

***PHYSICS GRADE NINE SHORT NOTE AND  
PRACTICE QUESTIONS  
PREPARED BY BORU BEJIGA***

***ADDRESS***

***Phone No:- 0947252967***

***EMAIL:- [borubejiga@gmail.com](mailto:borubejiga@gmail.com)***

***APRIL,2020***

## UNIT 5

### Simple machines

- Simple machines are often describes as elementary building block from which all other complex machines are made.

#### Purposes of machines

- Simple's machines are a device that makes works easier by performing one or more of the following:
  - It may increases the effect of force (Force multiplier)
  - It may increases the distance against the applied force (Speed multiplier)
  - It may change the direction of the applied force.

The term of simple machine is refers to:

- a device that only requires a single force to do work.
- a device for doing work that has only one part and
- a device that use a single effort to do work against a single load.

**Note that:** no machine can create extra energy. Because the law of conservation is obeyed in any machines or energy cannot be created or destroyed. This means the work output obtained from the machine is cannot be greater than the work input.

### Terms used to describe simple machines

#### 1. Input force ( Effort)

It is the force exerted to the machine.

#### 2. Output force (Load, weight...)

It is the force produced by the machine

#### 3. Mechanical advantage (MA)

It is the ratio between the load and the effort.

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}} = \frac{\text{outputforce}}{\text{inputforce}}$$

$$MA = \frac{FL}{FE}$$

There are two types of mechanical advantage

**3.1 Actual mechanical advantage (AMA)**

Is the ratio between the load and the effort taking account there is energy losses due to friction.

$$AMA = \frac{FL}{FE}$$

**3.2 Ideal mechanical advantage ( IMA)**

Is the ratio between the load and the effort, assuming there is no energy losses due to friction.

$$IMA = \frac{FL}{FE}$$

**4. Velocity ratio ( VR)**

Velocity ratio is the ratio between the distances moved by the effort and the distance moved the load.

Velocity ratio = distance moved by effort/ distance moved by load.

$$VR = SE/SL.$$

**5. Input work (W<sub>i</sub>)**

Input work is a work done on the machine.

In put work = effort force x distance moved by the effort.

$$W_i = FE \times SE$$

**6. Output work (W<sub>o</sub>)**

It is work done by the machine or work obtained from the machines.

In put work = load x distance moved by the load.

$$W_o = FL \times SL$$

**7. Efficiency (  $\eta$  )**

Efficiency is the work capacity on the performance of a machine.

Efficiency is defined as the ratio of the work output and work input.

$$\text{Efficiency} = \frac{\text{Output work}}{\text{input work}} \times 100$$

$$\eta = \frac{W_o}{W_i} \times 100 \text{ or}$$

$\eta = \frac{AMA}{VR} \times 100$
------------------------------------

If the machine was 100% efficient then  $\eta = AMA/VR = 1$ , in this case there are no energy losses then  $AMA = IMA = VR$

**Note that:**

- MA, VR and  $\eta$  have no units since its ratio
- $W_o$  and  $W_i$  are measured in joule (J)
- Wastage energy =  $W_i - W_o$
- If  $FL > FE$  and  $SE > SL$  then the machine considered as force multiplier
- If  $FL < FE$  and  $SE < SL$  then the machine considered as speed multiplier
- If  $MA \text{ \& } VR > 1$  then the machine considered as force multiplier
- If  $MA \text{ \& } VR < 1$  then the machine considered as speed multiplier
- If  $MA \text{ \& } VR = 1$  then the machine considered as direction changers.

**Example: 5.1**

A simple machine able to moves a load of 2000N through a distance of 40cm when a 100N moves through a distance of 10m. Calculate:

- A) The work on done the machine
- B) The work by done the machine
- C) The actual mechanical advantage
- D) The velocity ratio
- E) The efficiency of the machine
- F) The wasted energy
- G) The purposes of the machine.

**Solution: given**

$$F_L = 2000N$$

$$F_E = 100N$$

$$S_L = 40cm = 0.4m$$

$$S_E = 10m$$

**Required**

$$A) W_i = ?$$

$$B) W_o = ?$$

$$C) AMA = ?$$

$$D) VR = ?$$

$$E) E_w = ?$$

$$A) W_i = F_E \times S_E$$

$$= 100N \times 10m$$

$$= 1000J$$

$$B) W_o = F_L \times S_L$$

$$= 2000N \times 0.4m$$

$$= 800J$$

$$C) AMA = \frac{FL}{FE} = \frac{2000N}{100N} = 20$$

$$D) VR = \frac{SE}{SL} = \frac{10m}{0.4m} = 25$$

$$E) \eta = \frac{AMA}{VR} \times 100 = \frac{20}{25} \times 100 = 80\%$$

$$F) E_w = W_i - W_o = 1000J - 800J = 200J$$

Since  $FL > FE$ , then the machine is considered as force multiplier

**Types of simple machines**

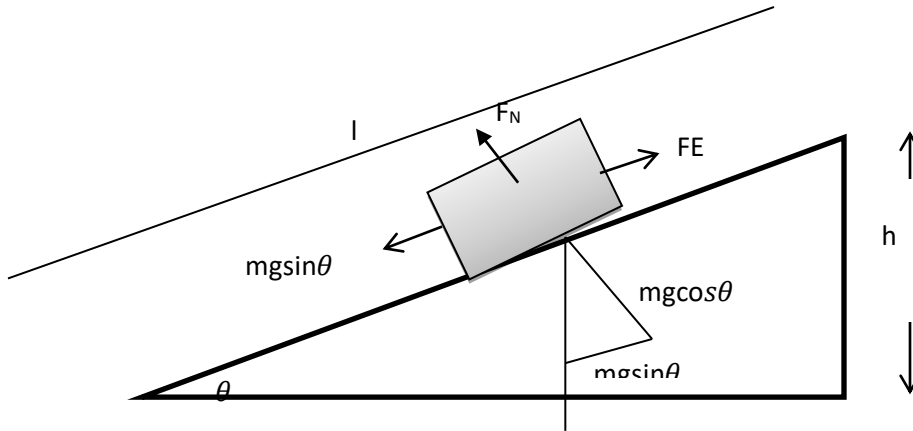
There are six types' simple machines. These are:

Inclined plane	lever
Wedge	pulley
Screw	wheel and axle

- Simple machine can be split into two groups.
- Wedge and screw can be thought of as special kinds of inclined plane.
- Pulley and wheel and axle can be considered to be as special kinds of lever.

### 1. Inclined plane

- Inclined plane is one of the simplest devices for raising heavy load.
- You know that its easier pushes a heavy object up ramp than its lift the same height. This is because inclined plane reduced the force necessary to move a load. e.g



If the block is moving along the inclined plane with constant then AMA is given by:

$$AMA = \frac{FL}{FE},$$

I. If there is no friction

$$AMA = \frac{FL}{FE}, \text{ in this case } FL = mg \text{ and } FE = mgsin\theta$$

$$AMA = \frac{mg}{mg \sin \theta}$$

$$\boxed{AMA = \frac{1}{\sin \theta}} \quad \text{or}$$

i. IF there is no friction  $AMA = VR$

$$AMA = \frac{SE}{SL}, \text{ but } SE = l \text{ and } SL = h$$

$$AMA = \frac{l}{h}, \text{ but } h = l \sin \theta$$

$$AMA = \frac{l}{l \sin \theta}$$

$$\boxed{AMA = \frac{1}{\sin \theta}}$$

- II. In reality when you push upward, you need to exert an effort greater than  $mg\sin\theta$  in order to overcome the force due to friction. So AMA is given by:

$$AMA = \frac{FL}{FE}, \text{ but } FL = mg \text{ and } FE = mg\sin\theta + fk$$

$$AMA = \frac{mg}{mg\sin\theta + fk} \text{ But } fk = \mu kmg\cos\theta$$

$$AMA = \frac{mg}{mg\sin\theta + \mu kmg\cos\theta}$$

$$AMA = \frac{mg(1)}{mg(\sin\theta + \mu k\cos\theta)}$$

$$AMA = \frac{1}{\sin\theta + \mu k\cos\theta}$$

Now the efficiency is given by

$$\eta = \frac{W_0}{W_i} \times 100, \text{ but } W_0 = mgh \text{ and } W_i = (mg\sin\theta + \mu kmg\cos\theta)l$$

$$\eta = \frac{mgh}{(mh\sin\theta + \mu kmg\cos\theta)l}$$

$$\eta = \frac{mg(h)}{mg(\sin\theta + \mu k\cos\theta)l}$$

$$\eta = \frac{h}{(\sin\theta + \mu k\cos\theta)l} \text{ But } h = l\sin\theta$$

$$\eta = \frac{l\sin\theta}{(\sin\theta + \mu k\cos\theta)l}$$

$$\eta = \frac{\sin\theta}{\sin\theta + \mu k\cos\theta} \times 100$$

**Example: 5.2**

A block of mass 20kg is pulled up with a constant velocity over an inclined plane surface 20m long that makes an angle of  $53^\circ$  with a horizontal surface. Calculate

A) The effort needed if the inclined plane is smooth

B) The input and output work

C) AMA and VR if  $\mu k = 0.2$

**Solution: given**

$$l = 20\text{m}$$

$$m = 20\text{kg}$$

$$\theta = 53^\circ$$

$$g = 10\text{m/s}^2$$

Required

$$\text{A) } FE = ?$$

$$\text{B) } W_i \text{ and } W_o = ? \quad W_o = mgh \quad \text{but, } h = l \sin \theta$$

$$\text{C) } \text{AMA and VR} = ? \quad = mgl \sin 53^\circ = 20\text{kg} \times 10\text{m/s}^2 \times 20\text{m} \times 0.8$$

$$W_o = 3200\text{J}$$

$$\text{D) } AMA = \frac{1}{\sin 53^\circ + \mu k \cos 53^\circ} = \frac{1}{0.8 + (0.2)(0.6)} = 1.1$$

$$VR = \frac{SE}{SL} \quad \text{In this case } SE = l \text{ and } SL = h \text{ but } h = l \sin \theta$$

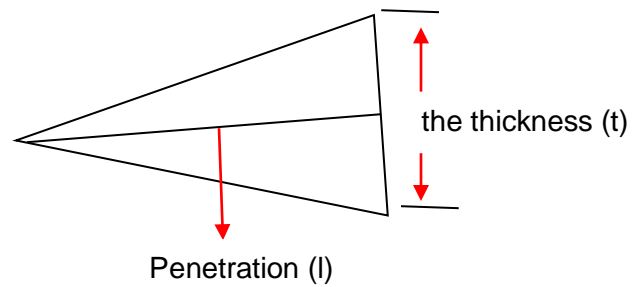
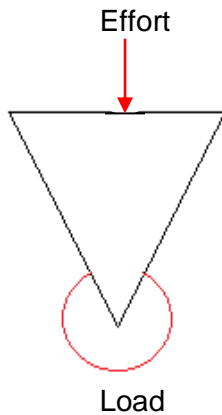
$$VR = \frac{1}{\sin \theta} = \frac{1}{0.8} = 1.25$$

### 1. The Wedge

- Wedge is a simple machine which are sharper at one edge than at another.
- Wedges are used to separate two objects or split objects apart.
- A wedge can be composed of two or one inclined plane.

#### **MA, VR and efficiency of a wedge**

Consider a wedge of length (l) and thickness (t) is used to split a log of wood by an effort (E)



i.  $\boxed{MA = \frac{F_L}{F_E}}$

ii.  $VR = \frac{S_E}{S_L}$  but  $SE = l$  and  $SL = t$

$\boxed{VR = \frac{l}{t}}$

iii. The efficiency is given by

$$\eta = \frac{AMA}{VR} = \frac{F_L}{F_E} \times \frac{l}{t}$$

$$\boxed{\eta = \frac{F_L \times t}{F_E \times l}}$$



## 2. The Screw

- The term screw refers to any cylindrical with thread around it.
- Screw is important (useful machine) that is used to:
  - ✓ load object together
  - ✓ dig into the ground
  - ✓ bore through the materials
- In one turn, the moves a distance equal to the separation between threads. This distance is called pitch (p) of the screw.
- The maximum theoretical mechanical advantage is given by

a.  $IMA = \frac{S_E}{S_L}$  but  $S_E = \pi d$  and  $S_L = p$

$$IMA = \frac{\pi d}{p} \text{ where } p = \text{the pitch of the screw in m}$$

$\pi d$  = circumference of the screw shaft

$$IMA = \frac{2\pi r}{p} \text{ or}$$

$$VR = \frac{2\pi r}{p}$$

### Example 5.3

In order to displace 50N of wood sideways a wedge of length 40cm and thickness 8cm is used. If the mechanical advantage of the machine is 4, then what is:

- A) The velocity ratio
- B) The efficiency
- C) Its purposes

### Solution:

Given:  $t = 8\text{cm}$ ,  $l = 40\text{cm}$ ,  $MA = 4$

Required:  $VR = ?$   $\eta = ?$

A)  $VR = \frac{l}{t} = \frac{40\text{cm}}{8\text{cm}} = \underline{5}$

B)  $\eta = \frac{AMA}{VR} \times 100 = \frac{4}{5} \times 100 = (0.8) (100) = \underline{80\%}$

C) Since  $S_E > S_L$  or  $F_L > F_E$ , then the machine is considered as force multiplier

**Example 5.4**

Suppose the pitch of the screw is 0.5cm and effort applied at the end of the spanner describes a circle of radius 10cm calculate the velocity ratio.

Given

$$r = 10\text{cm} = 0.1\text{m}$$

$$p = 0.5\text{cm}$$

Required

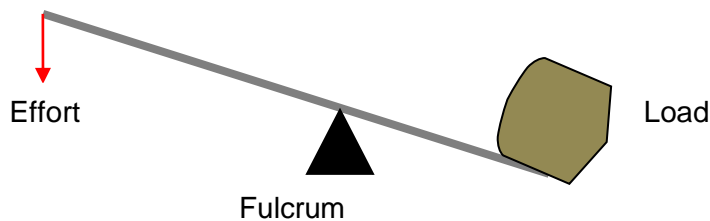
$$VR = ?$$

$$VR = \frac{2\pi r}{p} = \frac{2 \times 3.14 \times 10\text{cm}}{0.5\text{cm}} = \underline{124.6}$$

**5.3 Levers**

Lever is a bar which is free to turn around a fixed point. This fixed point is called the fulcrum (pivot).

E.g.

The MA, VR and efficiency of a lever

- The mechanical advantage of the lever is given by

$$MA = \frac{F_L}{F_E}$$

- The velocity ratio of the lever is given by

$$VR = \frac{S_E}{S_L} \quad \text{Where } S_E = \text{is the distance from the effort to the fulcrum}$$

$S_L =$  is the distance from the load to the fulcrum

- The efficiency of the lever is given by

$$\eta = \frac{AMA}{VR} \times 100$$

## **Different classes of lever**

There are three different classes (order) of lever depending on the relative position of the load, fulcrum and effort.

### **1. 1<sup>st</sup> class of lever**

Fulcrum is between the load and the effort.

Example:

- hammer
- see-saw
- scissors

### **2. 2<sup>nd</sup> class of lever**

Load is between the effort and the fulcrum.

