GRADE 11 PHYSICS SHORT NOTE

ON UNITS 5-8

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April, 2020

Unit-5
Work, Energy and Power

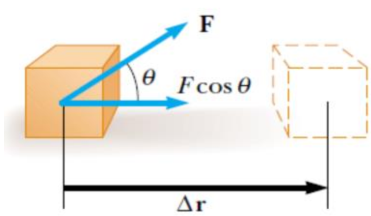
Learning competencies (objectives):

By the end of this unit you should be able to:

- ✓ Differentiate between work, energy, and Power.
- ✓ Determine work done by constant and variable force by using definition and graphical description of force versus displacement
- ✓ Proof the relationship between work and change in kinetic Energy and apply to solve related problems.
- ✓ Describe and explain the exchange among potential energy, kinetic energy and internal energy for simple mechanical systems, such as a pendulum, a roller coaster, a spring, a freely falling object.
- ✓ Apply the law of mechanical energy conservation to solve problems involving conservation of energy in simple systems with various sources of potential energy, such as springs
- ✓ Distinguish between conservative and non-conservative forces.
- ✓ Define and work out power.

Mechanical Work:

Work is the process of transforming energy. It is the scalar (dot) products of force and displacement.



$$\mathbf{W = \vec{F}.\vec{s} = \|\vec{F}\|\|\vec{s}\|\cos\theta \text{ or}}$$
$$\mathbf{W = \vec{F}.\vec{s} = F_x r_x + F_y r_y + F_z r_z}$$

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

If a number of forces $\vec{F}_1, \vec{F}_2, \ \ldots \ \vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then

$$W = \vec{F}_{net}.\Delta\vec{r} = (\vec{F}_1 + \vec{F}_2 + \ \ldots + \vec{F}_n).(\vec{r}_2 - \vec{r}_1)$$

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

It is a scalar physical quantity, Its SI unit is J and its dimensional formula is given by $[m][l]^2[t]^{-2} = ML^2T^{-2}$

Relation between different units:

$$1J = 10^7 erg \qquad 1eV = 1.6 \times 10^{-19} J$$
$$1KWh = 3.6 \times 10^6 J \qquad 1Calorie = 4.18J$$

Worked Examples:

1. A body of mass 4*kg* is placed at the origin, and can move only on the *x*-axis. A force of 10*N* is acting on it in a direction making an angle of 53° with the *x*-axis and displaces it along the *x*-axis by 12 *metres*. Find the work done by the force

   solution:

   $\boldsymbol{W = \vec{F}.\vec{s} = \|\vec{F}\|\|\vec{s}\|cos\theta}$

   $\boldsymbol{W = 10N * 12m * cos53°}$

   = (120J)(0.6)=72J

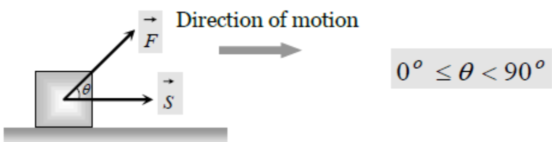
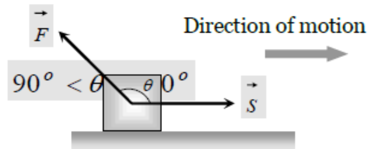
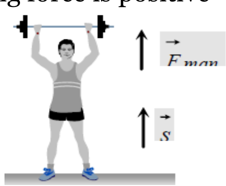
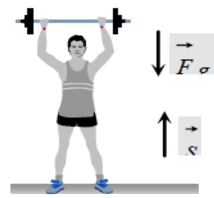
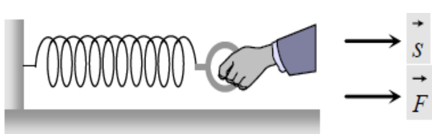
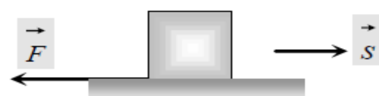
2. Find the work done if a particle moves from a position $\boldsymbol{\vec{r}_1 = i + j - 2k}$ to a position $\boldsymbol{\vec{r}_2 = 2i + 3j + 4k}$under the effect of force $\boldsymbol{\vec{F}} = 2i + 4j + 6k$.

   solution:

   $$\boldsymbol{W} = \boldsymbol{\vec{F}}.\Delta\boldsymbol{\vec{r}} = (2i + 4j + 6k).(\boldsymbol{\vec{r}_2 - \vec{r}_1})$$
   $$\boldsymbol{W} = \boldsymbol{\vec{F}}.\Delta\boldsymbol{\vec{r}} = (2i + 4j + 6k).(\boldsymbol{\vec{r}_2 - \vec{r}_1})$$
   $$\boldsymbol{W} = (2i + 4j + 6k).(\boldsymbol{i + 2j + 6k})$$
   $$\boldsymbol{W} = (2)(1) + (4)(2) + (6)(6)) = \boldsymbol{46J}$$

Activities:

1. A horizontal force of 5 *N* is required to maintain a velocity of 2 *m/s* for a block of 10 *kg* mass sliding over a rough surface. Calculate the work done by this force in one minute.
2. A box of mass 1 *kg* is pulled on a horizontal plane of length 1 *m* by a force of 8 *N* then it is raised vertically to a height of 2*m, Find* the net work done
3. A 10 *kg* satellite completes one revolution around the earth at a height of 100 *km* in 108 *minutes*. The work done by the gravitational force of earth will be

Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement	Negative work means that force (or its component) is opposite to displacement *i.e.*
 The positive work signifies that the external force favours the motion of the body.	 The negative work signifies that the external force opposes the motion of the body.
Example: **i**) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive 	*Example*: **i**) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative. 
ii) When a spring is stretched, work done by the external (stretching) force is positive. 	**(ii)** When a body is made to slide over a rough surface, the work done by the frictional force is negative. 

Zero Work Done:

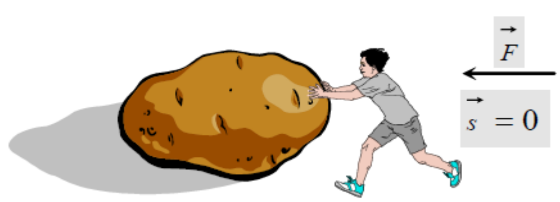
Under three condition, work done becomes zero $W = Fs\cos\theta = 0$

1. If the force is perpendicular to the displacement [$\vec{F} \perp \vec{s}$]

   Example: When a body moves in a circle the work done by the centripetal force is always zero.

2. If there is no displacement [$\vec{s}=0$]

Example: When a person tries to displace a wall or heavy stone by applying a force then it does not move, the work done is zero.

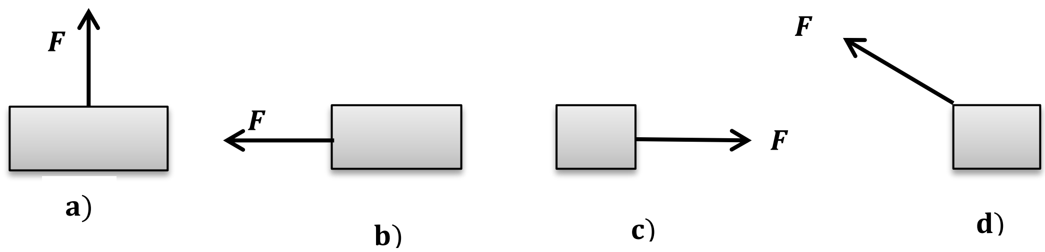


3. If there is no force acting on the body [$\vec{F} = 0$]

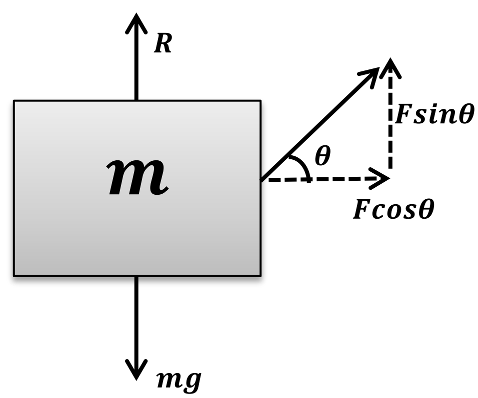
   Example: Motion of an isolated body in free space.

Activity:

Figure shown bellow shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.



Work Done by different constant forces on a body moving on rough surface:



Work Done by:

a) applied force (F)=$\|\vec{F}\|\|\vec{s}\|cos\theta$
b) friction(f)=$\|f\|\|\vec{s}\|cos180° = -\|f\|\|\vec{s}\|$
c) Reaction (R)= $\|\vec{R}\|\|\vec{s}\|cos90°$=0
d) Weight (mg)= $m\|\vec{g}\|\|\vec{s}\|cos90°$=0

Net force =$\left(\|\vec{F}\|cos\theta - \|f\|\right)\|\vec{s}\|$

Work Done by: $mg,\ Normal\ (R)\&\|\vec{F}\|sin\theta\ forces\ are\ zero\ work\ done$

Work done by a constant and variable force

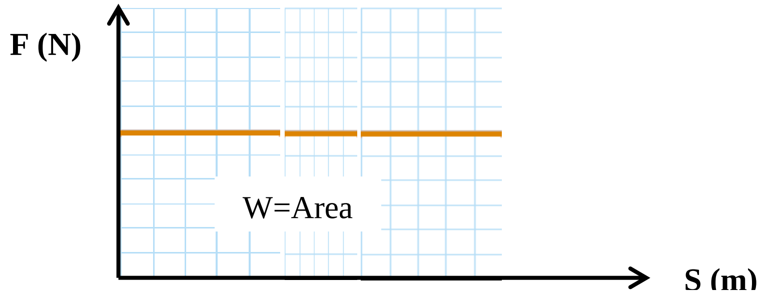
Work done by a constant force:

Work is said to be done by a constant force when a body is moving with a constant acceleration due to the force applied to displace the body through a certain distance in the direction of the net force applied.

$$\boldsymbol{W = \vec{F}.\vec{s}}$$

Work done by a force is equal to the area of the region bounded by component of the force parallel to the displacement axis.

Force Versus Distance Graph of Work Done by Constant force:



Work Done by a Variable Force

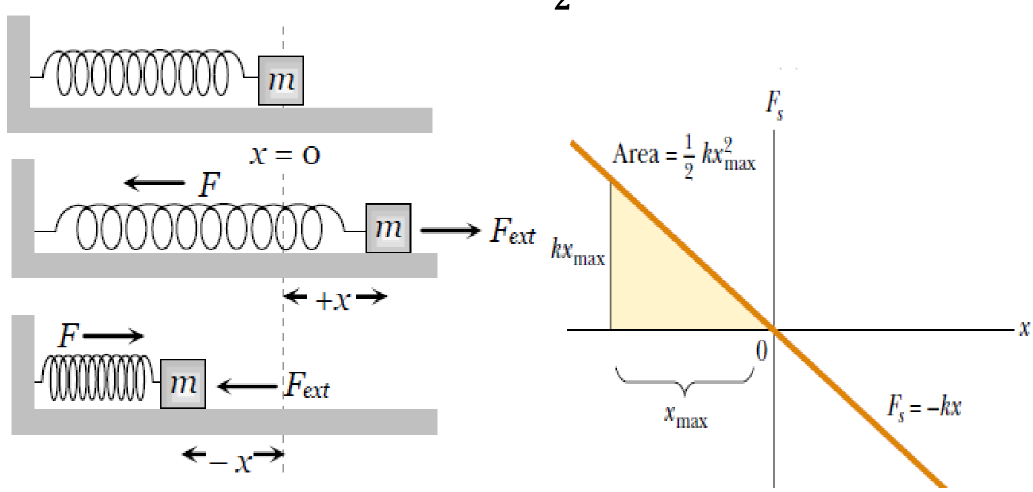
Variable force is a force that varies with time or position

a) Constantly varying force
   Example: Work done by a spring forces

Area under force displacement curve with proper algebraic sign represents work done by the force.

If spring is stretched from initial position x_1 to a final position x_2 then work done = Increment in elastic potential energy

$$\mathbf{W = \Delta EPE = \Delta U = \frac{1}{2}K(x_2{}^2 - x_1{}^2)}$$



Worked Example:

1) A force of 100N is required to stretch a spring that obeys Hook's law by 100cm. What is the work done in stretching the spring?

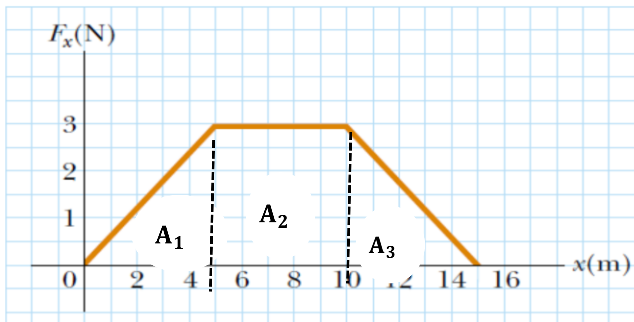
solution:

Applied force on stretched spring is linearly varying force with position

Work done =area under the graph

Work done =$(F_{average})\Delta x = \frac{0+F}{2} * \Delta x = \frac{0+100N}{2} * 1m = 50J$

2) A particle is subject to a force $\boldsymbol{F_x}$ that varies with position as shown in Figure below. Find the total work done by the force over the distance from x =0 to x = 15m?



solution:

Work done = area of the region bounded by component of the force

$$W=\mathbf{A_1+A_2+A_3}$$

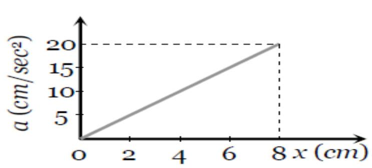
$$\mathbf{W=\frac{1}{2} * 3N * 5m + 3N * (10m - 5m) + \frac{1}{2} * 3N * (15m - 10m)}$$

$$\mathbf{W=30J}$$

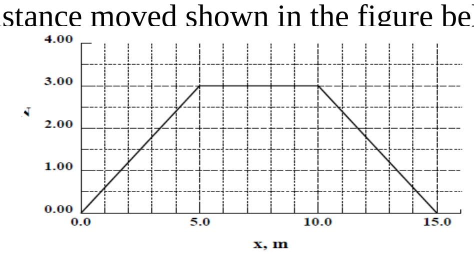
Activities:

Find the total work done by the force over the distance moved shown in the figure below:

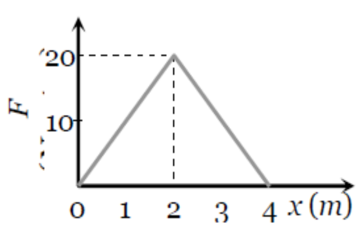
a)



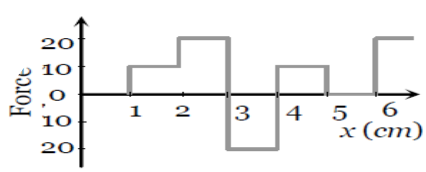
b)



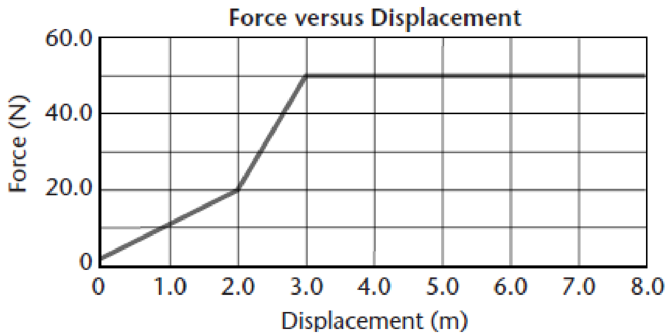
c)



d)

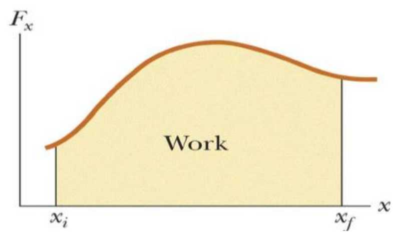


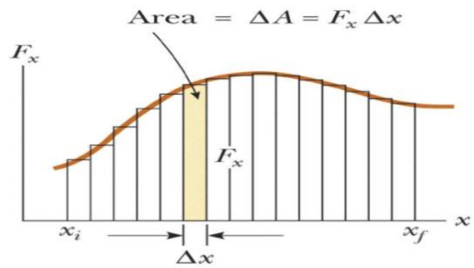
e)



Work Done by Non-Linear Variable Forces

To find work done by non-linear varying force we divide the region under in to very small rectangle and take the scalar sum of each area under the graph or integration.





$$W = \sum_{i}^{n} F_x \Delta x$$

Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.

Energy- is the capacity $(ablity)\ to\ do\ a\ work.$

i. It is a scalar physical quantity
ii. Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque
iii. Units: Joule (J) $[SI]$, erg $[cgs]$

Practical units: electron volt (eV), Kilowatt hour (KWh), Calories (Cal) Relation between different units:

$$1Joule = 10^7 erg$$
$$1eV = 1.6^{-19} J$$
$$1KWh = 3.6 \times 10^6 J$$
$$1Calorie = 4.15Joule$$

Forms of Energy:

There are several different forms of energy.
These include:

- Kinetic energy, gravitational potential energy, elastic potential energy, heat energy, sound energy, electrical energy… etc.

Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Let *m* = mass of the body, *v* = velocity of the body then $\mathrm{K.E} = \frac{1}{2}mv^2$

i. Kinetic energy depends on frame of reference: The kinetic energy of a person of mass *m*, sitting in a train moving with speed *v*, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

ii. **Work-energy theorem**: It states that work done by a force acting on a body sliding over a smooth horizontal surface is equal to the change produced in the kinetic energy of the body.

$$Work\ Done\ by\ Net\ Force = change\ in\ kinetic\ energy$$

$$W_{net} = \Delta\mathrm{K.E} = \frac{1}{2}m(v^2 - u^2)$$

Speed increase if the net work done is positive as speed decrease if the net work done is negative.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive *i.e.* body moves in the direction of the force (or field) and if kinetic energy decreases work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

Examples:

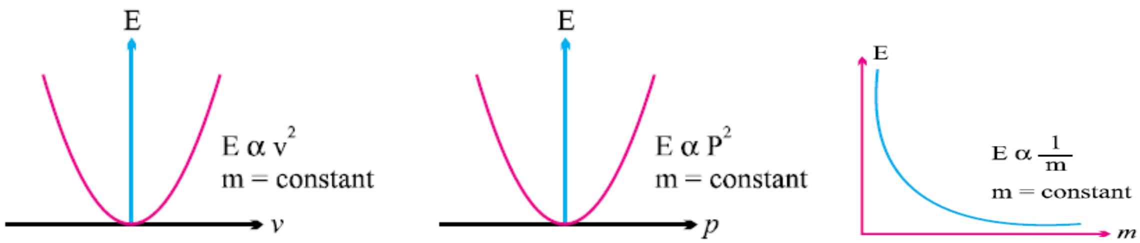
1. Write KE possessed by a body in terms of:
   a) Linear momentum and speed
   b) Linear momentum and mass

iii. Relation of kinetic energy with linear momentum : As we know

$$\mathrm{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}mv(v),\ p = mv \quad \& \quad v = \frac{p}{m}$$

$$\Rightarrow \mathrm{K.E} = \frac{\mathrm{p}^2}{2m} \qquad \& \qquad \mathrm{p} = \sqrt{2\mathrm{m}\,(\mathrm{K.E})}$$

iv. Various graphs of kinetic



Activities:

1. Use the definition of work done and Newton's second law, then show as work done by a force acting on a body is equal to the change produced in the kinetic energy of the body
2. A car of mass 1,400 kg accelerates from 2 m/s to 24m/s. The force of the engine acting on the car is 6,000N. Over what distance did the force act?
3. A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1*m/s* so as to have same *K.E.* as that of boy. Find the original speed of the man
4. A 3k*g* mass has a velocity of $(3\hat{i} + 4\hat{j})$*m/sec* at a certain instant. What is its kinetic energy?

Potential Energy:

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy is the energy possessed by an object because of its position or configuration.

Potential energy generally are of three types: Elastic potential energy, Electric potential energy and Gravitational potential energy *etc.*

Gravitational Potential Energy VS Work Done

When an object undergoes vertical displacement, the gravitational force does work on the object, work done against gravity is equal to the rise in gravitational potential energy of the object and the work done by the object is equal to the fall in its gravitational potential energy. Thus the work done by the object is the negative of the change in gravitational potential energy.

$$Work\ done\ by\ gravity = \vec{F}_{gravity}.\Delta\vec{h} = |\vec{F}_{gravity}||\Delta\vec{h}|\text{cose } 180°$$

$$W_{gravity} = -|\vec{F}_{gravity}||\Delta\vec{h}| \text{ (for an object rising up)}$$

$$\text{W} = \Delta GPE = \text{mg}\Delta h$$

- ✓ Work done by weight (gravity) on the ball as it falls a distance ‘h’ towards the earth is positive.

Activity: A body of mass 0.3 kg is taken up an inclined plane to length 10 m and height 8m and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.12. What is the

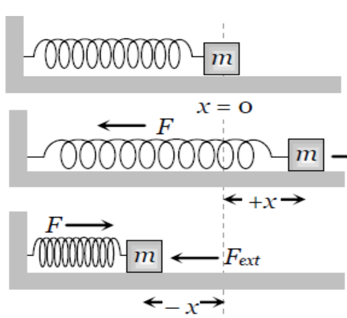
i. work done by the gravitational force over the round trip.
ii. work done by the applied force over the upward journey.
iii. work done by frictional force over the round trip.
iv. kinetic energy of the body at the end of the trip.

