Lecture 6: theoretical foundations of concurrency

timeline, events, precedence, 2-thread mutual exclusion, deadlock freedom, starvation freedom, N-thread mutual exclusion, sequential objects and specifications, concurrent objects, linearizability

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https://github.com/Svazars/parallel-programming/blob/main/slides/pdf/l6.pdf

In previous episodes

Concurrency is complicated due to:

- Independence of agents
  - combinatorial explosion of possible executions
- Uncontrollable execution speed of agents
  - spurious failures
- Uncertainty with definition of consistency
  - concurrent invariants
  - visibility
  - data race
  - atomicity

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Is it purely engineering issue or we have encountered some fundamental problem?

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At least, you will understand why proving concurrent algorithms could be really hard.

Supplementary materials

- Chapters 1-5 in "The Art of Multiprocessor Programming"
- Companion materials `https://booksite.elsevier.com/9780123973375`

Our course briefly introduces some important theoretical concepts, for better understanding consider looking through

- `chapter_02.ppt`
- `chapter_03.ppt`

Theoretical mode enabled

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Hint: focus on ideas rather mathematical formalisms.

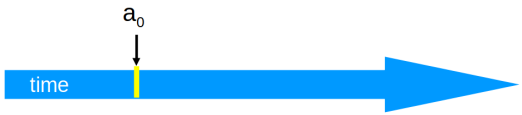
Lecture plan

1. Time, events, intervals, precedence
2. Mutual exclusion
   - 2 threads
   - N threads
   - Lower bounds on the Number of Locations
   - Optional: fairness
   - Mutual exclusion beyond read-write memory cells
3. Concurrent objects
   - Sequential and concurrent objects
   - Intuition behind consistency definitions
   - Optional: formal definitions and composability
4. Summary

Time and events

An **event** a_0 of thread `A` is

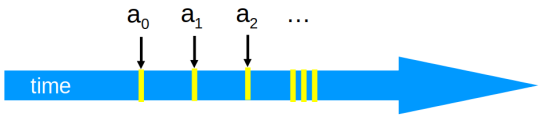
- Instantaneous
- No simultaneous events (break ties)



Threads

A thread A is (formally) a sequence a_0, a_1, ... of events

- "Trace" model
- Notation: $a_0 \rightarrow a_1$ indicates order



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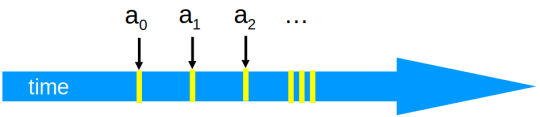
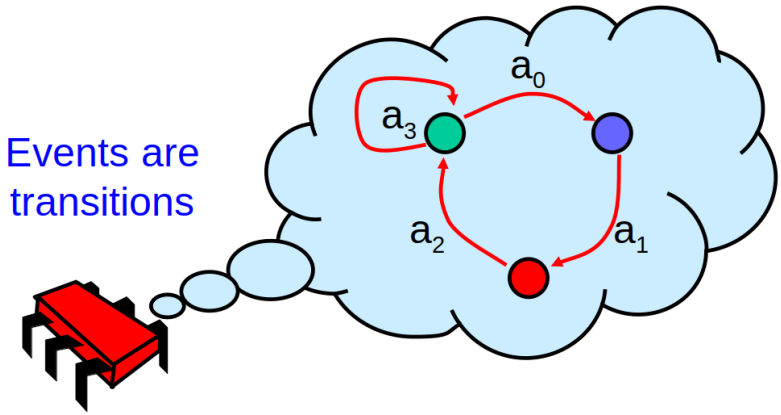


Example Thread events:

- Assign to shared or local variable
- Invoke method
- Return from method
- ...

Threads are state machines



Concurrent threads

Thread A

Thread B

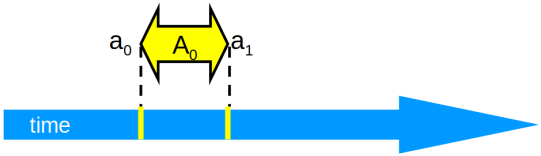
Events of two or more threads:

- Interleaved
- Not necessarily independent

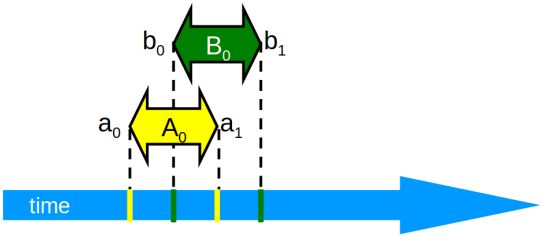
Methods are intervals

An **interval** $A_0 = (a_0, a_1)$ is

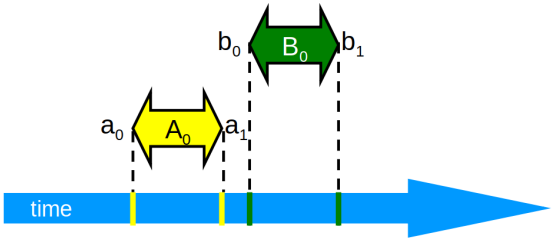
- Time between events a_0 and a_1



Overlapping intervals



Disjoint intervals



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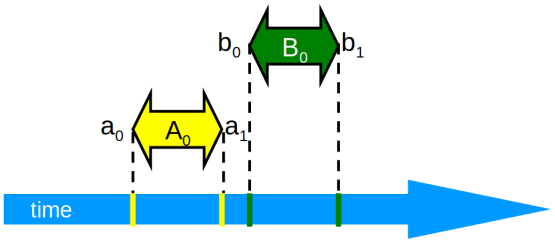

Precedence:

- Interval A_0 **precedes** interval B_0
- Notation: $A_0 \rightarrow B_0$

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Mutual exclusion

Thread is *well formed* if:

- each critical section is associated with a unique `Lock` object
- `Lock.lock` when trying to enter critical section
- `Lock.unlock` when leaving critical section

Let CS_A^j be the interval: thread `A` executes critical section for `j`-th time.

Mutual exclusion for 2 agents

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Theorem

Starvation freedom implies deadlock freedom.

Question time

Question: Mutual exclusion, Freedom from deadlock, Freedom from starvation. Which are safety properties, which are liveness properties?

Question time

Question: Restaurant "Concurrency pitfalls" claims that every philosopher will eat ordered meal before Sun explodes. Does it provide **Freedom from starvation**?

