信陽師範学院
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SCHOOL OF MATHEMATICS AND STATISTICS

第6章 MATLAB符号运算

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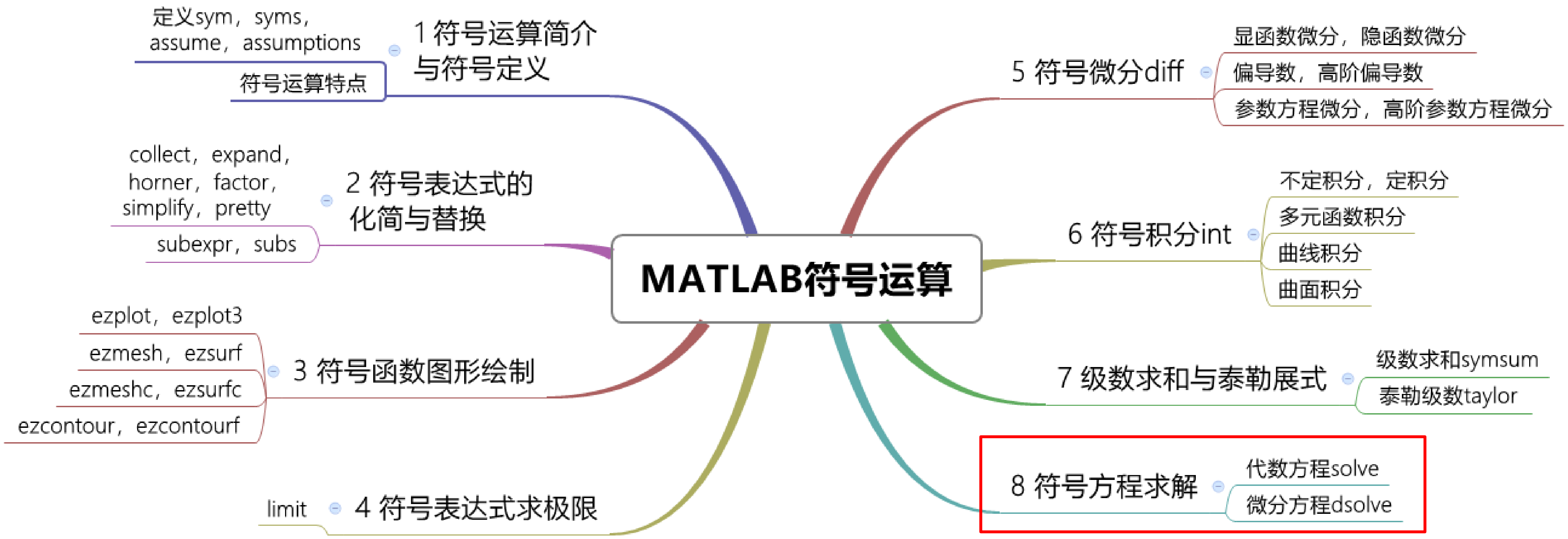
第7章 MATLAB符号运算思维导图



符号运算与数值运算的区别：数值计算的表达式、矩阵变量中不允许有未定义的自由变量，而符号计算可以含有未定义的符号变量。符号计算存放的是精确数据，耗存储空间，运行速度慢，但结果精度高； 数值计算则是以一定精度来计算的，计算结果有误差，但是运行速度快。

1. 代数方程符号求解函数介绍

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代数方程包括线性方程、非线性方程和超越方程等。函数 solve： 用于求解代数方程和方程组，其调用格式如下：

- S = solve(eqn,var)，求解方程 eqn 的解，对指定自变量求解。
- S = solve(eqns, Name,Value))，求由方程eqns 组成的系统， uses additional options specified by one or more Name,Value pair arguments；
- [y1,...,yN,parameters,conditions] = solve(eqns,vars,'ReturnConditions',true) ， returns the additional arguments parameters and conditions that specify the parameters in the solution and the conditions on the solution. 。

