Phase structure of the Quark-Meson model

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PACS numbers: 11.30.Rd, 11.10.Wx, 05.10.Cc, 12.38.Mh

I. INTRODUCTION

A

Recently, in Ref. [1],they resolve phase transitions with Discontinuous Galerkin methods at local potential approximation (LPA) truncation. However, Discontinuous Galerkin methods

II. 2 FLAVOR QUARK MESON MODEL

The effective action of the 2 flavor quark-meson model reads [2]

$$\Gamma_k[\Phi] = \int_x \{Z_{q,k}\bar{q}(\gamma_\mu\partial_\mu - \gamma_0\mu)q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2 + h_k\bar{q}(T^0\sigma + i\gamma_5\vec{T}\cdot\vec{\pi})q + V_k(\rho) - c\sigma\} \tag{1}$$

with the integral $\int_x = \int_0^{1/T} d_{x_0} \int d^3x$. T and μ denote the temperature and quark chemical potential, respectively. The meson/quark wave function $Z_{\phi,k}/Z_{q,k}$ are scale and field dependent. Moreover, in this work we ignore the longitudinal and transversal splitting of Z_k, i.e., $Z_k^{\parallel}$ and $Z_k^{\perp}$, respectively, and an interesting discussion is presented in Ref. [3]. T^0 and $\vec{T}$ are generators of $SU(N_f = 2)$ with $\mathrm{Tr}(T^iT^j) = \frac{1}{2}\delta^{ij}$ and $T^0 = \frac{1}{\sqrt{N_f}}\mathbb{1}_{N_f\times N_f}$. Quark and meson interaction is encoded in Yukawa coupling h_k in Eq. (1). The effective potential $V_k(\rho)$ is scale and field dependent and chirally invariant with chirally invariant variable $\rho = \frac{1}{2}\phi^2$. $\phi = (\sigma, \vec{\pi})$ is the meson field and its vacuum expectation value $\phi_0 = (\sigma, 0)$. The linear sigma term $-c\sigma$ explicitly breaks the chiral symmetry.

We introduce the renormalized field and renormalized coupling parameters

$$\bar{\phi} = Z_\phi^{1/2}\phi, \quad \bar{h} = \frac{h}{Z_\phi^{1/2}Z_q} \tag{2}$$

And the field dependent renormalized dimensionless meson and quark masses are obtained by

$$\bar{m}_\pi^2 = \frac{V_k'(\rho)}{k^2 Z_{\phi,k}} \tag{3}$$

$$\bar{m}_\sigma^2 = \frac{V_k'(\rho) + 2\rho V_k''(\rho)}{k^2 Z_{\phi,k}} \tag{4}$$

$$\bar{m}_q^2 = \frac{h_k^2\rho}{2k^2 Z_{q,k}^2} \tag{5}$$

the masses given above are curvature masses which distinguish from the pole and screening masses [4, 5]. the anomalous dimensions

$$\eta_{\phi/q,k} = -\frac{\partial_t Z_{\phi/q,k}}{Z_{\phi/q,k}} \tag{6}$$

The QM model at various truncations have been investigated in Ref. [2, 3, 6–8]. However, all of these investigation are based on Taylor expansion technology which only consider field dependent at the neighborhood of vacuum expectation value ϕ_0, while the full field dependent is not included. In [6], they show a good match of chiral phase diagram between Taylor expansion solution and grid solution within LPA truncations at low chemical potential. However, the Taylor expansion technology disables to resolve the problem at high chemical potential and low temperature.