LockOne

```
class LockOne implements Lock {
  boolean[] flag = new boolean[2];
  void lock() {
    int i = ThreadID.get(); // 0 or 1
    int j = 1 - i;
    flag[i] = true;         // I want to enter
    while (flag[j]) {}      // await other thread
  }
  void unlock() {
    int i = ThreadID.get();
    flag[i] = false;        // I do not want to enter any more
  }
}
```

LockOne: deadlock

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void lock() {
  int i = ThreadID.get();
  int j = 1 - i;
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}
```

$write_A(flag[A] = true)$ occur before $read_B(flag[A])$
$write_B(flag[B] = true)$ occur before $read_A(flag[B])$

LockOne: mutual exclusion

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void lock() {
  flag[i] = true;
  while (flag[j]) {}
}
void unlock() {
  flag[i] = false;
}
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From the code:

- $write_A(flag[A] = true) \rightarrow read_A(flag[B] = false) \rightarrow CS_A$
- $write_B(flag[B] = true) \rightarrow read_B(flag[A] = false) \rightarrow CS_B$

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Plan:

- Assume CS_A^j overlaps CS_B^k.
- Consider each thread's last (j-th and k-th) read and write in the `lock()` method before entering
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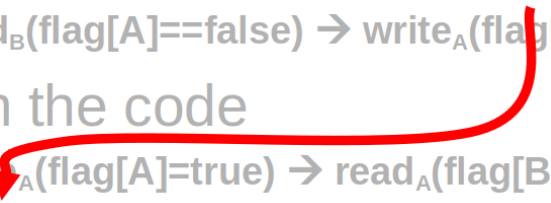

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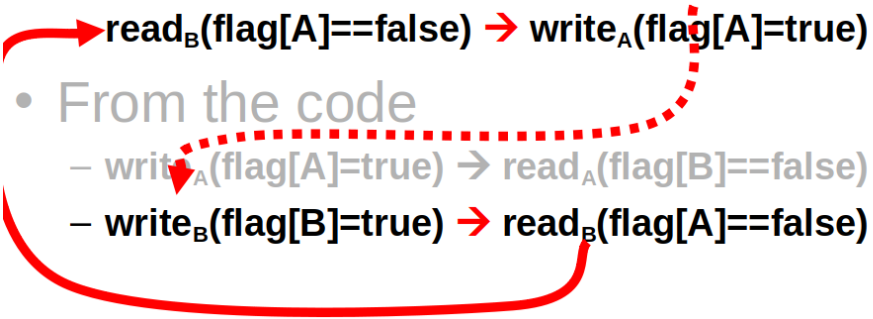

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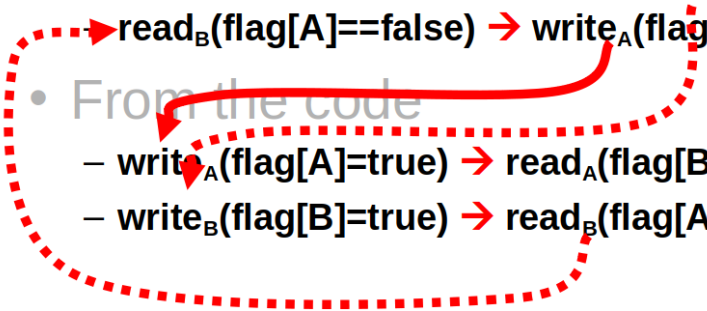

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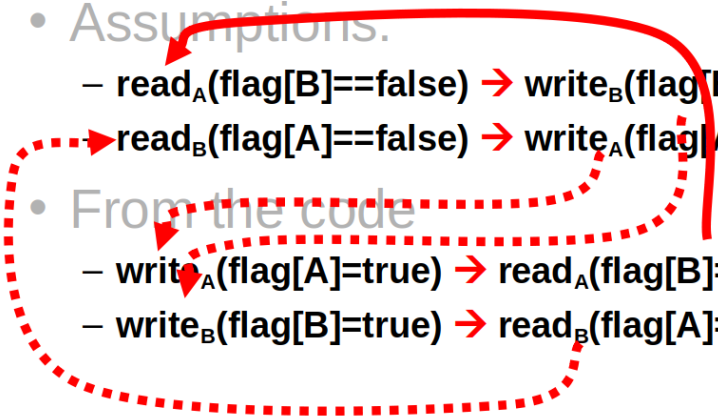

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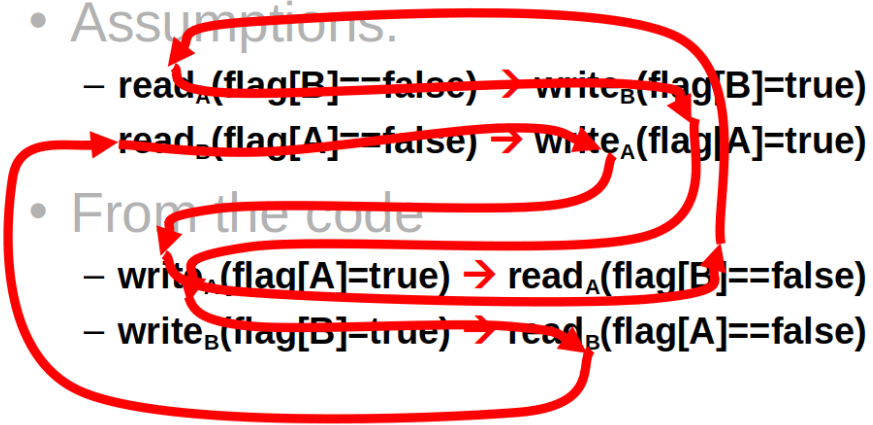

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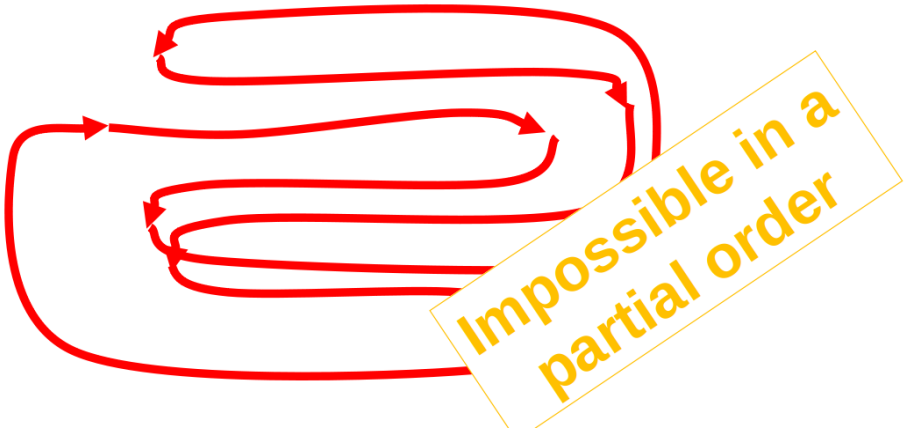

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flag[j] = true;
while (flag[i]) {}
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Properties

- Mutual exclusion
- Deadlock-freedom

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flag[i] = true;                flag[j] = true;
while (flag[j]) {}             while (flag[i]) {}
```

- Concurrent executions can deadlock
- Sequential executions OK

LockTwo

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public class LockTwo implements Lock {
 private int victim;
 public void lock() {
  int i = ThreadID.get(); // 0 or 1
  victim = i;             // let the other go first
  while (victim == i) {}; // wait
 }
 public void unlock() {}
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Satisfies Mutual Exclusion:

- if thread `i` in *CS*
- then `victim == j`
- cannot be both 0 and 1

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 public void lock() {
  int i = ThreadID.get(); // 0 or 1
  victim = i;             // let the other go first
  while (victim == i) {}; // wait
 }
 public void unlock() {}
}
```

LockTwo