Example:

- wheel barrow
- nut cracker

### **3. 3<sup>rd</sup> class of lever**

Effort is between the load and the fulcrum.

Example:

- ice tongs
- spade
- catapult
- fishing rod

### **Example 5.5**

In iron bar 4m long is used to lift a 1200kg block. The fulcrum is placed 1m from the load block.

Calculate: A) the velocity ratio

B) the input force required

**Solution: given**

$$SE=3m$$

$$SL=1m$$

$$FL = 12000N$$

$$A) VR = SE/SL = 3m/1m = 3$$

$$IMA = VR = 3$$

$$B) 3 = FL/FE$$

$$FE = FL/3 = 12000N/3$$

**Required**

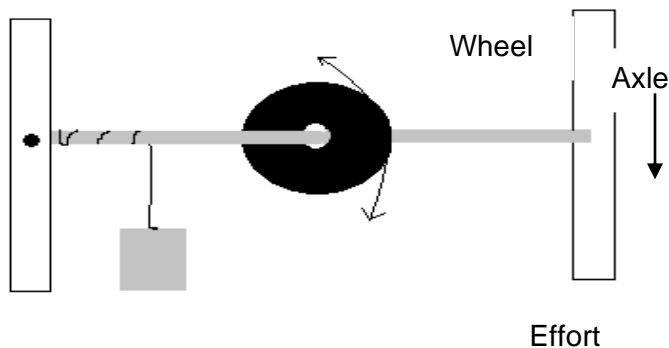
$$A) VR=?$$

$$B) FE=?$$

$$FE = 4000N$$

#### 4. Wheel and axle

A wheel is a circular disk attached to central rod called axle (see figure below)



- The actual mechanical advantage is given by

$$AMA = \frac{\text{Load}}{\text{Effort}} = \frac{F_L}{F_E}$$

- The VR of the wheel and axle is the ratio of radius of the wheel to the radius of the axle. Because when the wheel turns it cover a distance of  $2\pi r$  and in the same time the axle is cover a distance of  $2\pi r$

$$VR = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$VR = \frac{2\pi R}{2\pi r}$$

$$\boxed{VR = \frac{R}{r}}$$

If the machine was 100% efficient then

$$IMA = VR = AMA$$

$$\boxed{IMA = \frac{R}{r}}$$

- Depending on the relative radii, wheel and axle can be thought as force and speed multiplier.
  - When force is applied to wheel in the order to turn the axle, it is force multiplier. In this case VR is become
 
$$\boxed{VR = \frac{R}{r}}$$
  - When force is applied to axle in the order to turn the wheel, it is force multiplier. In this case VR is given

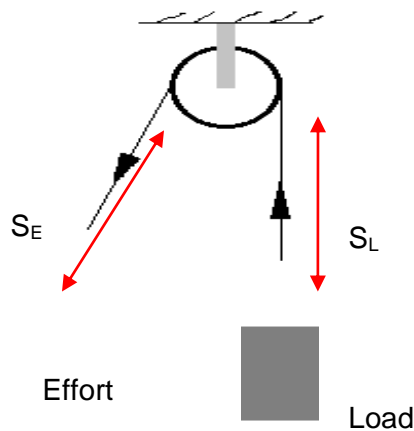
$$VR = \frac{r}{R}$$

## **Pulley system**

- A pulley is a device consisting of a wheel over which a rope or chain is pulled in order to lift heavy object.
- Pulley is a simple machine made with a rope or chain wrapped around a wheel.
- There are different types of pulley but the most common are:
  - ✓ fixed pulley
  - ✓ movable pulley
  - ✓ compound pulley (back and tackle pulley)
  - ✓ differential pulley (chain hoist)

### **1. Fixed pulley (simplest pulley)**

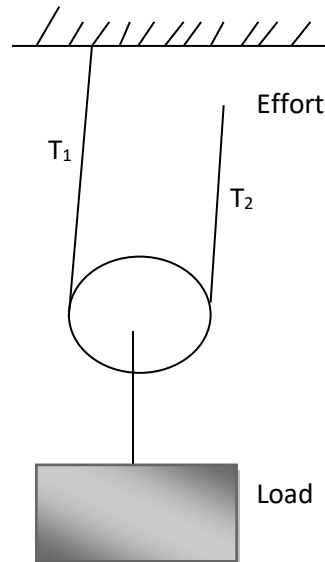
- This is the simplest form of pulley.
- Fixed pulley does not move with the load but it rotates about a fixed axis.
- The wheel on your school flag pole is a good example. E.g.



- ❖ the graph we observe that  $F_L = F_E$  and  $S_L = S_E$  then its velocity ratio is 1 ( $VR = 1$ ). Therefore a fixed pulley neither force or speed multiplier. Its advantage is only to change direction.

## 2. Movable pulley

- It is a kind of pulley where the pulley is moving together with the load and is not fixed with the stand. e.g.



The distance moved by effort twice the distance moved by the load.

$$SE = 2SL$$

$$FL = T_1 + T_2, \text{ but } T_1 = T_2 = FE$$

$$FL = FE + FE$$

$$FL = 2FE$$

Now if there is no losses of energy,  $IMA = VR$

$$VR = \frac{FL}{FE}, \text{ but } FL = 2FE$$

$$VR = \frac{2FE}{FE}$$

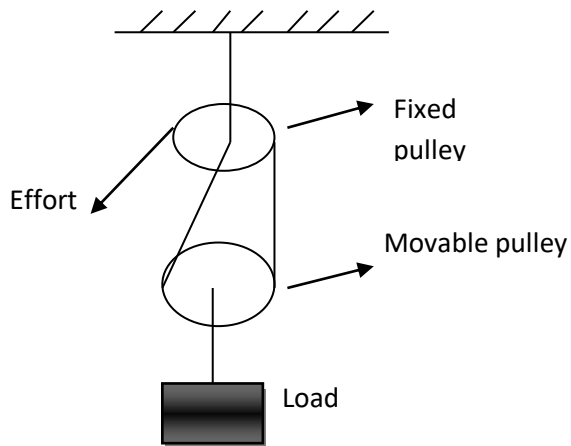
$$VR = 2$$

- Hence the movable pulley has a velocity ratio of two. This pulley system is used to half force the load in order to lift.

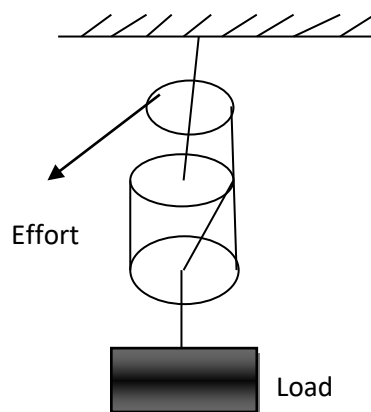
### 3. Compound pulley

- Compound pulley is the combination of fixed and movable pulley.
- The movable pulley provides the MA and whereas fixed pulley changes the direction of the force. This makes it easy to lift the load.
- The velocity of the compound pulley is obtained by simple counting the number of the ropes supporting the movable pulley.

$$VR = \text{Number of section of the rope that lifted the load}$$



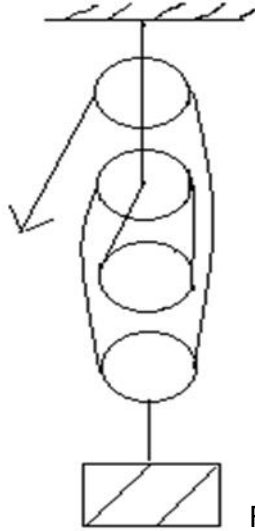
SE = 2SL and VR=2 because N=2



SE = 3SL and VR=3 because N=3

Example 5.5 for the following pulley system calculate

- A) The velocity ratio  
B) The force need to lift 8000N load



Given

$$N = 4$$

$$F_L = 8000\text{N}$$

Required

a)  $VR = ?$

b)  $F_E = ?$

Solution

a)  $VR = 4$  because  $N = 4$

b)  $F_L = T_1 + T_2 + T_3 + T_4$

but  $T_1 = T_2 = T_3 = T_4 = F_E$

$$F_L = F_E + F_E + F_E + F_E$$

$$F_L = 4F_E$$

$$F_E = \frac{F_L}{4}$$

$$F_E = \frac{8000\text{N}}{4}$$

$$F_E = 2000\text{N}$$

**Review exercise for unit 5**

- Which one of the following quantities cannot be increased by any machines?  
A. Distance      B. Speed      C. force      D. Energy
- Mechanical advantage is the amount by which a machine can multiply a force. The force that the machine applies to the object is called the ...  
A. input force      B. output force      C. force ratio      D. Newton force
- 16 cm thick and 40 cm long wedge is used to split a 6m long log of diameter 30cm. What is the velocity ratio of the wedge?  
A. 0.4      B. 5      C. 2.5      D. 0.2
- What is the ratio if the pitch of the screw is 0.8cm and the effort applied at the end of the spanner describes a circle of radius 10cm?  
A. 13.4      B. 78.5      C. 157      D. 234
- Which of the following devices is always a force multiplier?  
A. 1<sup>st</sup> class of lever      C. 2<sup>nd</sup> class of lever  
B. 3<sup>rd</sup> class of lever      D. Wheel and axle
- A simple machine has an efficiency of 75% and VR of 12. If an effort applied to the machine is 100N, then which of the following is **NOT** true about the machine?  
A. MA of the machine is 9  
B. Magnitude of the load is 900N



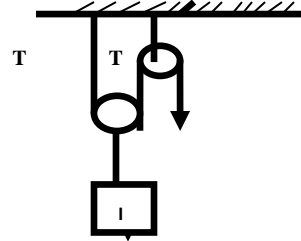
- C. 25% of energy lost due to friction  
D. The machine used as speed multiplier
7. Three of the following simple machines are basically the same. Choose the one that does not belong with the group.  
A. Pulley                      B. wedge                      C. lever                      D. wheel and axle
8. The diameter of the two upper pulleys of a chain hoist are 20cm and 16cm. if the efficiency of the hoist is 50 percent what force is required to lift a 600kg load? (use  $g=10\text{m/s}^2$ )  
A. 1200N                      B. 2200N                      C. 3200N                      D. 4200N
9. If the velocity ratio of a simple machine is greater than one, then the simple machine is called \_\_\_\_\_  
A. Speed multiplier                      B. Force multiplier  
C. Distance multiplier                      D. Power multiple
10. Usually the purpose of using a bicycle is:  
A. To multiply force                      C. To multiply power  
B. To multiply energy                      D. To multiply speed
11. An inclined plane of height 4m and a length of 10m are used to raise a 100N a load. If the surface is smooth, what is the effort required.  
A. 20N                      B. 30N                      C. 40N                      D. 50N
12. What is the purpose of using an inclined plane  
A. Multiplying distance                      C. Multiplying energy  
B. Multiplying force                      D. Multiplying speed
13. The difference in 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> class levers is  
A. The order the fulcrum, load and effort                      C. The amount of effort used  
B. The size of the load                      D. The distance between the effort and the load
14. A simple wheel and axle is used to lift a bucket of water out of a well. The radii of the wheel and axle are 20cm and 4cm, respectively. What is the theoretical effort required to lift a load of 300N when it is applied on the wheel, assuming no energy losses?  
A. 600N                      B. 150N                      C. 30N                      D. 60N
15. A 7cm thick and 20cm long wedge is used to pierce a 2cm long log of diameter 20cm. what is the velocity ratio of the wedge  
A. 10                      B. 2.14                      C. 0.1                      D. 0.47
16. Which one of the following is **NOT** the purpose of machines?  
A. Multiplying force                      C. Multiplying speed  
B. Transferring energy                      D. Multiplying energy
17. A force of 80N is needed to raise a 240N load with pulley system. The load goes up 2m for every 10m of the rope pulled through the pulleys. What is the efficiency of the pulley system?  
A. 48%                      B. 50%                      C. 60%                      D. 75%
18. The following would increase a wedge's mechanical advantage:  
A. Increase thickness and increase length  
B. Decrease thickness and increase length  
C. Increase thickness and decrease length  
D. Decrease thickness and decrease length

19. A simple wheel and axle is used to lift a bucket of water out of a well. The radii of the wheel and axle are 20cm and 4cm, respectively. What is the theoretical effort required to lift a load of 300N when it is applied on the wheel, assuming no energy losses?

A. 600N                      B. 150N                      C. 30N                      D. 60N

20. the pulley system shown bellow is a combination of movable and fixed pulley. what is AMA of the pulley?

A. 1                      C. 3  
B. 2                      D. 4



21. The following would decrease a wedge's velocity ratio:

A. increase thickness and increase length  
B. decrease thickness and increase length  
C. increase thickness and decrease length  
D. decrease thickness and decrease length

22. How is the efficiency of a simple machine defined?

A. the ratio of the ideal mechanical advantage to the actual mechanical advantage  
B. the ratio of the work input to the work output  
C. the ratio of the load to the force on machine  
D. the ratio of the actual mechanical advantage to the velocity ratio

23. Which of the following devices is always a speed multiplier?

A. 1<sup>st</sup> class of lever                      C. 2<sup>nd</sup> class of lever  
B. 3<sup>rd</sup> class of lever                      D. Wheel and axle

24. If the mechanical advantage of a simple machine is 10, then the

A. output force is 10 times the effort  
B. effort is 10 times the output force  
C. efficiency is 10%  
D. the work output is 10times the input

25. A differential pulley with radii  $r = 15\text{cm}$  and  $R = 18\text{cm}$  are fastened together and turn on the same axle. If a force of  $F = 200\text{N}$  is applied on the rope to lift a load  $W = 2000\text{N}$  then what is the efficiency?

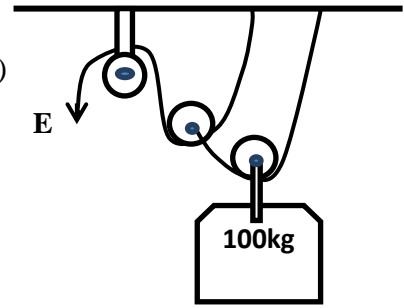
A. 63.3%                      B. 73.3%                      C. 83.3%                      D. 93.3%

26. A block of weight 4000N is pushed up a slope by a force of 400N. Assume there is no friction then calculate the velocity ratio

A. 5                      B. 10                      C. 15                      D. 20

27. A pulley system shown in the figure is used two movable pulley, one fixed and two mass less string over frictionless pulleys to lift a load of mass 100kg.

What is the effort required to lift this load? (Use  $g = 10m/s^2$ )



- A. 250N
- B. 500N
- C. 750N
- D. 1000N

28. On a 2.5m long first class lever a load is placed 0.5m away from the fulcrum. How large is the velocity ratio of the lever?