2. 求解代数方程

例1：求解代数方程（非线性，超越）

$$(1)\frac{1}{(x+1)}+\frac{4x}{x^2-4}=1+\frac{2}{x-2};\qquad (2)x-(x^3-4x-7)^{\frac{1}{3}}=1;$$

$$(3)x+xe^x-10=0;\qquad (4)2\sin(3x-\frac{\pi}{4})=1$$

```
>> syms x
>> eq1 = 1/(x+1) + 4*x/(x^2-4) == 1 + 2/(x-2);
>> sol1 = solve(eq1,x)   %sol1 =  2^(1/2)     -2^(1/2)
>> eq2 = x-(x^3-4*x-7)^(1/3) == 1;
>> sol2 = solve(eq3,x)    %sol2 = 3
>> eq3 = x+x*exp(x)-10 == 0;
>> sol3 = solve(eq4,x)  % vpasolve(eq4,x)
sol3 = 1.6335061701558463841931651789789
```

```
>> eq4 = 2*sin(3*x-pi/4) == 1;
>> [solx,parameters,conditions] =
solve(eq4,x,'ReturnConditions',true)
solx =
    (5*pi)/36 + (2*pi*k)/3
    (13*pi)/36 + (2*pi*k)/3
parameters =
   k
conditions =
    in(k, 'integer')
    in(k, 'integer')
```

2. 求解代数方程

例2：具有特殊要求的代数方程求解

$$(1)\ e^{\ln x \ln 3x} - 4 = 0; \quad (2)\ x^2 + 5x - 6 = 0; \quad (3)\ x^3 + x^2 + a = 0; \quad (4)\ \sin x + \cos 2x = 1$$

```
>> syms x
>> eqn = exp(log(x)*log(3*x)) == 4;
>> S = solve(eqn,x)
警告: Cannot solve symbolically. Returning a numeric approximation instead.
S = - 14.009379055223370038369334703094 - 2.9255310052111119036668717988769i
>> S = solve(eqn,x,'IgnoreAnalyticConstraints',true)   %忽略分析约束
S =
 (3^(1/2)*exp(-(log(256) + log(3)^2)^(1/2)/2))/3
  (3^(1/2)*exp((log(256) + log(3)^2)^(1/2)/2))/3
```

```
>> syms x positive
>> eqn = x^2 + 5*x - 6 == 0;
>> S = solve(eqn,x)
S =  1
%忽略设定的假设
>> S = solve(eqn,x,'IgnoreProperties',true)
S =  -6    1
>> assume(x,'clear')  %消除假设，以便影响其他方程求解
```

2. 求解代数方程

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例2：具有特殊要求的代数方程求解

$$(1)\ e^{\ln x \ln 3x} - 4 = 0; \quad (2)\ x^2 + 5x - 6 = 0; \quad (3)\ x^3 + x^2 + a = 0; \quad (4)\ \sin x + \cos 2x = 1$$

```
>> syms x a
>> eqn = x^3 + x^2 + a == 0;
>> S = solve(eqn, x)
S =
 root(z^3 + z^2 + a, z, 1)
 root(z^3 + z^2 + a, z, 2)
 root(z^3 + z^2 + a, z, 3)
%解的结果以root形式给出，设置MaxDegree参数获得显式解
>> S = solve(eqn, x, 'MaxDegree', 3)
>> Sval = vpa(subs(S,a,1),5)   %数值化显示三个虚根
```

```
>> syms x
eqn = sin(x) + cos(2*x) == 1;
S = solve(eqn,x)
S =
       0
     pi/6
 (5*pi)/6
%通过将"PrincipalValue"设置为true，只选择一个解。
>> S1 = solve(eqn,x,'PrincipalValue',true)
S1 =
0
```