```
public class LockTwo implements Lock {
 private int victim;
 public void lock() {
  int i = ThreadID.get(); // 0 or 1
  victim = i;             // let the other go first
  while (victim == i) {}; // wait
 }
 public void unlock() {}
}
```

Not deadlock free:

- Sequential execution deadlocks
- Concurrent execution does not

LockOne and LockTwo

LockOne, LockTwo:

- Mutual exclusion
- Deadlock-freedom

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LockOne, LockTwo:

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- Concurrent executions can deadlock
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LockOne:

- Concurrent executions can deadlock
- Sequential executions OK

LockTwo:

- Concurrent execution OK
- Sequential execution can deadlock

Idea: combine them!

Peterson's Algorithm

```
class Peterson implements Lock {
  private boolean[] flag = new boolean[2];
  private int victim;
  public void lock() {
    int i = ThreadID.get(); // 0 or 1
    int j = 1 - i;
    flag[i] = true;          // I am interested
    victim  = i;             // you go first
    while (flag[j] && victim == i) {}; // wait while other interested and
                                       // I am the victim
  }
  public void unlock() {
   flag[ThreadID.get()] = false; // I am not interested
  }
}
```

Peterson's Algorithm

Mutual exclusion

```
public void lock() {
  flag[i] = true;          // I am interested
  victim  = i;             // you go first
  while (flag[j] && victim == i) {}; // wait while other interested and
                                     // I am the victim
}
```

From the code:

- $write_B(flag[B] = true) \rightarrow write_B(victim = B)$
- $write_A(victim = A) \rightarrow read_A(flag[B]) \rightarrow read_A(victim)$

Assumption (both threads entered CS, latest to write victim is thread `A`):

- $write_B(victim = B) \rightarrow write_A(victim = A)$

Peterson's Algorithm

Mutual exclusion

- $write_B(flag[B] = true) \rightarrow write_B(victim = B)$
- $write_A(victim = A) \rightarrow read_A(flag[B]) \rightarrow read_A(victim)$
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Peterson's Algorithm

Mutual exclusion

- $write_B(flag[B] = true) \rightarrow write_B(victim = B)$
- $write_A(victim = A) \rightarrow read_A(flag[B]) \rightarrow read_A(victim)$
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Let's swap second and third lines.

Peterson's Algorithm

Mutual exclusion

- $write_B(flag[B] = true) \rightarrow write_B(victim = B)$
- $write_B(victim = B) \rightarrow write_A(victim = A)$
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Let's remove repeats.

Peterson's Algorithm

Mutual exclusion

- $write_B(flag[B] = true) \rightarrow$
- $write_B(victim = B) \rightarrow$
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Thread `A` read `flag[B] == true` and `victim == A` so it could not have entered the CS.

Peterson's Algorithm

Mutual exclusion

- $write_B(flag[B] = true) \rightarrow$
- $write_B(victim = B) \rightarrow$
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Thread `A` read `flag[B] == true` and `victim == A` so it could not have entered the *CS*. QED.

Peterson's Algorithm

Starvation freedom (implies deadlock freedom)

```
public void lock() {
  flag[i] = true;
  victim  = i;
  while (flag[j] && victim == i) {};
}
public void unlock() {
  flag[i] = false;
}
```

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```

- Thread `i` blocked only if `j` repeatedly re-enters so `flag[j] == true` and `victim == i`.

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  flag[i] = true;
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- Thread `i` blocked only if `j` repeatedly re-enters so `flag[j] == true` and `victim == i`.
- When thread `j` re-enters it sets `victim = j`.

Peterson's Algorithm

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  flag[i] = true;
  victim  = i;
  while (flag[j] && victim == i) {};
}
public void unlock() {
  flag[i] = false;
}
```

- Thread `i` blocked only if `j` repeatedly re-enters so `flag[j] == true` and `victim == i`.
- When thread `j` re-enters it sets `victim = j`.
- So thread `i` gets in.

Summary: 2-thread mutual exclusion

Formalize concurrent method execution:

- Single timeline
- Totally ordered atomic events
- Methods represented as partially ordered intervals
- *Precedence* is irreflexive, antisymmetric, transitive partial order

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Formalize mutex properties:

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- Single timeline
- Totally ordered atomic events
- Methods represented as partially ordered intervals
- *Precedence* is irreflexive, antisymmetric, transitive partial order

Formalize mutex properties:

- Mutual exclusion
- Deadlock freedom
- Starvation freedom (implies Deadlock freedom)

Prove theorems:

- Algorithm has property (*suppose not ...*)
- Algorithm lacks property (provide counterexample)

Lecture plan

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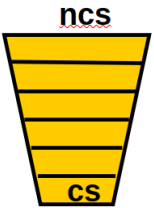
Filter Lock

Idea

There are n-1 “waiting rooms” called levels.

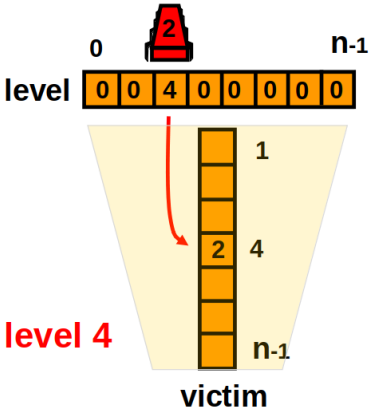
- At each level
  - At least one enters level
  - At least one blocked if many try
- Only one thread makes it through



Filter Lock

Level and victim

```
class Filter implements Lock {
  // level[i] for thread i
  int[] level;
  // victim[L] for level L
  int[] victim;
  public Filter(int n) {
    level  = new int[n];
    victim = new int[n];
    for (int i = 1; i < n; i++) {
      level[i] = 0;
    }
  }
}
```



Thread 2 at level 4

Filter Lock

```
class Filter implements Lock {
  …

  public void lock(){
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;
      while ((∃ k != i level[k] >= L) &&
             victim[L] == i ) {};
    }}
  public void unlock() {
    level[i] = 0;
  }}
```

Filter Lock

```
class Filter implements Lock {
  …

  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
            victim[L] == i) {};
    }}
  public void release(int i) {
    level[i] = 0;
  }}
```

One level at a time

Filter Lock

```
class Filter implements Lock {
  …

  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i)
    }}
  public void release(int i)
    level[i] = 0;
  }}
```

Announce intention to enter level L

Filter Lock

```
class Filter implements Lock {
  int level[n];
  int victim[n];
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
            victim[L] == i) {};
    }}
  public void release(int i)
    level[i] = 0;
  }}
```

Give priority to anyone but me

Filter Lock

Wait as long as someone else is at same or higher level, and I'm designated victim

```
public void lock() {
  for (int L = 1; L < n; L++) {
    level[i] = L;
    victim[L] = i;
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      victim[L] = i;
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            victim[L] == i) {};
    }}
```

Thread *enters* level L when it completes the loop

Filter Lock

Proof idea

- Start at level `L = 0`
- At most `n - L` threads enter level `L`
- Mutual exclusion at level `L = n-1`

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- No more than `n-(L-1)` at level `L-1`
- Induction step: by contradiction
- Assume all at level `L-1` enter level `L`
- Thread `A` last to write `victim[L]`
- Thread `B` is any other thread at level `L`

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Homework: slides 77-93 from `chapter_02.ppt`

Homework: Section 2.4 "The Filter Lock" (pages 28 - 31) from "The Art of Multiprocessor Programming"

Filter Lock

No starvation

Filter Lock satisfies properties:

- Just like Peterson Alg at any level
- So no one starves

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But what about fairness?