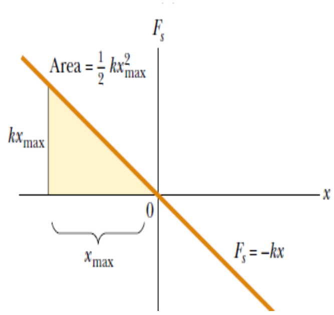
Conservation of energy

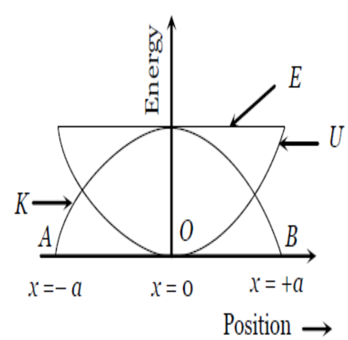
Work Done and Elastic Potential Energy:

If spring is stretched from initial position x_1 to a final position x_2 then work done = Increment in elastic potential energy

$$\mathbf{W = \Delta EPE = \Delta U = \frac{1}{2}K({x_2}^2 - {x_1}^2)}$$







Work done by applied force in stretched mass-spring system:

$$\text{W}_{applied\ force} = \text{positive work done}, and$$

$$\text{W}_{restoring\ force} = negative\ work$$

Work done by the elastic/restoring force in terms of change in elastic potential energy from x_o to x is given by:

$$W_{applied\ force} = -\Delta EPE = \Delta U = -\frac{1}{2}K(x^2 - {x_o}^2)$$

In the absence of external force, including friction total mechanical energy of the mas spring system at x_o and x are equal:

$$\frac{1}{2}m{v_o}^2 + \frac{1}{2}K{x_o}^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

At maximum displacement (extreme position, $v = 0$, x= x_{max}) and at the equilibrium postion ($v\ is\ maximum$, x= 0)

$$ME_{equilibrium} = ME_{extrem\ postion}$$

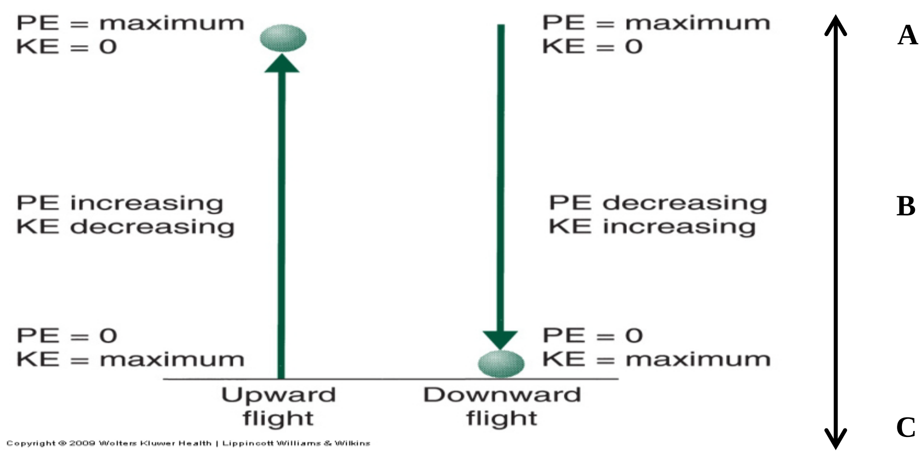
$$\frac{1}{2}m{v_o}^2 + \frac{1}{2}K{x_o}^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

$$\frac{1}{2}m{v_{max}}^2 + \frac{1}{2}K(0)^2 = \frac{1}{2}m(0) + \frac{1}{2}K{x_{max}}^2$$

$$\frac{1}{2}m{v_{max}}^2 = \frac{1}{2}K{x_{max}}^2$$

$$v_{max} = \sqrt{\frac{K}{m}({x_{max}}^2)} = x_{max}\sqrt{\frac{K}{m}}$$

Conservation of Mechanical Energy and Interchange between Kinetic and Gravitational potential Energy For An Object Rising up Falling Down:

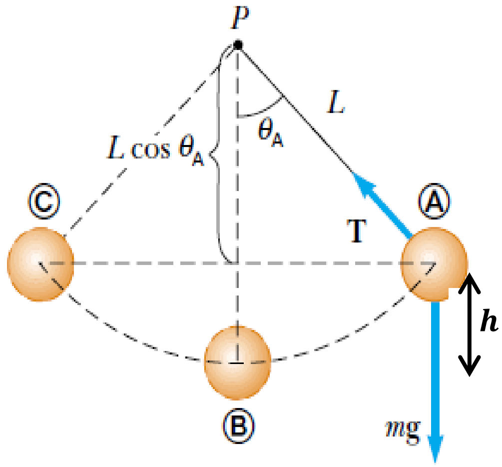


$v_A = 0$ and $h_A = maximum$, $h_C = 0\ and\ v_C = maximum$, but $ME_A = ME_B = ME_C$

If $ME_A = ME_C$, the $mgh = \frac{1}{2}mv^2$

$$v = \sqrt{2gh}$$

Conservation of Mechanical Energy and Interchange between Kinetic and Gravitational potential Energy For to-and-fro Oscillating Pendulum:



$$h = L - L cos\theta_A = L(1 - cos\theta_A)$$

$$ME_A = ME_B = ME_C$$

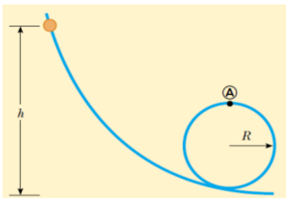
$$ME_A = ME_B$$

$$mgh = \frac{1}{2} m v_B{}^2$$

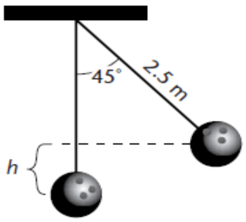
$$v_B = \sqrt{2gh} = \sqrt{2gL(1 - cos\theta_A)}$$

Activities

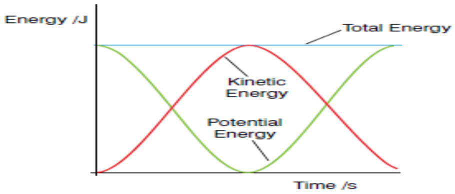
1. A ball is thrown on a frictionless wire with a speed of 10m/s at 'h' and a ball slides without friction around a loop-the- loop as shown below. The ball is thrown from h=2.5R. Find its speed at point A.



2. A 7.26-kg bowling ball hangs from the end of a 2.5-m rope. The ball is pulled back until the rope makes a 45° angle with the vertical. Find the gravitational potential energy of the system?



Graphical description showing how PE and KE of Oscillating system and vertical motion are related:



Conservative and dissipative forces

Work Done by Conservative Forces

Conservative Forces-*are path independent*
Example: Gravitational force, restoring forces, electrostatic forces
If the net work done by a force does not depend on the path taken between two points, we say that the force is a conservative force. For such forces it is also true that the net work done on a particle moving on around any closed path is zero.

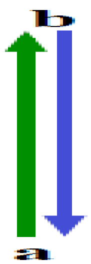
$$\Delta\text{ME} = \Delta\text{KE} + \Delta\text{PE} = 0$$

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical)

A force is a conservative force if the network it does on a particle moving around every closed path is zero.

Example: Work done by gravitation for a ball thrown upward that then falls back down



Solution:

$$W_{net} = \Delta\text{ME} = W_{gravity} = W_{aa} = W_{ab} + W_{ba}$$
$$= \Delta\text{KE} + \Delta\text{PE} = 0$$
$$\Delta\text{KE} = \frac{1}{2}m(v^2\text{-}v^2) = 0,\ v_o = v_f = v$$
$$\Delta\text{PE} = -\text{mgh} + \text{mgh} = 0$$

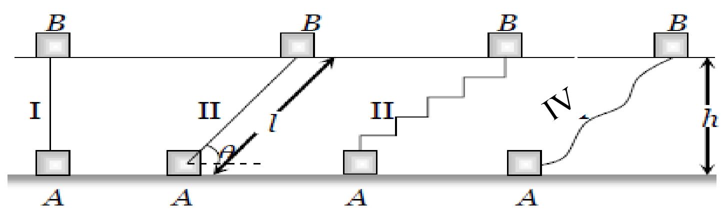
Non-conservative forces acting within a system cause a *change* in the mechanical energy of the system.

$$\Delta ME = \Delta KE + \Delta PE = W_{friction} = -fd$$

Note: Mechanical energy is only conserved when no dissipative or non-conservative force act on a body

Activity:

If a body of man *m* lifted to height *h* from the ground level by different path as shown in the Figure shown below. If $\mathbf{W}_{I}$, $\mathbf{W}_{II}$ $\mathbf{W}_{III}$and $\mathbf{W}_{IV}$ represent the work done in moving a particle from *A* to *B* along three different paths I, II, III AND IV respectively (as shown) in the gravitational field of a point mass *m*. Put the correct relation between the amount of work done on mass m in moving a particle from *A* to *B* along four different paths shown below.

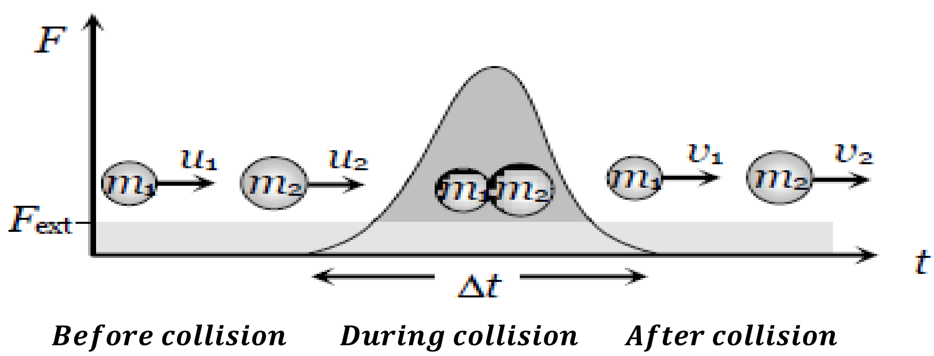


Collisions and Law of conservation of Kinetic Energy

Collision - is an interaction between two different masses in which momentum is conserved (particles may or may not come in real touch).

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

Stages of collision : Before, during and after collision



Momentum and energy conservation in collision:

i. **Momentum conservation**: In a collision average impulsive force is responsible and it is 'Internal' therefore the total momentum of system always remains conserved.

ii. **Energy conservation**: In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy, but this is not true for conservation of kinetic energy.

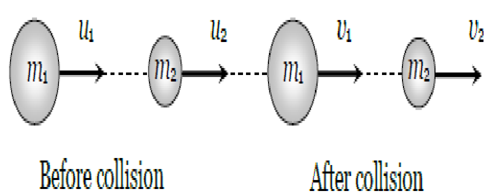
Types of collision:

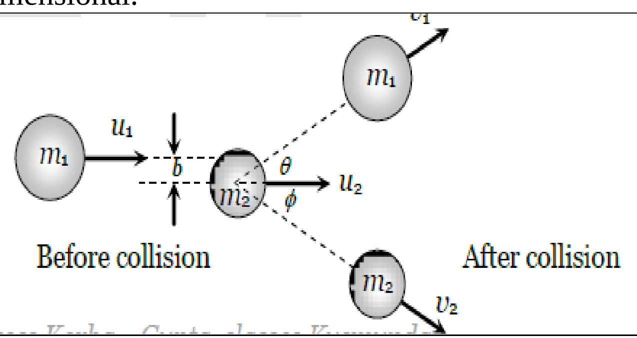
(i) On the basis of conservation of kinetic energy.

Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
(KE)final = (KE)initial	In some cases (KE)final <(KE)initial such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases (KE) final > (KE)initial	

(ii) On the basis of the direction of colliding bodies

Head on or one dimensional collision	Oblique collision`
In a collision if the motion of colliding particles before and after the collision is along the same line the collision is said to be head on or one dimensional.	If two particle collision is 'glancing' *i.e.* such that their directions of motion after collision are not along the initial line of motion, the collision is called oblique. If in oblique collision the particles before and after collision are in same plane, the collision is called 2-dimensional otherwise 3-imensional.





Perfectly Elastic Head on Collision:

Let two bodies of masses m_1 *and* m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are $\vec{v}_1$ *and* $\vec{v}_2$ respectively.

According to law of conservation of momentum

$$m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$m_1(\vec{u}_1 - \vec{v}_1) = m_2(\vec{v}_2 - \vec{u}_2) \qquad \text{(i)}$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$$

$$m_1({u_1}^2 - {v_1}^2) = m_2({v_2}^2 - {u_2}^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \qquad \text{(ii)}$$

Dividing equation (ii) by equation (i)

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1$$

Relative velocity of approach = Relative velocity of separation

Worked Examples:

1. The driver of the compact car suddenly applies the brakes hard for 2.0 s. As a result, an average force of $5 * 10^{-3}N$ is exerted on the car to slow it. What is the change in momentum, that is the magnitude and direction of the impulse, on the car?

Given:

$\Delta t = 2.0\text{sec}$

$\vec{\mathbf{F}}_{net} = 5 * 10^{-3}N$

Required:

$\Delta\mathbf{P} = ?$

solution:

$\Delta\mathbf{P} = \vec{\mathbf{F}}_{net} . \Delta\mathbf{t}$

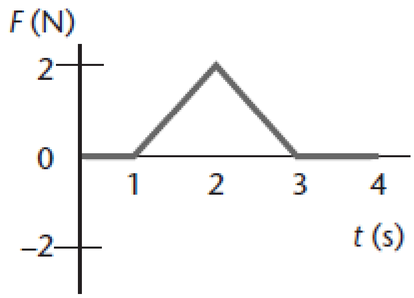
$\Delta\mathbf{P} = (5 * 10^{-3}N)(2\text{sec.})$

$\Delta\mathbf{P} = \mathbf{0.05}\boldsymbol{kgm/s}$

Activity:

1. In a perfectly elastic collision, both momentum and mechanical energy are conserved. Two balls with masses $\mathbf{m}_{\boldsymbol{A}}$ and $\mathbf{m}_{\mathbf{B}}$ are moving toward each other with speeds v_A and v_B, respectively. Solve the appropriate equations to find the speeds of the two balls after the collision.
2. A railway carriage of mass 9000 kg moving with a speed of 36 km/h collides with a stationary carriage of same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision? What type of collision is this ?

3. A 0.4kg ball, moving in the positive direction at 16.0 m/s, is acted on by the impulse shown in the graph shown below**.** What is the ball's speed at 4.0 s?



Mechanical Power

Power of a body is defined as the rate at which the body can do the work or time rate of energy is transferred.

Average power (P_{av})=$\frac{\Delta W}{\Delta t}$ &

Instantaneous power (P_{in})=$\frac{dW}{dt} = \frac{dE}{dt}$

P_{in}=$\vec{F}.\vec{v}_{av}$ (power is equal to the scalar product of force with velocity)

Worked Example:

A truck of mass 1000 kg accelerates uniformly from rest to a velocity of 15 m/s in 5 seconds. Calculate the average power of the engine during this period, neglect friction.

Given:	**Required**:	**solution**:
$m = 1{,}000kg$	**P=?**	$\mathbf{P} = \frac{\mathbf{W}}{t} = \frac{\Delta\mathbf{KE}}{t}$.
$\boldsymbol{u = 0m/s}$		$\mathbf{P} = \frac{\frac{1}{2}m(v^2-u^2)}{t} = \frac{\frac{1}{2}(1000)(15^2-0^2)}{5}J$
$v = \mathbf{15}\boldsymbol{m/s}$		$\mathbf{P} = \mathbf{22.5K}\boldsymbol{J} =$
$t = \mathbf{5}\boldsymbol{sec}$		

Review questions

1. A force F at an angle θ above the horizontal is used to pull a heavy suitcase of weight mg a distance d along a level floor at constant velocity. The coefficient of friction between the floor and the suitcase is μ. The work done by the frictional force is:

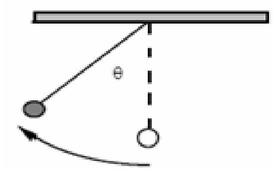
   (A) –Fd cos θ (B) – μ Fd cos θ (C) -μmgd (D) -μmgd cos θ

2. A 2 kg ball is attached to a 0.80 m string and whirled in a horizontal circle at a constant speed of 6 m/s. The work done on the ball during each revolution is:

   (A) 90 J (B) 72 J (D) 16 J (D) zero

3. A pendulum bob of mass m on a cord of length L is pulled sideways until the cord makes an angle θ with the vertical as shown in the figure to the right. The change in potential energy of the bob during the displacement is:

   A. mgL (1–cos θ)
   B. mgL (1–sin θ)
   C. mgLsinθ
   D. mgL cos θ



4. A softball player catches a ball of mass **m**, which is moving towards her with horizontal speed $\boldsymbol{v}$. While bringing the ball to rest, her hand moved back a distance **d**. Assuming constant deceleration, the horizontal force exerted on the ball by the hand is

   A. $\boldsymbol{\frac{mv^2}{2d}}$ B. $\boldsymbol{\frac{mv^2}{d}}$ C. $\boldsymbol{\frac{2mv}{d}}$ D. $\boldsymbol{\frac{mv}{d}}$

Questions 5-6: A car of mass m slides across a patch of ice at a speed v with its brakes locked. It the hits dry pavement and skids to a stop in a distance d. The coefficient of kinetic friction between the tires and the dry road is μ.

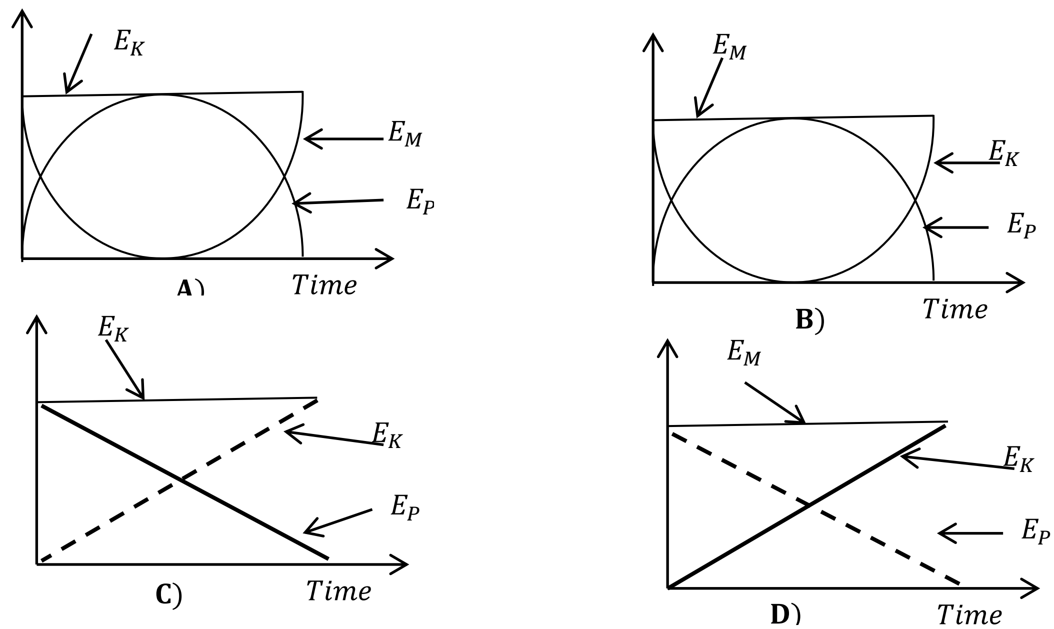
5. If the car has a mass of 2m, it would have skidded a distance of

   A. 0.5 d B. d C. 1.41 d D. 2 d

6. If the car has a speed of 2v, it would have skidded a distance of

   A. d B. 1.41 d C. 2 d D. 4 d

7. A pendulum is pulled to one side and released. It swings freely to the opposite side and stops. Which of the following might best represent graphs of kinetic energy (E_k), potential energy (E_p) and total mechanical energy (E_T)



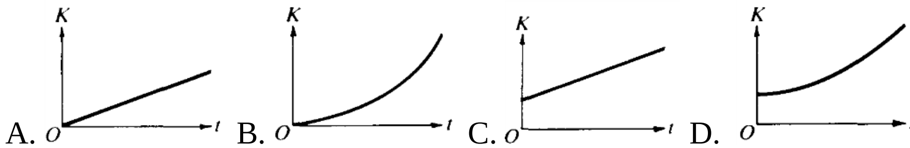
8. A ball is thrown vertically upwards with a velocity $\boldsymbol{v}$ and an initial kinetic energy $\boldsymbol{KE}$. When half way to the top of its flight, it has a velocity and kinetic energy respectively of

A. $\frac{v}{2}, \frac{KE}{2}$ B. $\frac{v}{\sqrt{2}}, \frac{\boldsymbol{KE}}{2}$ C. $\frac{v}{4}, \frac{\boldsymbol{KE}}{2}$ D. $\frac{v}{2}, \frac{\boldsymbol{KE}}{\sqrt{2}}$

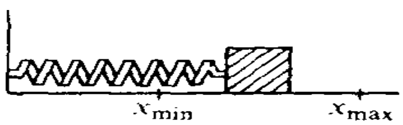
9. A football is kicked off the ground a distance of 50 yards downfield. Neglecting air resistance, which of the following statements would be **INCORRECT** when the football reaches the highest point?
   A. all of the balls original kinetic energy has been changed into potential energy
   B. the ball's horizontal velocity is the same as when it left the kickers foot
   C. the ball will have been in the air one-half of its total flight time
   D. the vertical component of the velocity is equal to zero

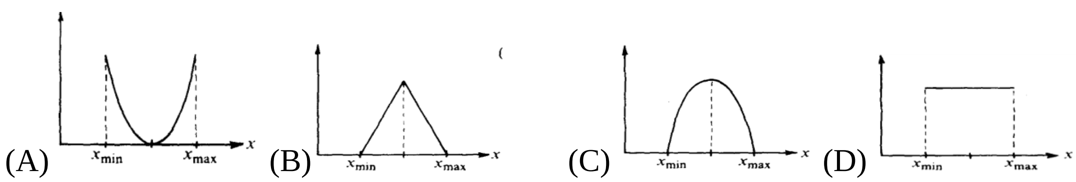
10. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
   A. the distance between the rocks increases while both are falling.
   B. the acceleration is greater for the more massive rock.
   C. they strike the ground more than half a second apart.
   D. they strike the ground with the same kinetic energy.