Notable that $Z_{\phi/q,k}$ should also be momentum depended. Due to the Matsubara The local potential approximation (LPA) truncation:

$$\partial_t Z_{\phi/q,k} = 0, \quad \partial_t h = 0 \tag{7}$$

$$\partial_t Z_{\phi,k} = 0, \quad \partial_t Z_{q,k} = 0, \quad \partial_t \bar{h} = 0 \tag{8}$$

Then with the Wetterich equation

$$\partial_t\Gamma_k[\Phi] = \frac{1}{2}\,\mathrm{Tr}\,G_{\phi\phi}[\Phi]\partial_t R_k^\phi - \mathrm{Tr}\,G_{q\bar{q}}[\Phi]\partial_t R_k^q \tag{9}$$

with the regulators R_k^ϕ, R_k^q are defined in Eq. (A1)

$$\partial_t V_k(\rho) = \frac{k^4}{4\pi^2}\{(N_f^2 - 1)l_0^B(m_{\pi,k}, \eta_{\phi,k}; T) + l_0^B(m_{\sigma,k}, \eta_{\phi,k}; T) - 4N_cN_f l_0^F(m_{q,k}, \eta_{q,k}; T, \mu)\} \tag{10}$$

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here $l_0^{B/F}$ are threshold functions which are defined in Eq. (A4,A5)

$$\eta_{\phi,k} = \frac{1}{6\pi^2}\Big\{\frac{4}{k^2}\rho(V_k'(\rho))^2\mathcal{BB}_{(2,2)}(\bar{m}_{\pi,k}^2,\bar{m}_{\sigma,k}^2;T) + N_c\bar{h}_k^2[(2\eta_{q,k}-3)\mathcal{F}_{(2)}(\bar{m}_{q,k}^2;T,\mu) - 4(\eta_{q,k}-2)\ \mathcal{F}_{(3)}(\bar{m}_{q,k}^2;T,\mu)]\Big\} \quad (11)$$

where $\mathcal{BB}_{(n,m)}$ and $\mathcal{F}_{(n)}$ are the threshold functions which are defined in Eq. A8A11)

III. RESULTS

In this work,we employ the grid on method to solve the flow equation Eq. (10 ,11). The UV cutoff is chosen as $\Lambda = 500\text{MeV}$, and the initial condiction The numerical

A. phase diagram at chiral limit

IV. SUMMARY AND DISCUSSIONS

In this work, we have studied

ACKNOWLEDGMENTS

The work was supported by the National Natural Science Foundation of China under Contracts Nos. 11775041.

Appendix A: Threshold Functions

In this work, we employ 3d Litim regulators R_k^ϕ, R_k^q[9, 10] for analytic and concise expressions of the threshold functions, which are defined as

$$R_k^\phi(q_0,\vec{q}) = Z_{\phi,k}\vec{q}^2 r_B\Big(\frac{\vec{q}^2}{k^2}\Big)$$

$$R_k^q(q_0,\vec{q}) = Z_{q,k} i\vec{\gamma}\cdot\vec{q} r_F\Big(\frac{\vec{q}^2}{k^2}\Big) \quad (A1)$$

with shape functions

$$r_B(x) = \Big(\frac{1}{x}-1\Big)\Theta(1-x)$$

$$r_F(x) = \Big(\frac{1}{\sqrt{x}}-1\Big)\Theta(1-x) \quad (A2)$$

With the regulators defined above, the scalar parts of the dimensionless propagators are

$$G_\phi(q,\bar{m}_{\phi,k}^2) = \frac{1}{q_0^2/k^2+1+\bar{m}_{\phi,k}^2}$$

$$G_q(q,\bar{m}_{q,k}^2) = \frac{1}{(q_0+i\mu)^2/k^2+1+\bar{m}_{q,k}^2} \quad (A3)$$

where the temperature is introduced via the Matsubara frequency $q_0 = 2\pi nT$ and $q_0 = 2\pi(n+1)T$ for bosons and fermmions, respectively.

The bosonic and fermionic loops are easily obtained as

$$l_0^B(\bar{m}_{\phi,k}^2,\eta_{\phi,k};T) = \frac{2}{3}\frac{1}{\sqrt{1+\bar{m}_{\phi,k}^2}}\Big(1-\frac{\eta_{\phi,k}}{5}\Big)\Big(\frac{1}{2}+n_B(\bar{m}_{\phi,k}^2;T)\Big) \quad (A4)$$