Filter Lock

No starvation

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But what about fairness?
Threads can be overtaken by others

Summary: N-thread mutual exclusion

N-level design:

- every step "filters"(prevents from progress) at least one thread
- reuse ideas from simpler design (2-thread Peterson's algorithm)

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  Lecture 8

[1] Historical name

Lecture plan

1. Time, events, intervals, precedence
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Mutual exclusion for N threads requires read-write of at least N locations

Optional homework:

> *Theorem 2.8.1. Any* `Lock` *algorithm that, by reading and writing memory, solves deadlock-free mutual exclusion for three threads must use at least three distinct memory locations.*

Key insights:

- Covering state

Generalized theorem: N-thread deadlock-free mutual exclusion requires N distinct locations.

Mutual exclusion: you know so far

Mutual exclusion could be solved:

- For `N` concurrent threads
- Using `N` boolean and `N` integer registers
- Provides starvation-freedom

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Few questions remain:

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- Could we "improve" registers with some additional operations to speed-up the algorithm?
- What if threads are created and deleted dynamically?

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Formalizing fairness

Optional homework: slides 95-99 from `chapter_02.ppt`
Optional homework: Section 2.5 "Fairness" (page 31)
Key insights:

- *doorway* section
- *waiting* section
- first-come-first-served ($D_A^j \rightarrow D_B^k$ then $CS_A^j \rightarrow CS_B^k$)

Bakery algorithm

Optional homework: slides 99-118 from `chapter_02.ppt`
Optional homework: Section 2.6 "Lamport's Bakery Algorithm" (pages 31 - 33)
Key insights:

- monotonically growing labels for operations
- proving fairness

Lecture plan

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Mutual exclusion using read/write locations

- Correct, starvation-free algorithm exists (filter lock)
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Before diving into classifications (Lecture 7, Lecture 8), let's study few more mathematical instruments.

Homework: before final exam

When you finish all lectures of second block, you will know many advanced concepts:

- Concurrent object consistency definitions
- Register constructions
- Atomic snapshots
- Consensus number

Revise all the correctness proofs from this lecture and be ready to answer some tricky questions:

- In order to prove correctness of Peterson's algorithm, should we use MRSW or MRMW register?
- In order to prove correctness of mutual exclusion primitives, should we require all registers to be linearizable or weaker guarantees will fit?
- Could you adapt `FilterLock` to allow dynamic thread creation and deletion?

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- Liveness (progress)

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We need to formalize

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for concurrent data structures and concurrent algorithms.

- How to define safety and liveness "universally"?
- How to reason about whole programs / software components in isolation?
- How to compose modules and correctness proofs?
- Are there "better" or "simpler" approaches?

Concurrent objects

Goals:

- **Describe** object
- **Implement** object
- **Check** for errors

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Check that **implementation** conforms to **specification**.

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Concurrent objects

Goals:

- **Describe** object
- **Implement** object
- **Check** for errors

Check that **implementation** conforms to **specification**.
Let's focus on descriptions:

- when implementation is **correct**
- the conditions under which it guarantees **progress**

Sequential objects and specifications

Sequential object:

- State
  - Set of fields
- Methods
  - The only way to change state

Sequential objects and specifications

Sequential object:

- State
  - Set of fields
- Methods
  - The only way to change state

Sequential specification:

- If (**precondition**)
  - the object is in such-and-such a state
  - before you call the method
- Then (**postcondition**)
  - the method will return a particular value
  - or throw a particular exception
- and (**postcondition**)
  - the object will be in some other state
  - when the method returns

Sequential specifications

- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

Sequential vs concurrent

Key difference (concurrency):

- Method call is not an event
- Method call is an interval

Sequential vs concurrent

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Sequential vs concurrent

Key difference (concurrency):

- Method call is not an event
- Method call is an interval

Implications:

- method calls could overlap, object might never be between method calls
  - in sequential specification we need to only define object state between method calls

Sequential vs concurrent

Key difference (concurrency):

- Method call is not an event
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Lecture plan

1. Time, events, intervals, precedence
2. Mutual exclusion
   - 2 threads
   - N threads
   - Lower bounds on the Number of Locations
   - Optional: fairness
   - Mutual exclusion beyond read-write memory cells
3. Concurrent objects
   - Sequential and concurrent objects
   - Intuition behind consistency definitions
   - Optional: formal definitions and composability
4. Summary

Consistency

What does it **mean** for a *concurrent* object to be correct?

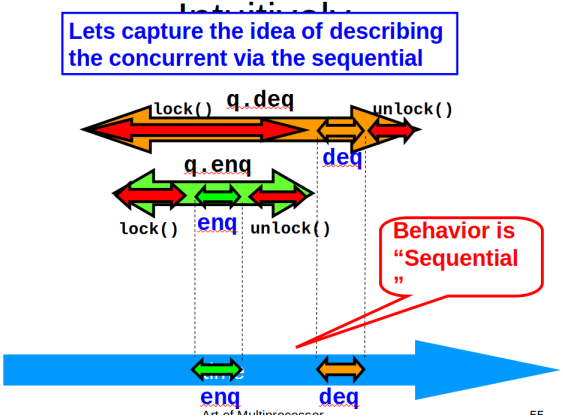
- What is a concurrent FIFO queue?

Consistency

What does it **mean** for a *concurrent* object to be correct?

- What is a concurrent FIFO queue?
- FIFO means strict temporal order
- Concurrent means ambiguous temporal order

Consistency



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Each method should

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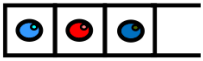
Sounds like a property of an execution ...
A **linearizable object**: one all of whose possible executions are linearizable

Examples



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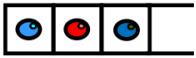

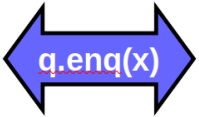

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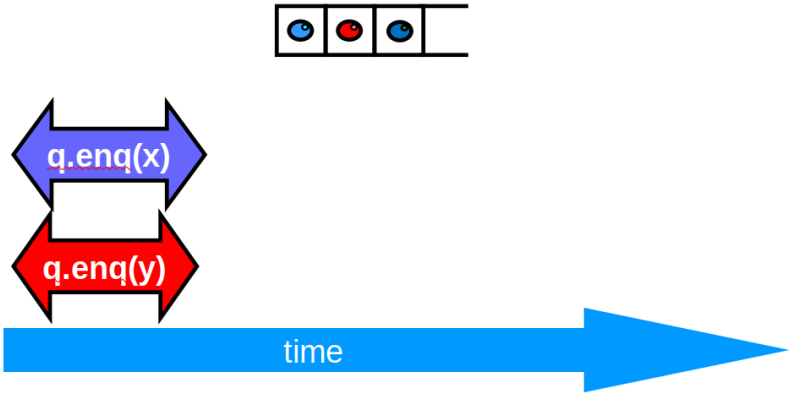