11. From the top of a high cliff, a ball is thrown horizontally with initial speed v_o. Which of the following graphs best represents the ball's kinetic energy K as a function of time t ?



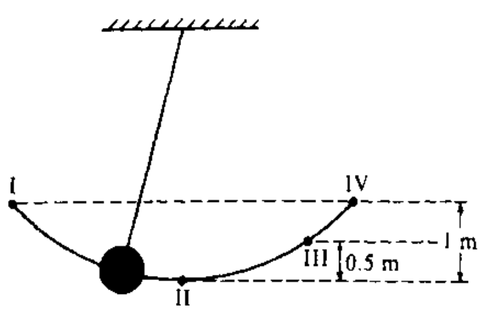
12. A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.





Which graph can represent the total mechanical energy of the block-spring system as a function of x ? (A) A (B) B (C) C (D) D

13. A ball swings freely back and forth in an arc from point I to point IV, as shown. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is I meter above II. Air resistance is negligible. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?

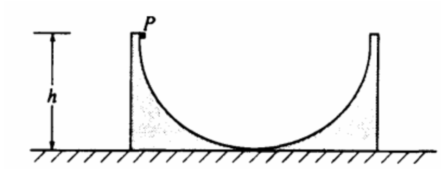


A. At point II
B. At some point between II and III
C. At point III
D. At some point between III and IV

The speed of the ball at point II is from question number 13 is most nearly

A. 3.0 m/s B. 4.5 m/s C. 9.8 m/s D. 14 m/s

14. The figure shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?



(A) It is equal to h/4 (B) It is equal to h/2 (C) It is equal to h

(D) It is between zero and h; the exact height depends on how much energy is lost to friction.

15. A child pushes horizontally on a box of mass *m* which moves with constant speed *v* across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

(A) μmgv (B) mgv (C) $\mu mg/v$ (D) $\mu mg/v$

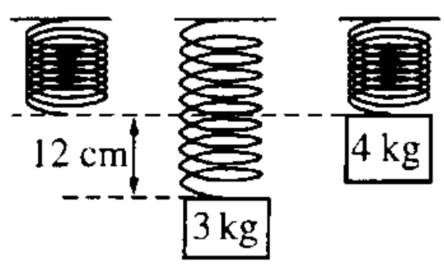
16. What is the kinetic energy of a satellite of mass m that orbits the Earth, of mass M, in a circular orbit of radius R?

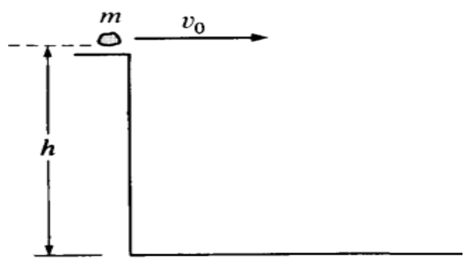
(A) $\frac{1}{2}\frac{GMm}{R}$ (B) $\frac{1}{4}\frac{GMm}{R}$ (C) $\frac{1}{2}\frac{GMm}{R^2}$ (D) $\frac{GMm}{R^2}$

17. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown, at which the spring is un stretched. How far will the 4.0 kg block fall before its direction is reversed?

(A) 18 cm (B) 16cm

(C) 32 cm (D) 48 cm





18. A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0. What is the kinetic energy of the rock just before it hits the ground?

(A) mgh (B) $\frac{1}{2} mv_0^2$ (C) $\frac{1}{2} mv_0^2 - mgh$ (D) $\frac{1}{2} mv_0^2 + mgh$

Unit-6

Rotational Motion

Learning competencies (objectives)

By the end of this unit you should be able to:

- ✓ Describe the motion of a rigid body about a pivot point.
- ✓ Derive equations of motion with constant angular acceleration.
- ✓ Solve problems involving angular quantities and apply the cross product definition of torque to solve problems.
- ✓ Apply the concepts of rotation dynamics and kinetic energy to solve problems.
- ✓ Identify factors affecting the moment of inertia of a body, state the parallel axis theorem and use it to solve problems involving the moment of inertia.
- ✓ Express angular momentum as a cross product of r and p.
- ✓ State the law of conservation of angular momentum.
- ✓ Apply the relationship between torque and angular momentum and the law of conservation of angular momentum in understanding various natural phenomena, and solving problems.
- ✓ Determine the location of position, velocity and acceleration of center of mass of point masses and location of a uniform rigid body.

Introduction

Rotational motion is motion along a curved path. Motion of wheels, gears, motors, planets, the hands of a clock, the rotor of jet engines and the blades of helicopters are some examples of rotational motion.

Rigid body: A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

System: A collection of any number of particles interacting with one another and are under consideration during analysis of a situation are said to form a system.

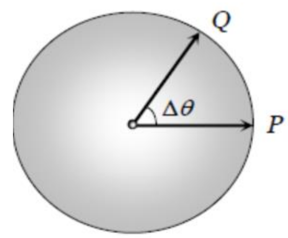
Internal forces: All the forces exerted by various particles of the system on one another are called internal forces. These forces are alone enable the particles to form a well-defined system. Internal forces between two particles are mutual (equal and opposite).

External forces: To move or stop an object of finite size, we have to apply a force on the object from outside. This force exerted on a given system is called an external force.

Angular Displacement

It is the angle described by the position vector *r* about the axis of rotation.

Angular displacement (θ)= $\frac{Linear\ displacement\ (s)}{Radius\ (r)}$



Unit : radian

Dimension : $[M^0L^0T^0]$

It is dimensionless quantity

Vector form: $\vec{s} = \vec{\theta} \times \vec{s}$

i.e., angular displacement is a vector quantity whose direction is given by right hand rule. It is also known as axial vector. For anti-clockwise sense of rotation direction of $\vec{s}$ is perpendicular to the plane, outward and along the axis of rotation and vice-versa.

$$2\pi radian = 360° = 1 \text{ revolution.}$$

If a body rotates about a fixed axis then all the particles will have same angular displacement (although linear displacement will differ from particle to particle in accordance with the distance of particles from the axis of rotation).

Angular Velocity ($\vec{\omega}$):

The angular displacement per unit time is defined as angular velocity.

If a particle moves from *P* to *Q* in time Δt, $\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$ where $\Delta\vec{\theta}$ is the angular displacement.

Instantaneous angular velocity $\vec{\omega} = \lim_{\Delta t\to 0} \frac{\Delta\vec{\theta}}{\Delta t}=\frac{d\vec{\theta}}{dt}$

Average angular velocity $\vec{\omega}_{av}=\frac{total\ angular\ displacement}{total\ time}=\frac{\theta_2-\theta_1}{t_2-t_1}$

Unit : *Radian/sec*

Dimension : $[M^0L^0T^{-1}]$ which is same as that of frequency.

Linear velocity in terms of angular velocity: $\vec{v} = \vec{\omega} \times \vec{r}$ (vector form)

Worked Example: If the position vector of a particle is $\vec{r}$=(3*i* + 4*j*) meter and its angular velocity is $\vec{\omega} =$ (*j* +2*k*) in *rad*/*sec* then its linear velocity is (in *m*/*s*)

Solution:

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 2 & 4 & 0 \end{vmatrix} = -8i \quad +4j \quad -2k$$

Activity:

1. A wheel completes 2000 rotations in covering a distance of 9.5 *km* . Find diameter of the wheel.
2. A wheel is at rest. Its angular velocity increases uniformly and becomes 60 rad/sec after 5 sec. Find total angular displacement of the wheel.
3. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds, (i) What is its angular acceleration (assume the acceleration to be uniform) (ii) How many revolutions does the wheel make during this time ?

Angular Acceleration

The rate of change of angular velocity is defined as angular acceleration.

If particle has angular velocity $\vec{\boldsymbol{\omega}}_{\mathbf{1}}$ at time $\boldsymbol{t_1}$ and angular velocity $\vec{\boldsymbol{\omega}}_{\mathbf{2}}$at time $\boldsymbol{t_2}$ then,

Angular acceleration: $\vec{\boldsymbol{\alpha}} = \frac{\vec{\boldsymbol{\omega}}_2 - \vec{\boldsymbol{\omega}}_1}{\boldsymbol{t_2} - \boldsymbol{t_1}}$

Instantaneous angular acceleration $\vec{\boldsymbol{\alpha}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{\omega}}}{\Delta t} = \frac{d\vec{\boldsymbol{\omega}}}{dt} = \frac{d^2\vec{\boldsymbol{\theta}}}{dt^2}$

Unit : *rad*/*sec*^{2}

Dimension : $[\mathrm{M^0L^0T^{-2}}]$.

If $\vec{\boldsymbol{\alpha}} = \mathbf{0}$, circular or rotational motion is said to be uniform.

Average Angular acceleration: $\vec{\boldsymbol{\alpha}}_{\boldsymbol{av}} = \frac{\vec{\boldsymbol{\omega}}_2 - \vec{\boldsymbol{\omega}}_1}{\boldsymbol{t_2} - \boldsymbol{t_1}}$

Relation between angular acceleration and linear acceleration: $\vec{a} = \vec{\boldsymbol{\alpha}} \times \vec{r}$ (vector form)

It is an axial vector whose direction is along the change in direction of angular velocity *i.e*. normal to the rotational plane, outward or inward along the axis of rotation (depends upon the sense of rotation).

Worked Examples:

1. The wheel of a car is rotating at the rate of 1200 revolutions per minute. On pressing the accelerator for 10 sec it starts rotating at 4500 revolutions per minute. Find the angular acceleration of the wheel.

Given:

$\boldsymbol{\omega_o}$ =1400rev/min

ω =5,400rev/min

$\boldsymbol{t}$**=10sec**

Req[d]

$\boldsymbol{\alpha}$ =?

Solution:

$$\boldsymbol{\alpha = \frac{\omega - \omega_o}{t}}$$

1rev/min= $\frac{\boldsymbol{2\pi rad}}{\boldsymbol{60sec}}$

$\boldsymbol{\omega_o}$= 1400rev/min

=1,400*$\frac{\boldsymbol{2\pi rad}}{\boldsymbol{60sec}}$=**146.53rad/se**

$\boldsymbol{\omega}$= 5400rev/min

=5,400*$\frac{\boldsymbol{2\pi rad}}{\boldsymbol{60sec}}$=**565.2rad/se**

$\alpha = \frac{\mathbf{(565.2 - 146.53)rad/sec}}{\mathbf{10}\boldsymbol{sec}}$

$\boldsymbol{\alpha = 41.87 \; {rad}/{sec^2}}$

2. A wheel is at rest. Its angular velocity increases uniformly and becomes 60 *rad*/*sec* after 5*sec*. The total angular displacement is

Solution:

Angular acceleration=$\alpha = \frac{\omega - \omega_o}{t} = \frac{60rad/sec - 0rad/sec}{5sec.}$=$\mathbf{12}\boldsymbol{{rad}/{sec^2}}$

Angular displacement=$\theta = 0 + \frac{1}{2}(12\, {rad}/{sec^2})(5sec)^2$=**150rad**

Equations of Linear Motion and Rotational Motion

Linear Motion	Rotational
If linear acceleration is 0, *u* = constant and *s*=*ut*.	If angular acceleration is $\boldsymbol{zero}$, $\boldsymbol{\omega}$ = constant and $\boldsymbol{\theta = \omega t}$
If linear acceleration *a* = constant, $\boldsymbol{s} = (\frac{\boldsymbol{u+v}}{\mathbf{2}})t$ $a = \frac{\boldsymbol{v-u}}{\boldsymbol{t}}$ $\boldsymbol{v = u + at}$ $\boldsymbol{s = ut + \frac{1}{2}at^2}$ $\boldsymbol{v^2 = u^2 + 2as}$ $\boldsymbol{s_{nth} = u + \frac{1}{2}a(2n-1)}$	If angular acceleration $\boldsymbol{\alpha}$ = constant then $\boldsymbol{\theta} = (\frac{\boldsymbol{u+v}}{\mathbf{2}})t$ $\boldsymbol{\alpha = \frac{\omega - \omega_o}{t}}$ $\boldsymbol{\omega = \omega_o + \alpha t}$ $\boldsymbol{\theta = \omega_o t + \frac{1}{2}\alpha t^2}$ $\boldsymbol{\omega^2 = {\omega_o}^2 + 2\,\alpha\theta}$ $\boldsymbol{\theta_{nth} = \omega_o + \frac{1}{2}\alpha(2n-1)}$
If acceleration is not constant, the above equation will not be applicable. In this case	If acceleration is not constant, the above equation will not be applicable. In this case
$\vec{\boldsymbol{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$ $\vec{\boldsymbol{a}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{s}}{dt^2}$	$\vec{\boldsymbol{\omega}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{\theta}}}{\Delta t} = \frac{d\vec{\boldsymbol{\theta}}}{dt}$ $\vec{\boldsymbol{\alpha}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{\omega}}}{\Delta t} = \frac{d\vec{\boldsymbol{\omega}}}{dt} = \frac{d^2\vec{\boldsymbol{\theta}}}{dt^2}$

Activity:

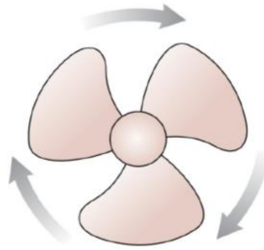
The fan blade is speeding up. What are the signs of $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{\alpha}}$?

A. $\vec{\boldsymbol{\omega}}$ is positive and $\vec{\boldsymbol{\alpha}}$ is positive.

B. $\vec{\boldsymbol{\omega}}$ is positive and $\vec{\boldsymbol{\alpha}}$ is negative.

C. $\vec{\boldsymbol{\omega}}$ is negative and $\vec{\boldsymbol{\alpha}}$ is positive.

D. $\vec{\boldsymbol{\omega}}$ is negative and $\vec{\boldsymbol{\alpha}}$ is negative

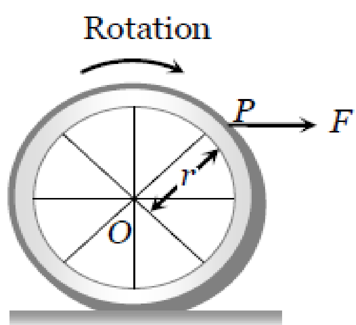


Rotation of a rigid body about a fixed axis

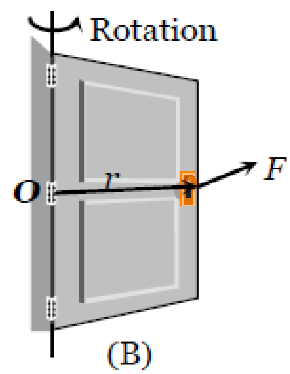
Axis of rotation *the axis about which a body rotates*

Torque and angular acceleration

Torque is the turning effect of force round a point.

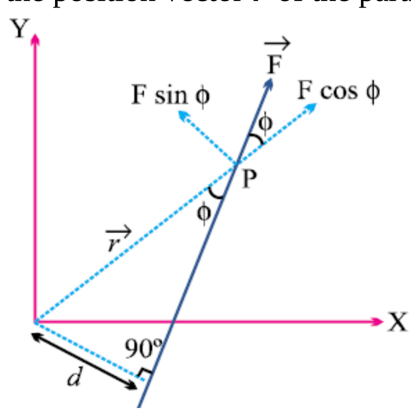


(A)



(B)

If the particle rotating in *xy* plane about the origin under the effect of force $\vec{\boldsymbol{F}}$ and at any instant the position vector $\vec{\boldsymbol{r}}$ of the particle is then,



Torque $\vec{\tau} = \vec{\boldsymbol{r}} \times \vec{F}$

$\vec{\tau} = |\vec{\boldsymbol{r}}||\vec{F}|sin\theta\hat{u}$

Where θ is the angle between the direction of $\vec{\boldsymbol{r}}$ *and* $\vec{F}$

Torque is an axial vector *i.e.*, its direction is always perpendicular to the plane containing vector $\vec{\boldsymbol{r}}$ *and* $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

Torque is also called as moment of force and d is called moment or lever arm.

Unit of torque: Newton. meter (M.K.S.) and Dyne.cm (C.G.S.)

Dimension : $[ML^2T^{-2}]$

A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.*, $\sum \vec{\tau} = 0$

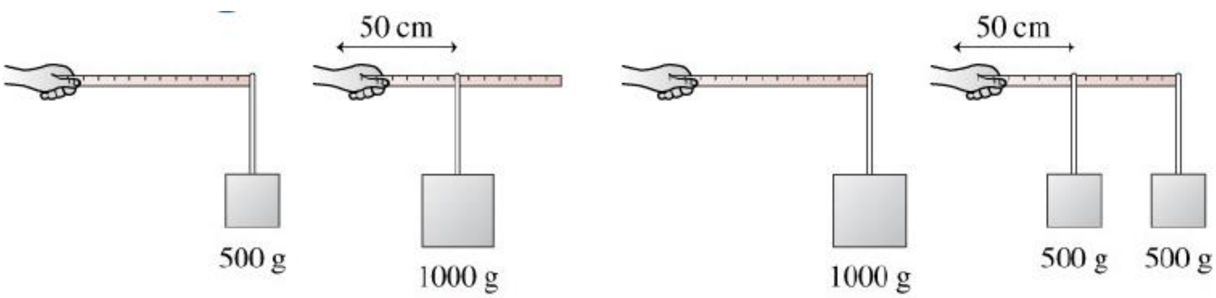
Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translational motion *i.e.*, torque is rotational analogue of force.

Worked Example: Find the torque of 7i-3j-5k about the origin which acts on a particle whose position vector is i+j-k.

Solution: $\vec{\tau} = \vec{\boldsymbol{r}} \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 7 & -3 & -5 \end{vmatrix} = -8i \quad -2j \quad -10k$

Activity:

A student holds a meter stick straight out with one or more masses dangling from it. Rank in order, from most difficult to least difficult, how hard it will be for the student to keep the meter stick from rotating.



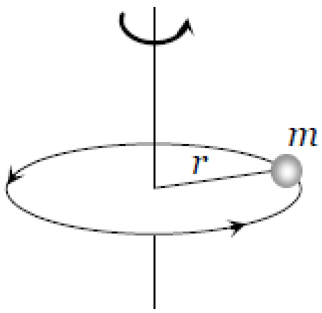
(a) (b) (c) (d)

A. c > b > d > a B. b = c = d > a C. c > d > b > a D. **c> d > a = b**

Rotational kinetic energy and rotational inertia

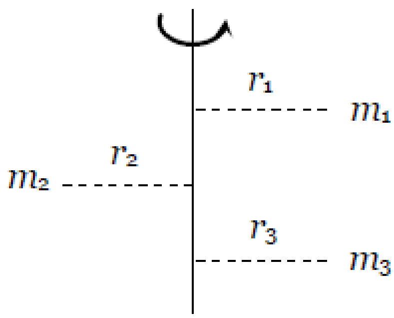
Rotational inertia *a measure of an object's resistance to changes in its speed of rotation over a certain time. Also known as moment of inertia.* It plays the same role in rotational motion as mass plays in linear motion.

1. Moment of inertia of a point mass at r distance from axis of rotation.



$$\mathbf{I = mr^2}$$

2. Moment of inertia of a body made up of number of particles (discrete distribution) is given by $\mathrm{I = m_1 r_1{}^2.+ m_2 r_2{}^2 + \cdots + m_n r_n{}^2} = \sum_{i=1}^{n} \mathrm{m}_n \mathrm{r}_n{}^2$



$$\mathbf{I = m_1 r_1{}^2.+ m_2 r_2{}^2 + m_3 r_3{}^2}$$

Dimension of I: $[ML^2T^0]$
S.I. unit of I: kgm^2.
Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.
Moment of inertia does not depend on angular velocity, angular acceleration, torque, angular momentum and rotational kinetic energy.

Worked Example:

Five particles of mass = $4kg$ are attached to the rim of a circular disc of radius $0.2m$ and negligible mass. Find the moment of inertia of the system about the axis passing through the center of the disc and perpendicular to its plane.

Given:

$m_1 = m_2 = m_3 = m_4 =$

$m_5 = 4kg$

$\mathbf{r_1 = r_2 = r_3 = r_4 = r_5 = \mathit{r}}$

$\mathbf{= 0.2\mathit{m}}$

Required:

I=?

solution:

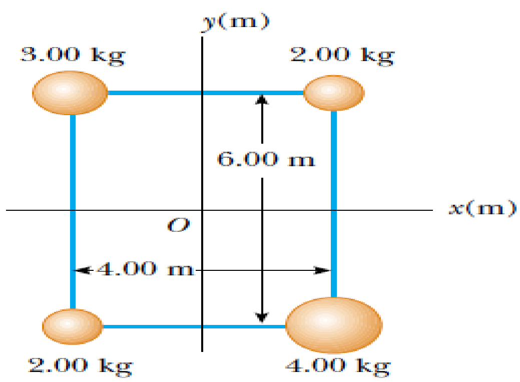
$\mathbf{I = m_1 r_1{}^2 . + m_2 r_2{}^2 + m_3 r_3{}^2 + m_4 r_4{}^2 + m_5 r_5{}^2}$

$\mathbf{= 5mr^2 = 5(4kg)(0.2)^2}$

$\mathbf{I = 0.8kgm^2}$

Activity:

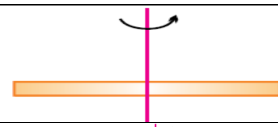
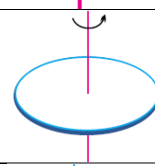
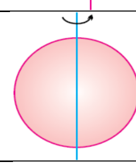
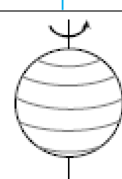
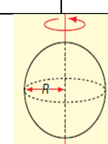
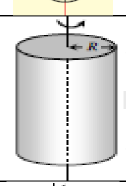
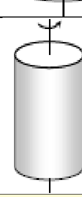
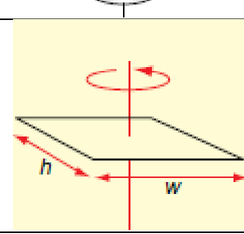
1. Moment of inertia is:
   A. the rotational equivalent of mass
   B. the point at which all forces appear to act.
   C. the time at which inertia occurs.
   D. an alternative term for moment arm.
2. Section of hollow pipe and a solid cylinder have the same radius, mass, and length. They both rotate about their long central axes with the same angular speed. Which object has the higher moment of inertia? (a) the hollow pipe (b) the solid cylinder (c) they have the same rotational kinetic energy (d) impossible to determine.
3. The four discrete particles in Figure shown below are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. If the system rotates in the xy plane about the z axis with an angular speed of 6.00 rad/s, calculate the moment of inertia of the system about the z axis



Moments of inertia of different rigid bodies:

Different rigid bodies have different moment of inertia depending on size mass, distribution of mass, shape of the body and on the position of axis of rotation.

