OROMIA EDUCATIONAL BUREAU

MATHEMATICS READING MATERIAL AND WORK SHEET FOR GRADE 9

UNIT FOUR, FIVE, SIX, SEVEN AND ANSWER KEY FOR WORK SHEET OF EACH UNIT.

PREPARED

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UNIT FOUR

RELATIONS AND FUNCTION

4.1 RELATIONS

Question: Define the meaning of set?

Set: Set is a collection of object which is well defined. So relation is a set whose elements are an ordered pairs.

Example: R= {(1, 5), (2.10)}

Relation is the way in which things are related to each other, The relating phrases are: 'is smaller than', 'is greater than', 'is multiple of', 'is factor of', 'is father of', 'is son of', 'is child of' and etc.

Definition 4.1 Let A and B be non-empty sets. A relation R from A to B is any subset of A×B. In other words, R is a relation from A to B if and only if $R \subseteq (A \times B)$.

Definition 4.2 **Let** R be a relation from a set A to a set B. Then

i. Domain of R = {x: (*x*, *y*) belongs to R for some *y*}

ii. Range of R = {y: (*x*, *y*) belongs to R for some *x*}

Example 1 Let A = { 1, 2, 3, 4} and B = { 1, 3, 5}

i $R_1 = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$ is a relation from A to B because $R_1 \subseteq (A \times B)$.

Example 2 Let A = {1, 2, 3} then observe that

$$R_1 = \{(1, 2), (1, 3), (2, 3)\},\ R_2 = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$$

and $R_3 = \{(x, y) \mid x, y \in A, x + y \text{ is odd}\}$ are relations on A.

Example 3: Given the relation R = {(1, 3), (2, 5), (7, 1), (4, 3)}, find the domain and range of the relation R.

Solution: Since the domain contains the first coordinates, domain = {1, 2, 7, 4} and

The range contains the second coordinates, range = {3, 5, 1}

Example 4: Given A = {1, 2, 4, 6, 7} and B = {5, 12, 7, 9, 8, 3} Find the domain and range of the relation

$R = \{(x, y): x \in A, y \in B, x > y\}$

Solution: If we describe R by complete listing method, we will find

R = {(4, 3), (6, 3), (7, 3), (6, 5), (7, 5)}.

This shows that the domain of R = {4, 6, 7} and the range of R={3, 5}

GRAPHY OF THE RELATION

The representation of a relation from set A to B by locating the ordered pairs in a coordinate system or by using arrows in a diagram displaying the members of both sets, or as a region on a coordinate system is called **graphs of a relations**.