and

$$l_0^F(\bar{m}_{q,k}^2,\eta_{q,k};T,\mu) = \frac{1}{3}\frac{1}{\sqrt{1+\bar{m}_{q,k}^2}}\Big(1-\frac{\eta_{q,k}}{4}\Big)\Big(1-n_F(\bar{m}_{q,k}^2;T,\mu)-n_F(\bar{m}_{q,k}^2;T,-\mu)\Big) \quad (A5)$$

here n_B and n_F are bosonic and fermionic distribution functions

$$n_B(\bar{m}_{\phi,k}^2;T) = \frac{1}{\exp\Big\{\frac{k}{T}(1+\bar{m}_{\phi,k}^2)^{1/2}\Big\}-1}$$

$$n_F(\bar{m}_{q,k}^2;T,\mu) = \frac{1}{\exp\Big\{\frac{1}{T}[k(1+\bar{m}_{q,k}^2)^{1/2}-\mu]\Big\}+1} \quad (A6)$$

To calculate the meson anomalous dimension Eq. (11), we also need the fermionic threshold functions $\mathcal{F}_{(n)}(\bar{m}_{q,k}^2;T,\mu)$ and two bosonic threshold functions $\mathcal{BB}_{(n,m)}(\bar{m}_{a,k}^2,\bar{m}_{b,k}^2;T)$. these are defined as follows:

$$\mathcal{F}_{(1)}(\bar{m}_{q,k}^2;T,\mu) = \frac{T}{k}\sum_{n_q} G(q,\bar{m}_{q,k}^2) = \frac{1}{2\sqrt{1+\bar{m}_{q,k}^2}}\Big(1-n_F(\bar{m}_{q,k}^2;T,\mu)-n_F(\bar{m}_{q,k}^2;T,-\mu)\Big) \quad (A7)$$

with

$$\mathcal{F}_{(n)}(\bar{m}_{q,k}^2;T,\mu) = \frac{(-1)^{n-1}}{(n-1)!}\frac{\partial^{n-1}\mathcal{F}_{(1)}(\bar{m}_{q,k}^2;T,\mu)}{\partial(\bar{m}_{q,k}^2)^{n-1}}. \quad (A8)$$

And the Matsubara summation of two meson propagators

$$\mathcal{BB}_{(1,1)}(\bar{m}^2_{a,k},\bar{m}^2_{b,k};T)=\frac{T}{k}\sum_{nq}G_\phi(q,\bar{m}^2_{a,k})G_\phi(q,\bar{m}^2_{b,k})$$

$$=\frac{1}{(\bar{m}^2_{b,k}-\bar{m}^2_{a,k})\sqrt{1+\bar{m}^2_{a,k}}}\Big(\frac{1}{2}+n_B(\bar{m}^2_{a,k};T)\Big)$$

$$+\frac{1}{(\bar{m}^2_{a,k}-\bar{m}^2_{b,k})\sqrt{1+\bar{m}^2_{b,k}}}\Big(\frac{1}{2}+n_B(\bar{m}^2_{b,k};T)\Big) \quad \text{(A9)}$$

Obviously, if $\bar{m}^2_{a,k}=\bar{m}^2_{b,k}$, the defination above will diverges to infinity. In this case, we define the threshold function as

$$\mathcal{BB}_{(1,1)}(\bar{m}^2_{a,k},\bar{m}^2_{a,k};T)=$$

$$-\frac{\partial}{\partial\bar{m}^2_{a,k}}\Big[\frac{1}{\sqrt{1+\bar{m}^2_{a,k}}}\Big(\frac{1}{2}+n_B(\bar{m}^2_{a,k};T)\Big)\Big] \quad \text{(A10)}$$

with

$$\mathcal{BB}_{(n,m)}(\bar{m}^2_{a,k},\bar{m}^2_{b,k};T)=\frac{(-1)^{n+m-2}}{(n-1)!(m-1)!}$$

$$\frac{\partial^{n+m-2}\mathcal{BB}_{(1,1)}(\bar{m}^2_{a,k},\bar{m}^2_{b,k};T)}{\partial(\bar{m}^2_{a,k})^{n-1}\partial(\bar{m}^2_{b,k})^{m-1}} \quad \text{(A11)}$$

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