3. 求解代数方程组

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例3：求解下列三个方程组（非线性，超越）

$$(1)\begin{cases} x^2+y^2=5 \\ 2x^2-3xy-2y^2=0; \\ x,y>0 \end{cases} \quad (2)\begin{cases} \sin x+y^2+\ln z=7 \\ 3x+2^y-z^3+1=0 \\ x+y+z=5 \end{cases}$$

```
>> syms x y z
>> eqn = [sin(x)+y^2+log(z)-7 ==0, 3*x+2^y-z^3+1 == 0, x+y+z-5 == 0];
>> sol = solve(eqn,[x,y,z]);   %vpasolve(eqn,[x,y,z])
sol.x = 5.1004127298867761621009050441017
sol.y = -2.6442371270278301895646143811868
sol.z = 2.5438243971410540274637093370 85
>> eqn = [x^2 + y^2 - 5 == 0, 2*x^2 - 3*x*y - 2*y^2 == 0];
>> sol = solve(eqn,[x,y])
sol =  包含以下字段的 struct:  x: [4×1 sym]    y: [4×1 sym]
```

```
>> eqn = [x^2 + y^2 - 5 == 0, 2*x^2
- 3*x*y - 2*y^2 == 0, x > 0, y > 0];
>> sol =
solve(eqn,[x,y],'ReturnConditions',true)
sol =
  包含以下字段的 struct:
          x: [1×1 sym]
          y: [1×1 sym]
    parameters: [1×0 sym]
    conditions: [1×1 sym]
sol.x =  2
sol.y = 1
```

代数方程应用

例4：抛物面$z = x^2 + y^2$ 被平面$x + y + z = 1$截成一个椭圆，求椭圆到原点的最长与最短距离。

解：这个问题实际上就是求函数 $f(x, y, z) = x^2 + y^2 + z^2$ 在条件$z = x^2 + y^2$及$x + y + z = 1$下的最大值和最小值问题。构造Lagrange函数

$$L(x, y, z) = x^2 + y^2 + z^2 + \lambda(x^2 + y^2 - z) + \mu(x + y + z - 1)$$

```
>> syms x y z u v
>> L = x^2+y^2+z^2+u*(x^2+y^2-z)+v*(x+y+z-1);
>> Lx = diff(L,x);  Ly = diff(L,y);  Lz = diff(L,z);  Lu = diff(L,u);  Lv = diff(L,v);
>> eqn = [Lx == 0, Ly == 0, Lz == 0, Lu == 0, Lv == 0];
>> sol = solve(eqn,[x,y,z,u,v])
>> for i = 1:4
    Lval(i) = vpa(subs(subs(subs(subs(subs(L,x,sol.x(i)),y,sol.y(i)),z,sol.z(i)),u,sol.u(i)),v,sol.v(i)),5);
end
```

```
Lval = [17.66, -0.25, -0.25, 0.33975]
%在实数意义上的最大点和最小点
Xmax =
[ -1.366, -1.366, 3.7321, -5.8868, -13.351]
Xmin =
[ 0.36603, 0.36603, 0.26795, -0.11325, -0.64915]
```

4. 微分方程符号求解

- S = dsolve(eqn,cond, Name,Value)：该函数用于求解常微分方程。

例5：求解微分方程的通解。

$$(1)\frac{dy}{dx}=\frac{x^2+y^2}{2x^2};\qquad (2)x^2\frac{dy}{dx}+2xy-e^x=0;\qquad (3)\frac{dy}{dx}=\frac{x}{y\sqrt{1-x^2}};\qquad (4)y^{(4)}-2y'''+5y''=0$$

```
>> syms x y(x)
>> eqn = diff(y,x) == (x^2+y^2)/2/x^2;
>> S = dsolve(eqn)
S =
    x,    -x*(1/(C + log(x)/2) - 1)
>> eqn2 = x^2*diff(y,x)+2*x*y-exp(x) == 0;
>> S2 = dsolve(eqn2)
S2 =
   -(C - exp(x))/x^2
```

```
>> eqn3 = diff(y,x) == x/y/sqrt(1-x^2);
>> S3 = dsolve(eqn3)
S3 =
    2^(1/2)*(C - (1 - x^2)^(1/2))^(1/2)
    -2^(1/2)*(C - (1 - x^2)^(1/2))^(1/2)
>> eqn4 = diff(y,x,4) - 2*diff(y,x,3) + 5*diff(y,x,2) == 0;
>> S4 = dsolve(eqn4)
S4 =
    (2*C1)/25 + C2 + (C1*x)/5 + C2*cos(2*x)*exp(x) + C3*sin(2*x)*exp(x)
```