- A. 0.5
- B. 2.5
- C. 4
- D. 5

29. An inclined plane is used to lift an object that weighs 360N by using 60N effort along a slope of length 20m and at  $60^\circ$  with the horizontal. What is the velocity ratio of this machine? (use  $\cos 60^\circ = \frac{1}{2}$ , and  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ )

- A.  $\frac{2\sqrt{3}}{3}$
- B.  $\frac{\sqrt{3}}{2}$
- C.  $\frac{3\sqrt{2}}{2}$
- D.  $\frac{1}{3}$

30. The following would increase a lever's mechanical advantage:

- A. Increase the load arm length
- B. Increase the effort arm length
- C. Double both the load and effort arm length
- D. Halve both the load and effort arm length

31. Scissors are a combination of what two machines?

- A. lever and wedge
- B. lever & inclined plane
- C. wheel and axle
- D. wedge and inclined plane

32. A wheelbarrow is an example of what simple machine?

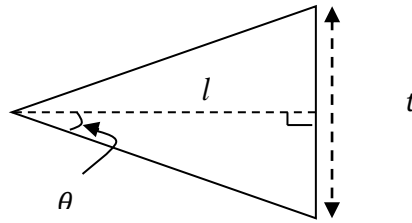
- A. Class 1 lever
- B. **Class 2 lever**
- C. Class 3 lever
- D. Wheel and Axle

33. An effort is applied on a block to raise a body on rough inclined plane of sloping at an angle of  $\theta$  with the horizontal. If the object is moving up with uniform velocity which one of the following is the expression of its efficiency?

- A.  $\tan \theta$
- B.  $\frac{1}{\sin \theta + \mu \cos \theta}$
- C.  $\mu g \cos \theta$
- D.  $\frac{\sin \theta}{\sin \theta + \mu \cos \theta}$

34. Consider active twin of the inclined planes joined back-to-back and their sloping sides are outside as shown below. Which one of the following is the correct expression of the velocity ratio of this wedge?

A.  $\frac{1}{2 \tan \theta}$       B.  $\frac{l}{\tan 2\theta}$       C.  $\frac{t}{2 \tan \theta}$       D.  $\frac{l}{2 \tan \theta}$



35. A lever used with the fulcrum at one end and the effort at the other is  $3m$  long. A load of  $600N$  is  $60cm$  from the fulcrum and is raised by an effort of  $200N$ . What is the efficiency of the lever?

A. 60%      B. 75%      C. 40%      D. 25%

36. Which one of the following **NOT** correct about the three classes of lever?

- A. For 1<sup>st</sup> class lever, velocity ratio could have a value greater than, less than or equal to 1.  
 B. For 2<sup>nd</sup> class lever, velocity ratio is always greater than 1.  
 C. For 3<sup>rd</sup> class lever, velocity ratio is always less than 1.  
 D. For 2<sup>nd</sup> class lever, velocity ratio is always less than 1.

## UNIT 6

### FLUID STATICS

Fluid is a substance that can flow. This includes both liquid and gasses.

The study about the density and pressure of the fluid at rest is known as fluid statics.

Air pressure

- Pressure is defined as the amount of force acting per unit area
- Pressure is the ratio of force to contact area that is perpendicular to the force.

Mathematically

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A}$$

Where, P= pressure in Pa

F = force in N

A = area in m<sup>2</sup>

**Note that:**

- I. When the area of contact is made small, the force per unit area is larger or pressure is large.
- II. When the area of contact is relatively large, the force per unit area is small or pressure is small.

Pressure is a scalar quantity and measured in Pascal (Pa)

$$1Pa = N/m^2 = kg/ms^2$$

#### Example 6.1

An elephant has a mass of 4000kg. Each of its feet covers an area equal to 0.8m<sup>2</sup>. What is the pressure from each foot?

Solution: Given  $P = \frac{F}{A}$  but,  $F = mg = 4000kg \times 10m/s^2 = 40,000N$

$m = 4000kg$   $P = \frac{40,000N}{0.8m^2}$

$A = 0.8m^2$   $p = 50000Pa$

**Required:** P=?

## Atmospheric pressure

- ✓ The atmosphere is the layer of air that surrounds (covers) the Earth.
- ✓ The pressure of the atmosphere is known as atmospheric pressure.

Atmospheric pressure is varies with:

- ❖ Temperature
- ❖ The altitude above the sea level
- ❖ The impact of weather change

The atmospheric pressure at sea level is  $1.01 \times 10^5 \text{Pa}$

$1 \text{atm} = 1.01 \times 10^5 \text{Pa} = 76 \text{mmHg} = 101 \text{kPa}$

- Barometer is a device used to measure the atmospheric pressure. There are different types of barometer. The most common used barometer are mercury barometer and aneroid barometer.

### Some uses of air pressure

Atmospheric pressure widely used in

- Drinking strew
- Force pump
- Suction pad
- Lifting pump
- Siphon

## 6.2 Fluid pressure

- ✓ Pressure in static fluid is due to the weight of the fluid.
- ✓ Liquid is one of the state of matter which has its own definite volume but not its own definite shape instead it take the shape of the container. E.g

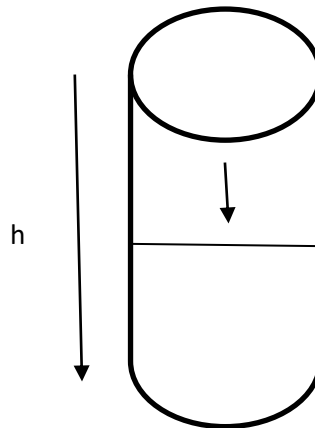
$$P = \frac{F}{A} \text{ but, } F = mg$$

$$P = \frac{mg}{A} \text{ but, } \rho = \frac{m}{V} \rightarrow m = \rho V$$

$$P = \frac{\rho Vg}{A} \text{ but, } V = Ah$$

$$P = \frac{\rho Ahg}{A}$$

$$P = \rho gh$$



- i. Pressure in fluid is depending up on:
  - ❖ The depth of the fluid
  - ❖ The density of the fluid
  - ❖ The acceleration of the gravity
- ii. Pressure increases with depth
- iii. Pressure is acts equally in all direction at the same point.
- iv. Pressure of the liquid does not depend on the shape of the object

### Example 6.2

What pressure is exerted by the water at bottom of a sea whose depth of 2000m?

Take (density of water =  $1010 \text{ kg/m}^3$  and  $g = 10 \text{ m/s}^2$ )

**Solution: Given**

$$\begin{aligned}
 h &= 2000 \text{ m} & P &= \rho gh \\
 \rho &= 1010 \text{ kg/m}^3 & &= 1010 \text{ kg/m}^3 \times 10 \text{ m/s}^2 \times 2000 \text{ m} \\
 g &= 10 \text{ m/s}^2 & &= 20200000 \text{ Pa} \\
 \text{Required} & & &= 2.02 \times 10^7 \text{ Pa}
 \end{aligned}$$

$P = ?$

### Fluid density

Density is mass of substance divided by its volume.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

Where,  $\rho$  = Greek letter (rho) stands for density

$m$  = mass of substance

$V$  = volume substance.

Density is a scalar quantity and its measure in  $\text{kg/m}^3$ . It also measured in  $\text{g/cm}^3$

$$1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$$

**Note that:** liquids have higher density than gases because the particles are closed together in liquid than gases.

### Relative density

- ✓ Relative density is the density of a substance to the density of a standard substance under specific condition.
- ✓ The relative density is the ratio between its density and the density of water.

$$\text{Relative Density} = \frac{\text{density of substance}}{\text{density of water}}$$

- ❖ Relative density sometimes called specific gravity and has no units since its ratio.

#### Example 6.3

The density of oil is  $800\text{kg/m}^3$  then what is the specific gravity when compared to water?

Take (density of water =  $1000\text{kg/m}^3$ )

**Solution: Given**

$$P_{\text{oil}} = 800\text{kg/m}^3 \quad R.D = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}}$$

$$P_{\text{water}} = 1000\text{kg/m}^3 \quad R.D = \frac{800\text{kg/m}^3}{1000\text{kg/m}^3}$$

**Required** R.D = 0.8

R.D = ?

### Total pressure

The total pressure is the sum of the pressure due to the fluid (gauge pressure) and atmospheric pressure. So

Total pressure = Atmospheric pressure + gauge pressure.

$$P_T = P_{\text{atm}} + \rho gh$$

### What is the difference between atmospheric gauge and absolute pressure?

#### 1. Absolute pressure ( $P_s$ )

It's a pressure exerted at a point by a fluid including atmospheric pressure. Absolute pressure is called total pressure of the system.

#### 2. Atmospheric pressure ( $P_{\text{atm}}$ )

It's a pressure of the surrounding air when measured at the surface of the earth.

#### 3. Gauge pressure ( $P_g$ )

It's a pressure exerted by a fluid without including atmospheric pressure.

**Absolute pressure = Atmospheric pressure + Gauge pressure**

$$P_s = P_{\text{atm}} + P_g$$

$$p_g = P_s - P_{\text{atm}}$$



**Example 6.4**

Water stands 10m deep in a storage tank whose top is open to the atmosphere. What are the absolute and gauge pressure at the bottom of the tank? Take ( $P_{\text{atm}} = 101\text{kPa}$ )

**Solution: Given**

$$h=10\text{m}$$

$$P_{\text{atm}} = 1.01 \times 10^5 \text{Pa}$$

$$\rho_w = 1000 \text{kg/m}^3$$

$$g = 10 \text{m/s}^2 \quad P_s = P_{\text{atm}} + P_g$$

**Required**

$$P_s \text{ \& } P_g = ?$$

$$P_g = \rho g h$$

$$= 1000 \text{kg/m}^3 \times 10 \text{m/s}^2 \times 10 \text{m}$$

$$= 100,000 \text{Pa} = 1.0 \times 10^5 \text{Pa}$$

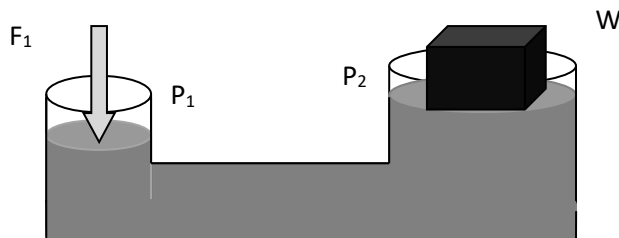
$$= 1.01 \times 10^5 \text{Pa} + 1.0 \times 10^5 \text{Pa}$$

$$= (1.01 + 1.0) \times 10^5 \text{Pa}$$

$$= 2.01 \times 10^5 \text{Pa}$$

**Pascal's principle**

- When a force is applied any part of confined fluids, it creates pressure inside the fluid. This pressure is transmitted throughout the fluid and is the same value everywhere.
- **Pascal's principle states: 'The pressure applied to an enclosed fluid is transmitted to every part of the fluid as well as to the wall of the container without reducing in value'**
- ✓ For example look at the following diagram



- If force  $F_1$  is applied to left hand piston ( $P_1$ ), it creates a pressure inside the fluid. This pressure is transmitted throughout the liquid as well as the same everywhere and exerts a force  $F_2$  on piston ( $P_2$ ).
- **According to the Pascal's principle pressure is constant.**

Mathematically

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\boxed{\frac{F_1}{2\pi r_1^2} = \frac{F_2}{2\pi r_2^2}}$$

This is known as Pascal's principle

### Application of Pascal's principle

- Pascal's principle widely applied to:
  - ❖ Hydraulic lift
  - ❖ Hydraulic presses
  - ❖ Hydraulic brake system of the car

#### Example 6.5

Two pistons are connected together to make hydraulic lift. The smaller piston has an area of  $0.05\text{m}^2$  and the larger piston has an area of  $4\text{m}^2$ . Calculate

- A) the pressure in the lift  
 B) the force at the larger piston if the force on the smaller piston is 400N

**Solution: Given**                      A) Pressure is constant

$$A_1 = 0.05\text{m}^2 \quad P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$A_2 = 4\text{m}^2 \qquad \qquad \qquad = \frac{F_1}{A_1} = \frac{400\text{N}}{0.05\text{m}^2} = 800\text{Pa}$$

$$F_1 = 400\text{N}$$

B) From Pascal's principle we have

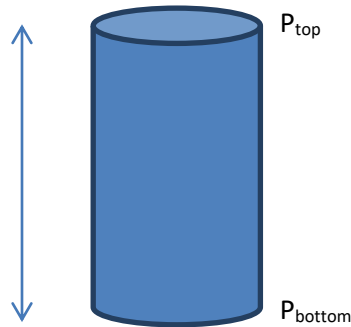
Required  $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$$\text{A) } P = ? \qquad \qquad \qquad F_2 = \left( \frac{F_1}{A_1} \right) A_2 = 800\text{Pa} \times 4\text{m}^2 = 3200\text{N}$$

$$\text{B) } F_2 = ?$$

## FORCE IN FLUID

- An object in water seems less heavy. This is due to a force from the water pushes it up against gravity. This force is called buoyant force (upthrust force).
- Buoyant force is a force from the water which is pushes a body up ward against gravity (weight). This force rises due to the fact that pressure is increases with depth. This means if you immersed an object in a fluid the pressure on the bottom will be greater than the pressure on the top. e.g



$$\Delta P = P \text{ at bottom} - P \text{ at top}$$

$$\Delta P = P_{\text{atm}} + \rho gh - P_{\text{atm}}$$

$$\Delta P = \rho gh$$

This implies difference in pressure means there is also a difference force acting on the top and the bottom of the object.

- The size of buoyant force depends on:
  - The density of the liquid
  - The volume of the object

## APPARENT WEIGHT

- As we mentioned, object seems to less heavy in water because the buoyant force pushing up ward against the object weight and so the weight appears to be drops. The weight of a body immersed in water (fluids) is known as apparent weight. Apparent weight of an object immersed in the fluids is the difference between the real weight and the buoyant force.
  - Apparent weight = real weight – buoyant force
  - Buoyant force = real weight – apparent weight
  - Real weight = apparent weight + buoyant force

### **Example 6.6**

A body that weight in 40N air is found to weight of 22N when immersed in water. What is buoyant force of the water?

