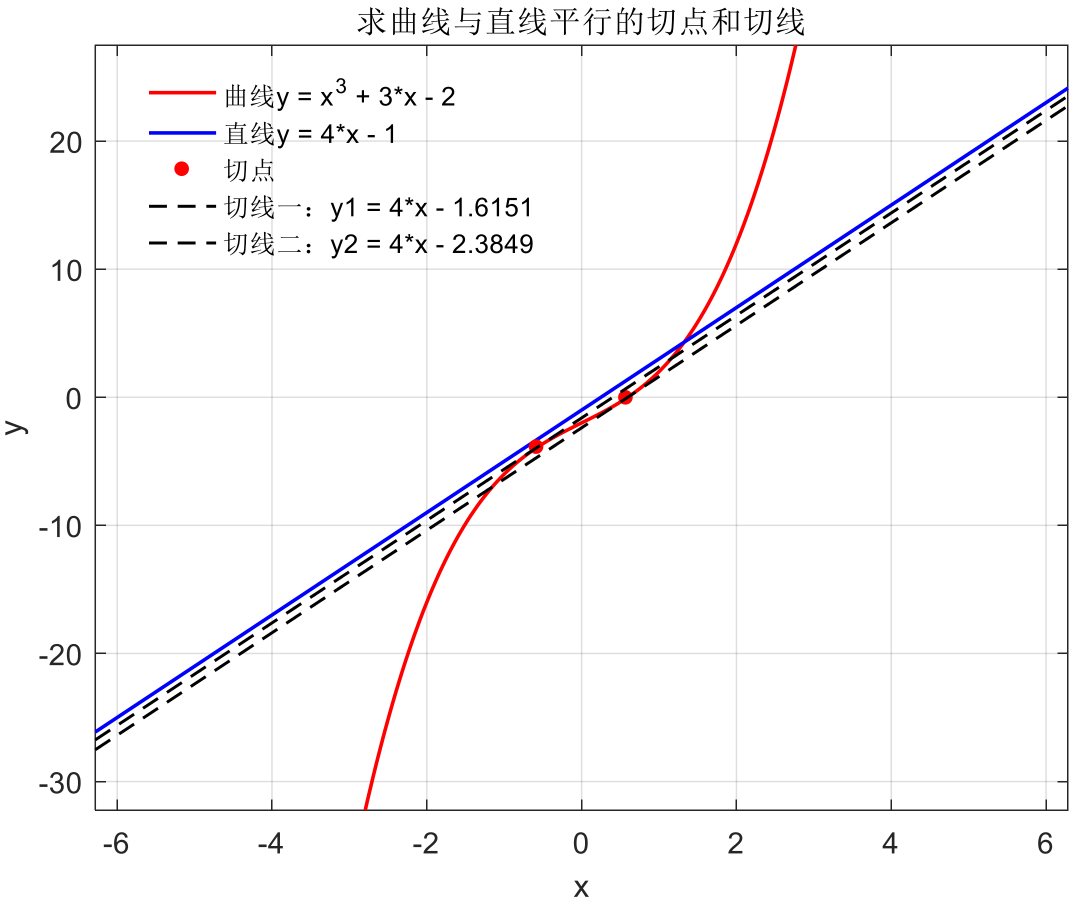
例6：已知参数方程 $y=\dfrac{\sin t}{(t+1)^3}$，$x=\dfrac{\cos t}{(t+1)^3}$，求 $\dfrac{dy}{dx}$ $\dfrac{d^3y}{dx^3}$

```
>> syms t
>> y = sin(t)/(t+1)^3;   x = cos(t)/(t+1)^3;   f = paradiff(y,x,t,1);
>> [n,d] = numden(f);    f = simple(n)/simple(d);
f =
  -(cos(t) - 3*sin(t) + t*cos(t))/(3*cos(t) + sin(t) + t*sin(t))
>> f = paradiff(y,x,t,3);
>> [n,d] = numden(f);        %提取分子与分母
>> f = simple(n)/simple(d)   %重写f，简化形式
f =
  (3*(t + 1)^7*(23*cos(t) + 24*sin(t) - 6*t^2*cos(t) - 4*t^3*cos(t) -  t^4*cos(t) + 12*t^2*sin(t) +
4*t^3*sin(t) - 4*t*cos(t) + 32*t*sin(t)))/(3*cos(t) + sin(t) + t*sin(t))^5
```

微分应用

例7：在曲线 $y = x^3 + 3x - 2$上哪一点的切线与直线$y = 4x - 1$平行。

```
>> x = sym('x');
>> y = x^3+3*x-2;    %定义曲线函数
>> df = diff(y);         %对曲线求导数
>> gh = df-4;
%求方程df-4=0的根，即求曲线何处的导数为4
>> sol = solve(gh)
sol = -3^(1/2)/3      3^(1/2)/3
>> fval = subs(y,x,sol)
fval = - (10*3^(1/2))/9 – 2       (10*3^(1/2))/9 - 2
>> plot(sol,fval,'r.','MarkerSize',15)
>> y1 = 4*(x-sol(1))+fval(1);   %切线一
>> y2 = 4*(x-sol(2))+fval(2);  %切线二
```

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