Mathematics' grade 10

Unit 5

Summarized note

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TRIGONOMETRIC FUNCTIONS

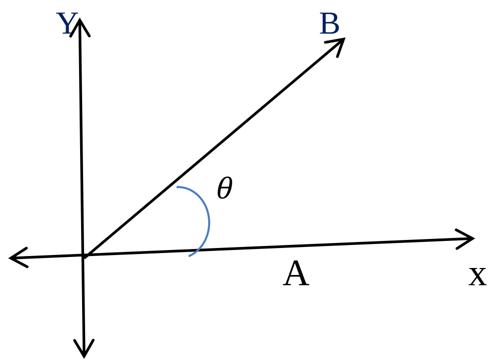
Main Contents

5.1 ***BASIC TRIGONOMETRIC FUNCTIONS***

5.1.1 The Sine, Cosine and Tangent Functions.

Basic terminologies

- ✓ An angle is determined by the rotation of a ray about its vertex from an initial position to a terminal position



- $\overrightarrow{OA}$ (Initial position) is called the initial side of θ.
- $\overrightarrow{OB}$ (Terminal position) is called the terminal side of θ.

Angles in standard position

- An angle in the coordinate plane is said to be in standard position, if
  1 its vertex is at the origin, and
  2 its initial side lies on the positive *x*-axis.
- ✓ The angle formed by a ray rotating anticlockwise is taken to be a positive angle.
- ✓ An angle formed by a ray rotating clockwise is taken to be a negative angle.

Radian measure of angles

- The angle θ subtended at the center of a circle by an arc equal in length to the radius is1 *radian*. That is $\theta = \frac{r}{r}$ **=** 1 *radian.*
- In general, if the length of the arc is *s* units and the radius is *r* units, then $\theta = \frac{s}{r}$ radians.

Rule 1

- To convert degrees to radians, multiply by $\frac{\pi}{180^0}$

  i.e., *radians* = *degrees x* $\frac{\pi}{180^0}$

. **Rule 1**

- To convert radians to degrees, multiply by $\frac{180^0}{\pi}$

  i.e., degrees = $radians\ x \frac{180^0}{\pi}$

Example Convert 60° to radians.

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Solution: A, $60° = 60°\ x\ \frac{\pi}{180^0}\ =\ \frac{\pi}{3}\ radians$

Example **convert** $\boldsymbol{\frac{\pi}{4} rad}$ **to degrees**

Solution: $\boldsymbol{\frac{\pi}{4} rad = \frac{\pi}{4}\ x\ \frac{180^0}{\pi} = 45°}$

Definition of the sine, cosine and tangent functions

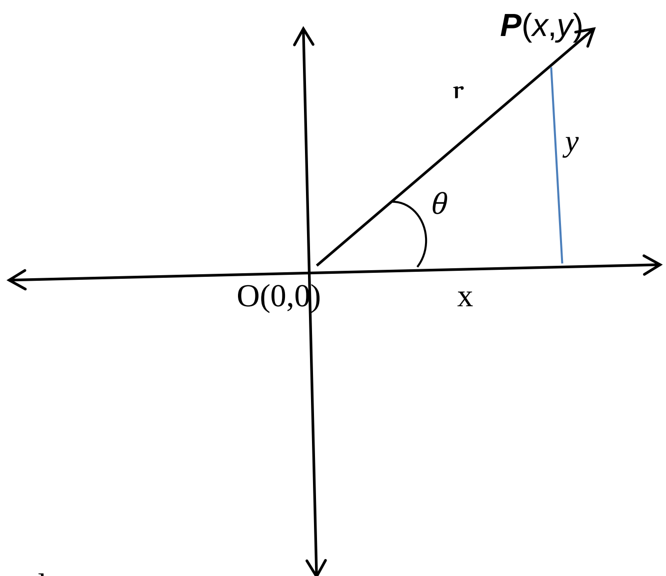
Definition 5.1

If θ is an angle in standard position and ***P***(*x*,*y*) is a point on the terminal side of θ, other than the origin **O**(0, 0), and *r* is the distance of point ***P*** from the origin **O**, then

sinθ = $\frac{OPP}{HYP} = \frac{y}{r}$

cosθ = $\frac{ADJ}{HYP} = \frac{x}{r}$

tanθ = $\frac{OPP}{ADJ} = \frac{y}{x}$



Remember that Δ*OPQ* is a right angle triangle.

