June 3, 2022

Dear Editor

We are submitting the revision of the manuscript "Waveform inversion via reduced order modeling" by Liliana Borcea, Josselin Garnier, Alexander V. Mamonov and Jörn Zimmerling for publication in Geophysics.

We wish to thank the associate editor and the two referees for the very careful and thoughtful reviews and also for the suggestions on how to improve the presentation of our results. We took into consideration these suggestions in our revision. The result is basically a complete rewrite of the theory part, a different organization of the material, which includes new appendices and the addition of new numerical results concerning the approximation of the array response matrix from tow streamer type measurements.

Here is a detailed list of the changes requested by the associated editor. We list in quotes the request and then explain how we addressed it

1. "Both reviewers thought the paper may be inaccessible to those without an advanced mathematical background. My suggestion is to open your manuscript with a high-level description of the key concepts and flow of ideas. I think the summary provided by reviewer #2 is an excellent example and starting point. It gives a person with a general applied mathematics background, such as myself, a "fighting chance" to understand your work. This still would leave out many journal readers, but will significantly enlarge the group able to grasp key concepts presented in your paper."

   We rewrote from scratch the theory part: We begin with the **Outline and motivation of the method** section where we present the key ideas in a style similar to that in the report of reviewer #2. We give the general description of the method, motivate the steps and explain why we expect the ROM based objective function to be better suited for velocity estimation. In the following **Technical details of ROM computation** section we give the flowchart and more technical details of the ROM construction. This is followed by the algorithms, where all the steps are spelled out again. Note that all this is for the ROM of the wave operator. The material on the ROM propagator, which is needed only for the regularization of the ROM construction, has been moved to Appendix E.

2. "Along the same lines, I found the algorithmic descriptions on pages 18-20 very helpful in understanding the methodology. But I still got lost in details. Would it be possible to make them even more self-contained by repeating formulas for computing various quantities involved and accompanying each line by an intuitive description which would speak to those who may not be able to keep track of various mathematical symbols?"

   We hope that after the flow chart given in the previous section, Algorithms 1 and 2 that now have all the pertinent formulas, should be easier to follow.

3. "Next, reviewers identified several aspects of your methodology which may limit its practical adoption. For example, the requirement for co-located sources and receivers excludes all

of conventionally acquired seismic data. It would be good to comment on this and other assumptions. For example, you could draw parallels with widely-practiced surface-related multiple attenuation which also requires full coverage by co-located sources and receivers. In this processing technique the required data is reconstructed on the fly either through Fourier interpolation or differential (partial) NMO. The key feature is that though such reconstruction is necessarily inexact, the methodology can tolerate small kinematic and amplitude errors. So the question is whether your method can do the same. You will find a list of other assumptions that need addressing when you read reviewer feedback."

   Thank you for pointing us in the right direction! We now added in the section **VELOCITY ESTIMATION WITH NOISY AND TOW STREAMER DATA** results with the matrix $\boldsymbol{\mathcal{M}}(t)$ approximated from tow streamer measurements using source-receiver reciprocity on the fly and interpolation.

4. "Finally, it would be preferable to update numerical examples to help readers better evaluate quality of your results. For example, could you show the results of applying a "conventional" multi-scale FWI, where one starts inversion from low frequencies and progresses to higher ones, thus avoiding cycle-skipping? You could also display the "true" velocity filtered down to the same frequency band as that used in your inversion, to be compared with your final inverted models. In my opinion, this set of items is desirable but is not a pre-requisite for publication."

   We considered displaying a low band pass filtered image of the velocity, but the paper is quite long already and we do not believe that this would add much to the understanding of our method.

Here is a detailed list of the changes requested by the first referee:

1. "Some knowledge of or background in the use of reduced order models (ROMs) seems necessary for the reader to acquire a good understanding of the material. The authors give a detailed description of their algorithm, and while the mathematics is not at a very difficult level, I find that not having prior exposure to ROMs makes it difficult to follow."

   We have rewritten the theory section in a much more explicit way, which motivates the steps, explains what the ROM is and gives a flow chart of the ROM computation. We do hope that this will make the material accessible.

2. "Geophysics articles are primarily published for the purpose of presenting some knowledge that can have practical applications, and that the reader can conceivably utilize in their own work. Again, it may be difficult for most geophysicists to apply the research presented in this paper."

   We believe that this comment is related to the next item, which we answer next.

3. "Some of the assumptions under which this algorithm is derived are quite limiting. At the least, the authors should discuss whether and how the algorithm could be extended. For example, as it is now, the sources and receivers are assumed to be located at the same places – this is never the case in reality, and in 3D does not even come close to reality. Also, you need to clarify the statement that the waveform must proceed deeper at each time step; I see that this just means that some portion of the waveform must go deeper at each step, but was

initially confused, thinking you might not allow turning waves, which are normally present."

We now show that the algorithm can work with measurements gathered with a tow streamer data acquisition setting. The comment about the waveform proceeding deeper at each step has been removed, as it created confusion and was not really needed. If it is the case that the snapshots that define the approximation space become linearly independent, then the regularization scheme will take care of that, so there is no real restriction on the algorithm.

4. "Overall, to understand your algorithm the reader must work hard to keep straight a number of subspaces, projections, matrix transformations, etc. This would be made easier if you present some background on the ROM technique, particularly if you could include plots that illustrate what a ROM actually is for some specific model and data set. However, even with that, I think the paper may be better suited to a journal that targets a more mathematically inclined audience already familiar with the necessary background."