Example 1: let A= {2, 3, 5} and

B= {6, 7, 10} then

The relation R from A to B be

"x is a factor of y" then

R= {(2, 6), (2, 10), (3, 6), (5, 10)} with

Domain x= {2, 3, 5} and

Range y= {6, 10}

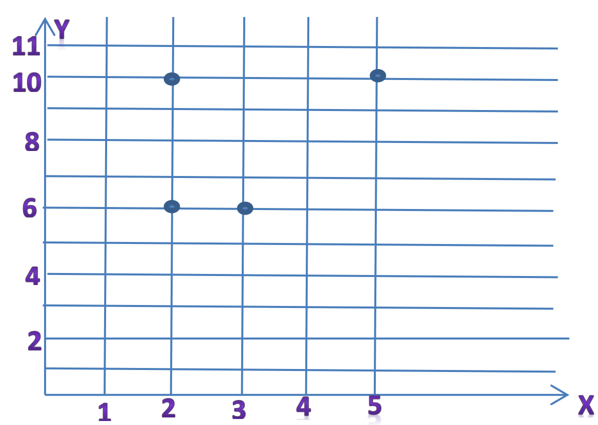


FIGURE 4:1

Example 2: sketch the graphs of the relation of

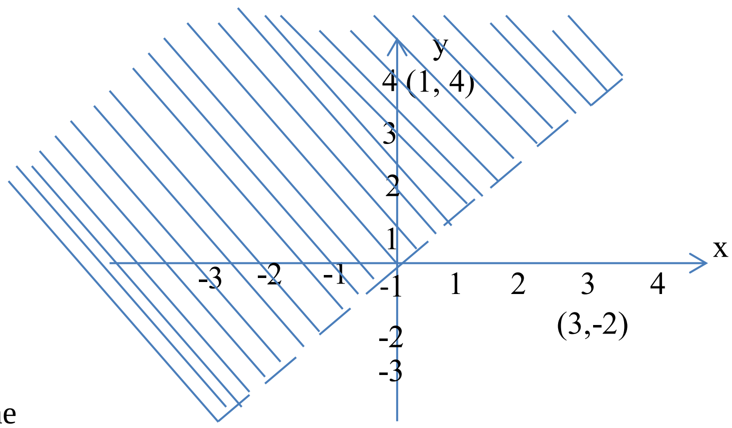
A. $R = \{(x, y): y > x; x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$

B. $R = \{(x, y): y \geq x + 1; x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$

Solution: A

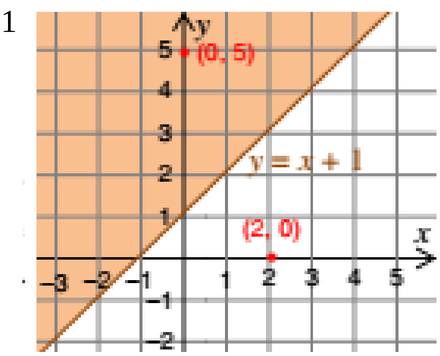
1. Draw the graph of the line y=x using broken line
2. Select two points one from one side and another from the other side. For example (1,4) and (3,-2) from the above and below the line y=x
3. The order pair (1, 4) the given relation.
   So the region where the point (1,4) is
   Contained the graphs of the relation
   In the inequality relation $y > x \Longrightarrow 4 > 1$ True
   So we shade the region of (1, 4)

The graphs of the relation is given as follows



- **Solution**: B

1. Draw the graph of the line y=x+1 using solid line
2. Select two points one from one side and another from the other side. For example (0, 5) and (2, 0) from the above and below the line y=x+1
3. The order pair (0, 5) the given relation.
   So the region where the point (0, 5) is
   Contained the graphs of the relation and we shade the region which contains (0, 5)



NOTE: The domain and range of
The relation is the set of real number.

To sketch the graph of a relation with two or more inequalities,

i Using the same coordinate system, sketch the regions of each inequality.

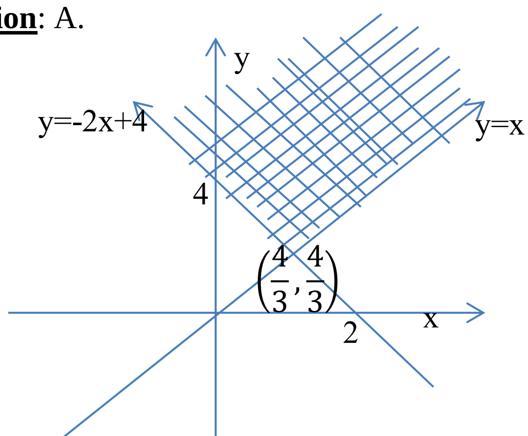
ii Determine the intersection of the regions.

Example: sketch the graphs of the following relation and determine the domain and range.

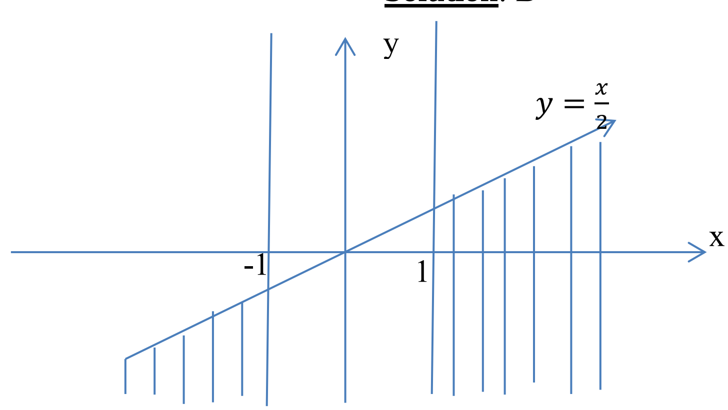
A. $R = \{(x, y): y \geq x \text{ and } y \geq -2x + 4\}$

B. $R = \{(x, y): x - 2y \leq 0 \text{ and } |x| \geq 1\}$

Solution: A.