5. 微分方程符号求解——初值条件

例6：求微分方程的特解。

$$(1)\frac{dy}{dx}=2xy^2,\quad y(0)=1;\quad (2)\frac{dy}{dx}=\frac{x^2}{1+y^2},\quad y(2)=1;\quad (3)\frac{dy}{dx}=ay,\quad y(0)=b;$$

$$(4)\frac{d^2y}{dx^2}=a^2y,\quad y(0)=1,\quad y'(\frac{\pi}{a})=0;\qquad (5)\left(\frac{dy}{dx}\right)^2+y^2=1,\quad y(0)=0.$$

```
>> syms x y(x) a b
>> eqn1 = diff(y,x) == 2*x*y^2;
>> cond1 = y(0) == 1;
>> ySol(x) = dsolve(eqn1,cond1)
ySol(x) =
-1/(x^2 - 1)
```

```
>> eqn2 = diff(y,x) == x^2/(1+y^2);
>> cond2 = y(2) == 1;
>> ySol2(x) = dsolve(eqn2,cond2)
ySol2(x) =
(((x^3/2 - 2)^2 + 1)^(1/2) + x^3/2 - 2)^(1/3) - 1/(((x^3/2 - 2)^2
+ 1)^(1/2) + x^3/2 - 2)^(1/3)
```

5. 微分方程符号求解——初值条件

例6：求微分方程的特解。

$$(1)\frac{dy}{dx}=2xy^2,\quad y(0)=1;\quad (2)\frac{dy}{dx}=\frac{x^2}{1+y^2},\quad y(2)=1;\quad (3)\frac{dy}{dx}=ay,\quad y(0)=b;$$

$$(4)\frac{d^2y}{dx^2}=a^2y,\quad y(0)=1,\quad y'(\frac{\pi}{a})=0;\qquad (5)\left(\frac{dy}{dx}\right)^2+y^2=1,\quad y(0)=0.$$

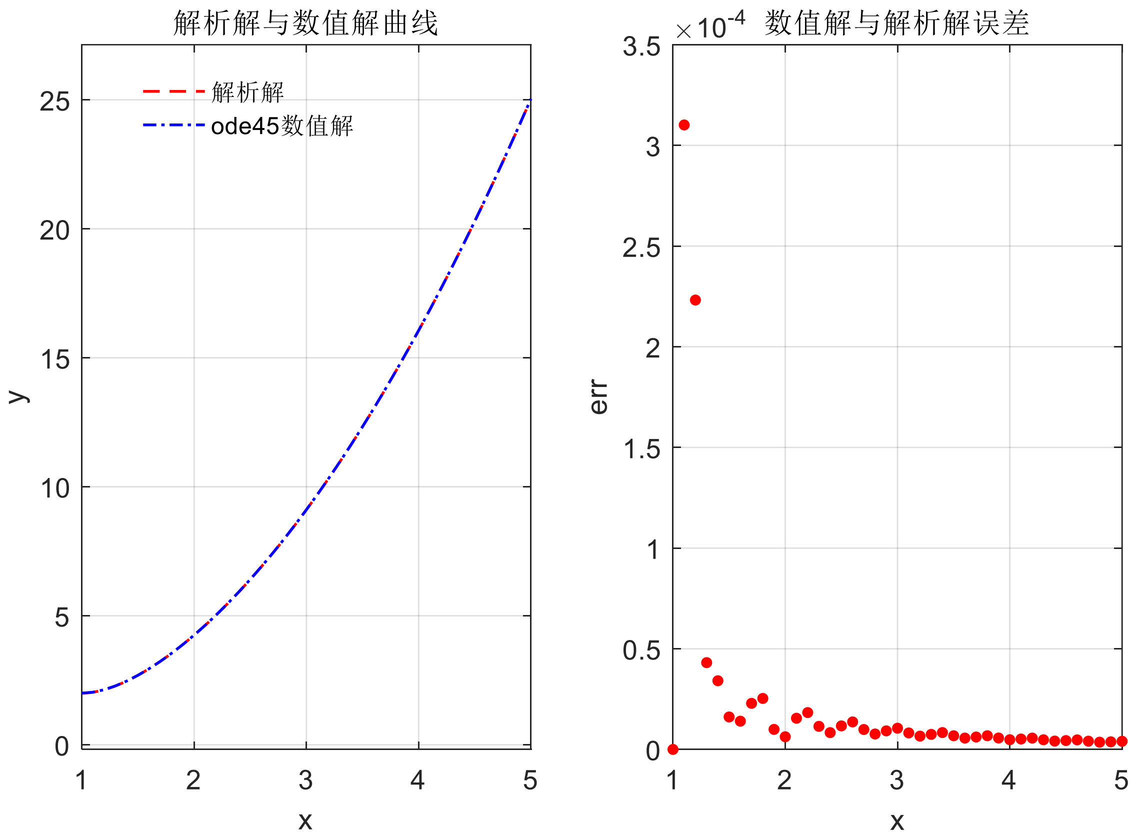
```
>> eqn4 = diff(y,x,2) == a^2*y;
>> Dy = diff(y,x);
>> cond4 = [y(0) == 1,Dy(pi/a) == 0];
>> ySol4(x) = dsolve(eqn4,cond4)
ySol4(x) =
exp(a*x)/(exp(2*pi) + 1) + (exp(2*pi)*exp(-a*x))/(exp(2*pi) + 1)
```

```
>> eqn5 = diff(y,x)^2+y^2 == 1;
>> cond5 = y(0) == 0;
>> ySol5(x) = dsolve(eqn5,cond5)
ySol5(x) =
 -(exp(- x*1i - (pi*1i)/2)*(exp(x*2i) - 1))/2
  -(exp(x*1i - (pi*1i)/2)*(exp(-x*2i) - 1))/2
```