   As we already mentioned above, we have rewritten the theory section to be more accessible. We have also provided a flow chart which should help in following the key steps.

5. "Abstract: You mention that the ROM misfit function is demonstrably convex – you need to make clear whether you can prove this or just show it in examples."

   Please see the material in section **Outline and motivation of the method**, immediately following equation (5).

6. " Pg. 2 – here you note that a pressure wave is generated by the sth sensor; this is a very confusing statement for a geophysicist to read, as waves are generated by sources. Your point is that your algorithm requires co-located sensors and sources, and you need to make this clear and explain whether this assumption can be eliminated."

   We now make it clear from the beginning, that for the theoretical discussion we assume co-located source-receivers. We also explain that the algorithm extends to such measurements approximated from tow streamer measurements.

7. "Pg. 5 – Theory section: to assist the reader, it would help if you could schematically illustrate the basics of the ROM algorithm. Could you use a flow chart and/or diagrams to relate the input data to the ROM and the ROM to the inverted model? For me, having never studied ROM, I have a hard time picturing the flow of data and the different projections on spaces. Can you include plots that show what the ROM actually is for a specific model?"

   We added the flow chart and now motivate each step in the **Outline and motivation of the method**.

8. "Pg. 5 – here you mention that you demonstrate convexity. You should give some discussion about why this is plausible with a reduced basis method (it certainly seems so), and if there is any likelihood of being able to prove this under certain conditions."

   Please see the answer at item 5 above.

9. "Pg 6, equation 5 – should give a reference for the Paley-Wiener theorem"

   Done. This is now just above eq. (31).

10. "Pg 8, equation at the bottom – make note that this operator equation holds because of the completeness properties of the eigenfunctions $y_l(x)$"

We do say we have a basis of eigenfunctions, which implies completeness.

11. "Pg 10, where you start to review the ROM construction of Borcea, this section is probably too brief to allow most readers to follow. If you want to address a wider audience, you will need to include more explanation of why you are performing each step."

    We moved the presentation of the ROM propagator in the appendix. The steps in the computation of the ROM operator, which is used mostly in this paper, are now motivated in the theory section.

12. " Pg 12 – you should make it clearer that you must factor the mass matrix (Cholesky) to allow you to separate out the component that propagates forward in time."

    Please see the discussion following eq. (19).

13. "Pp. 13-14 – You are introducing another basis V; I believe this is to project out the causal part of the wavefield, but again, there is little motivation given, and the reader would need more explanation of this step."

    Please see the discussion following eq. (24).

14. "Pg 17 - when you are comparing the two ROMs, it would be really helpful to include a schematic picture that indicates the differences between them, and what they represent."

    Since the propagator ROM has a marginal use in this paper, it has been moved to Appendix E, where we explain how we use it for regularization. The only difference between the ROMs that is relevant to this paper is that the propagator is a block tridiagonal matrix, whereas the other ROM is a full matrix. We now explain that in Appendix E.

15. "Pg 26-30 – It may be better to put the analysis for the noisy case into an appendix, and just discuss the general changes, and the numerical results, in the main body of the paper. This part is difficult to follow."

    Indeed, this is what we have done.

16. "Pg 32 – Conclusions: Again, you point out that sensors play the role of both sources and receivers – can this restriction be avoided? You should also add a discussion of the relative computational cost of traditional FWI and your ROM algorithm, estimate the asymptotic dependence of the cost on problem size (number of shots and receivers, model size, data frequency, etc.), for both 2D and 3D problems. Could the algorithm be practical in 3D?"

    We added the subsection **Computational cost** where we estimate the additional cost of using the ROM for inversion. Basically, this cost is smaller or at most comparable to that of solving the forward problem for all the sources.

17. "Figures, in general: the axis label spacings need to be selected better, yours typically have unusual spacings such 0.45. The figures will need to be larger, and the axes need to be labeled (e. g. with m/s)."

    We fixed the figures as suggested.

Here is a detailed list of the changes suggested by the second referee:

1. "Cycle-skipping mitigation - why?"

Please see the material in section **Outline and motivation of the method**, immediately following equation (5).

2. “Data restrictions”

   We now explain from the start that while the theory is written for collocated sources-receivers, we can also use approximations of the measurement matrix obtained from tow streamer type measurements. We also show inversion results obtained from such an approximation,

3. “Linear independence of snapshots”

   This linear independence is really only needed for the theoretical presentation. In practice we can have linearly dependent snapshots, in which case the mass matrix becomes singular and it must be regularized. The regularization is also needed to deal with noise and tow streamer data and it is described in Appendix E.

4. “Computational complexity”

   We added the subsection **Computational cost** where we estimate the additional cost of using the ROM for inversion. Basically, this cost is smaller or at most comparable to that of solving the forward problem for all the sources.

5. “The construction of the ROM propagator is really irrelevant, and could usefully be removed.”

   The ROM propagator is only used in the regularization scheme. We removed it from the main text and only give the relevant facts in Appendix E.

6. “Can this construction be applied to transmission data? For instance, what about one of the other Camembert experiments from the Gauthier et al. paper, in which the receivers are on the opposite side (in addition, to keep the coincident source-receiver hypothesis valid)?”

   The ROM that we have now does not apply to transmission data. We are thinking of how to do this, but it requires a complete rethinking and we do not have a solution, yet.

Thank you for considering our submission.

With best regards,

Jörn Zimmerling, on behalf of all the authors