Solution: B



Domain $\{x: x \in \mathbb{R}\}$ and

Range $\{y: y \geq \frac{4}{3}\}$

Domain $\{x: x \leq 1 \text{ } or \text{ } x \geq 1\}$and

Range $\{y: y \in \mathbb{R}\}$

In general, to sketch graphs of relations involving inequalities, do the following:

1. Draw the graph of a line(s) in the relation on the *xy*- coordinate system.
2. If the relating inequality is ≤ or ≥, use a solid line; if it is < or >, use a broken line.
3. Then take arbitrary ordered pairs represented by points, one from one side and the other from another side of the line(s), and determine which of the pairs satisfy the relation.
4. The region that contains points representing the ordered pair satisfying the relation will be the graph of the relation.

Note: A graph of a relation when the relating phrase is an inequality is a region on the coordinate system.

4.2 FUNCTIONS

Definition A function is:

- ✓ Relations such that no two ordered pairs have the same first-coordinates and different second-coordinates.
- ✓ It is a relation in which no two ordered pairs have the same first element.

Domain and range of a function

Domain: is the set of first element of the ordered pairs

Range: is the set of the 2nd element of the ordered pairs

Note: a relation is a function if and only if the domain is not repeated

Example: for each of the following relation identify whether it is function or not. If it is function determine the domain and range.

A. Consider the relation R = {(1, 2), (7, 8), (4, 3), (7, 6)}

solution: Since 7 is paired with both 8 and 6 the relation R is not a function.

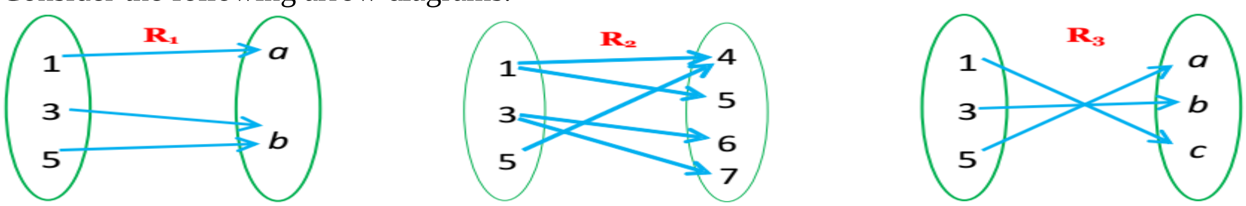
B. The relation R = {(*x*, *y*): *y* is the father of *x*}

Solution: is a function because no child has more than one father. So the domain is child and range is father of the child

C. Consider the relation R = {(*x*, *y*): *y* is a grandmother of *x*}.

Solution: This relation is not a function since everybody (*x*) has two grandmothers.

D. Consider the following arrow diagrams.



Which of these relations are functions?

Solution: $R_1$and R_3 are a function where R_2 is not a function, because 1 and 3 are paired with different the second elements.

Then domain and range of the functions are given as

Domain of R1= {1, 3, 5} and
Range of R1= $\{a, b\}$

Domain R3= {1, 3, 5} and
Range of R3= $\{a, b, c\}$

Functional notation

If *x* is an element in the domain of a function *f*, then the element in the range that is associated with *x* is denoted by *f*(*x*) and is called the image of *x* under the function *f*. This means *f* = {(*x*, *y*):*y*= *f* (*x*)}

The notation *f* (*x*) is called function notation. Read *f* (*x*) as "f of x"

Note: *f*, *g* and *h* are the most common letters used to designate a function. But, any letter of the alphabet can be used.

A function f is a mapping of a relation from set A to set B we write: A→B

- ✓ If $x \in A$ and $y \in B$,then the function is denoted by $y = f(x)$
- ✓ If $(x, y) \in f$ means $f = \{(x, y): y = f(x)\}$
- ✓ Domain of $f = \{x: x$ is the set of all possible input$\}$
- ✓ Range of $f = \{y: y$ is the set of all out puts$\}$

Domain of the function

- ✓ The domain of the function $f$is the largest set of real numbers for the value of $f(x)$ is a real number.
- ✓ If x is in the domain of a function f, we say that f is defined at x or $f(x)$ is exist
- ✓ If x is not in the domain of a function f, we say that f is not defined at x

Functional values

- ✓ If f: A → B is a function, then, for any $x \in A$ the image of x under f, $f(x)$ is called the functional value of f at x.
- ✓ $f(x)$ shows the y −value for the given x and shows that the value can be found when an x −value is known
- ✓ For example, if $f(x) = x – 3$, then the functional value of f at $x = 5$ is $f(5) = 5–3=2$. Finding the functional value of f at x is also called evaluating the function at x, *then when x=5, y=2*

Example1: for each of the following find domain and range.