微分方程符号解与数值解

例7：用微分方程的数值解法和符号解法解方程 $\frac{dy}{dx}=-\frac{2y}{x}+4x,\quad y(1)=2$

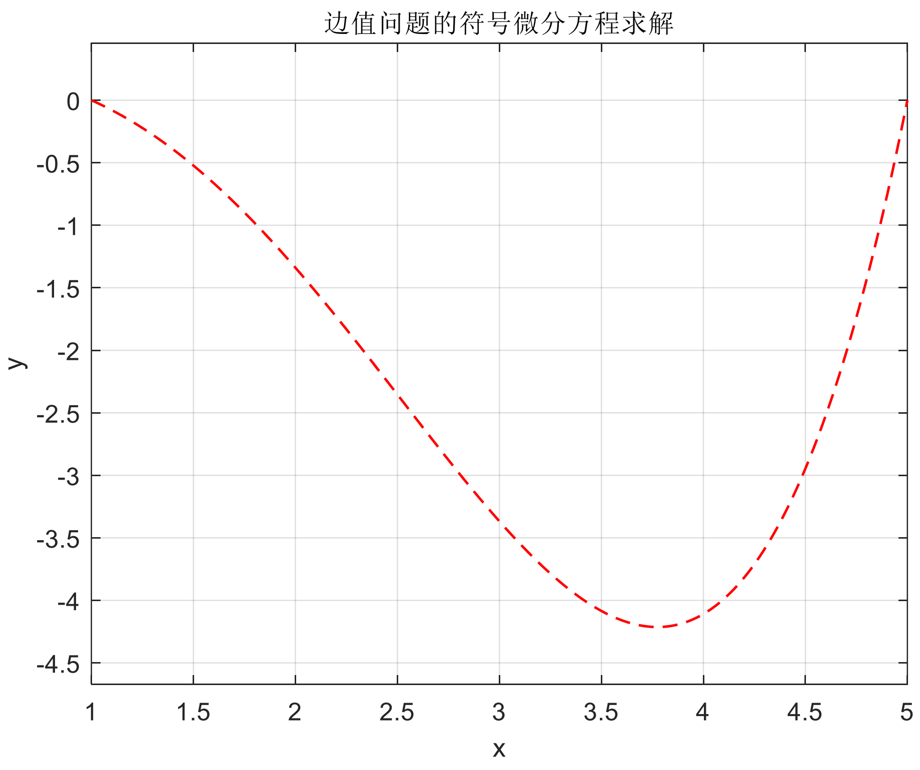
```
>> sol = dsolve('Dy+2*y/x-4*x','y(1)=2','x')
>> fh = @(x,y)(4*x^2-2*y)/x;
>>[t,y]=ode45(fh,[1,5],2);
>> subplot(1,2,1);
>> h = ezplot(sol,[1,5]);
>> set(h,{'Color','LineStyle','LineWidth'},{'r','--',1})
>> grid on;  hold on
>> plot(t,y,'b-.','LineWidth',1)
>> subplot(1,2,2)
>> soly = subs(sol,t);
>> err = abs(soly-y);
>> plot(t,err,'r.','MarkerSize',12)
```