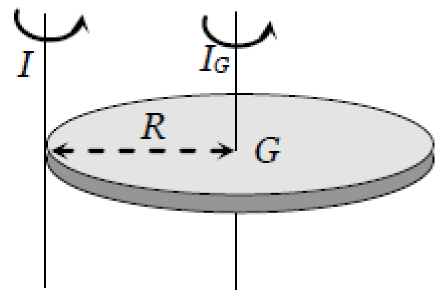
Moment of Inertia of Some Standard Bodies About Different Axes.

Body	Axis of Rotation	Figure	Moment of inertia
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$I = \frac{ML^2}{3}$
Long thin rod	About on axis passing through its centEr of mass and perpendicular to the rod.		$I = \frac{ML^2}{12}$
Disc	About an axis passing through C.G. and perpendicular to its plane		$I = \frac{Mr^2}{2}$
Disc	About its Diameter		$I = \frac{Mr^2}{4}$
solid sphere	About its Diameter		$I = \frac{2}{5}Mr^2$
Hollow sphere	About its Diameter		$I = \frac{2}{3}MR^2$
Solid cylinder	About its own axis		$I = \frac{1}{2}MR^2$
Thin cylindrical shell	About its own axis		$I = Mr^2$
Rectangular slab			$I = \frac{1}{12}M(h^2 + w^2)$

Parallel axis theorem

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through center of mass of the body $\mathbf{I}_g$ and $\mathbf{MR^2}$ where **M** is the mass of the body and **R** is the perpendicular distance between the two axes..

$$\boldsymbol{I} = \mathbf{I}_g + \mathbf{MR^2}$$



Worked Examples*:*

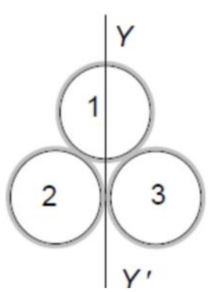
1. Moment of inertia of a solid sphere about an axis through its center is $\frac{2}{5}\mathbf{MR^2}$, Find moment of inertia of solid sphere about it tangential axis.

Solution:

$$\boldsymbol{I} = \mathbf{I}_g + \mathbf{MR^2}$$

$$\boldsymbol{I} = \frac{2}{5}\mathbf{MR^2} + \mathbf{MR^2} = \frac{7}{5}\mathbf{MR^2}$$

2. Three discs each of mass M and radius R are arranged as shown in the figure. Find the moment of inertia of the system about YY'. $\mathbf{I} = \frac{1}{4}\boldsymbol{MR^2}$



Solution:

$$\mathbf{I} = \mathrm{I}_1 + \mathrm{I}_2 . + \mathrm{I}_3$$

I_1 is about its diameter and, I_2 and I_3 are about tangential axis

$$I = \mathrm{I}_1 + \mathrm{I}_2 . + \mathrm{I}_3$$

$$\mathrm{I}_1 = \frac{1}{4}\boldsymbol{MR^2} \text{ and}$$

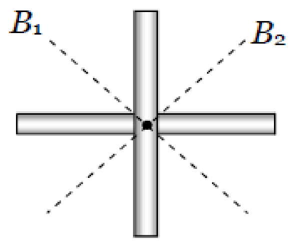
$$\mathrm{I}_1 + \mathrm{I}_2 . + \mathrm{I}_3$$

$$= \frac{1}{4}\boldsymbol{MR^2} + 2\left(\frac{1}{4}\mathrm{MR}^2 + \mathrm{MR}^2\right)$$

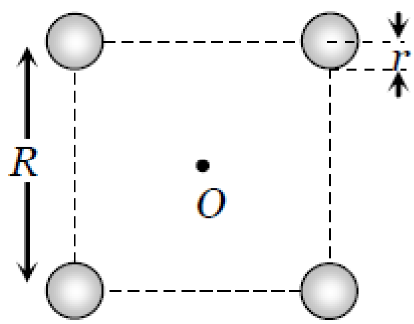
$$\mathbf{I} = \frac{11}{4}\mathbf{MR^2}$$

Activity:

1. Two identical rods each of mass ***M***. and length ***l*** are joined in crossed position as shown in figure bellow. Find the moment of inertia of this system about a bisector



2. Four solid spheres, each of mass **M** and radius **r** are situated at the four corners of square of side **R** as shown below. Determine the moment of inertia of the system about an axis perpendicular to the plane of square and passing through its center



Rotational Work Done

Rotational work done by torque is the scalar product of moment of force and angular displacement. It is given by:

$$W = \tau\theta \text{ (in Joule)}$$

Torque in terms moment of inertia and angular acceleration

Torque($\boldsymbol{\tau}$) is the vector product of lever arm and the force applied. It is given by:

$$\vec{\tau} = \vec{\boldsymbol{r}} \times \vec{F}$$

$$\boldsymbol{\tau = rF\sin\theta,\ \mathit{if}\ \theta = 90^\circ\ \sin\theta = 1,\ then}$$

$$\boldsymbol{\tau = rF = r(m a_t) = rm\alpha r = (mr^2)\alpha = I\alpha},$$

where **τ-torque, I-moment of inertia and α-angular acceleration**

Rotational kinetic energy

Rotational kinetic energy *the amount of kinetic energy a rigid body has from its rotational movement*. A body rotating about a fixed axis possesses kinetic energy because its constituent particles are in motion, even though the body as a whole remains in place. It is given by:

$$KE_{rotational}=\frac{1}{2}I\omega^2 \text{ (in Joule)}$$

where KE- rotational kinetic energy,

I-moment of inertia and ω-angular speed

Rotational kinetic energy	**Analogue to translational kinetic energy**
$KE_r=\frac{1}{2}I\omega^2$	$KE_r=\frac{1}{2}mv^2$
$KE_r=\frac{1}{2}L\ \omega$	$KE_r=\frac{1}{2}\mathbf{p}v$
$KE_r=\frac{L^2}{2I}$	$KE_r=\frac{P^2}{2m}$

Power : Rate of change of kinetic energy is defined as power

$$P=\frac{d}{d}(KE_{rot.}) = \boldsymbol{\tau}\,\omega$$

In vector form Power= $\vec{\boldsymbol{\tau}}\,.\,\vec{\omega}$ (Analogue to power in translational motion $\boldsymbol{P} = \vec{\mathbf{F}}\,.\,\vec{\boldsymbol{v}}$)

Worked Examples*:*

1. An automobile engine develops 200kW when rotating at a speed of 3,600 rev/min. What torque does it deliver?

Given:

$P = 200KW = 2 \times \mathbf{10^5 W}$

$\boldsymbol{\omega = 3,600 rev/min}$

$\boldsymbol{\omega = 3.6 \times 10^3 \times 2\pi rad/60sec}$

$\boldsymbol{\omega = 3.6 \times 10^3 \times 2\pi rad/60sec}$

ω = 376.8rad/sec

Required:

τ =?

solution:

$\mathbf{P = \tau\omega}$

$$\boldsymbol{\tau} = \frac{\boldsymbol{P}}{\boldsymbol{\omega}} = \frac{2 \times \mathbf{10^5 W}}{\mathbf{376.8 rad/sec}}$$

τ =530.79N.m.

2. The angular velocity of a body is $\vec{\omega} = (\hat{\imath} + 10\hat{\jmath} + 4\hat{k})$rad/sec and a torque $\vec{\boldsymbol{\tau}} = (\hat{\imath} - 4\hat{\jmath} + 4\hat{k})$Nm acts on it. Find the rotational power delivered.

Given:

$\vec{\omega} = (\hat{\imath} + 10\hat{\jmath} + 4\hat{k})$rad/sec

$\vec{\boldsymbol{\tau}} = (\hat{\imath} - 4\hat{\jmath} + 4\hat{k})$Nm

Required:

P =?

solution:

$\mathbf{P} = \vec{\boldsymbol{\tau}}.\vec{\omega}$

$\mathbf{P} = (\hat{\imath} - 4\hat{\jmath} + 4\hat{k}).\ (\hat{\imath} + 10\hat{\jmath} + 4\hat{k})$

$\mathbf{P} = (1)(1) + (-4)(10) + (4)(4)$

$\mathbf{P = 55\boldsymbol{Watt}}$

Activity:

To maintain a rotor at a uniform angular speed of 200 rad/s, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine? Assume that the engine is 100% efficient.

Rolling Without Slipping

In case of combined translational and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity $\boldsymbol{\omega}$.

By the law of conservation of energy

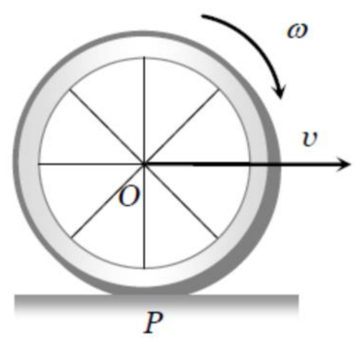
$$\mathrm{KE}_{Total} = \mathrm{KE}_{rot} + \mathrm{KE}_{tran}$$

$$\mathrm{KE}_{Total} = \frac{1}{2}\mathrm{m}v^2 + \frac{1}{2}\mathrm{I}\omega^2$$

$$\boldsymbol{v} = \omega \boldsymbol{r}$$

$$\mathrm{KE}_{Total} = \frac{1}{2}\mathrm{m}\omega^2 r^2 + \frac{1}{2}\mathrm{I}\omega^2$$

$$\mathrm{KE}_{Total} = \boldsymbol{\frac{1}{2}\omega^2(mr^2 + \mathrm{I})}$$



Rolling Down on Smooth Inclined Plane without Slipping

When a body of mass m and radius r rolling down on inclined plane without slipping when released from ‘h’ and angle of inclination with the horizontal is $\boldsymbol{\theta}$**,** it loses its potential energy. However it acquires both linear and angular speeds and hence, gains kinetic energy of translational and that of rotation.

Worked Example:

A ball of mass m and radius r is released from ‘h’ on a smooth inclined plane of inclination θ as shown below. Find it’s a) linear speed in terms of ‘g’ and ‘h’ b) its angular speed in terms of ‘g’, ‘h’ and radius ‘r’, and c) linear acceleration in terms of ‘g’ and ‘$\boldsymbol{\theta}$ ‘.

Solution:

If friction is zero, from the la of conservation of mechanical energy:

$$\mathrm{ME_{top}} = \mathrm{ME_{bottom}}$$

$$\mathrm{GPE_{top}} = \mathrm{KE_{Rtranslational\ at\ bottom}} + \mathrm{KE_{Rotational\ at\ bottom}}$$

$$\mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}\mathrm{I_{ball}}\omega^2,$$

$$\mathrm{I_{ball}} = \mathrm{I_{hollow\ sphere}} = \frac{2}{3}mr^2 \text{ and } \boldsymbol{v} = \omega r$$

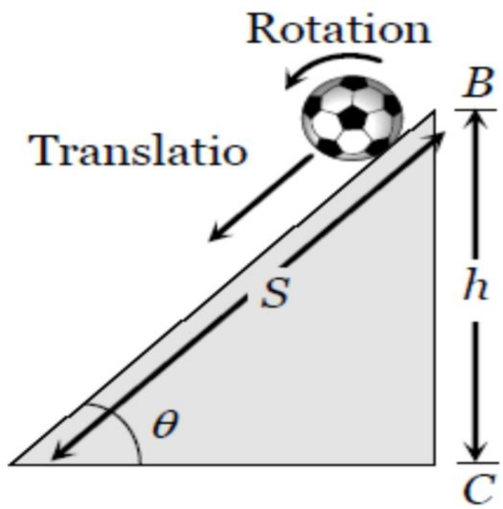
$$\Rightarrow \mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{3}mr^2)\omega^2$$

$$\Rightarrow \mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{3}m(\omega r)^2$$

$$\Rightarrow \mathrm{mgh} = \left(\frac{1}{2} + \frac{1}{3}\right)mv^2 = \frac{5}{6}mv^2$$

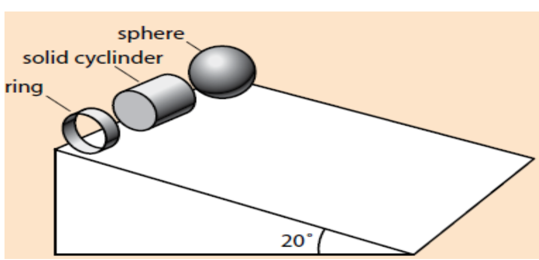
$$\mathrm{gh} = \frac{5}{6}v^2$$

$$\boldsymbol{v} = \sqrt{\frac{6}{5}\boldsymbol{gh}}, \quad \& \quad \boldsymbol{\omega} = \frac{1}{r}\sqrt{\frac{6}{5}\boldsymbol{gh}}$$



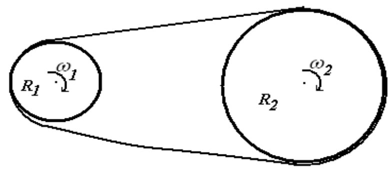
Activities:

1. A solid cylinder rolls down an inclined plane. Its mass is 3kg and radius 0.2 m. If the height of the include plane is 8m, what is rotational K.E. when it reaches the foot of the plane?
2. Consider a ring, sphere and solid cylinder all with the same mass. They are all held at the top of an inclined plane which is at 20° to the horizontal. The top of the inclined plane is 1 m high. The shapes are released simultaneously and allowed to roll down the inclined plane. Assume the objects roll without slipping and that they are all made from the same material. Assume the coefficient of static friction between the objects and plane to be 0.3.



3. A body of moment of inertia of $9kgm^2$ rotating with an angular velocity of $10 rad/sec$ has the same kinetic energy as a mass of $48kg$, find the magnitude of the velocity 48kg.
4. A hollow sphere and a thin cylinder of same mass are rolling and if their kinetic energies are equal, then find the ratio of their linear velocities.

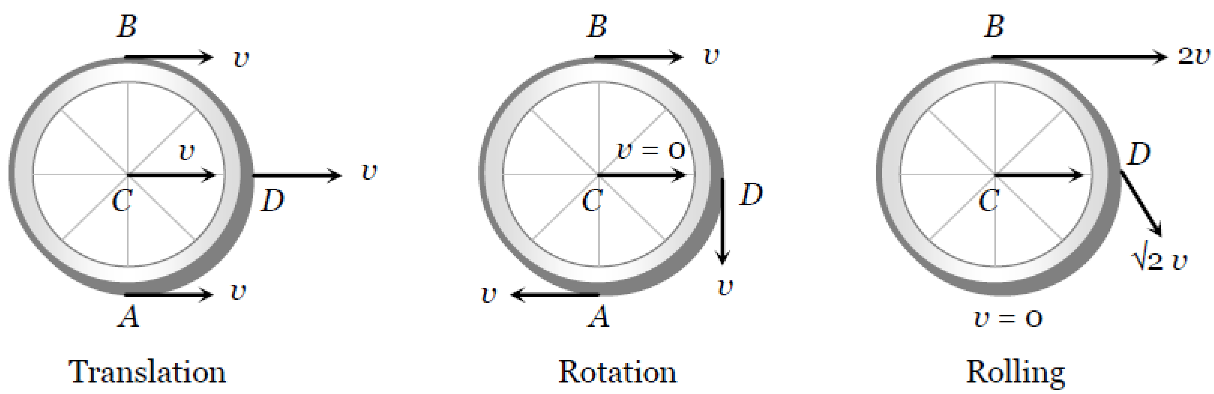
5. A solid sphere rolls down an inclined plane and its velocity at the bottom is v_1. Then same sphere slides down the plane (without friction) and let its velocity at the bottom be v_2. Find the ratio of $v_1 : v_2$.



6. The discs shown above are connected by a belt that rotates both discs at the same time. The belt does not slip as it rotates the discs. The discs have different radii as shown, with $R_1 < R_2$. What is the relation between the angular velocities of the discs?

A. $\omega_1 = \omega_2$ B. $R_1\omega_1 = \omega_2 R_2$ C. $\frac{\omega_1}{R_1} = \frac{\omega_2}{R_2}$ D. $\frac{R_1}{\omega_1} = \frac{R_2}{\omega_2}$

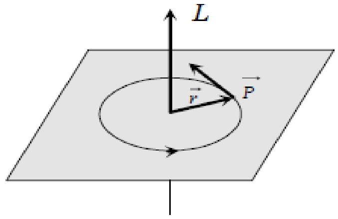
Linear velocity of different points in rolling : In case of rolling, all points of a rigid body have same angular speed but different linear speed. Let ***A, B, C*** and ***D*** are four points then their velocities are shown in the following figure.



Angular momentum and angular impulse

Angular momentum

The turning momentum of particle about the axis of rotation is called the angular momentum of the particle or the moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{\mathbf{P}}$ is the linear momentum of particle and $\vec{\mathbf{r}}$ its position vector from the point of rotation then angular momentum.

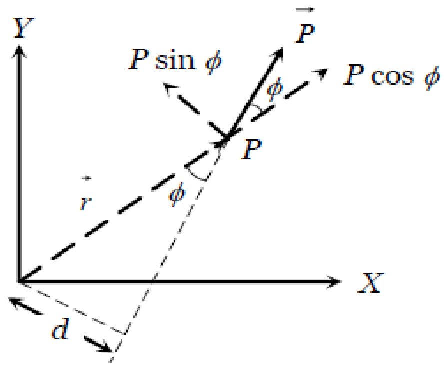


$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}}$

$\vec{\mathbf{L}} = rpsin\theta\hat{\boldsymbol{n}}$

Angular momentum is an axial vector *i.e.* always directed perpendicular to the plane of rotation and along the axis of rotation.

S.I. Unit of $\vec{\mathbf{L}}$: kgm^2s^{-1} *or* J.sec.



In case of circular motion angular momentum is given by:

$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = m(\vec{\mathbf{r}} \times \vec{\boldsymbol{v}}) = mvrsin\emptyset\hat{\boldsymbol{n}},$

if $\emptyset$ between is 90°, then

$L = mvr = m(\omega r)r = I\omega$

In vector form $\vec{\mathbf{L}} = I\vec{\omega}$, $I = mr^2$

Dimension : [ML^2T^{-1}]

In Cartesian co-ordinates if $\vec{\mathbf{r}} = xi + y$

$$\text{Then } \vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \\ \mathrm{x} & \mathrm{y} & \mathrm{y} \\ \mathrm{P_x} & \mathrm{P_y} & \mathrm{P_z} \end{vmatrix} = (y\mathbf{P}_z - \mathbf{z}\mathbf{P}_y)\hat{\imath} - (\mathbf{x}\mathbf{P}_z - z\mathbf{P}_x)\hat{\jmath} + (x\mathbf{P}_y - y\mathbf{P}_x)\hat{k}$$

Worked Example: The position of a particle is given by : $\vec{\mathbf{r}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and momentum $\vec{\mathbf{P}} = \mathbf{2}\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ .Find the direction of angular momentum

Solution:

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \mathrm{x} & \mathrm{y} & \mathrm{y} \\ \mathrm{P_x} & \mathrm{P_y} & \mathrm{P_z} \end{vmatrix}$$

Direction of $\vec{\mathbf{L}}$ is its unit vector given by: $\hat{\boldsymbol{u}} = \frac{\vec{\mathbf{L}}}{|\vec{\mathbf{L}}|} = \frac{\vec{\mathbf{L}}}{|\vec{\mathbf{L}}|}$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix} = \hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\hat{\boldsymbol{u}} = \frac{\vec{\mathbf{L}}}{|\vec{\mathbf{L}}|} = \frac{\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{1^2 + (-3)^2 + 2^2}} = \frac{\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{14}} \text{ (unit vector of } \vec{\mathbf{L}})$$

Angular impulse

Angular impulse *the change in angular momentum of a rotating body caused by a torque acting over a certain time.* It is given by:

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{L}} = \vec{\tau}_{av}\Delta t$$

The angular momentum of a system of particles is equal to the vector sum of angular momentum of each particle *i.e.*, $\vec{\mathbf{L}} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_n$

Analogy Between Translational Motion and Rotational Motion

Translational Motion		Rotational Motion	
Mass (*m*)		Moment of Inertia (*I*)	
Linear momentum	$\vec{\mathbf{P}} = \boldsymbol{m}\vec{\boldsymbol{v}}$ $\mathbf{P} = \sqrt{\mathbf{2}\boldsymbol{mE}}$	Angular Momentum	$L = I\omega$ $L = \sqrt{\mathbf{2}\boldsymbol{IE}}$
Force	**F=ma**	**Torque**	$\boldsymbol{\tau} = \mathrm{I}\boldsymbol{\alpha}$
Kinetic Energy	$\mathbf{KE}_{\boldsymbol{t}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{I}\boldsymbol{v}^{\mathbf{2}}$ $\mathbf{KE}_{\boldsymbol{t}} = \frac{\mathbf{P^2}}{\mathbf{2m}}$	Rotational Kinetic Energy	$\mathrm{KE} = \frac{1}{2}\mathrm{I}\boldsymbol{\omega}^{\mathbf{2}}$ $\mathbf{KE}_{\boldsymbol{R}} = \frac{\mathbf{L^2}}{\mathbf{2I}}$

The Law of Conservation of angular momentum

Newton's second law for rotational motion $\vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}$

So if the net external torque on a particle (or system) is zero then $\frac{d\vec{\mathbf{L}}}{dt}$=0, *i.e.* $\vec{\mathbf{L}} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_n$=*constant*

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As form $\vec{\mathbf{L}} = I\vec{\omega}$ so if $\vec{\boldsymbol{\tau}} = 0$ then $I\omega = constant$ $\mathbf{I} \sim \frac{\mathbf{1}}{\boldsymbol{\omega}}$

Since angular momentum $\mathbf{I}\boldsymbol{\omega}$ remains constant so when **I** decreases, angular velocity $\boldsymbol{\omega}$ increases and vice-versa.