(by the Pythagoras Theorem, $r = \sqrt{x^2 + y^2}$)

Example Evaluate the sine, cosine and tangent functions of θ, if θ is in standard position and its

Terminal side contains the given point P (*x* , *y*):

a, P (– 6, – 8)

Solution: The distance $r = \sqrt{x^2 + y^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10\ units$

sinθ = $\frac{OPP}{HYP} = \frac{-8}{10} = \frac{-4}{5}$

cosθ = $\frac{ADJ}{HYP} = \frac{-6}{10} = \frac{-3}{5}$

$\tan\theta = \frac{OPP}{ADJ} = \frac{-8}{-6} = \frac{4}{3}$

Activity

1. ***What is the value of*** $\sin\theta$ ***, if*** θ ***is in standard position and its terminal side contain the point p(-3 ,4).***

   A, $\frac{-3}{5}$ $B,$ $\frac{4}{5}$ $C,$ $\frac{5}{4}$ $D,$ $\frac{-5}{3}$ $E,$ $\frac{-4}{3}$

The unit circle

- The circle with center at (0, 0) and radius 1 unit is called the unit circle.
- unit circle is used to find the sine , cosine and tangent values of quadrantal angles.

Trigonometric values of 30°, 45° and 60°

- to find the trigonometric values of 45^0 use isosceles right angle triangle .
- to find the trigonometric values of 30^0 and 60^0 use equilateral triangle.

Signs of sine, cosine and tangent functions:

- In the first quadrant all the three trigonometric functions are positive.
- In the second quadrant only sine is positive.
- In the third quadrant only tangent is positive.
- In the fourth quadrant only cosine is positive.

Functions of negative angles:

- If θ is an angle in standard position, then

  $\sin(-\theta) = -\sin\theta$

  $\cos(-\theta) = \cos\theta$

  $\tan(-\theta) = -\tan\theta$

Complementary angles:

Two angles are said to be complementary, if their sum is equal to 90°. If θ and β are any two complementary angles, then

$\sin\theta = \cos\beta$

$\cos\theta = \sin\beta$

$\tan\theta = \frac{1}{\tan\beta}$

Reference angle θ_R:

- If θ is an angle in standard position whose terminal side does not lie on either coordinate axis, then the reference angle θ_R for θ is the positive acute angle formed by the terminal side of θ and the *x*-axis.

- The values of the trigonometric function of a given angle θ and the values of the corresponding trigonometric functions of the reference angle θ_R are the same in absolute value but they may differ in sign.

Supplementary angles:

- Two angles are said to be supplementary, if their sum is equal to 180°. If θ is a second quadrant angle, then its supplement will be (180° – θ).
- sin θ = sin (180° – θ),
- cosθ = – cos (180° – θ) ,
- tanθ = – tan (180° – θ)

Co–terminal angles

- ✓ Co-terminal angles are angles in standard position that have a common terminal side.

- ✓ Given an angle θ, all angles which are co–terminal with θ are given by the formula θ ± *n* (360), where *n* = 1 , 2, 3, . . .
- ✓ Co-terminal angles have the same trigonometric values.

Graphs of the Sine, Cosine and Tangent Functions

The sine and cosine functions.

- ✓ The domain of the sine and cosine functions are the set of all real numbers
- ✓ The range of the sine and cosine function are $\{y \mid -1 \leq y \leq 1\}$
- ✓ The graph of the sine and cosine functions repeats itself every 360° or 2π radians

The tangent function

- ✓ The range of the tangent function is the set of all real numbers
- ✓ The domain of the tangent function is $\{ \theta / \theta \neq n\frac{\pi}{2}$ where n is an odd integer$\}$
- ✓ The graph of the sine and cosine functions repeats itself every 180° or π radians

5.2, *THE RECIPROCAL FUNCTIONS OF THE BASIC TRIGONOMETRIC FUNCTIONS*

- ✓ The reciprocals of the sine, cosine and tangent functions, named respectively as cosecant, secant and cotangent functions.

  1, $\csc\theta = \frac{1}{\sin\theta}$

  2, $\sec\theta = \frac{1}{\cos\theta}$

  3, $\cot\theta = \frac{1}{\tan\theta}$

5.3 SIMPLE TRIGONOMETRIC IDENTITIES

Pythagorean identities:

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $\cot^2\theta + 1 = \csc^2\theta$