6. 微分方程符号求解——边值条件

例8：求微分方程边值问题 $x\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}=x^2$，　$y(1)=0,\ y(5)=0$

```
>> syms x y(x)
>> eqn = x*diff(y,x,2)-3*diff(y,x) == x^2;
>> cond = [y(1) == 0, y(5) == 0];
>> sol = dsolve(eqn,cond)
sol =
(31*x^4)/468 - x^3/3 + 125/468
>> h = ezplot(sol,[1,5]);
>> set(h,{'Color','LineStyle','LineWidth'},{'r','--',1})
>> grid on
>> title('边值问题的符号微分方程求解')
```



微分方程符号求解——未得到显式解

例9：求下列隐式微分方程组的解：

$$\begin{cases} x''\sin(y')+(y'')^2=-2xy+xx''y' \\ xx''y''+\cos(y'')=3x'y \end{cases}$$

初值条件：$x(0)=1, x'(0)=0, y(0)=0, y'(0)=1$

```
>> syms t x(t) y(t)
>> D2x = diff(x,t,2);  D2y = diff(y,t,2);
>> Dx = diff(x,t);  Dy = diff(y,t);
>> eqns = [D2x*sin(Dy)+D2y^2 == -2*x*y+x*D2x*Dy,x*D2x*D2y+cos(D2y) == 3*Dx*y];
>> cond = [x(0) == 1,Dx(0)==0,y(0)==0,Dy(0) == 1];
>> sol = dsolve(eqns,cond)
警告: Explicit solution could not be found.
> In dsolve (line 201)
sol =
[ empty sym ]
```

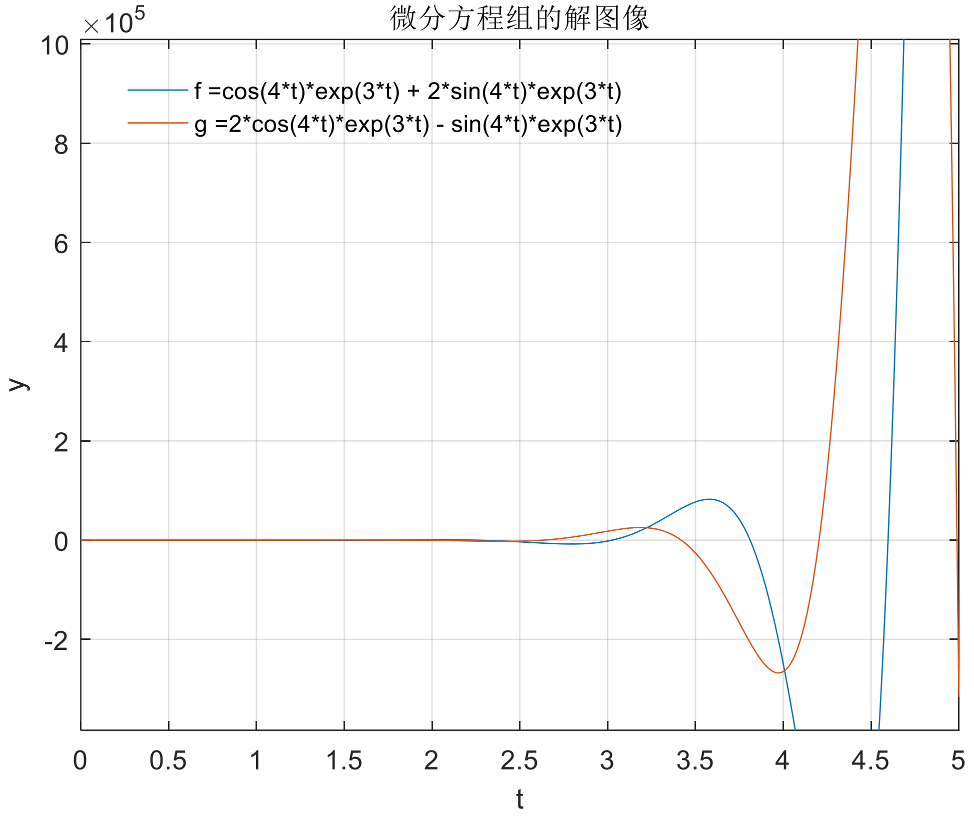
If dsolve cannot find an explicit or implicit solution, then dsolve issues a warning and returns an empty sym. Returning an empty symbolic object does not mean that no solutions exist.