Examples of law of conservation of angular momentum:

1. The angular velocity of revolution of a planet around the sun in an elliptical orbit increases when the planet come closer to the sun and vice-versa because when planet comes closer to the sun, it's moment of inertia *I* decreases there fore $\boldsymbol{\omega}$ increases.
2. A circus acrobat performs feats involving spin by bringing his arms and legs closer to his body or vice-versa. On bringing the arms and legs closer to body, his moment of inertia *I* decreases. Hence $\boldsymbol{\omega}$ increases.
3. A person-carrying heavy weight in his hands and standing on a rotating platform can change the speed of platform. When the person suddenly folds his arms. Its moment of inertia decreases and in accordance the angular speed increases.

Worked Example*:*

1. Two discs of moment of inertia 5kgm^2 and 10kgm^2 and angular speeds 12rad/sec and 4rad/sec are rotating along collinear axes passing through their center of mass and perpendicular to their plane. If the two are made to rotate together along the same axis, then what is rotational *KE* of system after they made to rotate together?

Solution:

$$\mathbf{L}_o = \mathbf{L}_f$$

$$\mathbf{I}_{1i}\boldsymbol{\omega}_{1i} + \mathbf{I}_{2i}\boldsymbol{\omega}_{2i} = \mathbf{I}_{1f}\boldsymbol{\omega}_{1f} + \mathbf{I}_{2f}\boldsymbol{\omega}_{2f}$$

$$\mathbf{I}_{1i}\boldsymbol{\omega}_{1i} + \mathbf{I}_{2i}\boldsymbol{\omega}_{2i} = (\mathbf{I}_{1f} + \mathbf{I}_{2f})\boldsymbol{\omega}_f$$

$$\boldsymbol{\omega}_f = \left(\frac{\boldsymbol{\omega}_{1i}\mathbf{I}_1 + \mathbf{I}_2\boldsymbol{\omega}_{2i}}{\mathbf{I}_1 + \mathbf{I}_2}\right) = \frac{12rad/sec * \mathbf{5kg}\boldsymbol{m}^2 + 4rad/sec * \mathbf{10kg}\boldsymbol{m}^2}{\mathbf{5kg}\boldsymbol{m}^2 + \mathbf{10kg}\boldsymbol{m}^2}$$

$$\boldsymbol{\omega}_f = \mathbf{6.67rad/sec}$$

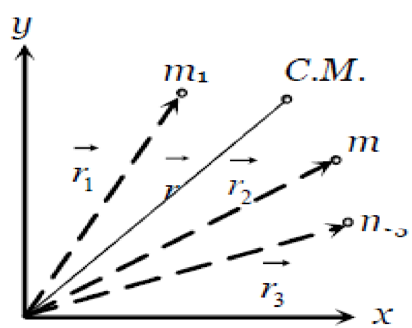
Activity:

1. Two discs of moments of inertia $\mathbf{I_1}$ and $\mathbf{I_2}$ about their respective axes (normal to the disc and passing through the center), and rotating with angular speed $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_2}$ are brought into contact face to face with their axes of rotation coincident, (i) What is the angular speed of the two-disc system ? (ii) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs.

Centre of mass of a rigid body (circular ring, disc, rod and sphere)

Every object has a balance point, referred to in physics as the center of mass. Centre of mass of a system (body) is a point that moves as though all the mass were concentrated there and all external forces were applied there.

(1) **Position vector of center of mass for *n* particle system :** If a system consists of *n* particles of masses $m_1, m_2, m_3, \ldots m_n$ whose positions vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_n$ respectively then position vector of center of mass the center of mass of *n* particles is a weighted average of the position vectors of *n* particles making up the system.



$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3, + \cdots + m_n\vec{r}_n}{m_1 + m_2 + m_{3+} \ldots + m_n}$$

(2) Position vector of center of mass for two particle system :

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

The center of mass lies between the particles on the line joining them.

If two masses are equal *i.e.*, $m_1 = m_2$ then position vector of center of mass $\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}$

(3) Important points about center of mass

i. The position of center of mass is independent of the co-ordinate system chosen.
ii. The position of center of mass depends upon the shape of the body and distribution of mass.

Example : The center of mass of a circular disc is within the material of the body while that of a circular ring is outside the material of the body.

iii. In symmetrical bodies in which the distribution of mass is homogenous, the center of mass coincides with the geometrical center or center of symmetry of the body.

iv. **Position of center of mass for different bodies**

S. No.	Body	Position of center of mass
i.	Uniform hollow sphere	Centre of sphere
ii.	Uniform solid sphere	Centre of sphere
iii.	Uniform rod	Centre of rod
iv.	A plane lamina (Square, Rectangle, Parallelogram)	Point of inter section of diagonals
v.	Rectangular or cubical block	Points of inter section of diagonals
vi.	Solid cylinder	Middle point of the axis of cylinder

The center of mass changes its position only under the translational motion. There is no effect of rotatory motion on center of mass of the body.

v. If the origin is at the center of mass, then the sum of the moments of the masses of the system about the center of mass is zero *i.e.* $\sum m_i\vec{r_i} = 0$

If a system of particles of masses $m_1, m_2, m_3, \ldots m_n$ move with velocities $\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots \vec{v}_n$ then the velocity of center of mass $\vec{v}_{cm} = \frac{\sum m_i\vec{v_i}}{\sum m_i}$

vi. If a system of particles of masses $m_1, m_2, m_3, \ldots m_n$ move with accelerations $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots \vec{a}_n$ then the acceleration of center of mass $\vec{a}_{cm} = \frac{\sum m_i\vec{a_i}}{\sum m_i}$

vii. *If* $\vec{r}$ is a position vector of center of mass of a system then velocity of center of mass $\vec{v}_{cm} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3, + \cdots + m_n\vec{r}_n}{m_1 + m_2 + m_{3+} \ldots + m_n}\right)$

viii. Acceleration of center of mass $\vec{a}_{cm} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2}{dt^2}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3, + \cdots + m_n\vec{r}_n}{m_1 + m_2 + m_{3+} \ldots + m_n}\right)$

ix. Force on a rigid body $\vec{\mathbf{F}} = m\vec{a}_{cm} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}$

x. For an isolated system external force on the body is zero $\vec{\mathbf{F}} = m\vec{a}_{cm} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2} = 0$ $\Rightarrow \vec{v}_{cm}$=constant *i.e.*, center of mass of an isolated system moves with uniform velocity along a straight-line path.

Worked Example:

1. Four point masses 2k, 4kg, 6kg and 8kg are placed at the corners of a square ABCD of 2m long respectively, find the position of center of mass of the system from corner a

Solution:

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3,+\cdots+m_n\vec{r}_n}{m_1+m_2+m_{3+}\ldots+m_n}$$

$$=\frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3,+m_4\vec{r}_4}{m_1+m_2+m_3+m_4}$$

$$=\frac{2kg(0)+4kg(2m)+6kg(2\sqrt{2})+8kg(2m)}{2kg+4kg+6kg+8kg}$$

$$\Rightarrow \vec{r}_{cm}=2.05\text{m}$$

2. The velocities of three particles of masses 20*g*, 30*g* and 50*g* are $2\hat{\imath}$,$10\hat{\jmath}$ and $10\hat{k}$ respectively. The velocity of the center of mass of the three particles is

Solution:

$$\vec{v}_{cm}=\frac{\sum m_i\vec{v_i}}{\sum m_i}=\frac{m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3}{m_1+m_2+m_3}=\frac{20(2\hat{\imath})+30(10\hat{\jmath})+50(10\hat{k})}{20+30+50}$$

$$\vec{v}_{cm}=\frac{40\hat{\imath}+300\hat{\jmath}+500\hat{k}}{100}$$

$$\vec{v}_{cm}=\mathbf{0.4\hat{\imath}+3\hat{\jmath}+5\hat{k}}$$

Activity:

The coordinates of the positions of particles of mass 7, 4 and 10*gm* are (1, 5, 3), (2,5,7) and (3,3,1)*cm* respectively. Find the position of the center of mass of the system

Self-Practice Questions and Problems

1. A child has a toy tied to the end of a string and whirls the toy at constant speed in a horizontal circular path of radius R. The toy completes each revolution of its motion in a time period T. What is the magnitude of the acceleration of the toy?

   A. $\frac{\pi R}{T^2}$ B. $\frac{4\pi^2 R}{T^2}$ C. g D. Zero

2. A centripetal force of 5N is applied to a rubber stopper moving at a constant speed in a horizontal circle. If the same force is applied, but the radius is made smaller, what happens to the speed, *v*, and the frequency, *f*, of the stopper?

   A. *v* increases & *f* increases C. *v* decreases & *f* increases

   B. *v* decreases & *f* decreases D. *v* increases & *f* decreases

3. A door is free to rotate about its hinges. A force **F** applied normal to the door a distance *x* from the hinges produces an angular acceleration of α. What angular acceleration is produced if the same force is applied normal to the door at a distance of 2*x* from the hinges?

   A. $\propto$ B. $2 \propto$ C. $\frac{1}{2} \propto$ D. $\frac{1}{4} \propto$

4. A solid disc has a rotational inertia that is equal to: $\mathbf{I} = MR^2$, where **M** is the disc's mass and **R** is the disc's radius. It is rolling along a horizontal surface without slipping with a linear speed of *v*. How are the translational kinetic energy and the rotational kinetic energy of the disc related?

   A. Rotational kinetic energy is equal to the translational.
   B. Translational kinetic energy is larger than rotational.
   C. Rotational kinetic energy is larger than translational.
   D. The answer depends on the density of the disc.

5. A child whirls a ball at the end of a rope, in a uniform circular motion. Which of the following statements is **NOT** true?

   A. The speed of the ball is constant
   B. The magnitude of the ball's acceleration is constant
   C. The velocity is of the ball is constant
   D. The acceleration of the ball is directed radially inwards towards the center

6. An automobile moves in a circle of radius 110 meters with a constant speed of 33m/s. What is the angular velocity of the car about the center of the circle in radians per second?

   A. 0.3rad/s B. 0.3πrad/s C. 0.6πrad/s D. 3.3πrad/s

7. A point mass of 500g started from rest rotates on a circular path of radius 2m and rotates with a uniform angular acceleration to attain angular velocity of 4rev/sec in 2seconds. What is the moment of force acted on it to rotate with this acceleration?

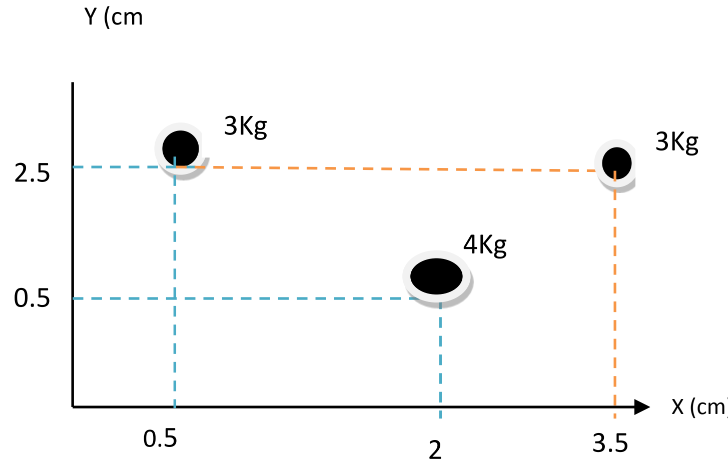
   A. $8\pi N.m$ B. $0.2\pi N.m$ C. $12 N.m$ D. $4.5 N.m$

8. In a uniform circular motion, the centripetal acceleration of a body moving in a circular path results from:
   A. Change in magnitude of tangential velocity.
   B. Change in direction of angular velocity
   C. Change in direction of tangential velocity
   D. Change in magnitude of tangential acceleration .
9. Which one of the following is **incorrect** about centripetal force?
   A) As a car makes a turn, the force of friction acting upon the turned wheels of the car provides centripetal force required for circular motion.
   B) As a bucket of water is tied to a string and spun in a circle, the tension force acting upon the bucket provides the centripetal force required for circular motion.
   C) The centripetal force for uniform circular motion alters the direction of the object by altering its speed.
   D) As the moon orbits the Earth, the force of gravity acting upon the moon provides the centripetal force required for circular motion.
10. A bowling ball of mass M and radius R. whose moment of inertia about its center is $\frac{2}{5}MR^2$, rolls without slipping along a level surface at speed v. The maximum vertical height to which it can roll if it ascends an incline is:

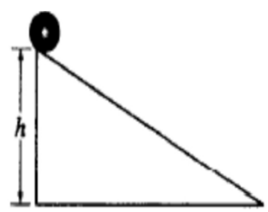
    A. $\frac{7v^2}{10g}$ **B.** $\frac{v^2}{2g}$ C. $\frac{v^2}{5g}$ D. $\frac{5v^2}{7g}$

11. Suppose that there are three point masses arranged as shown in the figure below. Where is the center of mass of this three- object- system with respect to the origin?

A. (2i +17j)cm
B. (2i +1.7j)cm
C. (-2i +1.7j)cm
D. (2i -1.7j)cm



12. A sphere of mass **M**, radius **r**, and rotational inertia I [illegible]eased from re[illegible]ie top o[illegible] inclined plane of height **h** as shown above.

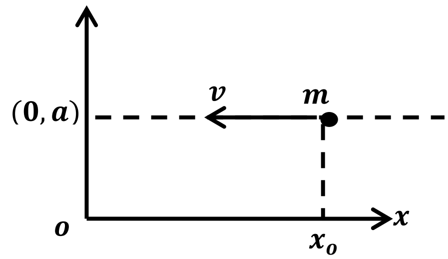


If the plane is frictionless, what is the speed v_{cm}, of the center of mass of the sphere at the bottom of the incline?

A. $\sqrt{\frac{2Mghr^2}{I}}$ B. $\sqrt{2gh}$ C. $\sqrt{\frac{2Mghr^2}{I+Mr^2}}$ D. $\frac{2Mghr^2}{I}$

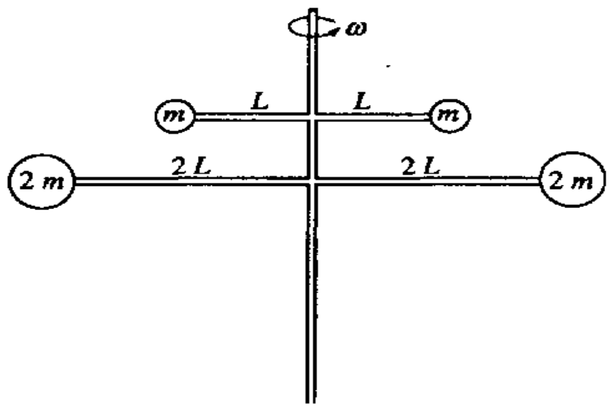
13. An ice skater is spinning about a vertical axis with arms fully extended. If the arms are pulled in closer to the body, in which of the following ways are the angular momentum and kinetic energy of the skater affected?

Angular Momentum	Kinetic Energy
A. Increase	Increase
B. Increase	Remains Constant
C. Remains Constant	Increase
D. Remains Constant	Increase



14. A particle of mass m moves with a constant speed $\boldsymbol{v}$ along the dashed line y = a. When the x-coordinate of the particle is $\boldsymbol{x_o}$, the magnitude of the angular momentum of the particle with respect to the origin of the system is

    A. $\boldsymbol{Zero}$ B. $\boldsymbol{mva}$ C. $\boldsymbol{mvx_o}$ D. $\boldsymbol{mv\sqrt{x_o^2 + a^2}}$



15. The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed $\boldsymbol{\omega}$. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?

    A. $\frac{2}{1}$ B. $\frac{1}{4}$ C. $\frac{1}{8}$ D. $\frac{4}{1}$

UNIT 7

7. EQUILIBRIUM

In this unit, you are more preferably expected:

- ✓ *Define the terms equilibrium, torque, concurrent force, coplanar force and couple*
- ✓ *Identify and label the forces and torques acting in problems related to equilibrium.*
- ✓ *Apply the first and second conditions of equilibrium to solve equilibrium problems.*
- ✓ *Draw free body diagrams to show all the forces acting.*
- ✓ *Describe the difference among the terms stable, unstable and neutral equilibrium.*

Equilibrium has different expressions in different contexts.

Equilibrium, in physics is the state of balance. Here by balance, it refers net force and net torque.

7.1. Conditions of equilibrium

Basically, there are two conditions of equilibrium in physics context.

First condition for equilibrium (Translational equilibrium)

If the sum of all the forces acting on a body in the x-direction is zero and the sum of the forces acting on a body in the y-direction is zero, the body is said to be in translational equilibrium.

Mathematical expression of First condition for equilibrium

$$\sum F_x = 0 \ \& \ \sum F_y = 0 \Rightarrow \sum F_{net} = 0$$

First condition of equilibrium is also known as *equilibrium of particles*.

Second condition of equilibrium (Rotational Equilibrium)

If the sum all torques about a specified pivot acting on a rotating body or system is zero, the body or system is in rotational equilibrium.

Here by torque, it means rotational effect of force.

Mathematical expression of Second condition for equilibrium

$$\sum \tau_{cw} + \sum \tau_{ccw} = 0 \Rightarrow \sum \tau_{cw} = \sum \tau_{ccw}$$

Second condition of equilibrium is also known as *equilibrium of rigid bodies*.

Note: *After all, when we say a body or a system is in equilibrium, it must fulfil both conditions of equilibrium.*

Static equilibrium and dynamic equilibrium

Static equilibrium is type of equilibrium that occurs when a body is at rest and there is *no net force or net torque* acting on it.

Dynamic equilibrium is also a type of equilibrium that occurs when a body is *moving at a steady velocity* and there is no net force or net torque acting on it.

7.2. Moment of force or torque

Torque is the rotational effect of force.

Mathematically, torque is defined as:

$\tau = r \times F \Rightarrow rF \sin\theta$, its direction can be either cw or ccw

States of equilibrium

There are three states of equilibrium:

- **Stable equilibrium**

When the center of gravity of a body lies below point of suspension or support, the body is said to be in stable equilibrium.

A book lying on a horizontal surface is an example of stable equilibrium. If the book is lifted from one edge and then allowed to fall, it will come back to its original position.

Other examples of stable equilibrium are bodies lying on the floor such as chair, table etc.

When the book is lifted its center of gravity is raised. The line of action of weight passes through the base of the book. A torque due to weight of the book brings it back to the original position

- **Unstable equilibrium**

When the center of gravity of a body lies above the point of suspension or support, the body is said to be in unstable equilibrium.

Examples:

- ✓ Pencil standing on its point or a stick in vertically standing position.
- ✓ If thin rod standing vertically is slightly disturbed from its position it will not come back to its original position. This type of equilibrium is called unstable equilibrium.
- ✓ Vertically standing cylinder and funnel etc.

When the rod is slightly disturbed its center of gravity is lowered. The line of action of its weight lies outside the base of rod. The torque due to weight of the rod toppled it down.

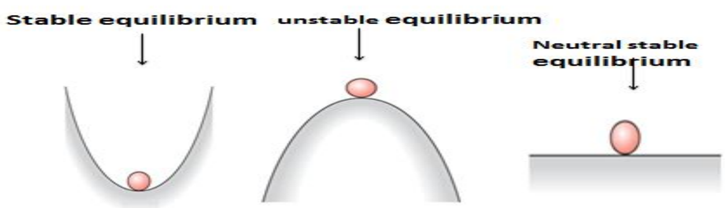
- **Neutral equilibrium**

When the center of gravity of a body lies at the point of suspension or support, the body is said to be in neutral equilibrium.

Example: rolling ball.

If a ball is pushed slightly to roll, it will neither come back to its original nor it will roll forward rather it will remain at rest. This type of equilibrium is called neutral equilibrium.

If the ball is rolled, its center of gravity is neither raised nor lowered. This means that its center of gravity is at the same height as before.



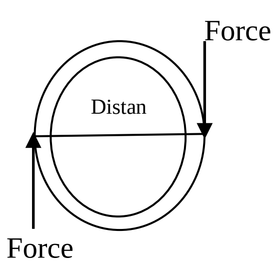
Notice that the center of gravity of a body is the point at which the body's entire weight can be regarded as being concentrated. A body can be suspended in any orientation from its center of gravity without tending to rotate.

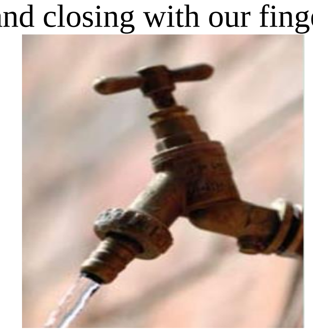
7.3. Couple

A couple is two forces of equal magnitude but acting in opposite direction along *different lines* on an object causing a rotation.

The best examples are:

- ✓ Force on a steering wheel of a car
- ✓ Forces on a tap, while opening and closing with our fingers





Note: As observed from the diagrams above, the two forces that are equal to each other in magnitude but acting in opposite direction act on the objects (tap and steering wheel). The points of applications of the two forces are separated by a distance d.

Hence the couple can produce torque. The torque produced can calculated as:

$\tau = Fd\cos\theta$, where;

- ✓ F is the magnitude of one of the forces

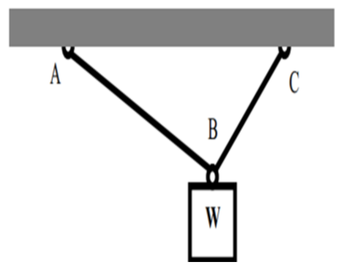
- ✓ d is the distance between the forces
- ✓ θ is an angle between F and d

Worked Examples:

1. Hayu is standing in an elevator, ascending at a constant velocity, what is the resultant force acting on her as a particle? **Answer:** *Assume that Hayu is considered as a particle*

For a particle at rest, or moving with constant velocity relative to an inertial frame, the resultant force acting on the isolated particle must be zero, must vanish.