A. $f(x) = 3$ **B.** $f(x) = 1 - 3x$ **C.** $f(x) = x + 4$ **D.** $f(x) = \sqrt{x - 1}$ **E.** $f(x) = \frac{1}{2x}$

Solution:

A. Domain = $\mathbb{R}$ and Range = $\{3\}$
B. Domain = $\mathbb{R}$ and Range = $\mathbb{R}$
C. Domain = $\{x: x \geq -4\}$ and Range = $\{y: y \geq 0\}$
D. Domain = $\mathbb{R}$ and Range = $\{y: y \geq -1\}$
E. Domain = $\mathbb{R}\backslash\{0\}$ and Range = $\mathbb{R}\backslash\{0\}$

EXAMPLE2: If $f(x) = 2x + \sqrt{x + 4}$, evaluate each of the following:

A. $f(-4)$ **B.** $f(5)$

Solution

a. $f(-4) = 2(-4) + \sqrt{-4+4} = -8$
b. $f(5) = 2(5) + \sqrt{4+5} = 10 + 3 = 13$

Exercise

A. Match each of the functions in column A with its corresponding domain in column

A	B
1. $f(x) = \sqrt{2-x}$	a $\{x: x \geq 3\}$
2. $f(x) = 2x – 1$	b $\{x: x \leq 2\}$
3. $f(x) = \sqrt{x - 3}$	c $\{x: x \in \mathbb{R}\}$

B. Match each of the functions in column A with its corresponding range in column B.

A	B
1. $f(x) = \sqrt{2-x}$	a $\{y: y \geq 0\}$
2. $f(x) = 2x – 1$	b $\{y: y \in \mathbb{R}\}$
3. $f(x) = \sqrt{x - 3}$	c $\{y: y \geq 10\}$

COMBINATION OF FUNCTION

Functions like numbers can be added, subtracted, multiplied and divided.

Definition

- If f and g are a functions, then $f+g$, $f-g$, $f.g$ and $\frac{f}{g}$ are the function defined by
- ✓ $(f+g)(x)=f(x)+g(x)$ ←sum of functions
- ✓ $(f-g)(x)=f(x)-g(x)$ ←difference of function
- ✓ $(f.g)(x)=f(x).g(x)$ ←product of function
- ✓ $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}, g(x)\neq 0$ ← Quotients of function.

Note: The domain of $f+g$, $f-g$, $f.g$ is the intersection of the domain of f and g and The domain of $\frac{f}{g}$ is the intersection of the domain of f and g but $g(x)\neq 0$

Example

A. Let $f(x)=\frac{2}{x-1}$ and $g(x)=\frac{2x-2}{3x+3}$ find **i .** $f+g$ **ii.** fg **iii .** domain of $(f+g)$ and fg

B. Let $f(x) = 3x - 3$ and $\frac{2}{x-1}$, then evaluate: i. $2fg(2)$ ii$\left(.\ \frac{f}{g}-2f\right)(3)$ iii. $(f-g)(4)$ and

iv. Determine the domain of

a. $f+g$ **b.** $f-g$ **c.** $f.g$ **d.** $\frac{f}{g}$

solution:

a. i. $f+g=\frac{2}{x-1}+\frac{2x-2}{3x+3}=\frac{2x^2+2x+8}{3x^2-3}$

ii. $fg=\left(\frac{2}{x-1}\right)\left(\frac{2x-2}{3x+3}\right)=\frac{4}{3x+3}$

iii. Domain of $f+g$ = Domain of fg = Domain of $f\cap$ Domain of $g=\mathbb{R}\backslash\{1\}\cap\mathbb{R}\backslash\{-1\}=\mathbb{R}\backslash\{-1,1\}$

b. Do as exercise

4.3 GRAPHS OF FUNCTIONS

A. Graphs of Linear Functions

Definition: If a and b are fixed real numbers, $a\neq 0$, then $f(x) = ax + b$ for $x\in\mathbb{R}$ is called a linear function. If $a = 0$, then $f(x) = b$ is called a constant function.