7. 微分方程组符号求解

例10：求解微分方程组

$$\begin{cases} \dfrac{df}{dt} = 3f + 4g \\ \dfrac{dg}{dt} = -4f + 3g \end{cases}, 其中 f(0) = 1, g(0) = 2$$

```
>> syms t f(t) g(t)
>> eqns = [diff(f,t) == 3*f+4*g,  diff(g,t) == -4*f+3*g];
>> cond = [f(0) == 1, g(0) == 2];
>> [f,g] = dsolve(eqns,cond)
   f = cos(4*t)*exp(3*t) + 2*sin(4*t)*exp(3*t)
   g = 2*cos(4*t)*exp(3*t) - sin(4*t)*exp(3*t)
>> ezplot(f,[0,5]);  hold on;  ezplot(g,[0,5]);  grid on
>> legend('f =cos(4*t)*exp(3*t) + 2*sin(4*t)*exp(3*t)','g =2*cos(4*t)*exp(3*t) - sin(4*t)*exp(3*t)')
>> title('微分方程组的解图像')
```



8. 复合方程

复合方程通过函数 compose 进行：

- compose(f,g)，返回函数 f(g(y))，其中 f = f(x)，g = g(y)，x 是 f 的默认自变量，y 是 g 的默认自变量；
- compose(f,g,z)，返回函数 f(g(z))，自变量为 z；
- compose(f,g,x,z)，返回函数 f(g(z))，指定 f 的自变量为 x；
- compose(f,g,x,y,z)，返回函数 f(g(z))，f 和 g 的自变量分别指定为 x 和 y。

例11：复合下列函数

$$f=\frac{1}{1+x^2},\quad g=\sin y,\quad h=x^t,\quad p=e^{-\frac{y}{u}}$$

3. 复合方程

```
>> syms x y z t u
>> f = 1/(1 + x^2);
>> g = sin(y);
>> h = x^t;
>> p = exp(-y/u);
>> a = compose(f,g)   %默认自变量
a =
   1/(sin(y)^2 + 1)
>> b = compose(f,g,t)  %指定自变量为t
b =
   1/(sin(t)^2 + 1)
>> c = compose(h,g,x,z)  %指定h的自变量为x，且复合之后指定变量为z
c =
   sin(z)^t
```

```
>> e = compose(h,p,x,y,z)   %指定h自变量为x，p自变量为y，且复合之后指定变量为z
e =
    exp(-z/u)^t
>> f = compose(h,p,t,u,z)   %指定h自变量为t，p自变量为u，且复合之后指定变量为z
f =
      x^exp(-y/z)
```

9. 反方程

反方程通过函数 finverse 求得：

- g = finverse(f)，在函数 f 的反函数存在的情况下，返回函数 f 的反函数，自变量为默认自变量；
- g = finverse(f,v)，在函数 f 的反函数存在的情况下，返回函数 f的反函数，自变量为 v。