2. What do you need to know to determine the force in cables AB and BC?



Answer:

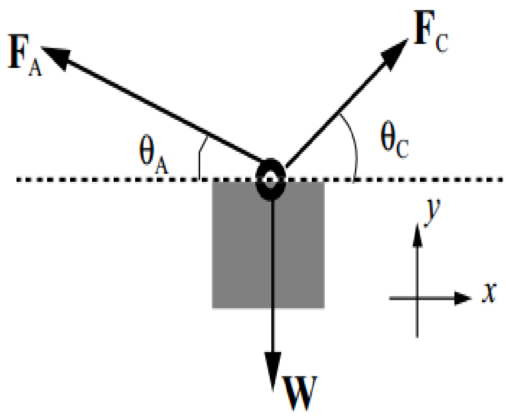
- ✓ You need to know that the cables support only tension.
- ✓ You need to know the weight of the block.
- ✓ You need to know the angles AB and BC each make with the horizontal.
- ✓ You also must know how to isolate the system as a particle.
- ✓ You must know the laws of static equilibrium for an isolated particle.
- ✓ You must suppose that the force on the block due to gravity acts vertically downward.

3. Referring to worked example number 2, Show that the forces in cable AB and BC are given by:

$$F_c = \frac{W \cos\theta_A}{\sin(\theta_A + \theta_C)} \text{ and } F_A = \frac{W \cos\theta_C}{\sin(\theta_A + \theta_C)}$$

Solution:

First isolate the system, assuming it as a particle. Point *B*, where the *line of action* of the weight vector intersects with the lines of action of the tensions in the cables becomes our particle.



Obviously there are three force vectors F_A, F_C & W . Since the system is static equilibrium, the *resultant force on the isolated particle must vanish*.

The sum all component forces along x, $\sum F_X = 0$

$$-F_A \cos\theta_A + F_C \cos\theta_c = 0 \ldots (1)$$

And for the sum of y components, $\sum F_Y = 0$:

$$F_A \sin\theta_A + F_C \sin\theta_c - W = 0 \ldots (2)$$

Even though there are various ways to proceed at this point, multiply the first equation by $\sin\theta_A$, the second by $\cos\theta_A$ and add the two we obtain:

$$\sin\theta_A \left(-F_A \cos\theta_A + F_C \cos\theta_c = 0\right) \ldots (1.1)$$

$$\cos\theta_A \left(F_A \sin\theta_A + F_C \sin\theta_c - W = 0\right) \ldots (2.1)$$

$$F_C \left(\sin\theta_A \cos\theta_C + \sin\theta_C \cos\theta_A\right) = W \cos\theta_A$$

Making use of an appropriate trigonometric identity, we can write:

$$F_c = \frac{W \cos\theta_A}{\sin(\theta_A + \theta_C)}$$

$$F_A = \frac{W \cos\theta_C}{\sin(\theta_A + \theta_C)}$$

4. If the vector sum of the external forces acting on it is zero, a particle is in equilibrium. What criterion must be satisfied for a particle in equilibrium in this situation?

Answer:

a) It can be at rest and remains at rest. *Static Equilibrium*

b) It can move with constant velocity. *Dynamic Equilibrium*

5. If there are only two forces acting on a particle that is in equilibrium, what can we say about these two forces?

Answer:

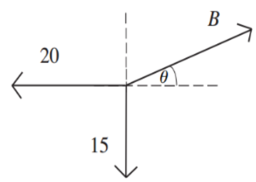
The two forces must be equal in magnitude and opposite in direction to each other.

6. If three forces act on a particle that is in equilibrium, then what can we say about these three forces?

Answer:

- ✓ When the three forces are placed end to end, they must form a triangle.
- ✓ Problems involving 3 or more forces can also be solved in a variety of ways, including the sine and cosine rules (Resolving the forces as **i**, **j** unit vector notation)

7. The three forces in the diagram are in equilibrium. What are the values of B and θ?



Solution:

$\sum F_x = 0 \Rightarrow B\cos\theta - 20 = 0 \ldots(1)$

$\sum F_y = 0 \Rightarrow B\sin\theta - 15 = 0 \ldots(2)$

Solving these equations (1) and (2) simultaneously, we obtain:

$$\frac{\sin\theta}{\cos\theta} = \frac{15}{20} = 0.75$$

$$\Rightarrow \theta = 37^0 \text{ And, } B = \frac{15}{\sin\theta} = \frac{15}{\sin 37^0} = 25N$$

2.When opening a door of your class ro
a distance of 0.850m from the hinges.

Solution:

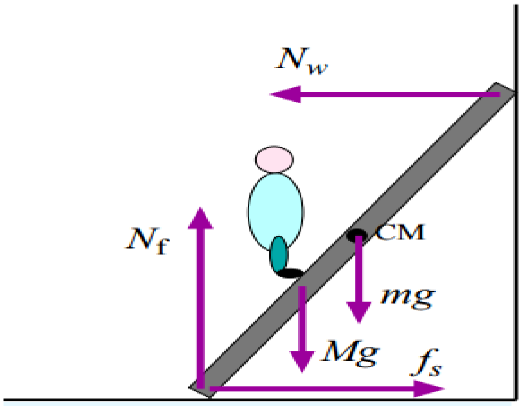
To calculate the torque we use $\tau = r \times F \Rightarrow \tau = rF\sin\theta = rF$, where the perpendicular distance is 0.850m, the force is 55N, and the hinges are the pivot point.

So, $\tau = rF = (0.85\text{ m})(55\text{ N}) = 46.75\text{ N.m}$

3. A ladder having a uniform density and a mass m rests against a frictionless vertical wall at an angle of 60^0. The lower end rests on a flat surface where the coefficient of static friction is 0.40. A student whose mass is twice the ladder's mass attempts to climb the ladder. What fraction of the length L of the ladder will the student have reached when the ladder begins to slip?

Solution:

Let the problem is clearly described with the diagram and geometry shown below.



- ✓ L is the length of the ladder
- ✓ CM is the center of mass of the ladder
- ✓ mg is the weight of the ladder
- ✓ Mg is the weight of the student
- ✓ N_f is normal force from the floor
- ✓ N_w is normal force from the wall
- ✓ f_s is static friction

Since the system is in equilibrium, now we apply the conditions for static equilibrium as:

$\sum F_{net} = 0$

$f_s - N_w = 0 \ldots (1)$

$N_f - mg - mg = 0 \Rightarrow N_f - 2mg - mg = 0 \Rightarrow N_f - 3mg = 0 \ldots (2)$

Since the ladder is not rotating about any pivot, we can apply second condition of equilibrium. We note that the gravity forces from the student and the ladder's CM make an angle of 30^0 with the line joining the axis to the application points; they give a clockwise (negative) torque. The normal force from the wall makes an angle of 60^0 with the line from the axis, and it gives a positive torque.

Let the student is at a distance of X from the bottom end of the ladder.

The torque due to all the forces about the bottom end can be calculated as:

$$-(X)(2mg)\cos 30^0 - \frac{L}{2}(mg)\cos 30^0 + L(N_w)\sin 30^0 = 0$$

$$-(X)(2mg)\frac{\sqrt{3}}{2} - \frac{L}{2}(mg)\frac{\sqrt{3}}{2} + L(0.4mg)\frac{1}{2} = 0$$

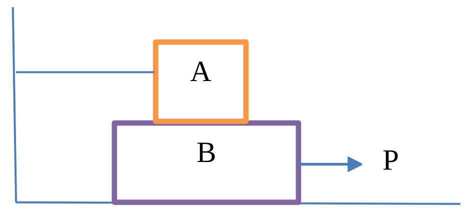
$$\frac{X}{L} = 0.134 \text{ m}$$

The student can climb a fraction of 0.134 (that is, nearly 13.44%) of the length of the ladder before it starts to slip.

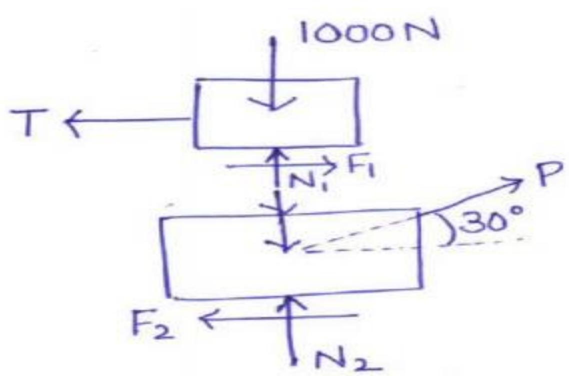
4. Block A weighing 1000N rests over block B which weighs 2000N as shown in figure below. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is $\frac{1}{3}$, what should be the value of P to move the block (B), if:

a) P is horizontal.

b) P acts at 30^0 upwards to horizontal.



Solution:



From 1^{st} condition of equilibrium, we have:

$$\sum F = 0$$

This implies that:

$\sum V = 0$, where V is vertical force

From the above diagram N_1=1000N and F_1 i

Hence;

$$\mu = \frac{F_1}{N_1} = 0.25 \Rightarrow F_1 = 0.25 N_1 = 250 \ N$$

And similarly,

$\sum H = 0$, where H is horizontal force

$F_1 - T = 0 \Rightarrow F_1 = T_1 = 250 \ N$

Considering equilibrium of block B,

$\sum V = 0$

$N_2 - 2000 - N_1 = 0$

$N_2 = 2000 + N_1 = (2000+1000)$ N=3000N

$\frac{F_2}{N_2} = \frac{1}{3} = \mu$ =0.3(1000)=3000N

$F_2 = 0.3N_2$

$\sum H = 0$

$P = F_1 + F_2 = (250 + 1000) N = 1250 \ N$

(b) When P is inclined:

$$\sum V = 0$$

$$N_2 - 2000\ N - N_1 + P\sin(30^0) = 0$$

$$N_2 = 3000\ N - 0.5P$$

From law of friction,

$$F_2 = \frac{1}{3}N_2 = \frac{1}{3}(3000 - 0.5P) = 1000 - \frac{0.5}{3}P$$

$$\sum H = 0$$

$$P\cos 30 = F_1 + F_2$$

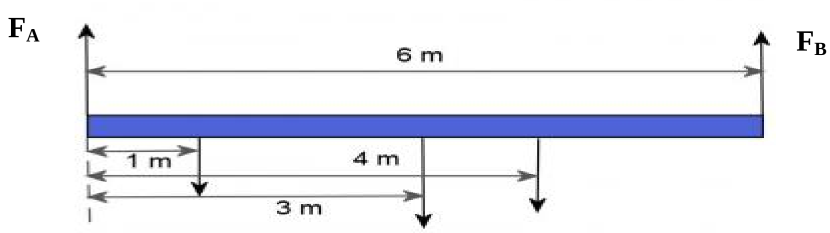
$$\Rightarrow P\cos 30 = 250 + \left(1000 - \frac{0.5}{3}P\right)$$

$$\Rightarrow P\left(\cos 30 + \frac{0.5}{3}P\right) = 1250$$

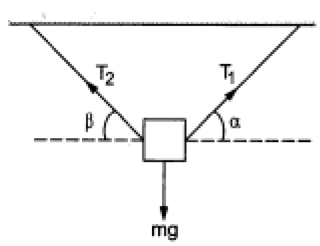
$$\Rightarrow P = 1210.43N$$

Self-Practice Questions and Problems

1. The point of intersection of the lines of action of the weight of all the points of body is called
   A) center of the body C. center of mass of the body
   B) center of gravity of the body D. none of the above
2. Two forces which are equal in magnitude but opposite in direction and not acting along the same line is a
   A) Couple B) Rotation C) Torque D)Motion
3. A body is said to be in dynamic equilibrium when it
   A) possess instantaneous velocity.
   B) is in uniform motion along a straight line.
   C) is moving with changing acceleration
   D) is in uniform motion along a circular path.
4. The torque is the vector product of two vectors force and displacement and is a vector
   A) at 180° to the plane of the force and displacement.
   B) at 45 ° to the plane of the force and displacement.
   C) at 90° to the plane of the force and displacement.
   D) at 0° to the plane of the force and displacement.
5. A plank, 6 m long and weighing 400 N has its center of gravity 4 m from one end as shown in figure below. It is supported near each end and two painters are standing on it. One weighs 500 N, and is at the center. The other weighs 350 N, and is standing 1 m from the light end. What is the value of F_B?



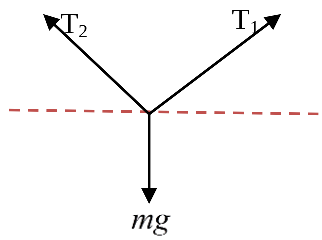
6. A body of mass m is suspended by two strings making angles α and β with the horizontal. What are the tensions in the strings?
   A) $T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}, T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$
   B) $T_2 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}, T_1 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$
   C) $T_2 = \frac{mg\sin\beta}{\cos(\alpha+\beta)}, T_1 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$
   D) $T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}, T_2 = \frac{mg\sin\alpha}{\sin(\alpha+\beta)}$



Solution

Using the free body diagram as shown below:

Since the body is in equilibrium:



$$T_1 \sin(\alpha) + T_2 \sin(\beta) = mg \quad ....(1)$$

$$T_1 \cos(\alpha) = T_2 \cos(\beta) ...(2)$$

$$T_1 = T_2 \frac{\cos(\beta)}{\cos(\alpha)} ....(*)$$

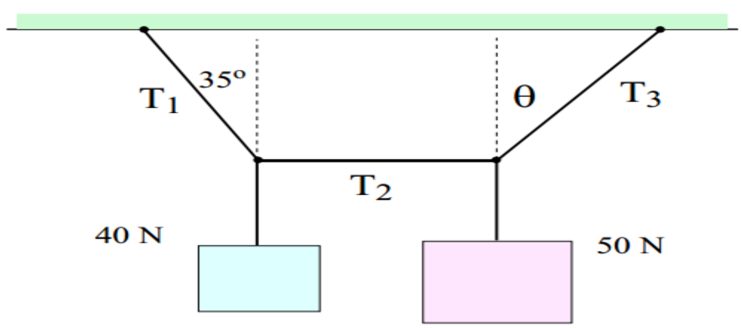
$$(T_2 \frac{\cos(\beta)}{\cos(\alpha)}) \sin(\alpha) + T_2 \sin(\beta) = mg$$

$$T_2 = \frac{mg \cos(\alpha)}{(\cos(\beta)\sin(\alpha) + \cos(\alpha)\sin(\beta))}$$

$$T_2 = \frac{mg \cos(\alpha)}{\sin(\alpha + \beta)} \; and T_1 = \frac{mg \cos\beta}{\sin(\alpha + \beta)}$$

Note that: $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

7. The system in figure below is in equilibrium with the string in the center exactly horizontal. Find (a) tension T_1, (b) tension T_2, (c) angle θ and (d) tension T_3.



a) $T_1 = \frac{40\ N}{\cos 35^0} = \frac{40\ N}{0.82} = 48.78\ N$

b) $T_2 = T_1 \sin 35^\circ = 48.78N\ (0.5735) = 27.98N$

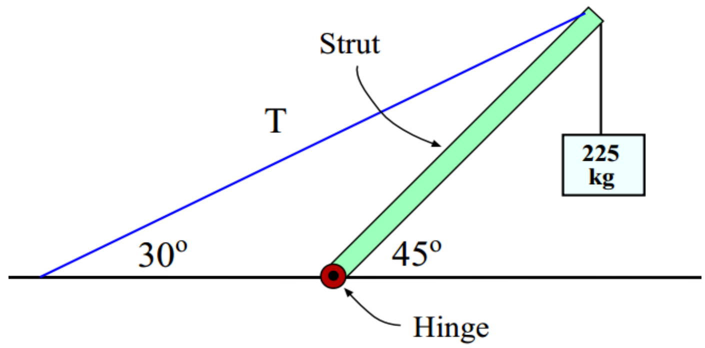
c) $\tan\theta = \frac{27.98\ N}{50\ N} = 0.5596 \Rightarrow \theta = \tan^{-1}(0.5596) = 29.23^0$

d) $T_3 = \frac{50\ N}{\cos 29.23^0} = \frac{50\ N}{0.8726} = 57.3\ N$

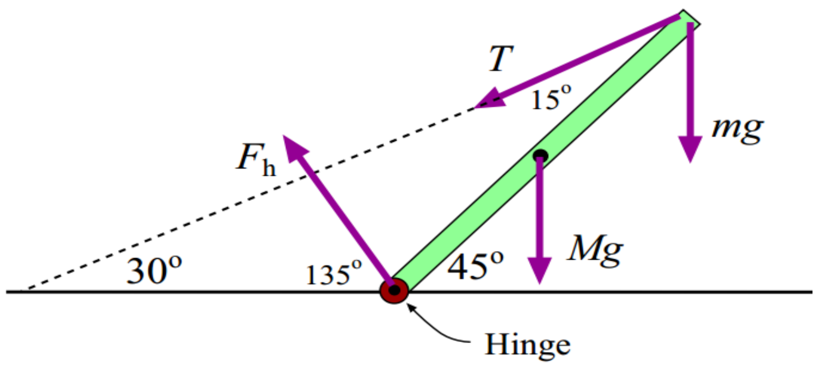
8. The system in figure below is in equilibrium. A mass of 225Kg hangs from the end of the uniform strut whose mass is 45Kg. Find

   a) the tension T in the cable

   b) the horizontal and vertical force components exerted on the strut by the hinge.



Hint: The forces on the system are clearly visualized as shown in figure below.



Answer

a) $T = \frac{1749.825}{0.2588} = 6761.3\,N$

b) The horizontal and vertical components of the force of the hinge on the strut are:

$F_{h,x} = 5855.46\text{ N} \;\&\; F_{v,x} = 6080.65\text{ N}$

UNIT 8
PROPERTIES OF BULK MATTER

In this unit, you are preferably expected to:

- ✓ *Define the terms elastic limit, stress, strain, Young's modulus, shear modulus, density, atmospheric pressure, absolute pressure, surface tension, turbulent flow and streamline flow.*
- ✓ *State and apply Hooke's law, Archimedes's principle, Pascal's principle, Bernoulli's principle*
- ✓ *Use equation of continuity, Bernoulli's equation and Stokes's law to solve problems.*
- ✓ *Distinguish between heat, temperature, internal energy and work.*
- ✓ *Solve problems involving thermal conductivity and heat exchange.*

8.1.Introduction to Bulk matter

By bulk matter, it means matter in a quantity to be seen, touched or weighed.

At a given temperature and pressure a substance is either a solid, liquid or gas. For instance, water, the best-known substance, is a solid (ice) at a pressure of 1atm and temperature of 0˚C and below. Between 0˚C and 100˚C water is a liquid, and above 100˚C it is a gas (in the form of steam or water vapor). The solid, liquid and gaseous states of a substance are called **phases**.

When a substance melts or freezes and boils or condenses it is said to undergo a **phase change**.

Basically, there are the phases of matter known as *solid, liquid and gas*.

Solid

A solid is a rigid macroscopic system with a definite shape and volume. It consists of particle-like atoms connected together by molecular bonds. Each atom vibrates about an equilibrium position, but an atom is not free to move around inside the solid. A solid is nearly **incompressible**, meaning that the atoms are about as close together as they can get.

Liquid

Like a solid, liquid is **nearly incompressible** too.

A liquid flows and deforms to fit the shape of its container. The molecules are held together by *weak molecular bonds* and are therefore free to move around.

Gas

Each molecule in a gas moves through space as a free, non-interacting particle until, on occasion, it collides with another molecule or with the wall of the container. A gas, like liquid, is a fluid. It is also **highly compressible**, meaning that a lot of space exists between the molecules.

- *The fundamental building block of all matter is an atom.*

8.2. Elastic behavior

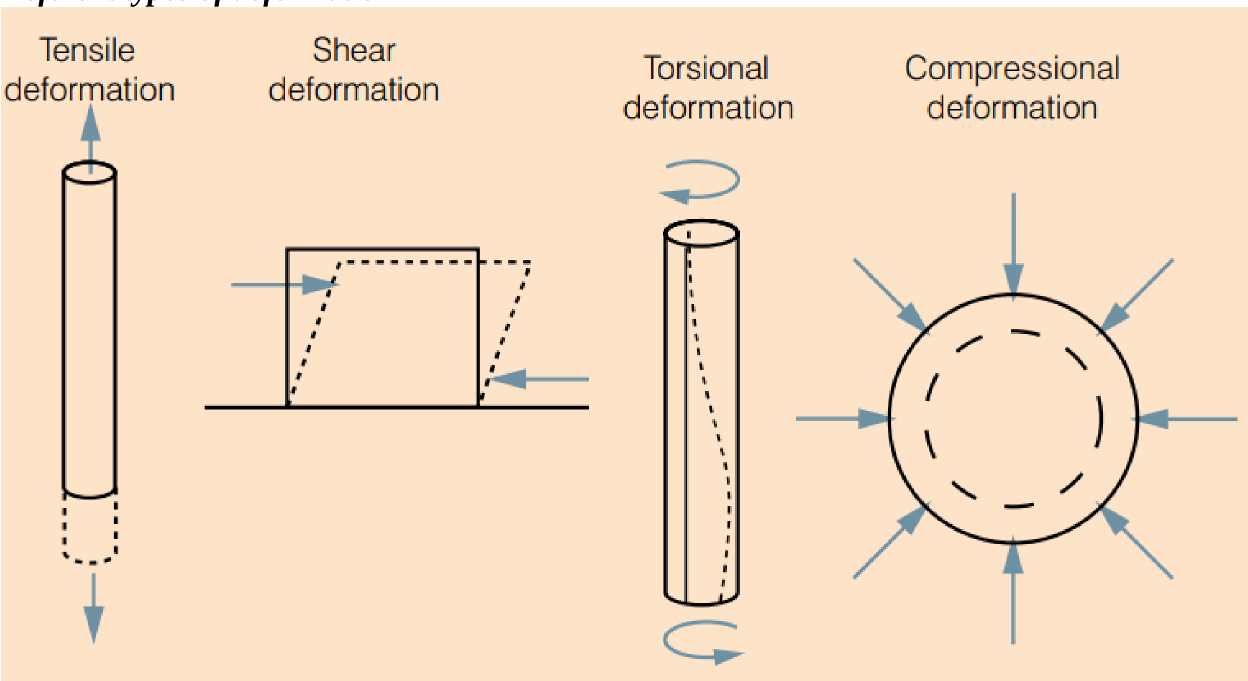
Under this subtopic we will discuss about mechanics of solid and liquid.