**Solution:** Given  $F_b = W_r - W_{ap}$

$$W_r = 40\text{N} \quad F_b = 40\text{N} - 22\text{N}$$

$$W_{ap} = 22\text{N} \quad F_b = 18\text{N}$$

**Required**

$$F_b = ?$$

## ARCHIMEDE'S PRINCIPLE

- Archimedes' realize that when an object immersed in a liquids it displace a certain volume of the liquids.
- **Archimedes principle states: ‘an object wholly or partially immersed in fluid buoyant up by force equal to the weight of the fluid displaced by the object’ in other words, the buoyant force acting on the object is equal to the weight of displaced liquids.**

$$\text{Buoyant force} = \text{Weight of displaced fluid}$$

**Note that:** the greater the value of liquids displaced the greater the buoyant force

The apparent weight in light of Archimedes principle

Apparent weight = real weight – buoyant force

Apparent weight = real weight – weight of displaced fluid

Because buoyant force = weight of displaced fluid

When an object is immersed totally in fluid the volume of the displaced fluid is equal to the volume of immersed object

$$\text{Volume of object} = \text{volume of displaced fluid}$$

### **Application Archimedes principle**

- Determine the volume of solid
- Determine the density of liquid

### **Example 6.7**

A solid has a weight of 20N in air when submerged in water its weight drops into 15N and when it's submerged in unknown liquid its weight become 10N. Calculate:

- The buoyant force
- The volume of the water displaced
- The density of the solid
- The density of the unknown liquids

### **Solution: Given**

$$W_r = 20\text{N}$$

$$W_{ap} = 15\text{N}$$

$$W_{ap} = 10\text{N}$$

$$\rho_w = 1000\text{kg/m}^3$$

$$\text{A) } F_b = W_r - W_{ap}$$

$$= 20\text{N} - 15\text{N}$$

$$= 5\text{N}$$

$$\text{B) } F_b = W_d$$

**Required**

$$5N = \rho_w V_d g$$

A)  $F_b = ?$ 

$$V_d = 5N / \rho_w g$$

B)  $V_d = ?$ 

$$= \frac{5N}{1000 \text{ kg/m}^3 \times 10 \text{ m/s}^2}$$

C)  $P_s = ?$ 

$$= 0.0005 \text{ m}^3$$

D)  $P_{un} = ?$ 

C) For submerged solid the volume of liquid displaced is equal to the volume the solid.

$$W = mg \Rightarrow m = \frac{W}{g} = \frac{20N}{10 \text{ kg/s}^2} = 2 \text{ kg}$$

$$\rho_s = \frac{m}{V} = \frac{2 \text{ kg}}{0.0005 \text{ m}^3} = 4000 \text{ kg/m}^3$$

D)  $F_b = W_r - W_{ap}$ 

$$= 20N - 10N = 10N$$

 $F_b$  = the weight of displaced in fluid

$$10N = W_d \text{ but, } W_d = \rho_l V_d g$$

$$10N = \rho_l V_d g$$

$$\rho_l = \frac{10N}{V_d g} = \frac{10N}{(0.0005)(10 \text{ m/s}^2)}$$

$$\rho_l = 2000 \text{ kg/m}^3$$

**Floating or sinking**

- An object floats on or sink in a fluid depend on:
  - The weight of an object
  - The size of the buoyant force acting on the object and
  - The relative density between the object and the fluid

The law of the floatation states:

“If the buoyant force is equal to the weight of the object then the object will floats” or

“If the weight of the volume of the fluid displaced is equal to the weight of an object then the object will float” moreover, we can think of the following.

- i. If the density of the object is greater than the density of the fluid , it will sink  
(Relative density > 1)
- ii. If the density of the object is less than the density of the fluid , it will float  
(Relative density < 1)

➤ Generally if an object is floating

$$\text{Buoyant force} = \text{weight of displaced fluid} = \text{weight of object}$$

$$\Rightarrow \rho_{ob} V_{ob} g = \rho_l V_l g$$

This equation  $\rho_{ob} V_{ob} = \rho_l V_l$  is floating

iii. If the weight of the object is greater than the buoyant force an object will sink

$$\Rightarrow \rho_{ob} V_{ob} g > \rho_l V_l g$$

### Example 6.7

A floating wooden block has a volume of  $1\text{m}^3$  and displaces  $0.75\text{m}^3$  of water. What is the density of the block?

**Solution; Given** from the law of floatation, we have

$$\rho_w = 1000\text{kg/m}^3 \quad \rho_{ob} V_{ob} = \rho_w V_w$$

$$V_w = 0.75\text{m}^3 \quad \rho_{ob} = \frac{\rho_w V_w}{V_{ob}}$$

$$\begin{aligned} V_{ob} &= 1\text{m}^3 \\ g &= 10\text{m/s}^2 \\ \rho_{ob} &= \frac{1000\text{kg/m}^3 \times 0.75\text{m}^3}{1\text{m}^3} \\ &= 750\text{kg/m}^3 \end{aligned}$$

**Required**

$$\rho_{ob} = ?$$

### Example 6.8

A solid cube of material is 10cm on each side. If it floats on oil of density  $800\text{kg/m}^3$  with  $2/3$  of the block out of oil then, what is the density of the material?

**Solution:**

**Give:**  $h = 10\text{cm}$ ,  $\rho_{oil} = 800\text{kg/m}^3$ ,  $V_d = V_{oil} = 1/3 V_{solid}$

**Required:**  $\rho_m = ?$

Use the law of floatation

$$\rho_m V_m = \rho_{oil} V_{oil} \text{ but } V_{oil} = V_m / 3$$

$$\rho_m V_m = \rho_{oil} \left( \frac{V_m}{3} \right)$$

$$\rho_m = \rho_{oil} / 3 = 800\text{kg/m}^3 / 3$$

$$\rho_m = 266.6\text{kg/m}^3$$

**Review exercise for unit 6**

- If an object is rising up through water, which one of the following relation between buoyant force of water and weight of the object is **correct**
  - Buoyant force > weight
  - Buoyant force < weight
  - Buoyant force = weight
  - they do not have any relation
- At what depth of a sea water of density  $1000\text{kgm}^{-3}$  would the pressure be  $4.01 \times 10^5 \text{ Pa}$ ? ( $p_{\text{atm}} = 1.01 \times 10^5 \text{ Pa}$ ,  $g = 10\text{m/s}^2$ )
  - 20m
  - 40m
  - 30m
  - 50m
- A floating wooden block has a volume of  $0.4\text{m}^3$  and displaces  $0.3\text{m}^3$  of water. What is the relative density of the block?
  - 1
  - 0.5
  - 0.75
  - 0.85
- A solid cube of material is 20cm on each side. If it floats in one oil density  $850\text{kg/m}^3$  with  $2/3$  of the block out of oil. What is density of the solid cube of material?
  - $450\text{kg/m}^3$
  - $283.3\text{kg/m}^3$
  - $566.6\text{kg/m}^3$
  - $1000\text{kg/m}^3$
- In a hydraulic press a force of 24N is applied to a smaller piston of area  $0.4\text{m}^2$ . What load will be supported by the larger piston whose area is  $0.7\text{m}^2$ ?
  - 42N
  - 24N
  - 48N
  - 84N
- Which one of the following is the unit of pressure?
  - mmHg
  - Pascal
  - $\text{Nm}^2$
  - 1atm
- Pressure in a liquid at rest **doesn't** depend on:
  - Depth
  - Density of liquid
  - acceleration due to gravity
  - shape of container
- What pressure is exerted by the water at the bottom of a sea whose depth is 100m ( use density of sea water  $= 1030\text{kg/m}^3$  &  $g = 10\text{m/s}^2$ )
  - $1.03 \times 10^6 \text{ pa}$
  - $2.03 \times 10^6 \text{ pa}$
  - $3.06 \times 10^6 \text{ pa}$
  - $4.08 \times 10^6 \text{ pa}$
- A body that weight 40N in air is found to be weight 32N when immersed in water what buoyant force of water?
  - 72N
  - 40N
  - 30N
  - 8N
- What is the buoyant force on 5kg solid object has a density of  $2 \times 10^4 \text{kg/m}^3$  when immersed in fluid having a density of  $5 \times 10^3 \text{kg/m}^3$ ?
  - 12.5N
  - 20N
  - 24.5
  - 40
- Which one of the following are **not** the properties of pressure of the liquid at rest?
  - It independent on the shape of the container
  - It depend of the mass of the liquid above the surface
  - It increase as depth increase
  - It depends on the acceleration of gravity.
- “ when a body is immersed either totally or partially in a fluid, it experiences a buoyant force equal to the weight of the fluid displaced “ this principle is:
  - Pascal's principle
  - Archimedes' principle
  - Bernoulli's principle
  - Continuity principle

13. Which one of the following does the buoyant force depend on?
- A. The volume of the object                      C. The density of the fluid  
B. The density of the object                      D. A and C are correct
14. A container holds water of density  $1000\text{kg/m}^3$ . Taking the gravitational acceleration as  $10\text{m/s}^2$ , the pressure at a depth of 100 mm is:
- A. 1kPa                      B. 1MPa                      C. 100Pa                      D. 1Pa
15. If the absolute pressure at the bottom of a freshwater pond is  $2.313 \times 10^5\text{Pa}$ , what will be the depth of the pond?
- A. 33m                      B. 23.13m                      C. 13m                      D. 10.13m
16. A plastic cube having a side of length 10cm floats on freshwater. If the volume of the cube that's submerged in water is 60 percent of its total volume. What is the density of the cube? ( $\rho_{\text{water}} = 1000\text{kg/m}^3$ )
- A.  $1000\text{kg/m}^3$                       B.  $800\text{kg/m}^3$                       C.  $700\text{kg/m}^3$                       D.  $600\text{kg/m}^3$
17. A very large tank is full of oil whose density is  $800\text{kg/m}^3$ . If the absolute pressure at bottom of the tank is  $1.81 \times 10^5\text{Pa}$ , then what is the depth of the oil? (take that atmospheric pressure is  $1.01 \times 10^5\text{Pa}$ )
- A. 100m                      B. 75m                      C. 10m                      D. 9.5m
18. If you feel a pressure three times that of atmospheric pressure when you are deep under water. How high is the water level from your position? ( $1\text{atm} = 1 \times 10^5\text{Pa}$ , &  $\rho_w = 1000\text{kg/m}^3$ )
- A. 10m                      B. 20m                      C. 30m                      D. 40m
19. The gauge pressure of fluid in a pipe is 70kPa and the atmospheric pressure is 100kPa. The absolute pressure of the fluid in the pipe is:
- A. 7 MPa                      B. 30 kPa                      C. 170kPa                      D. 10/7kPa
20. A certain objects float in the three fluids  $F_1$ ,  $F_2$  and  $F_3$  of densities  $0.9\rho$ ,  $\rho$  and  $1.2\rho$  respectively. The volume of the displaced fluid by the object is
- A. Least  $F_1$                       B. Least  $F_3$                       C. Least  $F_2$                       D. greatest for  $F_2$
21. An object weight is 4N in air and has a density of  $8\text{g/cm}^3$  when totally immersed in the water what will be the apparent weight?
- A. 1N                      B. 2N                      C. 3N                      D. 4N
22. A book rest on a table which it's face having a sides 30cm by 25cm. if it exerts a pressure of 200Pa then determine the mass of the block.
- A. 3kg                      B. 2.5kg                      C. 2kg                      D. 1.5kg
23. An open tank filled with water to depth of 3m. What is the absolute pressure at the point 1.2m below the upper surface of water? (take  $P_{\text{atm}} = 1.01 \times 10^5\text{Pa}$ )
- A.  $1.22 \times 10^5\text{Pa}$                       B.  $1.13 \times 10^5\text{Pa}$                       C.  $2.11 \times 10^5\text{Pa}$                       D.  $2.13 \times 10^5\text{Pa}$



24. The relative density of a substance is defined as the
- product of the density of water and the density of a substance
  - difference between the density of a substance and the density of water
  - ratio of the density of a substance to the density of water
  - Ratio of the density of a water to the density of substance.
25. Atmospheric pressure increases as
- Altitude decreases.
  - Air density decreases.
  - air temperature increases
  - latitude decreases
26. As the temperature increases air pressure \_\_\_\_\_
- Increases
  - decreases
  - stay the same
  - doesn't change
27. A cubic block of wood of relative density 0.85 floats in water. If  $v_b$  represents the volume of the block, then what is the volume of the submerged portion of the block?  
(Use  $\rho_{H_2O} = 10^3 \text{ kg/m}^3$ )
- $v_b$
  - $0.85v_b$
  - $0.15v_b$
  - $0.5v_b$
28. An object weight 40N in air and has a density of  $10\text{g/cm}^3$ . When totally immersed in water then what will be the apparent?
- 20N
  - 30N
  - 36N
  - 40N
29. A solid of cube of material is 10cm on each edge. If it's floats in an oil of density  $900\text{kg/m}^3$  with one - third of the block is out of oil. What is the density of the materials?
- $300\text{kg/m}^3$
  - $600\text{kg/m}^3$
  - $900\text{kg/m}^3$
  - $1200\text{kg/m}^3$
30. Which one of the following is the property of sound wave that is related to frequency?
- Loudness
  - Intensity
  - Pitch
  - Timber
31. Which physical quantity increase as sound travel from air to water?
- Frequency
  - Period
  - Seed
  - Wavelength
32. A ray of light enters from air to a glass medium whose index of refraction is  $3/2$ . If the angle of incidence is  $30^\circ$ , what is the sin of the angle of refraction? (use  $\sin 30^\circ=0.5, \cos 30^\circ=0.86$ )
- $3/2$
  - $1/3$
  - $3/4$
  - $4/3$
33. In what temperature will the speed of sound in air be  $348\text{m/s}$ ? ( use  $v = 330\text{m/s}$ )
- $30^\circ\text{C}$
  - $50^\circ\text{C}$
  - $70^\circ\text{C}$
  - $90^\circ\text{C}$
34. An electromagnetic wave propagates in vacuum at speed  $v$  & wave length  $\lambda$  as the wave enter some unknown medium its speed decreases to  $0.6v$ . What is the wavelength of unknown medium
- $0.4\lambda$
  - $0.5\lambda$
  - $0.6\lambda$
  - $0.7\lambda$
35. In high air pressure the molecules are \_\_\_\_\_
- Warm and moving slow
  - Cloth together and moving slow
  - far apart and moving fast
  - hot and moving rapidly
36. Rectangular box measures  $10\text{m} \times 5\text{m} \times 20\text{m}$  and its weight is  $800\text{N}$ . The least pressure it can exert when it rests on a flat surface is:
- 4N
  - 16Pa
  - 8Pa
  - 2Pa

## UNIT 7

### Temperature and heat

- ✓ Temperature and heat are two important and interrelated but they are different physical quantities. The main differences between them are the following:
- ✓ Temperature is a measure of the degree of hotness and coldness of substances while heat is a form energy that transferred from one object to another.
- ✓ Heat is flow energy from hotter region to colder region while temperature indicates the direction of heat flow.
- ✓ Temperature is a basic quantity but heat is a derived physical quantity.
- ✓ The heat added to a body is the sum of the kinetic energy and potential energy of the particles.

That is,

$$\text{Heat} = \sum \text{Kinetic energy} + \sum \text{Potential energy}$$

$$Q = \sum KE + \sum U$$

Both heat and temperature are scalar quantities.

Heat is measured in joule (J) and temperature is measured in kelvin (K)

- Heat can flow from one region to another in three different ways.
  - Conduction
  - Convection
  - Radiation

### Temperature scales.

Temperature scales are used in measuring temperature. These are Celsius scale ( $^{\circ}\text{C}$ ), Kelvin scale (K) and Fahrenheit scale ( $^{\circ}\text{F}$ ). To design a temperature scale two fixed reference points have to be used. These points are freezing point of water (ice point of temperature) and boiling point of water (steam point temperature).

Absolute zero is the temperature at which a substance has no thermal energy.

#### 1. Celsius scale ( $^{\circ}\text{C}$ )

- The temperature scale where the freezing point of the water is fixed at  $0^{\circ}\text{C}$  and the boiling point of water is fixed at  $100^{\circ}\text{C}$ .

#### 2. Kelvin scale(K)

It uses absolute zero as one of its fixed point.

In Kelvin scale the freezing point of water is 273K and the boiling point of water is 373K.

#### 3. The Fahrenheit scale ( $^{\circ}\text{F}$ )

In Fahrenheit scale, the freezing point of water is  $32^{\circ}\text{F}$  and the boiling of the water is  $212^{\circ}\text{F}$ .