Sometimes linear functions are written as $y = ax + b$. where its domain is the set of real number.

Example

In each of the following given linear function, determine

i. slope ii. Whether increasing or decreasing

iii. x and y intercept IV. Sketch the graphs

a. $y-3x-5=-2$

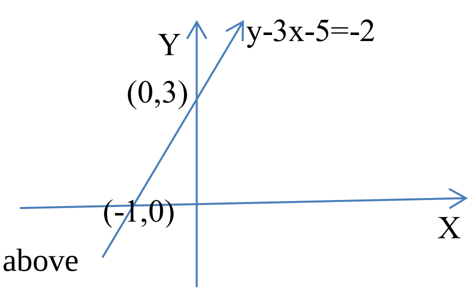
b. $2x+y=6$

c. $f(x)-6=-2$

Solution

a. $y-3x-5=-2\rightarrow y=3x+3$

Then $a=3$ and $b=3$ so the slope is $a=3$ and increasing because $a=3>0$, x –intercept $\left(-\frac{b}{a},0\right)$= (-1,0) and y –intercept (0, b)=(0,3) then the graphs of the function is gives as above



Grade 9 Mathematics Note And Some Practice Question For Second Semester

Note: The graphs of $f(x) = ax + b$ is straight line

- ✓ a is the slope of the line
- ✓ If $a > 0$ the graph of $f(x) = ax + b$ is increasing
- ✓ If $a < 0$the graph of $f(x) = ax + b$ is decreasing
- ✓ If $a = 0$the graph will be $f(x) = b$ horizontal line, it is called constant linear function.
- ✓ If $x = 0$, then $f(x) = b \leftarrow y$ −intercept (0,b) where the graph crosses the y-axis

If f(x)= 0, then $ax + b = 0 \rightarrow x = -\frac{b}{a} \leftarrow x$ −intercept which is $\left(-\frac{b}{a}, 0\right)$

B and C do as exercise

Question: if a linear function f satisfies the given condition, find f(x)

a. f(2)=5 and f(1)=-7
b. f(1)=-3 and f(2)=-8
c. In the x-y plane, the line with ax+by=1, where a and b are a constants, intersects the y-axis where y=2, what is the value of b?

GRAPHS OF QUADRATIC FUNCTIONS

Definition: A function defined by $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$ and $a \neq 0$ is called a quadratic function. a is called the leading coefficient.

Example 1: $f(x) = 2x^2 + 3x + 2$ is a quadratic function with $a = 2,\ b = 3,\ c = 2$

Note: Any function that can be reduced to the form $f(x) = ax^2 + bx + c$ is also called a quadratic function

Example 2:$f(x) = (x-2)(x+2)$ is expressed as $f(x) = x^2 - 4$ with $a = 1,\ b = 0,\ and\ c = -4$

Sketching graphs of quadratic function using a table of values

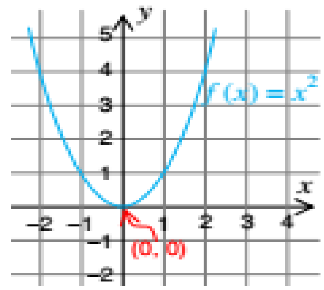
Example3: Draw the graphs of each of the following

a. $f(x) = x^2$. c. $f(x) = 2x^2$.
b. $f(x) = -x^2$. d. $f(x) = -2x^2$.

Solution:

A. Evaluating the function values in the interval of $-2 \le x \le 2$

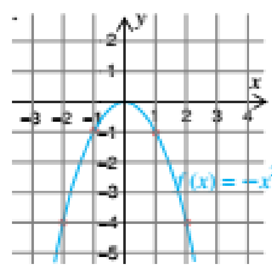
x	-2	-1	0	1	2
$f(x) = x^2$	4	1	0	1	4



Then the graph is given as

B. Evaluating the function values in the interval of $-2 \le x \le 2$

x	-2	-1	0	1	2
$f(x) = -x^2$	-4	-1	0	-1	-4



Similarly the graph of the function is given as

c and d do as exercise

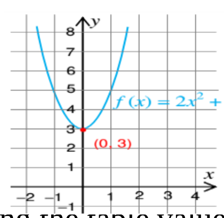
Example 4: sketch the graphs of the following using table value.