Mechanics of solid

- **Deforming force**: - A force acting on a body which produces change in its shape instead of its state of rest or uniform motion of the body.

Basically, there are broadly two types of deformations:

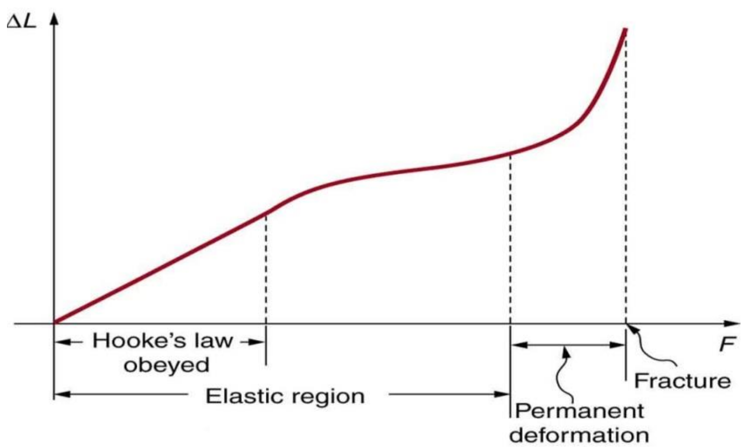
- ***Elastic deformation***. When the deforming force is removed the material returns to the dimension it had before the load was applied. This type of deformation is reversible or non-permanent.
- ***Plastic deformation***. This occurs when a large deforming force is applied to a material. The deforming force is so large that when removed, the material does not spring back to its previous dimension. There is a permanent, irreversible deformation. The minimal value of deforming force which produces plastic deformation is known as the *elastic limit* for the material.
- ***Deferent types of deformation***



- **Hooke's law**: - Within certain limits named elastic limit, the force required to stretch an elastic object such as a metal spring is directly proportional to the extension of the spring. This is known as Hooke's law and commonly written:

$F = -k\Delta L$, where F is the applied force, ΔL is the length of extension/compression and k is a constant of proportionality known as the *spring constant* which is usually given in N/m.

Note that the minus sign indicates that applied force and restoring force are equal in magnitude and oppositely directed.



The graph shown above is a graph of deformation versus applied force.

The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is $1/k$. For larger forces, the graph is curved but the deformation is still elastic. ΔL will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

Stress, Strain and Young Modulus

Deformation is experienced by objects under the action of external force.

For example, this may be squashing, squeezing, ripping, twisting, shearing, or pulling the objects apart.

In the language of physics, two terms describe the forces on objects undergoing deformation: *stress* and *strain*.

- **Stress** is a quantity that describes the magnitude of forces that cause deformation. Stress is generally defined as *force per unit area*. When forces pull on an object and cause its elongation, like the stretching of an elastic band, we call such stress a **tensile stress**.
- When forces cause a compression of an object, we call it a **compressive stress**.

- When an object is being squeezed from all sides, like a submarine in the depths of an ocean, we call this kind of stress a **bulk stress** (**volume stress**). In other situations, the acting forces may be neither tensile nor compressive, and still produce a noticeable deformation.

  For example, suppose you hold a book tightly between the palms of your hands, then with one hand you press-and-pull on the front cover away from you, while with the other hand you press-and-pull on the back cover toward you. In such a case, when deforming forces act tangentially to the object's surface, we call them 'shear' forces and the stress they cause is called **shear stress**.

  The SI unit of stress is the Pascal (Pa).

  Mathematically, stress can be expressed as: $\delta = \frac{F}{L}$

- **Strain** is given as a fractional change in either length (under tensile stress) or volume (under bulk stress) or geometry (under shear stress). Therefore, strain is a dimensionless number.
- Strain under a tensile stress is called **tensile strain**.
- Strain under bulk stress is called **bulk strain**.
- Strain caused by shear stress is called **shear strain**.

  The greater the stress, the greater the strain; however, the relation between strain and stress does not need to be linear.

  Mathematically, strain can be expressed as: $\varepsilon = P \frac{\Delta L}{L_o}$

- Only when stress is sufficiently low is the deformation it causes in direct proportion to the stress value. The proportionality constant in this relation is called the **elastic modulus**. In the linear limit of low stress values, the general relation between stress and strain is:

$stress = (K) \times strain$, where the constant K is known as elastic modulus.

Therefore, within elastic limit, stress is proportional to strain.

Note that, the elastic modulus:

- for tensile stress is called **Young's modulus** (Y).
- for the bulk stress is called the **bulk modulus** (K).
- for shear stress is called the **shear modulus (G)**.

- **Compressibility:** the reciprocal of bulk modulus of a material is called its compressibility.

  Compressibility $= \frac{1}{K}$

- **Proportionality limit (P)** – The stress at the limit of proportionality point P is known as proportionality limit.
- **Elastic limit** - the maximum stress which can be applied to a wire so that on unloading it return to its original length is called the elastic limit.
- **Yield point(Y) -** The stress, beyond which the length of the wire increase virtually for no increase in the stress.
- **Plastic region**- the region of stress- strain graph between the elastic limit and the breaking point is called the plastic region.
- **Fracture point or Breaking point (B) -** the value of stress corresponding to which the wire breaks is called breaking point.

8.3.Mechanics of fluid

Here under this sub topic we will see about fluid statics and fluid dynamics separately.

8.3.1. Fluid statics

Fluid Statics deals with fluids at rest. A fluid at rest has no shear stress. Consequently, any force developed is only due to normal stresses i.e., pressure.

Pressure: - The force/threat acting per unit area is called pressure.

The S.I Unit of pressure is N/m^2 or Pascal (Pa)

Pascal's law: - Pressure applied to an enclosed fluid is transmitted to all part of the fluid and to the wall of the container.

Application of Pascal's law

(1) Hydraulic lifts

(2) Hydraulic brakes

- **Pressure exerted by liquid column**: - $P = \rho g h$, where h = depth of liquid, ρ = density of the fluid, g = acceleration due to gravity.
- **Variation of pressure with depth**:

  $P = P_a + \rho g h$, where P_a = atmospheric pressure
- **Atmospheric pressure**: - The pressure exerted by atmosphere is called atmospheric pressure.
- At sea level, atmospheric pressure = 0.76mmHg.

  Mathematically, 1atm = 1.013 x 10^5 Nm^{-2}

- **Archimedes' principle**: - It states that when a body is immersed completely or partly in a fluid, it is buoyed up by a force equal to the weight of the fluid displaced.
  The force that buoyed up the object is known as up thrust or buoyant force (F_B).

Mathematically $F_B = W_{fd}$, where W_{fd} is the weight of fluid displaced

Where V is volume of fluid displaced, ρ is its density

- **Torricelli's theorem/speed of efflux**:-It states that the velocity of efflux i.e. the velocity with which the liquid flows out of an orifice (a narrow hole) is equal to that which is freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.
  Quantitatively velocity of efflux is given by:

$$v = \sqrt{2gh}$$

- **Surface tension (T):-** It is the property of a liquid by virtue of which, it behaves like an elastic stretched membrane with a tendency to contract so as to occupy a minimum surface area.

Mathematically, $T = \frac{F}{L}$. The SI unit is Nm^{-1}

Surface Energy: The potential energy per unit area of the surface film is called the surface energy. Surface tension is numerically equal to surface energy.

$$\text{Surface energy} = \frac{\textit{Work done in increasing the surface area}}{\textit{Increase in area}}$$

- **Angle of contact**: - The angle which the tangent to the free surface of the liquid at the point of contact makes with the wall of the containing vessel, is called the angle of contact For liquid having convex meniscus, the angle of contact is obtuse and for having concave meniscus, the angle of contact is acute.
- **Capillary tube**: - is a tube of very fine bore.
- **Capillarity**:-is the rise or fall of liquid inside a capillary tube when it is dipped in it.
- **Ascent formula**:- when a capillary tube of radius 'r' is dipped in a liquid of density *s* and surface tension T, the liquid rises or depresses through a height,

$$h = \frac{2T\cos(\theta)}{r\rho g}$$

In this formula,

- ✓ T represents the surface tension in a liquid-air environment,
- ✓ θ is the angle of contact or the degree of contact,
- ✓ ρ is the density of the liquid in the representative column,
- ✓ g is the acceleration due to the force of gravity and
- ✓ r is the radius of the tube in which the liquid is presented in.

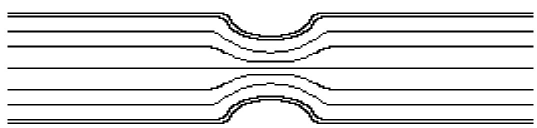
8.4.Fluid Dynamics

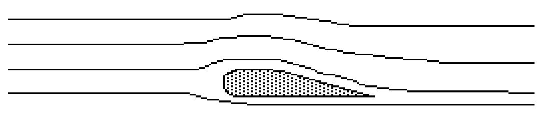
In physics and engineering, **fluid dynamics** is a sub discipline of fluid mechanics that describes the flow of fluids (liquids and gases). It has several sub disciplines, including aerodynamics (the study of air and other gases in motion) and **hydrodynamics** (the study of liquids in motion).

Fluid Flow

Fluid flow can be described in terms of two main types. **Streamline** flow and **turbulent** flow.

Streamline flow, also known as **laminar** flow, is illustrated below, flow through a pipe and flow around an airplane wing.





In streamline flow, the motion of a particle after it passes a particular point is the same as the motion of the particle that preceded it at that point. The path that a particle takes is called a **stream line**. Every particle that passes any particular point will follow the stream line that goes through that point. A bundle of stream lines, like the ones here, is known as a **stream tube**. Fluid never crosses the surface of a stream tube.

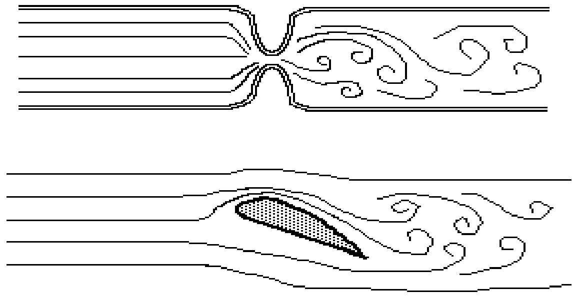
Turbulent flow is flow in which the fluid undergoes irregular fluctuations, or mixing, in contrast to laminar **flow**, in which the fluid moves in smooth paths or layers.

In **turbulent flow** the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction.

In turbulent flow, the motion of a particle after it passes a particular point may be quite different from the motion of the particle that preceded it at that point.

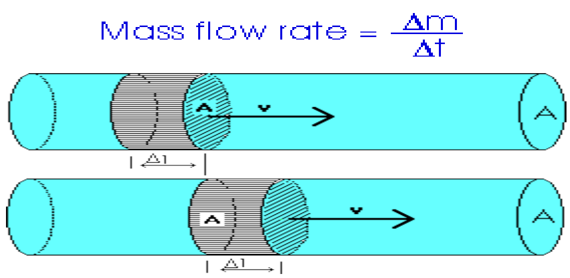
Turbulent flow is characterized by randomness or irreproducibility of the motion of individual particles.

It usually occurs in fluids moving at high speeds. As you might expect, friction is far greater in turbulent flow. We will concentrate most of our attention on streamline flow.



Equation of Continuity

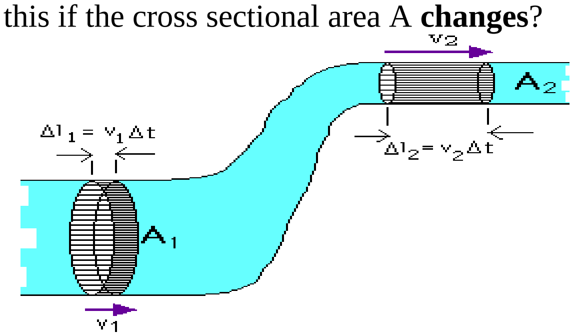
When a fluid is in motion, it must move in such a way that mass is conserved. To see how mass conservation places restrictions on the velocity field, consider the **steady** flow of fluid through a duct (that is, the inlet and outlet flows do not vary with time). The inflow and outflow are **one-dimensional**, so that the velocity V and density \rho are constant over the area A.



$\Delta V = A\Delta l$, where $\Delta l = v\Delta t$

$$\text{Massflowra te} = \frac{\Delta m}{\Delta t} = \rho Av$$

Now, how can we handle this if the cross sectional area A **changes**?



$$\Delta m_1 = \rho_1 A_1 v_1 \Delta t$$

$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t$$

We are not creating nor destroying the already existing mass. The mass Δm_1 that flows **into** a region must equal the mass Δm_2 that flows **out of** the region. That is, $\Delta m_1 = \Delta m_2$

$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Assuming that the fluid we are considering is **incompressible fluid**, the above equation can be reduced to:

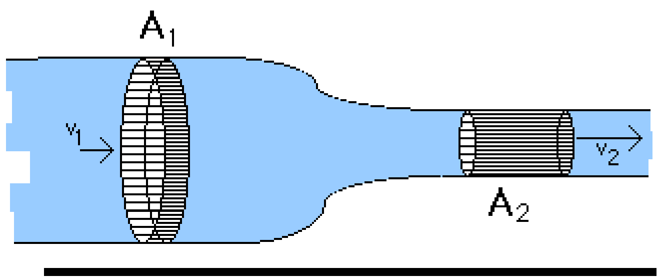
$A_1 v_1 = A_2 v_2$

Bernoulli's Equation

The relationship between pressure and velocity in fluids is described quantitatively by *Bernoulli's equation*, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an *incompressible, frictionless fluid,* the following sum is constant:

$\frac{1}{2}\rho v^2 + P + \rho g y = cons\tan t$

To prove this equation consider the flow of fluid in the flow tube shown below.

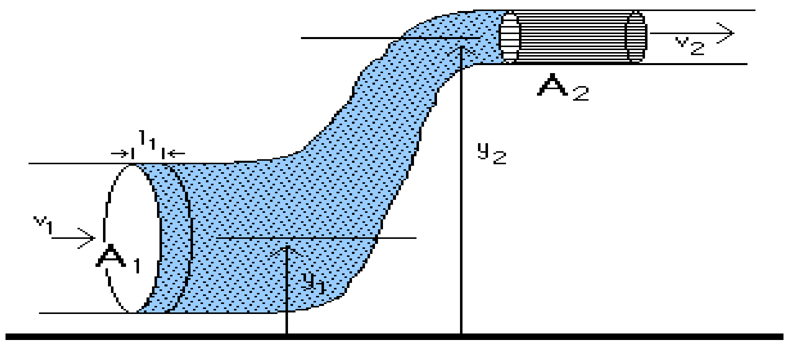


From the Equation of Continuity,

$$A_1 v_1 = A_2 v_2$$

We know that the fluid must be moving slower at position 1 where the cross section A_1 is larger and it must be moving faster at position 2 where cross section A_2 is smaller. That is, the fluid must **accelerate** as is moves from position 1 to position 2. That means the **pressure** on the fluid at position 1 must be greater than the **pressure** at position 2 in order to provide a net force to cause this acceleration. This is an example of **Bernoulli's Principle** that the **pressure** exerted by a moving fluid is **greater** where the speed of the fluid is **smaller** and the pressure is **smaller** where the speed of the fluid is **greater.**

Now consider fluid that flows along a stream tube with a change in cross sectional area **and** a change in height. Work must be done on the fluid to change its **kinetic energy** and its **potential energy**.



At position 1, the force on the shaded portion of the fluid is

$$F_1 = P_1 A_1$$

Likewise, at position 2,

$$F_2 = P_2 A_2$$

The **work** done at the two positions is:

$$W_1 = F_1 l_1 = P_1 A_1 l_1$$

And,

$$W_2 = -F_2 l_2 = -P_2 A_2 l_2$$

Gravity also does work,

$$W_{grav} = mgy_1 - mgy_2 = -mg(y_2 - y_1)\text{, where}$$

$$m = \rho_1 A_1 l_1 = \rho_2 A_2 l_2$$

So that,

$$W_{net} = W_{grav} + W_2 + W_1$$

We **know** that the network on anything equals the **change in kinetic energy**,

$$W_{net} = \Delta KE = \tfrac{1}{2} m (v_2^2 - v_1^2)$$

$$W_1 + W_2 + W_{gravity} = \tfrac{1}{2} m (v_2^2 - v_1^2)$$

$$P_1 A_1 l_1 - P_2 A_2 l_2 - mg(y_2 - y_1) = \tfrac{1}{2} m v_1^2 + \tfrac{1}{2} m v_2^2$$

$$P_1 A_1 l_1 + mgy_1 + \tfrac{1}{2} m v_1^2 = P_2 A_2 l_2 + mgy_2 + \tfrac{1}{2} m v_2^2$$

$$\tfrac{1}{2}\rho_1 A_1 l_1 v_1^2 + P_1 A_1 l_1 + \rho_1 A_1 l_1 g y_1$$

$$= \tfrac{1}{2}\rho_2 A_2 l_2 v_2^2 + P_2 A_2 l_2 + \rho_2 A_2 l_2 g y_2$$

Recall that

$$A_1 l_1 = A_2 l_2 = V$$

$$\tfrac{1}{2}\rho_1 V v_1^2 + P_1 V + \rho_1 V g y_1 = \tfrac{1}{2}\rho_2 V v_2^2 + P_2 V + \rho_2 V g y_2$$

From the above equation V is canceled out, and we get:

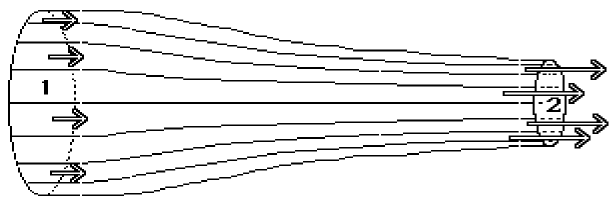
$$\tfrac{1}{2}\rho_1 v_1^2 + P_1 + \rho_1 g y_1 = \tfrac{1}{2}\rho_2 v_2^2 + P_2 + \rho_2 g y_2$$

This means, $\tfrac{1}{2}\rho v^2 + P + \rho g y = cons\tan t$

This equation is specifically known as **Bernoulli's Equation.**

If the vertical height y does not change, the equation can be reduced to a form:

$$\frac{1}{2}\rho v^2 + P = constant$$



Application of Bernoulli's principle

(i) Working of Bunsen burner

(ii) Lift of an air foil

(iii) Spinning of ball (Magnus effect)

(iv) Sprayer

(v) Ping pong ball in air jet.

- **Viscosity**: - It is the property of liquid (or gases) due to which a backward dragging force acts tangentially between two layers of liquid when there is relative motion between them.
- **Newton's formula for Viscous force**: - the viscous force between two liquid layer each of area A and having a velocity gradient $\frac{dv}{dx}$ is :

  $F = \eta A \frac{dv}{dx}$**,** where **η** is coefficient of viscosity.
- **Coefficient of viscosity**: - It is define as the tangential viscous force which maintains a unit velocity gradient between two parallel layers each of unit area S.I unit of coefficient of viscosity is poiseuille or pascal-second.
- **Poiseuille's equation**:- when a liquid of coefficient of viscosity flows through a tube of length 'l' and radius r, then the volume of liquid following out per second is given by:

  $\left[v = \frac{\pi P r^4}{8\rho l}\right]$, Where P is the difference of pressure between the two ends of the tube.

- **Stokes's law and terminal velocity**

When a small spherical body falls through a viscous fluid then its velocity first goes on increasing. After certain time, a stage is reached in which the total upward force (viscous force + up thrust) just balances the total downward force (weight of the body) then the body moves with a constant speed called *terminal velocity*.

Stoke found out that that the viscous force 'F' acting on spherical body of radius 'r' moving with a terminal velocity 'v' in a fluid of coefficient of viscosity 'η' is given by:

$F = 6\pi\eta rv$. This is known as Stokes's law.

When a solid sphere of radius r and density ρ_s is dropped in to a fluid of density ρ_f and coefficient of viscosity η , its terminal velocity is given by:

$$v_T = \frac{2}{9} r^2 \left[\frac{\rho_s - \rho_f}{\eta} \right] g$$

- **Reynolds number**: - It is a pure number which determines the nature of flow of liquid through a pipe. It does not have any units. Quantitatively Reynold's number is described as:

$$N = \frac{\rho D V_c}{\eta},$$

Where **η** is coefficient of viscosity of liquid, **ρ** is density of liquid D is the diameter of the tube, V_c is critical velocity

For stream line flow, Reynold's number <2000

For turbulent flow, Reynold's number > 3000

For uncertain flow, 2000<Reynold's number<3000

- **Critical velocity**: - It is that velocity of liquid flow, up to which the flow of liquid is streamlined and above which its flow becomes turbulent. Critical velocity of a liquid (V_c) flowing through a tube is given by:

$$V_c = \frac{K\eta}{\rho r}$$

8.5.Heat, temperature and thermal expansion

- ✓ Heat- it is a form of energy, which produce in us the sensation of warmth.
- ✓ Temperature: - The degree of hotness or coldness of a body is called temperature.
- ✓ Thermometer- It is a device used to measure the temperature of a body.

✓ Scales of temperature: - there are four scales of temperature. Given below are scales of temp with lower and upper fixed point Temperature scales.

Scales	Lower fixed point (Melting point of ice	Upper fixed point (Boiling point of water)
Celsius	0℃	100℃
Fahrenheit	32°F	212°F
Kelvin	273K	373K
Reaumur	0°R	80°R

Specific heat: - It is the amount of heat required to raise the temperature of unit mass of substance through unit degree Celsius.

$$C = \frac{\Delta Q}{m \Delta T}$$

Latent heat: - It is define as the quantity of heat required to change the unit mass of the substance from its one state completely to another state at constant temperature.