Note that: kelvin and Celsius scales are often used together as they have the same scale division. This means the change in the temperature in both scales are the same.

$$(\Delta T_c = \Delta T_k).$$

To convert Celsius temperature to kelvin temperature or Fahrenheit temperature and vice-versa, the following relations are used.

$$1. T_K = T_C + 273K \quad \text{or} \quad T_C = T_K - 273K$$

$$2. T_C = \frac{5}{9}(T_F - 32) \quad \text{or} \quad T_F = \frac{9}{5}(T_C) + 32$$

### What happen when a substance absorbs heat energy?

As a substance absorbs heat energy, the particles vibrate move in solid and move faster in fluid because the heat energy converted into kinetic energy as temperature is rises.

This means the substance (particles) will expand (increases in size).

The expansion of substances on heating is called thermal expansion.

### Thermal equilibrium

Thermal equilibrium is a situation where is no net movement of heat between the two bodies. If two bodies are in thermal equilibrium, they will also have the same temperature.

The details of how two bodies are obtained thermal equilibrium are governed by the 1<sup>st</sup> law and 2<sup>nd</sup> law thermodynamics.

### Frist law of thermodynamics

Thermodynamics is about the way system exchange energy with its environments.

The 1<sup>st</sup> law of thermodynamics is state that, “the increase in internal energy of the system is equal to the sum of the energy entering through the heating and the work done on the system.

Mathematically

$$\Delta U = \Delta Q + \Delta W$$

Where,  $\Delta U$  = increase in internal energy of the system

$\Delta Q$  = the amount of heat energy transferred to the system

$\Delta W$  = the amount of work done on the system

**Note:** internal energy of the system is the sum of the kinetic energy and potential energy

### Second law of thermodynamics

The 2<sup>nd</sup> law of thermodynamics concerns the direction of heat flow between two bodies.

### Example 7.1

Supposes 2000J of heat added to water in boiling without doing work done on the water. What is the internal energy of the water?

**Solution: Given** from the 1<sup>st</sup> law of thermodynamics we have,

$$\Delta Q = 2000J \quad \Delta U = \Delta Q + \Delta W$$

$$\Delta W = 0 \quad \Delta U = 2000J + 0$$

**Required:**  $\Delta U = ?$ 

$$\Delta U = 2000\text{J}$$

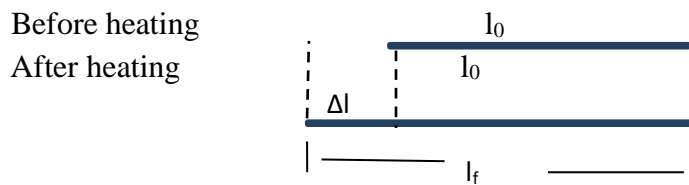
## 7.2 Expansion of solid liquid and gases

### The e expansion of solids

- When a solid heated its particles move further apart causing the solid to expand. The amounts of expansion in a solid depend on:
  - The nature of the solid
  - The rise of the temperature
- There are three types of expansion of solid. These are
  - ❖ Linear expansion ( one dimension expansion)
  - ❖ Area expansion ( two dimension expansion)
  - ❖ Volume expansion( three dimension expansion)

#### 1. Linear expansion of solid

When a metal rod is heated it expands and increases in length. This expansion is called linear expansion. The diagram in figure below represents a metal rod of length  $l_0$  before heating and  $l_f$  after heating.



As the temperature of the rod increases from  $T_0$  to  $T_F$  its length changes by  $\Delta l = l_f - l_0$

According the law of expansion on, the change in length of the rod is directly proportional to both the original length ( $l_0$ ) and the rise of the temperature ( $\Delta T$ ).

Mathematically

$$\Delta l \sim l_0 \text{ and } \Delta l \sim l_0 \Delta T$$

$$\text{➤ } \Delta l \sim l_0 \Delta T$$

$$\text{➤ } \frac{\Delta l}{l_0 \Delta T} = \text{constant} = \alpha$$

$$\text{➤ } \alpha = \Delta l / (l_0 \Delta T) \text{ Where } \alpha \text{ is called the coefficient linear expansion and is the increase in the length of a 1m rod of given substance when its temperature is increase by 1K.}$$

$$\alpha = \Delta l / (l_0 \Delta T)$$

$$\Delta l = l_0 \alpha \Delta T$$

$$\text{but, } \Delta l = l_f - l_0$$

$$l_f - l_0 = l_0 \alpha \Delta T$$

$$l_f = l_0 + l_0 \alpha \Delta T$$

$$l_f = l_0 (1 + \alpha \Delta T)$$

**Example 7.2**

If a 4m copper wire is heated and its temperature rises from 10°C to 110°C if ( $\alpha = 1.7 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$ ) then calculate

A) the change in the length of the wire

B) the final length of the wire

**Solution: Given**

$$l_0 = 4$$

$$\Delta T = T_f - T_0$$

$$\Delta T = 110^\circ\text{C} - 10^\circ\text{C} = 100^\circ\text{C}$$

$$\alpha = 1.7 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

Required

A)  $\Delta l = ?$   $l_f = l_0 + \Delta l$

B)  $l_f = ? = 4\text{m} + 0.0068\text{m}$   
 $= 4.0068\text{m}$

A)  $\Delta l = l_0 \alpha \Delta T$

$$\Delta l = 4\text{m} \times 1.7 \times 10^{-5} \text{ } ^\circ\text{C}^{-1} \times 100^\circ\text{C}$$

$$\Delta l = (4 \times 1.7) \times 10^{-3} \text{m}$$

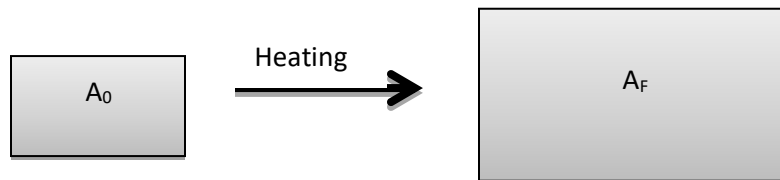
$$= 6.8 \times 10^{-3} \text{m}$$

$$= 0.0068\text{m}$$

B)  $\Delta l = l_f - l_0$

**Surface (area) expansion of solid (superficial)**

When metal plate is heated it expands and increases in size. This expansion is called surface or area expansion.



If the metal plate of original ( $A_0$ ) is heated to a temperature of ( $\Delta T$ ) then, the increase or change in area ( $\Delta A$ ) of the plate after heating is given by

$$\Delta A = A_0 \beta \Delta T$$

**Where,**  $\beta$  (beta) is coefficient of surface expansion and is equal to  $\beta = 2\alpha$

The final area of the plate after heating is given by;

- $\Delta A = A_F - A_0$
- $A_F = A_0 + \Delta A$
- $A_F = A_0 + A_0 \beta \Delta T$

$$A_F = A_0(1 + \beta \Delta T)$$

•

**Example 7.3**

Calculate the change in the surface of copper plate with an area  $8.85\text{m}^2$  at  $20^\circ\text{C}$  when heated to temperature of  $120^\circ\text{C}$ . ( $\beta = 3.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ )

Given

$$A_0 = 085\text{m}^2$$

$$\beta = 3.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$$

$$\Delta T = 100^\circ\text{C}$$

$$\Delta A = A_0 \beta \Delta T$$

$$= (085\text{m}^2)(3.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(100^\circ\text{C})$$

$$= 2.89 \times 10^{-3} \text{m}^2$$

Required

$$\Delta A = ?$$

**Volume expansion**

Volume expansion is take place in three dimensions

Volume expansion is the increase in length, width and height of a substance due to heating. For example consider a solid cube of side  $l_0$  at temperature  $T_0$  before heating as shown in fig. (a). then the cube is heated to a final temperature  $T_f$  and after heating the cube attain the side  $l_f$  as show in fig. (b)

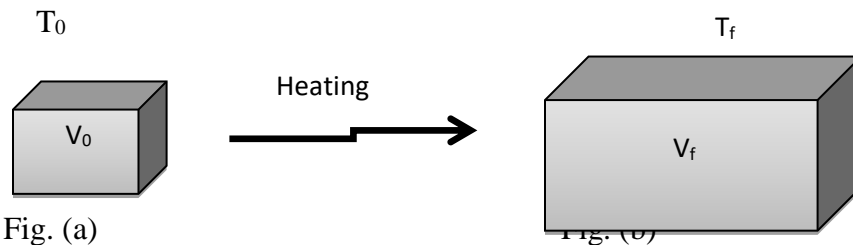


Fig. (a)

Fig. (b)

As the temperature of the cube increases from  $T_0$  to  $T_f$  it expands in all sides and its volume changes from  $V_0$  to  $V_f$  hence  $\Delta V = V_f - V_0$

According to the law of expansion the change in volume of substances depend on the original volume and the rise of temperature. Mathematically

$$\triangleright \Delta V \sim V_0 \Delta T$$

$$\triangleright \frac{\Delta V}{V_0 \Delta T} = \text{Constant}$$

$$\triangleright \frac{\Delta V}{V_0 \Delta T} = \gamma$$

$$\triangleright \boxed{\Delta V = V_0 \gamma \Delta T}$$

$$\boxed{\gamma = 3\sigma}$$

Where,  $\gamma$  (gamma) is called coefficient volume expansion and equal to,

$$\triangleright \Delta V = V_0 \gamma \Delta T$$

$$\triangleright \Delta V = V_f - V_0$$

$$\triangleright V_f - V_0 = V_0 \gamma \Delta T$$

$$\triangleright V_f = V_0 + V_0 \gamma \Delta T$$

$$\triangleright \boxed{V_f = V_0 (1 + \gamma \Delta T)}$$

#### Example 7.4

A rectangular block 50cm by 40cm and 20cm at 20°C is heated to temperature of 220°C. If the linear coefficient of the solid is  $1.5 \times 10^{-6} 1/^\circ\text{C}$  then what is the increase in volume?

**Solution: Given**

$$\Delta V = V_0 3\alpha \Delta T$$

$$V_0 = l \times w \times h = (4 \times 10^4 \text{ cm}^3) (3 \times 1.5 \times 10^{-6} 1/^\circ\text{C}) (200^\circ\text{C})$$

$$V_0 = 50 \text{ cm} \times 40 \text{ cm} \times 20 \text{ cm} = 14.4 \text{ cm}^3$$

$$V_0 = 4 \times 10^4 \text{ cm}^3$$

$$\Delta T = 200^\circ\text{C}$$

$$\alpha = 1.5 \times 10^{-6} 1/^\circ\text{C}$$

**Required**

$$\Delta V = ?$$

#### Application of thermal expansion

Thermal expansion widely uses in the following

- ❖ Bimetallic strips
- ❖ Bimetallic strip as a switch and Thermostat

#### Bimetallic strips

- Bimetallic strip is a strip made of two different metals bonded together along their length. For example if iron and brass bonded together, the coefficient of linear expansion of iron ( $1.1 \times 10^{-5} 1/^\circ\text{C}$ ) is less than that of the brass ( $1.9 \times 10^{-5} 1/^\circ\text{C}$ ). When the strip is heated the brass expands more than the iron and the strip bend.
- **Note that** the bimetallic strip bends towards the metal which expands less when heated and bends toward the metal which contrast most when cooled.

## Expansion of liquids

Liquids increase in volume with increasing temperature and have larger coefficient of volume expansion than solid. For liquid we have two types of expansion

1. Real expansion: is the actual increases (expansion) of the size of the liquids.
2. Apparent expansion: is the observed expansion of liquid which may affected by the expansion the container.

The law of volume expansion can applied to the liquids.

$$\Delta V = V_0 \gamma \Delta T$$

Real expansion of liquid = apparent expansion + volume expansion of the container

$$\Delta V_{\text{real}} = \Delta V_{\text{app}} + \Delta V_{\text{cont}}$$

$$(\gamma_{\text{re}})V_0\Delta T = (\gamma_{\text{ap}})V_0\Delta T + (\gamma_{\text{con}})V_0\Delta T$$

$$(\gamma_{\text{re}})V_0\Delta T = (\gamma_{\text{ap}} + \gamma_{\text{con}})V_0\Delta T$$

$$\gamma_{\text{re}} = \gamma_{\text{ap}} + \gamma_{\text{con}}$$

### Example: 7.4

Calculate the real and apparent expansion of 1000cm<sup>3</sup> water in glass when it's heated from 20°C to 120°C. Take ( $\gamma_{\text{glass}} = 9.9 \times 10^{-6} \frac{1}{^\circ\text{C}}$  and  $\gamma_{\text{water}} = 2.1 \times 10^{-4} 1/^\circ\text{C}$ )

Solution: Given  $\Delta V_{\text{re}} = V_0 \gamma_w \Delta T$

$$\Delta T = 100^\circ\text{C} = 1.0 \times 10^3 \text{cm}^3 \times 2.1 \times 10^{-4} 1/^\circ\text{C} \times 100^\circ\text{C}$$

$$V_0 = 1000 \text{cm}^3 = 1.0 \times 10^3 \text{cm}^3 \quad = 21 \text{cm}^3$$

$$\gamma_{\text{glass}} = 9.9 \times 10^{-6} 1/^\circ\text{C} \quad \Delta V_{\text{ap}} = V_0 \gamma_{\text{ap}} \Delta T \text{ but, } \gamma_{\text{ap}} = \gamma_{\text{re}} - \gamma_{\text{con}}$$

$$\gamma_{\text{water}} = 2.1 \times 10^{-4} 1/^\circ\text{C} \quad = 1.0 \times 10^3 \text{cm}^3 (2.1 \times 10^{-4} - 9.9 \times 10^{-6}) \times 100^\circ\text{C}$$

$$\text{Required} \quad = (1.0 \times 10^3 \text{cm}^3) (2.0 \times 10^{-4} 1/^\circ\text{C}) (100^\circ\text{C})$$

$$\Delta V_{\text{rel}} \& \Delta V_{\text{app}} = ? = 20 \text{cm}^3$$



## Expansion of gases

The expansion of gases is much larger than that of liquids and solids. This is because gas molecules are very far apart and can move freely than the molecules of liquids and solids.

We know that there is no volume expansion coefficient for gases. This is because the volume of the gases is detected by a number of factors. These are:

- Temperature
- Pressure
- The amount of number of molecules present in the gases.

The relationship between the volume of the gases and its temperature can be expressed as

$$PV = nRT$$

Where, P = pressure of the gas in Pa

V = volume of the gas in m<sup>3</sup>

n = number of the moles of gases

T = absolute temperature

R = universal gas constant

If a change in temperature occurs at constant pressure it possibly determines the change in volume of the gases using the ideal gas equation.

$$\Delta V = nR \Delta T / P$$

## Quantity of heat, specific heat capacity and heat capacity

The amount of heat energy required to raise the temperature of substance is depend on:

- The types (nature) of substance being heated.
- The mass of the substances
- The temperature rise required

Mathematically

$$Q = mc\Delta T$$

The unit of energy is called joule (J). Another unit is called calorie (cal).

One calorie is the quantity of heat energy required to increase the temperature of 1g of water by 1°C. 1cal = 4.2J

What is meant by the specific heat capacity heat capacity?

Specific heat capacity is the amount of heat required to raise the temperature of 1kg of given substance by one 1k.