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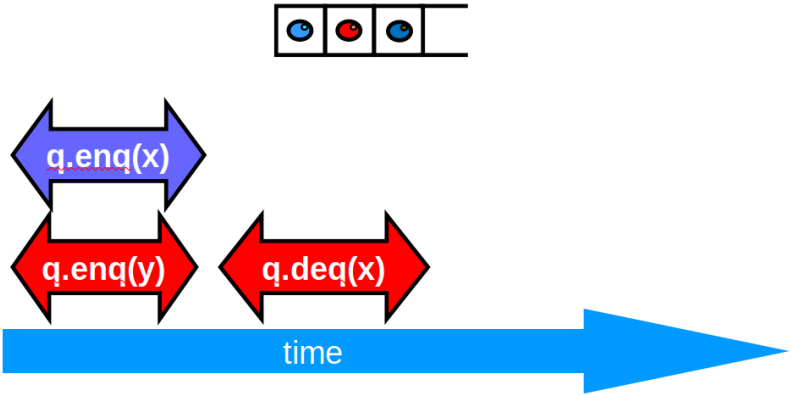

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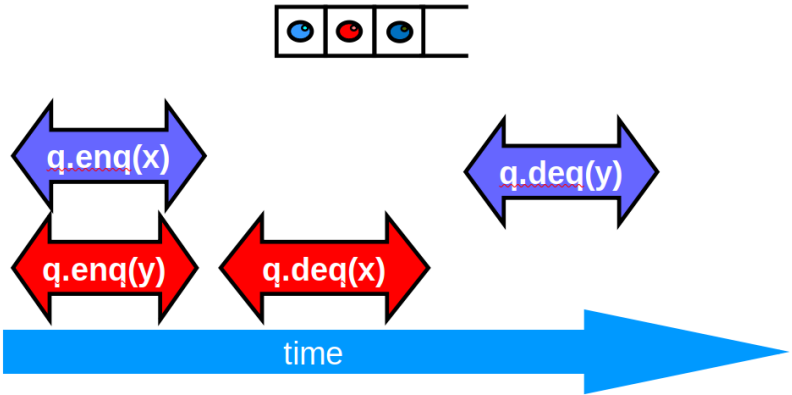
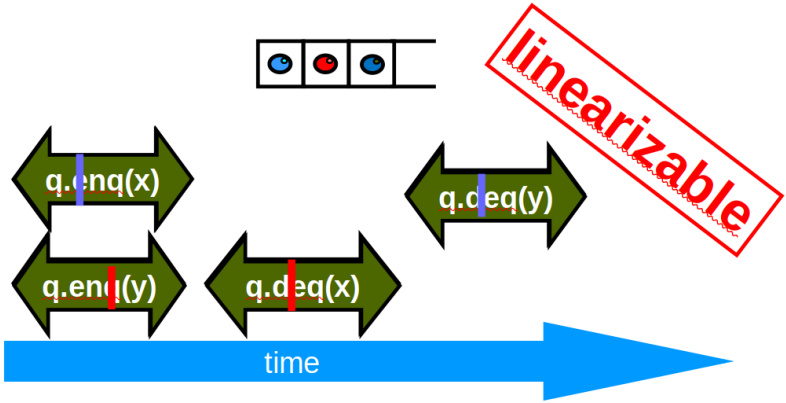
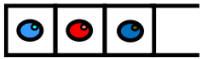


Examples



Examples



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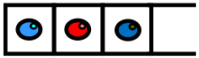

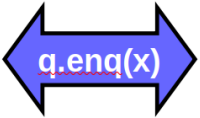

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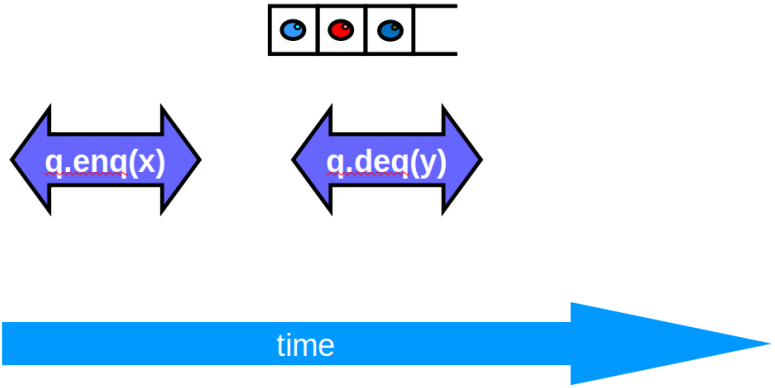
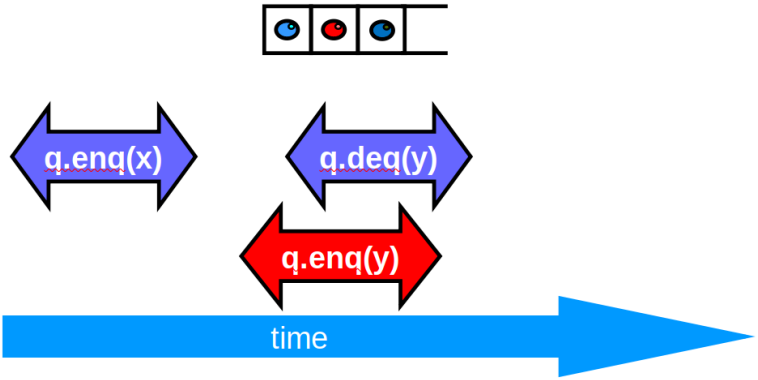
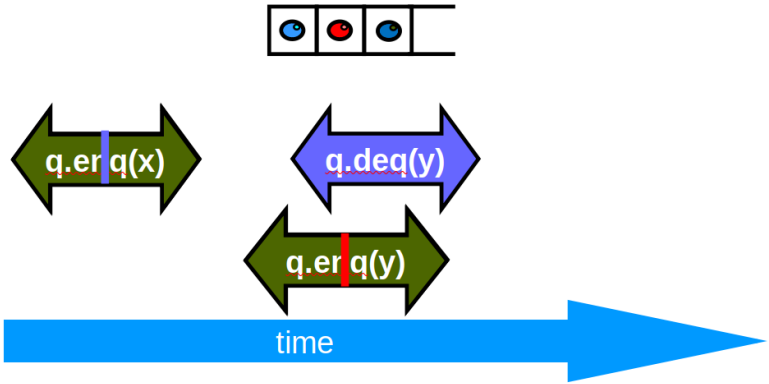