Mathematically, $Q = ML$

Calorimeter: - A device used for measuring heat.

Principle of calorimetry: - It is a principle in which $Q_{Lost} = Q_{gained}$

Transfer of heat: - there are three modes by which heat transfer takes place.

ii. **Conduction**: - It is the process by which heat is transmitted from one point to another through a substance in the direction of fall of temperature without the actual motion of the particles of the substance. When two opposite faces of a slab, each of cross section A and separated by a distance d are maintained at temperature T_1 and T_2 ($T_1 > T_2$), then amount of heat that flows in time t.

$Q = \frac{KA(T_1 - T_2)t}{d}$, Where K is coefficient of thermal conductivity of the matter

Coefficient of thermal conductivity: - It may be defined as the quantity of heat energy that flows in unit time between the opposite faces of a cube of unit side, the faces being kept a one degree difference of temperature.

The SI unit of coefficient of thermal conductivity: $J\,S^{-1}m^{-1}K^{-1}$ or $W\,m^{-1}K^{-1}$

iii. **Convection**: - It is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of the heated particles of the substance.

iv. **Radiatio**n: - It is the process by which heat is transmitted from one place to another without heating the intervening medium.

Perfect black body: - It is a body which absorbs heat radiations of all the wavelengths, which fall on it and emits the full radiation spectrum on being heated.

Stefan's law: - It states that the total amount of heat energy radiated per unit time per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the substance of the body.

Mathematically; $\frac{P}{A} \alpha T^4$

$P = \delta A T^4$, where δ is called Stefan's constant. Its value is 5.67 x 10^{-8} $J.S^{-1}m^{-2}K^{-4}$

Newton's laws of cooling: - It states that the rate of loss of heat or rate of cooling of a body is directly proportional to the temperature difference between the body and the surrounding, provided the temperature difference is small.

Mathematically, $-\frac{dQ}{dt} = K(T - T_o)$ the minus sign is due to cooling (decrease in temperature)

Worked Examples

1. A wire is stretched by a force such that its length becomes double. How will the Young's modulus of the wire be affected?

   Answer:

   Young's modulus remains the same.

2. How does the Young's modulus change with rise in temperature?

   Answer:

   Young's modulus of a material decreases with rise in temperature.

3. Which of the three modulus of elasticity Y, K and η is possible in all the three states of matter (solid, liquid and gas)?

   Answer:

   Bulk modulus (K)

   The **bulk modulus (K)** of a substance is a measure of how resistant to compression that substance is.

It is defined as the ratio of the infinitesimal pressure increase to the resulting *relative* decrease of the volume. Other moduli describe the material's response (strain) to other kinds of stress.

4. Which of the two forces; deforming or restoring is responsible for elastic behavior of substance?

Answer:

Restoring force

Deforming force is the force which displaces a body from its original position or configuration either by changing location or shape.

Restoring force tends to move the place of the body back to its original position or configuration.

5. A boat carrying a number of large stones is floating in a water tank. What will happen to the level of water if the stones are unloaded into the water?

Answer:

The level of water will **fall,** because the volume of the water displaced by stones in water will be less than the volume of water displaced when stones are in the boat.

6. A rain drop of radius r falls in air with a terminal velocity v. What is the terminal velocity of a rain drop of radius 3r?

Solution

$$v = \frac{2}{9} r^2 \left[\frac{\rho_f - \rho_s}{\eta} \right] g$$

Here *v* is directly proportional to r^2

$$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1} \right)^2 \Rightarrow v_2 = \left(\frac{3r}{r} \right) v_1^{\,2} = 9 v_1$$

7. Explain that why Steel is more elastic than rubber?

Answer:

Consider two wires, one made of steel and another made of rubber having equal length L and cross sectional area A. When subjected to the same deforming force F, the extension produced in steel is ΔL_s and in rubber is ΔL_R ; such that $\Delta L_R \rangle \Delta L_s$.

Then: $$Y_s = \frac{FL}{A\Delta L_s}$$

And; $$Y_s = \frac{FL}{A\Delta L_R}$$

$$\frac{Y_s}{Y_R} = \frac{\Delta L_R}{\Delta L_s}$$

As $\Delta L_s \langle \Delta L_R$ implying that $Y_s \langle Y_R$

Hence steel is more elastic.

8. A wire stretches by a certain amount under a load. If the load and radius are both increased to four times, find the stretch caused in the wire.

Solution:

For a wire of radius r stretched under a force F,

$$Y = \frac{FL}{r^2 l(\pi)}$$

$$l = \frac{FL}{r^2 Y(\pi)}$$

Let l' be the extension when both the load and the radius are increased to four times, then:

$$l' = \frac{4(FL)}{(4r)^2 Y(\pi)} = \frac{L}{4}$$

9. Calculate the percentage increase in the length of a wire of diameter 2mm stretched by a force of $1KgF$. Young's modulus of the material of wire is:

$Y = 1.5 \times 10^{11} N/m^2$

Solution:

$W = mg$, here $g = F$ in the body of the problem.

$W = mg \Rightarrow W = 1Kg\ (10\ N/Kg) = 10\ N \quad r = 1mm = 10^{-3} m$

Cross section of wire , $\pi r^2 = \pi \times (10^{-3})^2 = \pi \times 10^{-6} m^2$

$$Y = \frac{FL}{Al} \Rightarrow \frac{l}{L} = \frac{F}{AY} = \frac{10\ N}{10^{-6}\pi m^2 x15\ x10^{10}\ N/m^2} = 0.0212\ x10^{-3} = 2.12\ x10^{-5}$$

- Percentage increase = $2.12 \times 10^{-5} \times 100$ = **0.00212%**

10. The pressure of a medium is changed from $1.01x10^5$Pa to $1.165x10^5$ Pa and the change in volume is 10% keeping temperature constant. Find the bulk modulus of the medium.

Solution:

$$\Delta P = (1.165 - 1.01)10^5 Pa = 0.155x10^5 Pa$$

$$\frac{\Delta v}{v} = 10\% = 0.1$$

Now, $K = \frac{\Delta P}{\Delta v / v} = 1.55 x10^5 Pa$

11. Twenty seven (27) identical drops of water are falling down vertically in air each with a terminal velocity of 0.15m/s. If they combine to form a single bigger drop, what will be its terminal velocity?

Solution:

Let r be radius of each drop whose v = 0.15 m/s

$$v = \frac{2}{9} r^2 \left[\frac{\rho_s - \rho_f}{\eta} \right] g$$

Let R be the radius of the big rain drop.

Volume of big drop = Volume of 27 small drops.

$$V = \frac{4}{3} \pi R^3 = 27x \frac{4}{3} \pi r^3$$

$$R = 3r$$

Let v_1 be the terminal velocity of bigger drop

$$v_1 = \frac{2}{9} R^2 \left[\frac{\rho_s - \rho_f}{\eta} \right] g \quad \text{and}$$

$$\frac{v_1}{v} = \frac{R^2}{r^2} = 9$$

$$v_1 = 9v = 1.35 \, m/s$$

12. Water flows through a horizontal pipe line of varying cross section at the rate of $0.2m^3/s$. Calculate the velocity of water at a point where the area of cross section of the pipe is $0.02m^2$.

Solution

Rate of flow (Q) =Av $v = \frac{Q}{A} = \frac{0.2m^3/s}{0.02\,m^2} = 10\,m/s$

13. A cylinder of height 20m is completely filled with water. Find the efflux speed of water through a small hole on the side wall of the cylinder near its bottom.

Solution:

The problem can be managed using **Torricelli's law**, which is also known as **Torricelli's theorem.**

The law states that the speed *v* of efflux of a fluid through a sharp-edged hole at the bottom of a tank filled to a depth *h* is the same as the speed that a body (in this case a drop of water) would acquire in falling freely from a height.

Efflux is defined as something that is flowing out or the process of flowing out.

i) When water is flowing out of a river inlet and into a larger stream that is nearby, this is an example of efflux.
ii) The water that is flowing out of a river and into a larger body of water is an example of efflux.

Now, $v = \sqrt{2gh} = \sqrt{2(10\,m/s^2)(20\,m)} = 20\,m/s$

14. Water at a pressure of $4x10^4$ Nm^{-2} flows at $2ms^{-1}$ through a pipe of $0.02m^2$ cross sectional area which is reduced to $0.01m^2$. What is the pressure in the smaller cross section of the pipe?

Solution

From equation of continuity, we have, $A_1v_1 = A_2v_2$

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{0.02}{0.01}(2m/s) = 4m/s$$

Since the pipe is purely horizontal $(h_1 = h_2 = 0)$, we can use the reduced form of Bernoulli's equation as:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$P_1 = P_2 + \tfrac{1}{2}\rho(v_2^2 - v_1^2)$ $P_1 = 4x10^4\,N/m^2 + \tfrac{1}{2}(1000)Kg/m^3(16\,m^2/s^2 - 4m^2/s^2) = 3.4x10^4\,Pa$

15. A nozzle with a radius of 0.250cm is attached to a garden hose with a radius of 0.9cm. The flow rate through hose and nozzle is 0.5L/s. Calculate the speed of the water:

(a) in the hose. b) in the nozzle.

Solution

Let r_1 = 0.9cm be the radius of the hose

r_2 = 0.25cm be the radius of the nozzle)

Q = 0.5L/s (flow rate)

From the concept of flow rate and equation of continuity, we have:

$$Q = A_1v_1 = A_2v_2$$

Where $A_1 = \pi(r_1)^2 = 0.81\pi cm^2$ and $A_2 = \pi(r_2)^2 = 0.0625\pi cm^2$

$1L = 10^{-3}m^3$

Now, $v_2 = \frac{Q}{A_1} = \frac{0.5L/s}{(0.0625)(3.14)cm^2} = 2.55L/s.cm^2 = 25.5m/s$

$$v_1 = \frac{Q}{A_2} = \frac{0.5L/s}{(0.81)(3.14)cm^2} = 0.196L/s.cm^2 = 1.96m/s$$

16. In Example 20 above, we found that the speed of water in a hose increased from 1.96m/s to 25.5m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is 1.01×10^5 N/m^2 (atmospheric, as it must be) and assuming level, frictionless flow.

Solution:

Level flow means constant depth, so Bernoulli's principle applies. We are asked to find P_1

Solving Bernoulli's principle for P_1 yields:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2).$$

$$P_1 = 1.01\times10^5 \text{ N/m}^2 + 500\text{Kg/m}^3[(25.5\text{m/s})^2-(1.96\text{m/s})^2]$$

$$P_1 = 1.01\times10^5 \text{ N/m}^2 + 3.23\times10^5 \text{ N/m}^2$$

$$P_1 = 4.24\times10^5 \text{ N/m}^2$$

Discussion:

This absolute pressure in the hose is greater than in the nozzle, as expected since *v* is greater in the nozzle. The pressure P_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

17. The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body.

(a) Calculate the average speed of the blood in the aorta if the flow rate is 5L/min and aorta has a radius of 10 mm.

(b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5L/min, the speed of blood in the capillaries is about 0.33mm/s. Given that the average diameter of a capillary is 8μm, calculate the number of capillaries in the blood circulatory system.

We can use $Q = A\,v$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

(a) The flow rate is given by $Q = A\,v$

$$v = \frac{Q}{A} = \frac{0.5L/\min}{(3.14)(100\ x10^{-6}\,m^2)} = 0.0016\ x10^{6}\ L/m^2.\min = 16\ m/\min = 0.27\ m/s$$

(b) $n_1A_1v_1=n_2A_2v_2$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives:

Thus, $n_2 = 5\,x10^{9}$ capillaries

$$n_2 = \frac{A_1v_1}{A_2v_2}n_1 = \frac{(100\ x10^{-6})(0.27)}{(16\ x10^{-12})(0.00033)}(1) = 5\,x10^{9}$$

Here in the above calculation π and the units cancel each other.

Discussion:

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting.

18. A hollow plastic sphere of radius 3.5 cm is lowered below the surface of water reservoir by a rope. Calculate the buoyant force exerted by the water on the sphere.

Solution:

The buoyant force exerted on any object in liquid (water) is determined by the following expression: $F_b = \rho_w gV_{in}$

where V_{in} - volume of a solid inside water.

ρ_w = 1000Kg/m^3- is the density of liquid (water).

In the present problem the whole sphere is in the water. It means that V_{in} is the volume of the whole sphere:

$$V_{in} = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}(3.14)(0.035\ m)^3 = 0.00018\ m^3$$

Here the radius (r = 3.5 cm) of the sphere is converted into meters.

With the known value of V_{in}, we can find the buoyant force.

$$F_b = \rho_w gV_{in} = (1000\text{Kg/m}^3)(10\text{N/Kg})(0.00018\text{m}^3)\ F_b = 1.8N$$

Self-Practice Questions and Problems

1. In a hydraulic press a force of 24N is applied to a smaller piston of area $0.4m^2$. What load will be supported by the larger piston whose area is $0.7m^2$?

   A) 42N B) 24N C) 48N D) 84N

2. A plastic sphere floats in water with 50% of its volume submerged. This same sphere floats in glycerin with 40% of its volume submerged. What are the densities in $kg.m^{-3}$ of the glycerin and the sphere respectively?

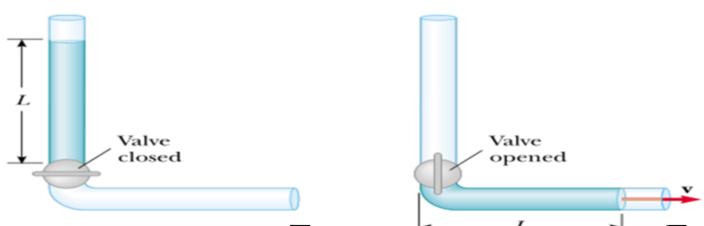
   A) 500, 1250 B) 1250,500 C) 5,12 D) 1000,500

3. Aluminum of volume $10^{-3}m^3$ is completely submerged in water and displaces some amount of water. What is the magnitude of the buoyant force acting on the aluminum?

   $\rho_{Al} = 2.7x10^3\ kg/m^3$.

   A) 10N B) $2\sqrt{5}$N C) $2\sqrt{3}$N D) 26.5N

4. An incompressible, non-viscous fluid is initially at rest in the vertical portion of the pipe shown in figure a, where L =2m. When the valve is opened, the fluid flows into the horizontal section of the pipe. What is the speed of the fluid when all of it is in the horizontal section, as shown in figure b? Assume the cross-sectional area of the entire pipe is constant.



   A) $2\sqrt{10}$ m/s (a) B) $\sqrt{5}$m/s (b) C) $\sqrt{2}$m/s D) $20\sqrt{5}$m/s

Solution

GPE of water in the vertical portion of the pipe = KE of water in the horizontal portion of the pipe (after the valve is opened)

$$mgh = \frac{1}{2}mv^2 \Rightarrow v^2 = 2gh$$

$$v = \sqrt{2(10\ m/s^2)2m} = 2\sqrt{10}\ m/s$$

5. Let a wire obeying Hooke's law be stretched under tensile force F and having Young's modulus Y. Which one of the following mathematical expression gives the force constant of the wire where the original length of the wire is L_o?

   A) $\frac{YA}{L_o}\Delta l$ B) $\frac{AL_o}{Y}$ C) $\frac{L_o}{YA}$ D) $\frac{YA}{L_o}$

6. "The speed of water spraying from the end of a garden hose increases as the size of the opening is decreased with the thumb.'' This statement best describes:

   A) Archimedes's principle C) Pascal's principle
   B) Equation of continuity D) Bernoulli's equation

7. Water flows through a horizontal pipe of diameter 2cm with a speed of 5m/s. What is the speed of the water as it passes through a narrow section of the pipe of diameter 1cm?

   A) 10m/s B) 2.5m/s C) 20m/s D) 1.25m/s

8. Hydraulic brakes and hydraulic jacks work on:

   A) Bernoulli's principle C) Archimedes's principle
   B) Pascal's principle D) Equation of continuity

9. Water enters a house through a pipe 2cm in inside diameter at an absolute pressure of 4atm. The pipe leading to the second floor bathroom 5m above is 1cm in diameter. When the flow velocity at inlet pipe is 4cm/s, what is the flow speed of water exiting of the pipe in the bath room?

   A) 16cm/s B) 6cm/s C) 1.6cm/s D) 2.3cm/s

10. The pressure at the bottom of a cylindrical tube filled with water was measured to be 5Kpa.If the water in the tube were replaced with ethyl alcohol , what would be the new pressure at the bottom of the tube? Use the density of ethyl alcohol 0.8g.cm^{-3}

   A) 4Kpa B) 5Kpa C)4.8Kpa D) 6250Pa

11. The Velocity of air at the top of the airplane wing is 220m/s, under the wing it is 180m/s. The wing size is 8m by 3m and the density of air is 1kg.m^{-3}. Calculate the net force on the wing. Note that the thickness of the wing is negligible.

   A) 8000N B) 192KN C) 32.4KN D) 48.4KN

12. What causes surface tension in water?

   A) The intermolecular force of attractions between water molecules
   B) Surface tension is the natural force existing in a liquid which holds its surface together
   C) The intermolecular force of repulsions between water molecules
   D) Adhesive forces between water molecules

13. When a solid sphere of radius r and density ρ_s is dropped in to a fluid of density ρ_f and coefficient of viscosity η , which one of the following expression correctly shows its terminal velocity?

   A) $\frac{4}{3}r^2\left[\frac{\rho_s - \rho_f}{\eta}\right]g$ C) $\frac{2}{9}r^2\left[\frac{\rho_f - \rho_s}{\eta}\right]g$

   B) $\frac{2}{9}r^2\left[\frac{\rho_s - \rho_f}{\eta}\right]g$ D) $\frac{2}{9}r3\left[\frac{\rho_f + \rho_s}{\eta}\right]g$

14. What is the flow rate of water through a 20cm long capillary tube that has a diameter of 0.15cm if the pressure difference across the tube is 4KPa? Use η for water $=8.01\text{x}10^{-2}$ Pa.s

A) $3.1\text{x}10^{-11}m^3/s$ B) $3.1cm^3/s$ C) $760cm^3/s$ D) $13cm^3/s$

15. What does the value of Reynolds number in fluid dynamics describe?
   A) It describes the strength of surface tension of liquids
   B) It describes the viscosity of liquids
   C) It describes whether the fluid flow is horizontal or vertical
   D) It describes whether the fluid flow is stream lined or turbulent

16. What is the rate of energy radiation per unit area of a black body (e = 1) at a temperature of 2000K?

Use $\delta = 5.67\text{x } 10^{-8}W.m^{-2}. K^{-4}$

A) $90.72W. m^{-2}$ C) $90.72KW.m^{-2}$

B) $907200J.m^{-2}$ D) none of the above

17. By how much should the pressure on a liter of water be changed to compress it by 0.10%? Use Bulk modulus of water $=2.2\text{x}10^9$Pa

A) $2.2\text{x}10^{-6}$Pa B) $2.2\text{x}10^7$Pa C) $2.2\text{x}10^6$Pa D) 0.001GPa

18. What is the buoyant force on a 78.74N of iron immersed in water? Use $\rho_{iron} = 7874\ Kg/m^3$

A) 1N B) 8.37N C) 2N D) 10N

19. The energy required to melt 10g of copper at its melting point is: Use L_f = 209KJ/Kg

A) 2090J B) 2900J C) 9020J D) 2.95KJ

20. What is the amount of heat required to raise the temperature of 4Kg of water from 40^0c to 60^0c? [Use c_w = 4200J/Kg.K]

A) 336KJ B) 4200J C) 336J D) 633KJ

21. A 5Kg of water at an initial temperature of 20^0c is mixed with a 2Kg of water at 95^oC. What is the common final temperature of the mixture in degree Celsius?

A) 73.57 B) 41.43 C) 33 D)100

22. A tank holding 60Kg of water is heated by a 3KWatt electric immersion heater. Find the time for the temperature to rise from 10^0C to $60\ ^0C$

A) 70minutes B) 70sec C) 4200minutes D) 439sec

23. A copper cylinder of mass 0.5Kg is heated up from room temperature (20^0C) by supplying it with heat energy of 11,700J. What is the final temperature of the cylinder?

(Use s.h.c. of copper 390J/kg.K)

A) 30^0C B) 80^0C C) 60^0C D) 8^0C

24. Venturi effect is one of applications of
   A) equation of continuity
   B) Bernoulli's equation
   C) light equation
   D) speed equation

Venturi effect is the reduction in fluid pressure that results when a fluid flows through a constricted section of a pipe. The **Venturi** effect is named after Giovanni Battista, an Italian physicist.

Answer Key for Self-Practice Questions and Problems

Unit-5: Work, Energy and Power

1. **A**	**6. D**	**11. D**	**16. A**
2. **D**	**7. C**	**12. D**	**17. B**
3. **A**	**8. B**	**13. C**	**18. D**
4. **A**	**9. A**	**14. D**	
5. **B**	**10. A**	**15. A**	

Unit-6: Rotational Motion

1. B	**6. A**	**11. B**
2. C	**7. A**	**12. B**
3. C	**8. C**	**13. C**
4. A	**9. C**	**14. B**
5. C	**10. A**	

Unit-7: Equilibrium

1. **B**	6. **A**	8. **a) 6761.3N**
2. **A**	7. **a) 48.78N**	**b) 6080.65N**
3. **B**	**b) 27.98N**	
4. **C**	**c) 29.23°**	
5. **575N**	**d) 57.3N**	

Unit-8: PROPERTIES OF BULK MATTER

1. **A**	**6. B**	**11. B**	**16. D**	**21. B**
2. **B**	**7. C**	**12. A**	**17. B**	**22. A**
3. **A**	**8. B**	**13. B**	**18. C**	**23. B**
4. **A**	**9. A**	**14. A**	**19. A**	**24. B**
5. **D**	**10. A**	**15. D**	**20. A**	

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