Mathematically

$$c = \frac{Q}{m\Delta T}$$

Where, m= mass of substance

C= specific heat capacity

$\Delta T$  = change in temperature.

The unit of specific heat capacity is  $\frac{J}{kg^{\circ}C}$  or  $\frac{J}{kgk}$

Heat capacity is the heat energy required to raise the temperature of a body by 1k.

Mathematically

$$C = \frac{Q}{\Delta T}$$

The unit of specific capacity is  $\frac{J}{^{\circ}C}$  or  $\frac{J}{k}$

Specific heat capacity (c) and heat capacity (C) can be related as follows

$$\triangleright C = \frac{Q}{\Delta T} \text{ but } Q = mc\Delta T$$

$$\triangleright C = \frac{mc\Delta T}{\Delta T}$$

$$\triangleright \boxed{C = mc}$$

### Example 7.5

Calculate the quantity of heat energy required to heat 2kg block of iron from 20°C to 140°C. ( $c_i = 470 \frac{J}{kg^{\circ}C}$ )

**Solution: Given**

$$Q = mc\Delta T$$

$$M = 2kg$$

$$= 2kg \times 470 \frac{J}{kg^{\circ}C} \times 120^{\circ}C$$

$$\Delta T = 120^{\circ}C$$

$$= 112,800J = 1.128 \times 10^5 J$$

$$c_i = 470J/kg^{\circ}C$$

Require

$$Q = ?$$

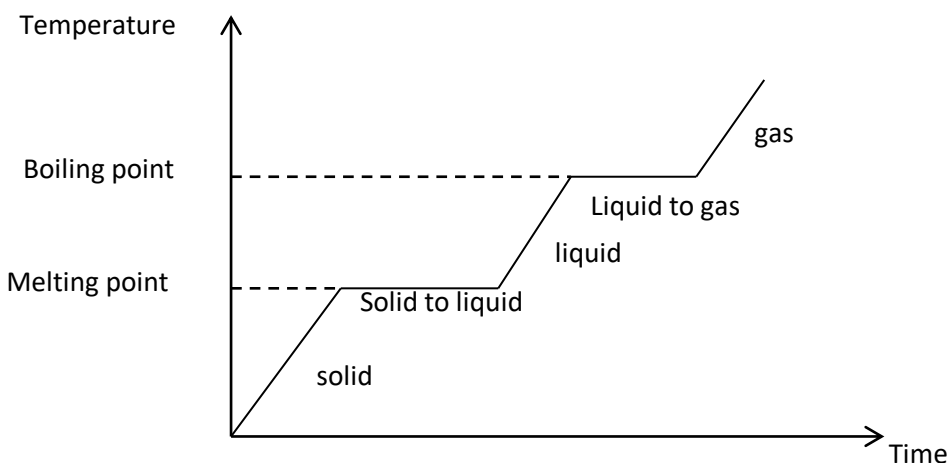
### 7.4 Change of state

- ✓ State or phase, the distinct form of a substance under different condition, example solid, liquid and gas.
- ✓ The heat required to change the state of the substance without change the temperature is known as latent heat (hidden heat).

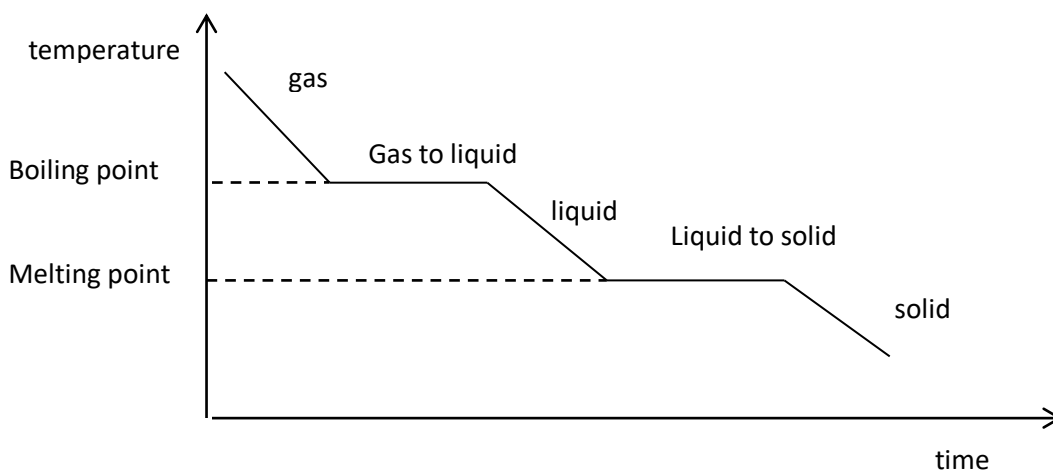
## Heating and cooling curves

**Heating curve:** is a graph showing the temperature of a substance against time as heat energy applied and it change state.

For example look at the following graph.



**Cooling curve:** is a graph showing the temperature of a substance against time as heat energy losses and it change state. For example in the figure below shows a gas cooling t eventually becomes a solid.



### Definition of specific latent heat

The specific latent heat ( $L$ ) is the quantity of heat energy required to change 1kg of a substance from one state to another at constant temperature. Mathematically

$$\text{Specific latent heat} = \frac{\text{Heat energy required}}{\text{mass melted or mass boiled}}$$

$$L = \frac{Q}{m} \quad Q = mL$$

The unit of specific latent heat is  $\frac{J}{kg}$

However there are two change of state to be considered, liquid to gas and solid to liquid. So we use two different version of latent heat, latent heat fusion (melting) and latent heat of vaporization (boiling)

Latent heat of fusion: is the amount of heat energy required to change the state of substance from a solid to a liquid at constant temperature.

Latent heat of vaporization: is the amount of heat energy required to change the state of substance from a liquid to a gas at constant temperature

Specific latent heat of fusion ( $L_f$ ) is the amount of heat energy required to change 1kg of a substance from a solid to a liquid at constant temperature

Mathematically

$$L_f = \frac{Q}{m} \rightarrow Q = mL_f$$

Specific latent heat of vaporization ( $L_v$ ) is the amount of heat energy required to change 1kg of a substance from a liquid to a gas at constant temperature.

Mathematically

$$L_v = \frac{Q}{m} \rightarrow Q = mL_v$$

We can use specific heat capacity and specific latent heats to calculate the total energy required when substances are heated.

state

Total energy required = energy required increase temperature + energy required to change

$$Q_{\text{total}} = mc\Delta T + mL$$

### Example: 7.5

Calculate the amount of heat required to convert 200g of water at 100°C completely to steam at 100°C. Take ( $L_v = 2.258 \times 10^6 \text{ J/kg}$ ).

**Solution: given**

$$m = 200\text{g} = 0.2\text{kg} \quad Q = mL_v$$

$$L_v = 2.258 \times 10^6 \text{ J/kg} \quad = 0.2\text{kg} \times 2.258 \times 10^6 \text{ J/kg}$$

**Required**  $Q = 4.516 \times 10^5 \text{ J}$

$$Q = ?$$

**Example: 7.6**

How much energy is needed required increasing to change 200g of ice at 0°C to water of 80°C?

Take ( $c = 4200\text{J/kg}^\circ\text{C}$  and  $L_f = 3.336 \times 10^5\text{J/kg}$ )

**Solution: there are two stages**

- The heat energy required to melt the water  
 $Q_1 = mL_f = 0.2\text{kg} \times 3.336 \times 10^5\text{J/kg}$   
 $Q_1 = 6.672 \times 10^4\text{J} = 66,720\text{J}$
- The heat energy required to heat the water from 0°C to 80°C  
 $Q_2 = mc\Delta T = 0.2\text{kg} \times 4200\text{J/kg}^\circ\text{C} \times 80^\circ\text{C}$   
 $Q_2 = 67,200\text{J}$   
 Thus, the total amount of heat required is  
 $Q_{\text{total}} = Q_1 + Q_2 = 66,720\text{J} + 67,200\text{J}$   
 $Q_{\text{total}} = 133,920\text{J}$

**Review questions for unit 7**

1. Which of the following is the best definition of thermal expansion?
  - A. Molecules moving faster
  - B. An object increasing in size due to increased temperature.
  - C. An object decreasing in size due to increased temperature
  - D. An object remaining the same in size no matter the temperature
2. Thermal expansion of material has units as
 

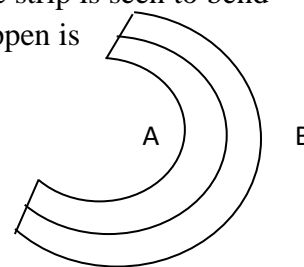
A. J/kg.k	B. J/mol.k	C. W/m.k	D. 1/k
-----------	------------	----------	--------
3. When two bodies are in thermal equilibrium, they will have equal:
 

A. Thermal conductivity	C. temperature
B. Specific heat	D. heat energy
4. What is the ratio of the coefficient of linear expansion to coefficient of area expansion?
 

A. 1: 2	B. 1: 3	C. 1: 4	D. 1 : 5
---------	---------	---------	----------
5. when an object contract up on cooling
  - A. Volume increasing and density increasing
  - B. Volume decreasing and density decreasing
  - C. Volume increasing and density decreasing
  - D. Volume decreasing and density increasing

6. If a 10m copper wire is heated and its temperature rises from 20°C to 120°C if ( $\alpha = 1.8 \times 10^{-4} \text{C}^{-1}$ ) then what is the change in the length of the wire?  
A. 5.18m                      B. 0.18m                      C. 15.18m                      D. 20.18m
7. Which of the following term correctly describes the mechanism of heat transfer by which you get heat holding a piece of metal?  
A. Radiation      B. Convection      C. Conduction      D. Thermal equilibrium.
8. A physics student derived the formula for the volume expansion,  $V$ , of a cubic box in terms of its linear expansion coefficient,  $\alpha$ . Which of the following formulas it possibly derived by the student? ( assume  $T$  is the change in temperature of the box and  $V_0$  is its initial volume)  
A.  $\Delta V = 3V_0 \alpha \Delta T$       B.  $\Delta V = 2V_0 \alpha \Delta T$       C.  $\Delta V = 3/2 V_0 \alpha \Delta T$       D.  $\Delta V = V_0 \alpha \Delta T$
9. If the coefficient of linear expansion of solid is  $5 \times 10^{-4} \text{C}^{-1}$  what is coefficient of surface and coefficient volume expansion will be:  
A. 0.0001 And 0.00015                      C. 0.001 and 0.0015  
B. 0.002 And 0.005                      D. 0.01 and 0.015
10. The temperature at which a substance has no thermal energy is \_\_\_\_\_  
A. Absolute value      B. Absolute zero      C. Celsius scale      D. Kelvin scale
11. A solid block requires 3000J of heat energy to increase its temperature by 60K. What is the block's heat capacity?  
A. 150 J/K                      B. 50 J/K.                      C. 1550 J/K                      D. 350 J/K
12. The heat capacity of object B twice that of A. initially A is at 200°C and B is 350°C. They are placed in thermal contact and the combination is isolated. What is the final temperature?  
A. 300°C                      B. 350°C                      C. 400°C                      D. 450°C
13. Suppose 1000J of heat added to a gas and at same time of 400J of work is done in compressing then calculate the change in the internal energy of the gas?  
A. 600J                      B. 400J                      C. 1000J                      D. 1400J
14. 8000J of heat is required to raise the temperature of a body from 20°C to 100°C. What heat is needed to raise the temperature of the same body at 60°C?  
A. 2000J                      B. 4000J                      C. 6000J                      D. 8000J
15. What is the heat energy required to melt 4kg of copper at its melting point? ( $L_f = 2.09 \times 10^5 \text{J/kg}$ )  
A. 418000J                      B. 836000J                      C. 234000J                      D. 812000J
16. Which physical quantity remain a constant during a change of state?  
A. Volume                      B. Heat energy                      C. Temperature                      D. Density
17. gaps are left in railway track to compensate thermal expansion during  
A. Rain season                      B. Winter                      C. Hot season                      D. wind
18. Coefficient of volume expansion of solid is  
A. greater than liquids and gases                      C. less than liquids and gases  
B. equals to gases                      D. equal to liquids

19. When a bimetallic strip made of two metals A and B is cooled the strip is seen to bend toward A as shown figure below. The likely reason for this to happen is
- Metal A is stronger than metal B
  - Metal B is stronger than metal A
  - Metal A has greater coefficient of expansion than metal B.
  - Metal B has** greater coefficient of expansion than metal A



20. Specific capacity is
- measured in J/kg
  - the heat energy required to raise the temperature of a body by  $1^{\circ}\text{C}$
  - determine the rate at which heat flows between two bodies in contact
  - different for different substances.
21. molecules of solid vibrates with large amplitude at
- zero temperature
  - lower temperature
  - higher temperature
  - pressure
22. Thin strips of iron and zinc are riveted together to form a bimetallic strip that bend when heated. the iron is on the inside of the bend because
- it has a higher specific heat
  - it has a lower specific heat
  - it has a higher coefficient of linear expansion
  - it has a lower coefficient of linear expansion
23. A hot allow with specific heat capacity of  $200\text{J/kg}\cdot^{\circ}\text{C}$  cooled to  $50^{\circ}\text{C}$  when added in to water. What amount of energy is given off by the allow if its mass is  $0.3\text{kg}$ ?
- 3000J
  - 1000J
  - 1500J
  - 5000J
24. When a substance changes state from gas to liquids state , which one of the following occurs,
- Heat is absorbed by the substance
  - Heat **is** given off by the substance
  - The temperature of the substance increases.
  - The temperature of the substance decreases
25. The most suitable material for cooking is the one having
- High specific heat and low conductivity
  - Low specific heat and low conductivity
  - Low specific heat and high conductivity
  - High specific heat and low conductivity
26. A body "A" is ten times the mass and half times the specific heat capacity of body "B". If they applied with equal amount of heat, how do their temperature changes compared?
- Change of temperature in A is 10 times greater
  - Change of temperature in A is 5 times greater
  - Change of temperature in B is 10 times greater
  - Change of temperature in B is 5 times greater

27. A sheet of silver with dimension of 40cmx50cm that is at room temperature of 25°C is heated to a temperature of 125°C. What will the new of the sheet be? ( $\alpha = 1.9 \times 10^{-5} 1/^\circ\text{C}$ )  
A. 7.6cm<sup>2</sup>      B. 76cm<sup>2</sup>      C. 3.8cm<sup>2</sup>      D. 38cm<sup>2</sup>
28. Which one of the following method of transfer of heat is effectively used for liquid and gases?  
A. Conduction      B. Convection      C. Radiation      D. Expansion
29. The temperature of an object rises by 60°C. By how much degree has its kelvin temperature increased?  
A. 60      B. 212      C. 333      D. 273
30. The density of water is maximum at  
A. 0°C      B. 10°C      C. 100°C      D. 4°C



## UNIT 8

### WAVE MOTION AND SOUND

#### Wave propagation

A wave is the process of propagation of oscillation in a medium.

A wave is disturbance that produced at some part of the medium and transporting energy or sometimes information from one place to another place.

Waves are a series disturbance (vibration) that travels through a medium.