A. $f(x) = 2x^2 + 3$ B. $f(x) = 2x^2 - 3$ C. $f(x) = -2x^2 + 3$ D. $f(x) = -2x^2 - 3$

Solution :

A. The table value of the function $f(x) = 2x^2 + 3$

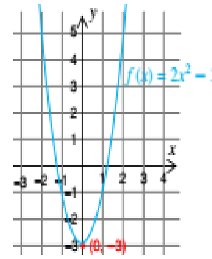
x	-2	-1	0	1	2
$f(x) = 2x^2 + 3$	11	5	3	5	11



Then the graphs of the function is given as shown above using the table value given above

B. The table value of the function $f(x) = 2x^2 - 3$

x	-2	-1	0	1	2
$f(x) = \mathbf{2x^2 - 3}$	5	-1	-3	-1	5



Then the graphs of the function is given as follows using the table value given above

<u>c and d do as exercise</u>

In general from the above graphs we summarize the following

Case 1: If $a > 0$

1. The graph opens upward.
2. The vertex is (0, 0) for $f(x) = ax^2$ and (0, c) for $f(x) = ax^2 + c$
3. The domain is all real numbers.
4. The range is $\{y: y \geq 0\}$ for $f(x) = ax^2$ and $\{y: y \geq c\}$ for $f(x) = ax^2 + c$
5. The vertical line that passes through the vertex is the axis of the parabola (or the axis of symmetry).

Case 2: If $a < 0$

I. The graph opens downward.
II. The vertex is (0, 0) for $f(x) = ax^2$ and (0, c) for $f(x) = ax^2 + c$
III. The domain is all real numbers.
IV. The range is $\{y: y \leq 0\}$ for $f(x) = ax^2$ and $\{y: y \leq c\}$ for $f(x) = ax^2 + c$
V. The vertical line that passes through the vertex is the axis of the parabola (or the axis of symmetry).

Sketching graphs of quadratic functions, using the shifting rule

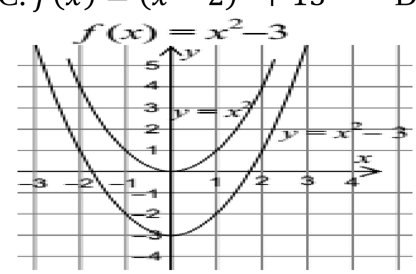
To sketch the graphs of quadratic function in the above we have used tables of values to sketch graphs of quadratic functions. Now we shall see how to use the shifting rule to sketch the graphs of quadratic functions.

Example: sketch the graphs of each of the following by shifting rule

A. $f(x) = x^2 - 3$ B. $f(x) = (x-5)^2$ C. $f(x) = (x-2)^2 + 13$ D. $f(x) = (x+1)^2 - 7$

<u>SOLUTION</u> :

A. To sketch the graphs of **f (x)** = x^2**–3** by shifting 3 unit downward from the graphs of $f(x) = x^2$

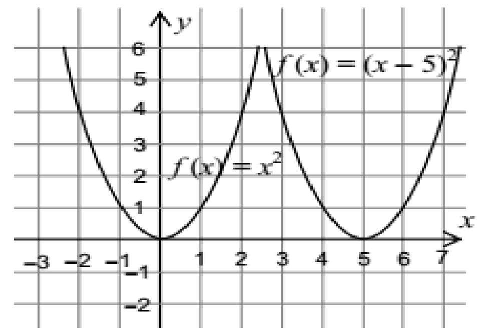


Note:

Let $k > 0$, then the graph of $f(x) = (x - k)^2$ is obtained by shifting the graph of $f(x) = x^2$ by k units to the right and the graph of $f(x) = (x + k)^2$ is obtained by shifting the graph of $f(x) = x^2$ by k units to the left

B. By constructing a table of values, you can draw the graph of $f(x) = (x-5)^2$ and see that it is a shifting of the graph of $f(x) = x^2$ by 5 units to the right.