Examples



Examples



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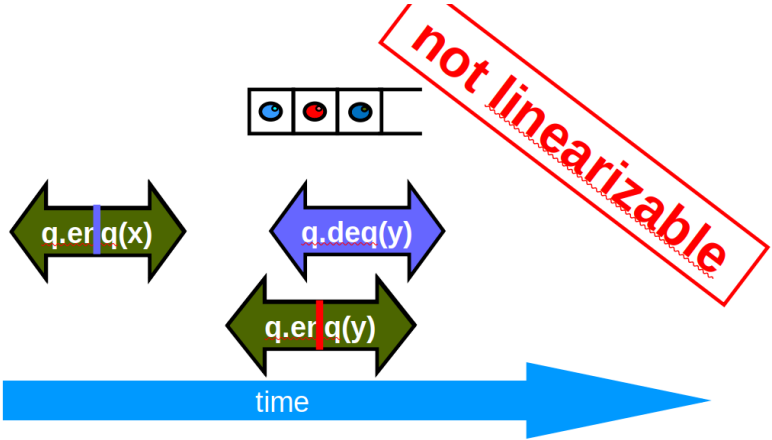

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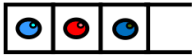

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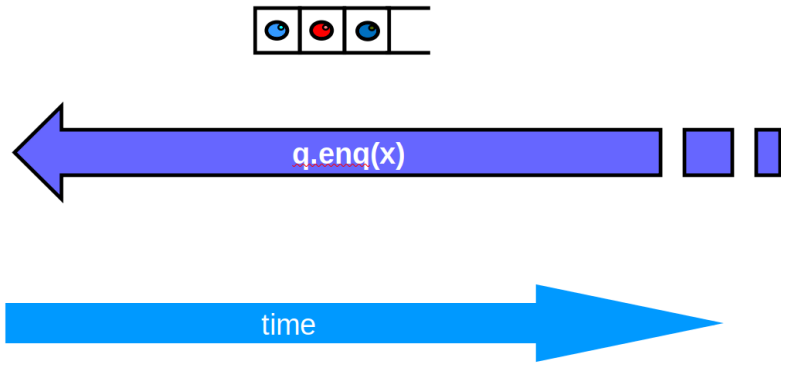

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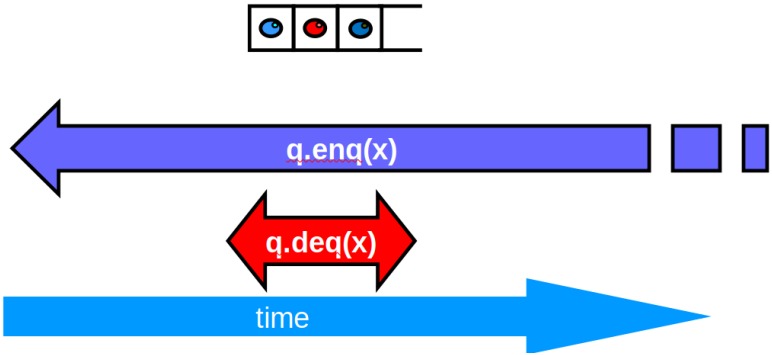
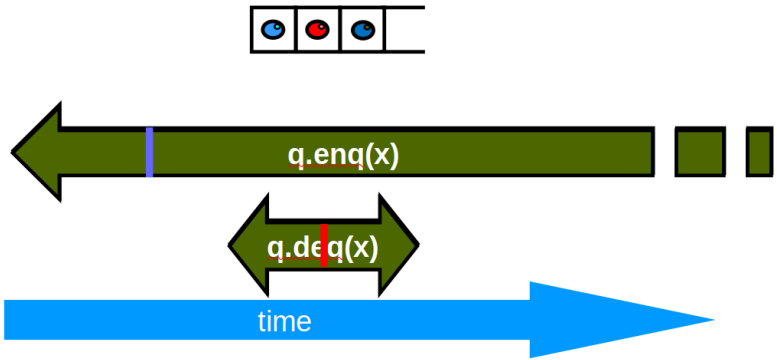


Examples



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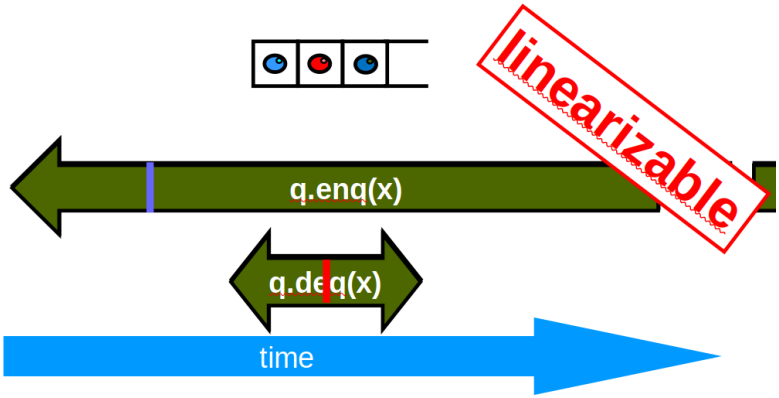

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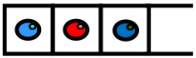

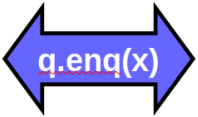

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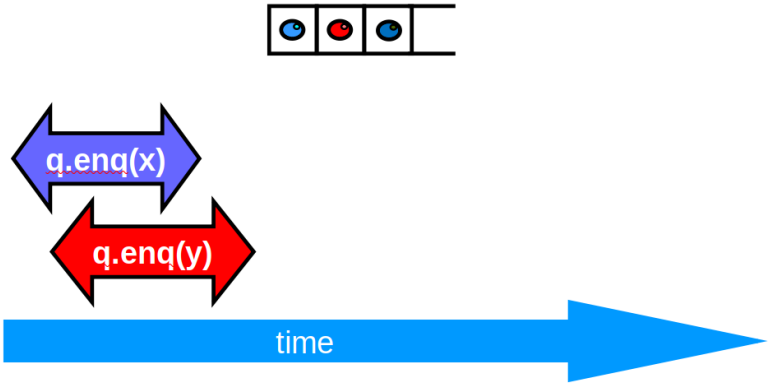

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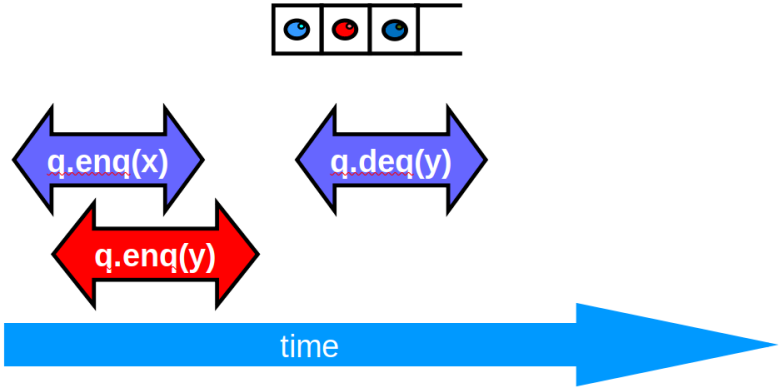