Wave is transporting energy without transporting matter

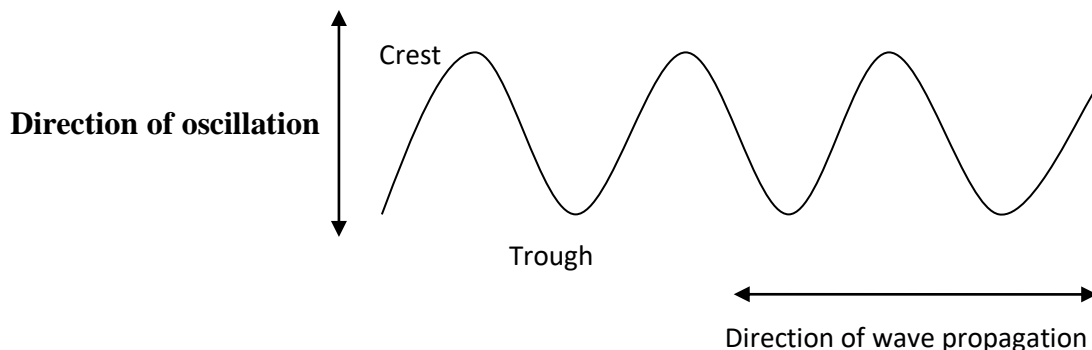
#### Types of waves

Based on the direction of energy propagation or direction of vibration, in relative to the direction of wave movement, wave classified into two. These are

- Transvers wave
- Longitudinal wave

#### Transverse wave

- ✓ Transverse waves are where the oscillation of the particles is perpendicular to the direction of wave motion.
- ✓ In transverse waves the direction of the vibration and the direction of wave propagation are right angle to each other.
- ✓ Wave direction means the direction in which wave is travelling or the direction in energy propagating. e.g



- ✓ Transverse waves can be represented by a series up and down movement of particles.

Crest is the maximum point of the transverse wave

Troughs is the minimum point of the transverse wave

- Examples of transverse waves are:
  - ❖ All electromagnetic waves
  - ❖ Waves on string particles are parallel to the direction of wave motion?
  - ❖ S- wave in earth quakes

### Longitudinal wave

- ✓ Longitudinal wave is a wave where the oscillations of the particles forwards and backwards along the wave motion.
- ✓ In longitudinal waves, the direction of the vibration and the direction of the wave motion are parallel.
- Examples of longitudinal waves are:
  - Sound waves
  - Pressure waves
  - P- Wave in earth quakes

### Wave characteristics

The following terms are used to express any type's waves

#### 1. Speed waves

Wave speed is the speed at which a wave moves (propagating).

#### 2. Amplitude

Amplitude is the maximum distance from equilibrium position.

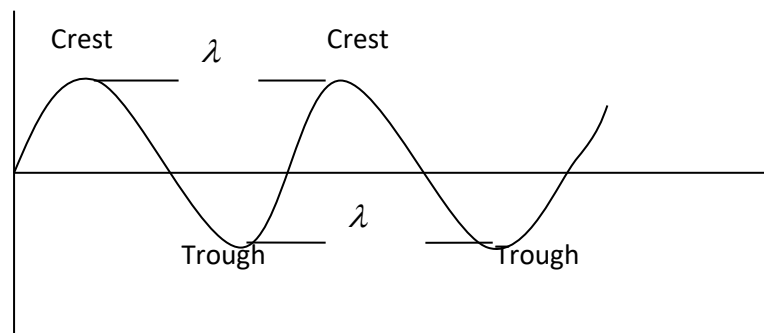
Amplitude is highest crest or trough measured from equilibrium position.

#### 3. Wavelength ( $\lambda$ )

The wave length is distance of one complete wave cycle.

The wavelength is the distance between two identical points' on adjacent waves.

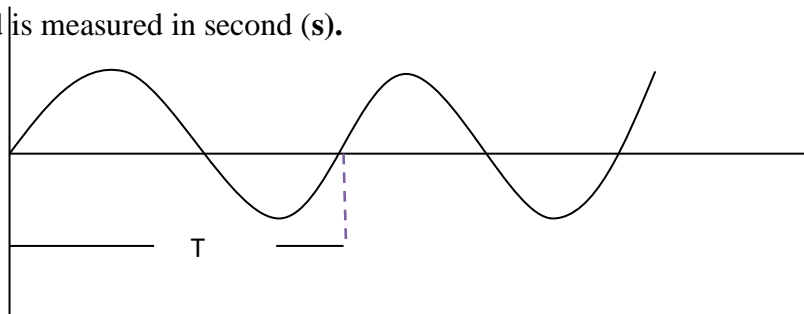
Wavelength is the distance between two successive crests or trough.



#### 4. Period (T)

Period is the time required to complete one full cycle of oscillation.

Period is measured in second (s).



### 5. frequency (f)

A wave frequency is the number of oscillation or cycles per unit time.

The frequency tells us how frequently or rapidly an oscillation takes place.

For example 20Hz means 20 complete oscillations per second.

Mathematically

$$\text{Frequency} = \frac{\text{Number of complete cycle}}{\text{time taken}}$$

Frequency is measured in hertz (Hz) and  $1\text{Hz} = \text{s}^{-1}$

Frequency related to the time period by

$$\boxed{f = \frac{1}{T}} \quad \text{Or} \quad \boxed{T = \frac{1}{f}}$$

### The wave equation

The equation that related, wave speed, wavelength and frequency of the wave is known as wave equation.

$$\text{wave speed} = \frac{\text{wavelength}}{\text{time period}}$$

$$v = \frac{\lambda}{T} \text{ but } f = \frac{1}{T}$$

$$\boxed{v = \lambda f}$$

#### Example: 8.1

An FM radio station broad cast at a frequency of 100MHz. What is the period?

**Solution: given**

$$f = 100\text{MHz} = 1 \times 10^8 \text{Hz}$$

$$T = 1/f$$

**Require**

$$= 1/1 \times 10^8 \text{Hz}$$

T=?

$$= 1 \times 10^{-8} \text{sec}$$

#### Example: 8.2

A string makes 20 vibration in 50 seconds. Calculate:

A) the period

B) frequency of the oscillation

**Solution: given**

$$\text{Number cycle oscillation} = 20$$

$$\begin{aligned} \text{A) } T &= \frac{\text{time taken}}{\text{number of oscillation}} \\ &= 50/20 = 2.5 \text{sec} \end{aligned}$$

$$\text{Time} = 50 \text{ sec}$$

$$\text{Required B) } f = \frac{\text{number of oscillation}}{\text{time taken}}$$

A) T=?

$$= 20/50 = 0.4 \text{ Hz}$$

B) f=?

## Mechanical vs electromagnetic waves

The wave categorized into wave based on the matter they are able to travel through it. These are:

- mechanical wave
- electromagnetic wave

### 1. mechanical wave

- ❖ Mechanical wave is a wave that requires a material medium or a substance for propagation and production.
- ❖ A wave that is not capable of propagating energy through a vacuum.
- Examples of mechanical waves are;
  - Sound wave
  - Water wave
  - Pressure wave

### 2. Electromagnetic wave

- ❖ A wave characterized by the oscillation of an electromagnetic field
- ❖ A wave can propagate through vacuum as well as through matter (medium).
- ❖ All electromagnetic waves propagate with the speed of light ( $v=3 \times 10^8 \text{ m/s}$ ) in vacuum.
- Examples of electromagnetic waves are:
 

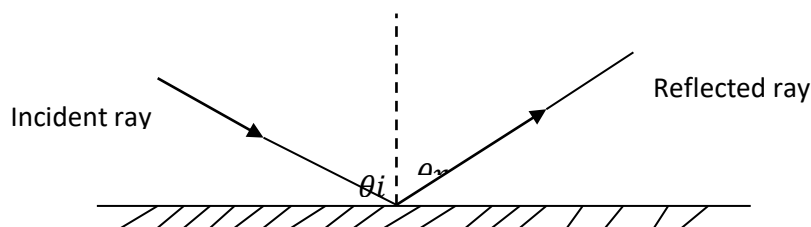
○ Radio wave	ultraviolet wave
○ Micro wave	x-ray
○ Infrared wave	gamma-ray
○ Visible wave	

## Wave behavior

All types of waves have four properties in common. These are:

### 1. Reflection

- ❖ Reflection occurs when the waves bounce off a fixed surface and change direction
- ❖ A turning back of the wave from the given surface
- ❖ A ray coming to the reflecting surface is called incident ray and that leaves the reflecting surface is called reflected ray. e.g



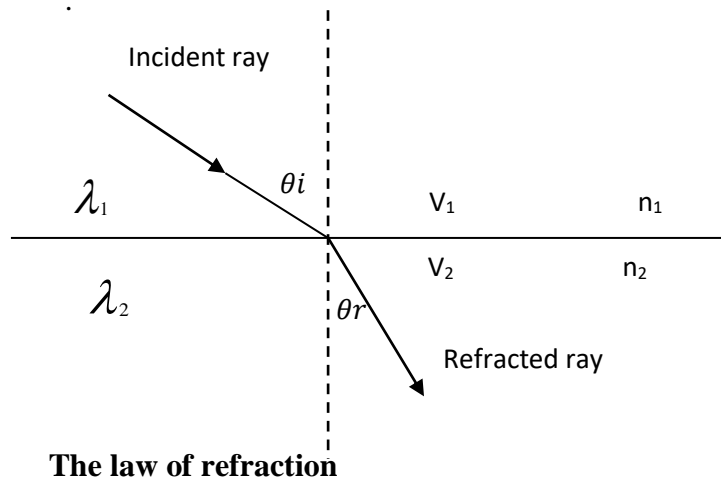
### The law of reflection

The law of reflection states that: 'the angle of incidence ( $\theta_i$ ) of a wave is equal to the angle reflected ( $\theta_r$ ) of a wave.'

$$\theta_i = \theta_r$$

### 1. Refraction

- As the waves enters different medium, its speed may change so the wave bends in one particular direction. This bending of the wave is called refraction.
- The light ray (wave) is bent away from the normal line as its pass from lighter to denser medium.
- The light ray (wave) is bent away from the normal line as its pass from denser to lighter medium. E.g



#### The law of refraction

- The law of refraction state that “the ratio of the sine angle of incident to the sine angle refraction is constant”.

Mathematically

$$\frac{\sin \theta_i}{\sin \theta_r} = \text{constant}$$

$$\frac{\sin \theta_i}{\sin \theta_r} = n \text{ where, } n = \text{index of refraction and } n = \frac{n_2}{n_1}$$

$$\boxed{\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}} \rightarrow \text{This is called smell's law}$$

**Note that;** the refractive index of the light in the vacuum is 1

### 2. Diffraction

Diffraction is the spread out of the wave when the wave passes through gap or around obstacle.

### 3. Interference

Interference is the mixing up of two or more waves either to add up or cancel out each other. There are two types of interference. These are:

#### 4. Constructive interference

- ❖ It occurs where two waves that are in phase combine to make a large amplitude.
- ❖ It occurs when two identical waves meet their crest to crest or trough to trough this give a large oscillation

#### 5. Destructive interference

- ❖ It occurs where two waves that are out of phase combine to cancel each other.
- ❖ It occurs when a crest of one wave coincide with a trough of another waves, a wave small or zero oscillation is established

#### Example: 8.3

The speed of the light in a glass is  $2 \times 10^8 \text{ m/s}$ . What is the refractive index of the glass?

**Solution: given**

$$n_1 = 1$$

From Snell's law we have,

$$V_2 = 2 \times 10^8 \text{ m/s}$$

$$\frac{n_2}{n_1} = \frac{V_1}{V_2} \rightarrow n_2 = \frac{V_1}{V_2}$$

$$V_1 = 3 \times 10^8 \text{ m/s}$$

$$n_2 = 3 \times 10^8 \text{ m/s} / 2 \times 10^8 \text{ m/s}$$

**Required**

$$n_2 = 3/2 = 1.5$$

$$n_2 = ?$$

### 8.4 Sound wave

- Sound is a longitudinal mechanical wave which is created by oscillation of object and audibly perceived through a sense of hearing

#### Hearing

- The human ear is capable detecting sound with a range of frequencies between 20Hz to 20,000Hz. This is referred to as audible range.
  - Any sound frequency below 20Hz is known as infrasound or infrasonic.
  - Any sound frequency above 20,000Hz is known as ultrasound or ultrasonic

#### The speed of sound

- Sound propagates at different speed through different medium. For example sound waves travels faster in solid than in liquid and faster in liquid than in gases.
  - The speed of sound across a medium (matter) depend on:
    - The nature of the medium
    - The temperature of the medium.

The experiment shows that the speed of the sound is increases by  $0.6 \text{ m/s}$  for every degree Celsius increases in temperature of gases. So

$$V = 330 \text{ m/s} + 0.6 \text{ m/s}(T_c)$$

**Example: 8.4**

What is the speed of sound in air at temperature of 60°C?

**Solution: given**

$$V = 330 \text{ m/s at } 0^\circ\text{C}$$

$$V = 330 \text{ m/s} + 0.6 \text{ m/s}(T_c)$$

$$T_c = 60^\circ\text{C}$$

$$V = 330 \text{ m/s} + 0.6 \text{ m/s}(60)$$

**Required**

$$V = 330 \text{ m/s} + 36 \text{ m/s}$$

$$V = ? \text{ at } 60^\circ\text{C}$$

$$V = 366 \text{ m/s}$$

**How do we describe sound waves?**

- **Loudness**

- Loudness is audible strength of sound which depend on the amplitude of the sound wave

- **Pitch**

- A characteristic of sound wave which distinguishes a sharp or shrill sound from a grave or dull sound. It depends upon frequency.

- **Timber**

A quality of sound (tone)

- **Echoes**

- A reflecting of sound is called echoes. If the reflecting surface is “s” distance from the source and “t” is the time taken by the observer to hear echo, the velocity of sound v is calculate as follow

$$v = \frac{\text{total distance}}{\text{time taken}} \text{ but, } S_{\text{total}} = 2s$$

$$v = \frac{2s}{t}$$

**Example; 8:5**

A person is standing 100m away from the wall. The person bangs a drum and hears the echoes 0.5 second later. What is the speed of the sound?

**Solution: Given**

$$ST = 2 \times 100 \text{ m} = 200 \text{ m and } t = 0.5 \text{ sec}$$

**Required: v=?**

$$v = \frac{S_T}{t} = \frac{200 \text{ m}}{0.5 \text{ sec}} = 400 \text{ m/s}$$

### The intensity of sound wave

The intensity of sound is the energy received by each square meter per second.

Mathematically

$$\text{intensity} = \frac{\text{energy}/\text{time}}{\text{area}}$$

$$I = \frac{E/t}{A} \text{ but, } E/t = P$$

$$I = \frac{P}{A} \text{ but, } A = 4\pi r^2 \text{ Because sound is transmitted in all direction so the area will be spherical}$$

$$I = \frac{P}{4\pi r^2}$$

The intensity of sound is measured in  $W/m^2$

The intensity of  $I_1$  at a distance  $r_1$  from the source and the intensity of  $I_2$  at another distance  $r_2$  from the source related by the formula of the following.

$$I_1 = P/4\pi r_1^2 \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad I_2 = P/4\pi r_2^2$$

$$\rightarrow I_1 = I_2 \left( \frac{r_2^2}{r_1^2} \right) = \text{and } I_2 = I_1 \left( \frac{r_1^2}{r_2^2} \right)$$

#### Example: 8.5s

A sound produced from a certain source has an intensity of  $4 \times 10^{-4} W/m^2$  at 20m from the source. What will be the intensity of sound at distance of 60m?