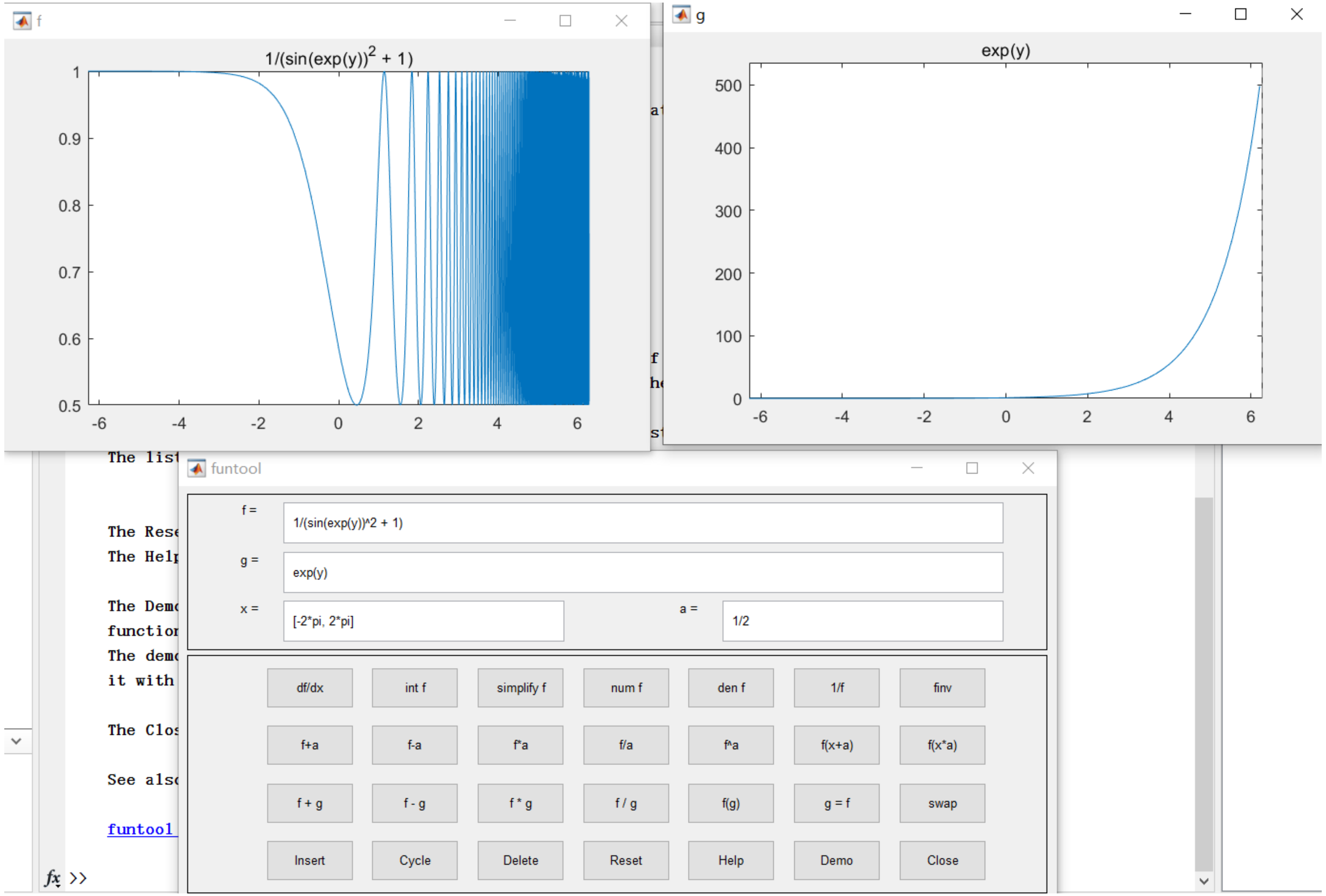
例12：求函数 $f(x)=\dfrac{1}{\tan x}$， $g(u,v)=e^{u-2v}$ 的反函数

```
>> syms x u v
>> fh1 = 1/tan(x);
>> fh2 = exp(u-2*v);
>> ffh1 = finverse(fh1)
ffh1 =  atan(1/x)
```

```
>> ffh2 = finverse(fh2,u)
ffh2 =  2*v + log(u)
>> ffh3 = finverse(fh2,v)
ffh3 =  u/2 - log(v)/2
```

符号函数计算器

- 在命令窗口中执行funtool即可调出单变量符号函数计算器。

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