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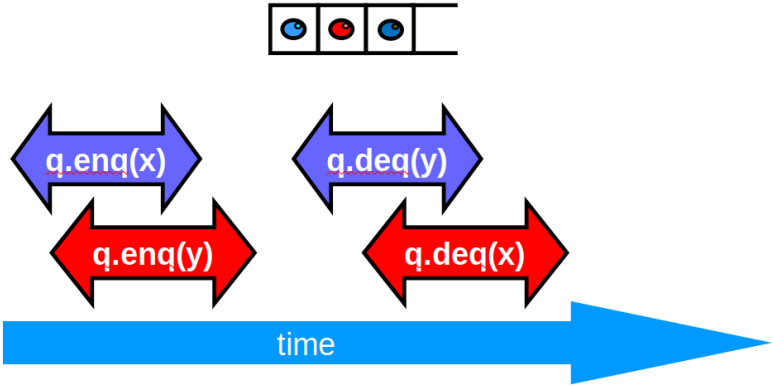

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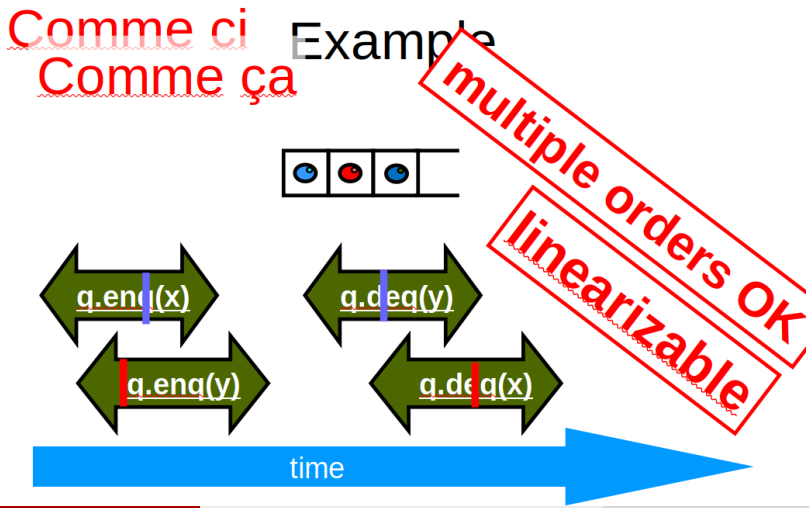
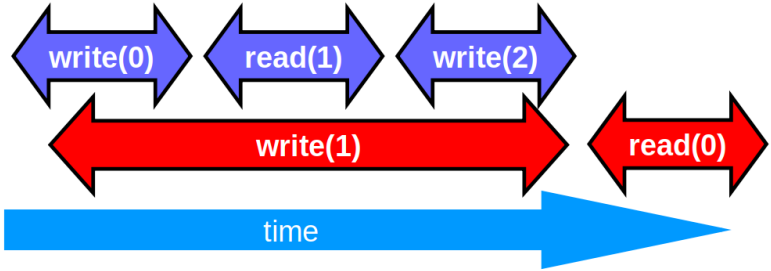
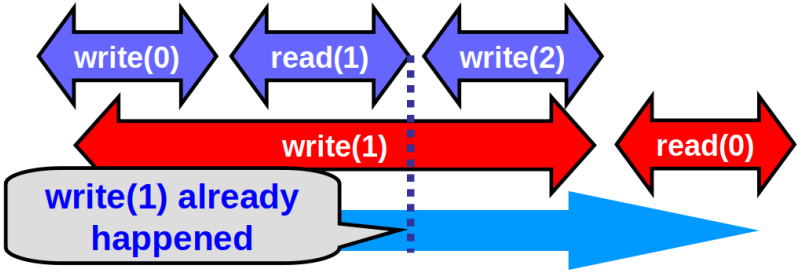

Comme ci
Comme ça
Example
multiple orders OK
linearizable
q.enq(x)
q.deq(y)
q.enq(y)
q.deq(x)
time

Examples



Examples



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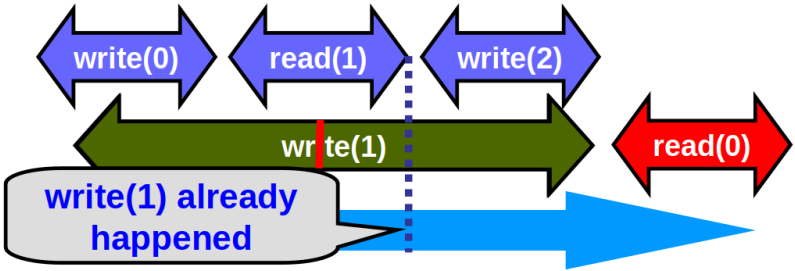

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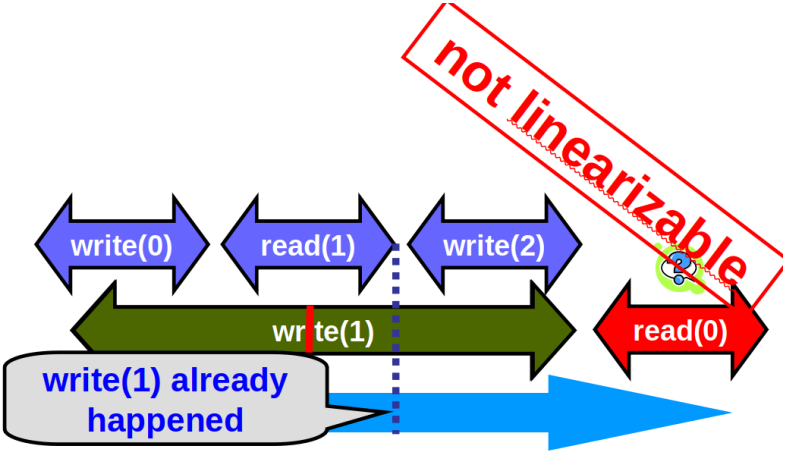
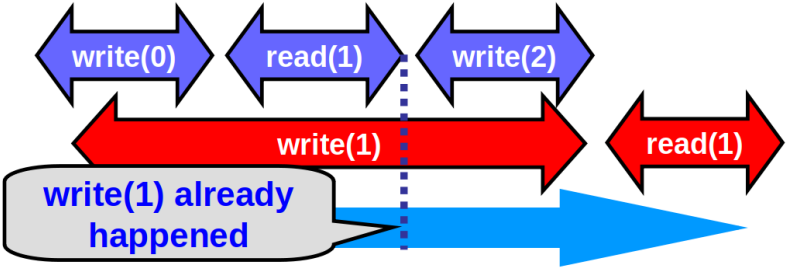

not linearizable
write(0)
read(1)
write(2)
write(1)
read(0)
write(1) already happened