**Solution: given**

$$I_1 = 4 \times 10^{-4} W/m^2, \quad r_1 = 20m \text{ and } r_2 = 60m$$

Required:  $I_2 = ?$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \rightarrow I_2 = I_1 \left( \frac{r_1^2}{r_2^2} \right) = (4 \times 10^{-4} W/m^2) \left( \frac{20^2}{60^2} \right)$$

$$I_2 = 4.4 \times 10^{-5} W/m^2$$



### Application of sound wave

- Sound waves have many uses in addition to their uses in communication and music

The following are some application of sound.

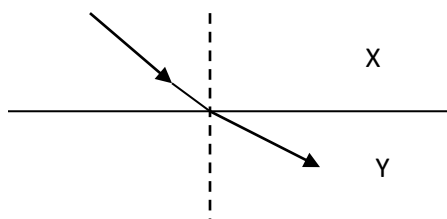
- To determine the depth of sea or ocean
- For medical diagnosis
- For searching food
- Uses for blind person
- For find oil(mineral) underground

### Practice questions for unit 8

- What do wave transport from one place to another?  
A. Energy    B. Wavelength    C. mass    D. Amplitude
- Which one the following does **not** belong to the common properties of the wave?  
A. Interface    B. Polarization    C. Refraction    D. Diffraction
- Which physical quantities decrease as sound travels from water to air?  
A. Frequency and period    C. Frequency and wavelength  
B. Speed and frequency    D. Wavelength and speed
- What is the wavelength in a vacuum of microwave of a frequency of  $15 \times 10^9 \text{ Hz}$ ?  
A. 5m    B. 2.5m    C. 0.2m    D. 0.02m
- Refraction is the  
A. bending the light as it passes through a small gap  
B. Change in the intensity of light as it crosses the boundary between two transparent media.  
C. Bending of light as it crosses the boundary between two transparent media.  
D. Change in the frequency of light as it crosses the boundary between two transparent media
- What is the velocity of a wave whose wavelength is 2m and whose frequency is 5 Hz?  
A. 20 m/s    B. 2.5 m/s    C. 0.4 m/s    D. 10 m/s
- the frequency of a wave is the  
A. number of complete waves passing a given point per second  
B. time taken for one complete wave to pass a given point  
C. distance the wave travels in one second  
D. minimum distance between identical point on adjacent waves
- The phenomena of bending of light round corners is called  
A. Interference of light    C. Diffraction  
B. Polarization    D. Refraction

9. When a ray of light passes from a rarer to a denser medium it's
  - A. Wavelength decreases
  - B. Frequency decreases
  - C. Wavelength and frequency unchanged
  - D. Frequency increases
10. An electromagnetic wave can be produced by
  - A. A steady current
  - B. Electromagnetic fields
  - C. Any moving charge
  - D. Any accelerating charge
11. What is the basic difference between mechanical and electromagnetic waves?
  - A. Mechanical waves are longitudinal waves while electromagnetic waves are transverse.
  - B. Mechanical waves require material medium while electromagnetic waves do not.
  - C. Mechanical waves are compression while electromagnetic waves rarefaction.
  - D. Mechanical waves travel in one direction while electromagnetic waves do in all direction.
12. A constructive interference is a wave phenomenon that occurs when two waves moves in
  - A. The same direction in phase
  - B. The same directions out of phase
  - C. opposite directions out of phase
  - D. opposite directions in phase
13. All electromagnetic waves travel with the same speed in
  - A. Air
  - B. Water
  - C. Mirror
  - D. Vacuum
14. Which one of the following cannot pass through the vacuum?
  - A. Ultraviolet
  - B. Infrared rays
  - C. x- ray
  - D. Sound wave.
15. In a certain wave oscillation of the particles is right angle to the direction of the wave propagation. Which one is **not** true about wave?
  - A. It is a longitudinal wave
  - B. It is a transverse wave
  - C. It is electromagnetic wave
  - D. It is a radio wave
16. A ray of light reflected from the smooth surface. Which of the following about the ratio of the angle of incidence to the angle of reflection is true?
  - A. It is less than one
  - B. It is greater than one
  - C. It is equal to one
  - D. It is impossible to determine.
17. The property of a sound wave that is related to the loudness is:
  - A. The wavelength
  - B. the pitch
  - C. The speed
  - D. The intensity
18. Which one of the following activities is possible without electromagnetic radiation?
  - A. Speaking with a friend.
  - B. Reading a book
  - C. Satellite communication
  - D. Watching a television program
19. As wind blows across a field of grain, the tops of the plants move back and forth when a waves travels across the field. This wave is a
  - A. Transverse wave
  - B. Polarized wave
  - C. electromagnetic wave
  - D. longitudinal wave

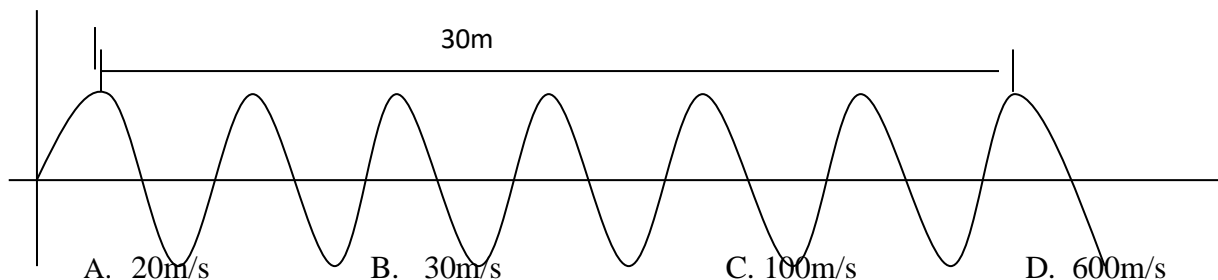
20. The intensity of a sound 2m away from the speaker  $100\text{W/m}^2$ . What will be the intensity at a distance of 10m away from the speaker?
- A.  $16\text{W/m}^2$       B.  $4\text{W/m}^2$       C.  $32\text{W/m}^2$       D.  $100\text{W/m}^2$ .
21. An electromagnetic wave propagates in vacuum at speed  $v$  & wave length  $\lambda$  as the wave enter some unknown medium its speed decreases to  $0.6v$ . What is the wavelength of unknown medium?
- A.  $0.4\lambda$       B.  $0.5\lambda$       C.  $0.6\lambda$       D.  $0.7\lambda$
22. A wave has a speed of  $50\text{m/s}$  and a wavelength of  $25,000\text{m}$ . What is the frequency of the wave?
- A.  $2 \times 10^{-3}\text{Hz}$       B.  $6 \times 10^2\text{Hz}$       C.  $5 \times 10^2\text{Hz}$       D.  $1.8 \times 10^6\text{Hz}$
23. The distance between the adjacent crests or troughs in a transverse wave is  $20\text{cm}$ . if the frequency of the wave is  $800\text{Hz}$ , then what is the speed of the wave?
- A.  $16\text{m/s}$       B.  $32\text{m/s}$       C.  $160\text{m/s}$       D.  $320\text{m/s}$
24. The property of a sound wave that is related to the loudness is:
- A. The wavelength      B. The pitch      C. The speed      D. The intensity
25. A sound wave is sent to a rigid wall  $75\text{m}$  from the source. If the reflected sound is received after  $0.5\text{sec}$ , then what is the speed of the sound waves?
- A.  $150\text{m/s}$       B.  $300\text{m/s}$       C.  $330\text{m/s}$       D.  $165\text{m/s}$
26. light travels from x to medium y as shown in the diagram below



Which one of the following is correct?

- A. Speed decreases and the wavelength increases
- B. Both the speed and wavelength decreases
- C. Both the speed and wavelength increases
- D. Both the speed and wavelength unchanged
27. The distance between two nearby crest of the water wave in a pond is measured to be  $0.8\text{m}$ . If ten crests are passing at point every second without considering the reference crest, what is speed of the wave?
- A.  $4.8\text{m/s}$       B.  $24\text{m/s}$       C.  $8\text{m/s}$       D.  $80\text{m/s}$

28. What will be the effect of temperature on speed of sound?
- The speed of sound decreases with the increases of temperature of the medium.
  - The speed of sound decreases with the decrease of temperature of the medium.
  - The speed of sound increases with the decrease of temperature of the medium.
  - The speed of sound increases with the increase of temperature of the medium.
29. A light travel in a block of plastic that has an index of refraction of 2 then what is the speed of light in the plastic?
- $1 \times 10^8 \text{ m/s}$
  - $1.5 \times 10^8 \text{ m/s}$
  - $2 \times 10^8 \text{ m/s}$
  - $3 \times 10^8 \text{ m/s}$
30. Tola shouts into a hard dark cave; after fraction of second, he hears the sound of his own voice. This phenomena is:
- Interference
  - Diffraction
  - Reflection
  - refraction
31. Student counts 40 complete oscillations in 8second. What is its period?
- 5 sec
  - 0.25 sec
  - 0.2 sec
  - 0.4 sec
32. The distance between consecutive crests of a water wave is 0.5m. what is the frequency of the wave, its speed 40m/s?
- 20 Hz
  - 50Hz
  - 60Hz
  - 80Hz
33. The frequency of the wave on a rope shown in the figure below is 20Hz. what is the speed of this wave?



34. The distance between two successive crest and trough of a transverse wave is equal to:
- $2\lambda$
  - $\lambda$
  - $\lambda/2$
  - $\lambda/4$
35. The speed of sound through sea water is about 4000m/s and a wave pulse is sent from the ship and takes 0.8 sec to return back. What is the depth of the sea water?
- 800m
  - 1600m
  - 2000m
  - 4000m
36. the frequency range to which a normal ear can detect varies from \_\_\_\_\_ to \_\_\_\_\_
- 20Hz to 20,000Hz
  - 20Hz to 200,000Hz
  - 20Hz to 40,000Hz
  - 100Hz to 10,000Hz

## Answer of review exercise

**Unit five**

- |                 |                  |                  |                  |
|-----------------|------------------|------------------|------------------|
| 1. <u>  D  </u> | 10. <u>  D  </u> | 19. <u>  D  </u> | 28. <u>  C  </u> |
| 2. <u>  B  </u> | 11. <u>  C  </u> | 20. <u>  B  </u> | 29. <u>  A  </u> |
| 3. <u>  B  </u> | 12. <u>  B  </u> | 21. <u>  C  </u> | 30. <u>  B  </u> |
| 4. <u>  B  </u> | 13. <u>  A  </u> | 22. <u>  D  </u> | 31. <u>  A  </u> |
| 5. <u>  C  </u> | 14. <u>  D  </u> | 23. <u>  C  </u> | 32. <u>  B  </u> |
| 6. <u>  D  </u> | 15. <u>  B  </u> | 24. <u>  A  </u> | 33. <u>  D  </u> |
| 7. <u>  B  </u> | 16. <u>  D  </u> | 25. <u>  C  </u> | 34. <u>  A  </u> |
| 8. <u>  A  </u> | 17. <u>  C  </u> | 26. <u>  B  </u> | 35. <u>  B  </u> |
| 9. <u>  B  </u> | 18. <u>  B  </u> | 27. <u>  A  </u> | 36. <u>  D  </u> |

**Unit six**

- |                 |                  |                  |                  |
|-----------------|------------------|------------------|------------------|
| 1. <u>  A  </u> | 10. <u>  B  </u> | 19. <u>  C  </u> | 28. <u>  C  </u> |
| 2. <u>  C  </u> | 11. <u>  B  </u> | 20. <u>  B  </u> | 29. <u>  B  </u> |
| 3. <u>  C  </u> | 12. <u>  B  </u> | 21. <u>  B  </u> | 30. <u>  C  </u> |
| 4. <u>  B  </u> | 13. <u>  D  </u> | 22. <u>  D  </u> | 31. <u>  D  </u> |
| 5. <u>  A  </u> | 14. <u>  A  </u> | 23. <u>  B  </u> | 32. <u>  B  </u> |
| 6. <u>  B  </u> | 15. <u>  C  </u> | 24. <u>  C  </u> | 33. <u>  A  </u> |
| 7. <u>  D  </u> | 16. <u>  D  </u> | 25. <u>  A  </u> | 34. <u>  C  </u> |
| 8. <u>  A  </u> | 17. <u>  C  </u> | 26. <u>  B  </u> | 35. <u>  B  </u> |
| 9. <u>  D  </u> | 18. <u>  B  </u> | 27. <u>  B  </u> | 36. <u>  A  </u> |

**Unit seven**

- |                 |                  |                  |                  |                  |
|-----------------|------------------|------------------|------------------|------------------|
| 1. <u>  B  </u> | 7. <u>  C  </u>  | 13. <u>  D  </u> | 19. <u>  D  </u> | 25. <u>  C  </u> |
| 2. <u>  D  </u> | 8. <u>  A  </u>  | 14. <u>  C  </u> | 20. <u>  D  </u> | 26. <u>  D  </u> |
| 3. <u>  C  </u> | 9. <u>  C  </u>  | 15. <u>  B  </u> | 21. <u>  C  </u> | 27. <u>  A  </u> |
| 4. <u>  A  </u> | 10. <u>  B  </u> | 16. <u>  C  </u> | 22. <u>  D  </u> | 28. <u>  B  </u> |
| 5. <u>  D  </u> | 11. <u>  B  </u> | 17. <u>  C  </u> | 23. <u>  A  </u> | 29. <u>  C  </u> |
| 6. <u>  B  </u> | 12. <u>  A  </u> | 18. <u>  C  </u> | 24. <u>  B  </u> | 30. <u>  D  </u> |

**Unit eight**

- |                 |                  |                  |                  |
|-----------------|------------------|------------------|------------------|
| 1. <u>  A  </u> | 10. <u>  B  </u> | 19. <u>  D  </u> | 28. <u>  D  </u> |
| 2. <u>  B  </u> | 11. <u>  B  </u> | 20. <u>  B  </u> | 29. <u>  B  </u> |
| 3. <u>  D  </u> | 12. <u>  A  </u> | 21. <u>  C  </u> | 30. <u>  C  </u> |
| 4. <u>  D  </u> | 13. <u>  D  </u> | 22. <u>  A  </u> | 31. <u>  C  </u> |
| 5. <u>  C  </u> | 14. <u>  D  </u> | 23. <u>  C  </u> | 32. <u>  A  </u> |
| 6. <u>  D  </u> | 15. <u>  A  </u> | 24. <u>  D  </u> | 33. <u>  C  </u> |
| 7. <u>  A  </u> | 16. <u>  C  </u> | 25. <u>  B  </u> | 34. <u>  A  </u> |
| 8. <u>  C  </u> | 17. <u>  D  </u> | 26. <u>  A  </u> | 35. <u>  B  </u> |
| 9. <u>  A  </u> | 18. <u>  A  </u> | 27. <u>  C  </u> | 36. <u>  A  </u> |