Examples



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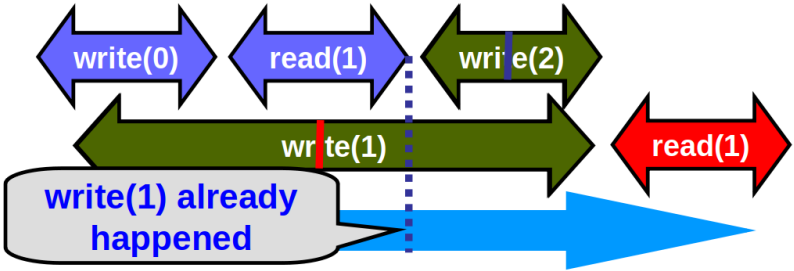

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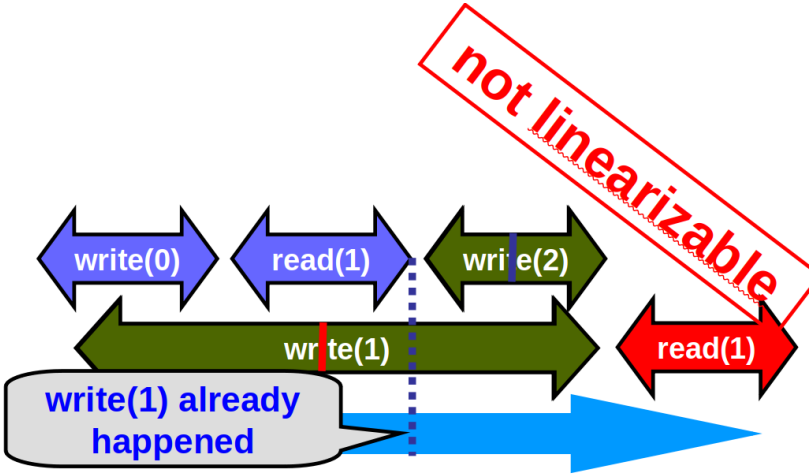
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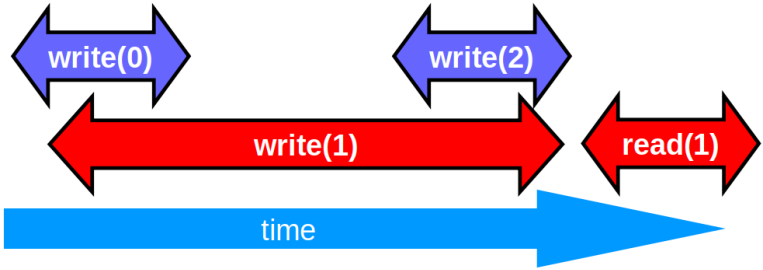

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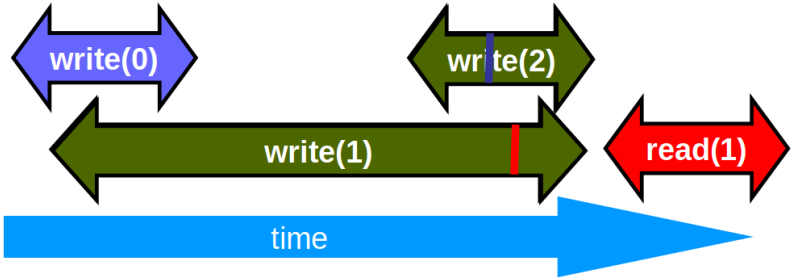

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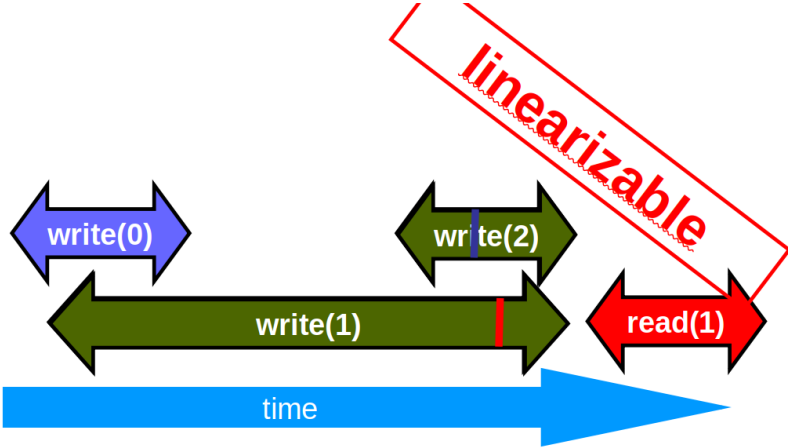

Linearizability: summary

Each method should

- "take effect"
- instantaneously
- between invocation and response events

Linearization point: when method "commit changes to object state"

- Could be different code points for different executions!

Linearizability

- requires to analyze executions, not just methods
- compositional – result of composing linearizable objects is linearizable[2]

[2] Optional homework: Theorem 3.6.1 in "The Art of Multiprocessor Programming"

Linearizability is not the only one

There exists:

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- Sequential consistency
  - Provides other guarantees, harder to reason (no linearization points)
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  - One of the weakest (yet understandable) consistencies
  - Compositional
  - Complicated performance-oriented concurrent data structures may use it to formalize possible and impossible results

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Linearizability is useful yet could be not applicable (by design[3] or by implementation[4])

[3] RCU: advantages and disadvantages https://en.wikipedia.org/wiki/Read-copy-update

[4] ConcurrentLinkedDeque is non-linearizable https://bugs.openjdk.org/browse/JDK-8256833

Intuition behind consistency definitions: takeaways

- **Describe** object first, then implement and test it
- Sequential specifications totally rock
  - Easy to understand and document
  - Modular and extensible
- Concurrent methods are not instantaneous and could overlap

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- Every concurrent execution have order among object-specific operations: linearizable object
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Solution 2: use more fragile and complicated consistency definitions
Remember to check if your approach is compositional.

Lecture plan

1. Time, events, intervals, precedence
2. Mutual exclusion
   - 2 threads
   - N threads
   - Lower bounds on the Number of Locations
   - Optional: fairness
   - Mutual exclusion beyond read-write memory cells
3. Concurrent objects
   - Sequential and concurrent objects
   - Intuition behind consistency definitions
   - Optional: formal definitions and composability
4. Summary

Formal definitions

Optional homework: slides 95-176 from `chapter_03.ppt`
Optional homework: Chapter 3 "Concurrent Objects" up to section 3.6 "Formal Definitions" (pages 45 - 58)
Key insights:

- quiescent consistency
- sequential consistency
- linearizability
- formal definitions: history, projections
- compositional linearizability
- nonblocking property

Summary

First steps towards formalization of concurrent execution

- Timeline, Event, Interval, Precedence

Rigorously prove mutual exclusion properties:

- Mutual exclusion, Deadlock-freedom, Starvation-freedom

Non-trivial results for mutual exclusion

- Peterson's algorithm
- FilterLock
- Lower bounds on the number of locations

Formalizing allowed behaviour of concurrent object:

- Sequential object, sequential specification
- Concurrent object, consistency
- Linearizability, linearization points
- Non-linearizable executions

Summary: homework

Your second critical task will be related to the theoretical concurrency.
Maybe you would like to re-read slides and check out companion slides for "The Art of Multiprocessor Programming"