$f(x)=(x-5)^2$



x	-2	-1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
$f(x)=(x-5)^2$	49	36	25	16	9	4	1	0	1

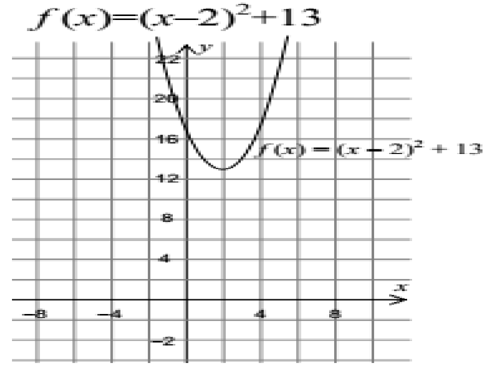
Note:

1 The graph of $f(x) = (x+k)^2 + c$ opens upward.

2 The vertex of the graph of $f(x) = (x+k)^2 + c$ is (–k, c) and the vertex of the graph of $f(x) = (x+k)^2 - c$ is (k, –c). Similarly the vertex of the graph of $f(x) = (x+k)^2 - c$ is (–k, –c) and the vertex of the graph of $f(x) = (x+k)^2 + c$ is (k, c).

C. To construct the graphs of $f(x) = (x-2)^2 + 13$, by drawing the graphs of $f(x) = x^2$, and shifting on the line of x-axis by 2 unit to the right direction and shifting 13 units on the axis of symmetry $x = 2$ then the graphs is given as follows.

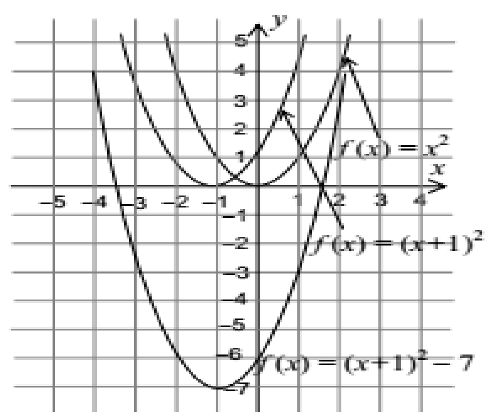
$f(x)=(x-2)^2+13$



X	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
$f(x) = (x-2)^2$	25	16	9	4	1	0	1
$f(x) = (x-2)^2 + 13$	38	29	22	17	14	13	14

D. Similarly to draw the graphs of $f(x) = (x+1)^2 - 7$ by shifting from the graphs of $f(x) = x^2$ to the left direction by one unit we get the graphs of $f(x) = (x+1)^2$, lastly we shift 7 unit downward on the line of (x=-1) axis of symmetry. Then using table value.

X	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
$f(x) = (x+1)^2$	4	1	0	1	4	9	16
$f(x) = (x+1)^2 - 7$	-3	-6	-7	-6	-3	2	9



Generally:

- ✓ Quadratic function is a function of the form $f(x) = ax^2 + bx + c,\ with\ a \neq 0$
- ✓ The domain of the quadratic function is the set of all real number.
- ✓ The graph of all quadratic function is called parabolas.
- ✓ The lowest or the highest point of the parabola called the vertex.
- ✓ The vertical line passing through the vertex in each parabola is called axis of symmetry.
- ✓ To graph and quadratic function $f(x) = ax^2 + bx + c$,

$\Rightarrow$ $f(x) = ax^2 + bx + c$ by completing the square, expressed as:

$= a\left(x^2 + \frac{b}{a}x\right) + c$ ⟵Factoring a form of $ax^2 + bx$

$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a^2}$

$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$, from these we conclude the following

If $h = -\frac{b}{2a}$, and $k = \frac{4ac-b^2}{4a}$, then $f(x) = ax^2 + bx + c$ is expressed as $f(x) = a(x-h)^2 + k$, then

- ➢ Vertex $(h,\ k)$
- ➢ Axis of symmetry, $x = h$
- ➢ If $a > 0$, the graph is open upward
- ➢ If $a < 0$, the graph is open downward
- ➢ The graph is narrows as $|a|$ is increase
- ➢ To obtain the graph of $f(x) = a(x-h)^2 + k$, shifting the graphs of $f(x) = ax^2$

i. We shift h unit to the right if $f(x) = a(x-h)^2 + k$ h unit to the left if $f(x) = a(x+h)^2 + k$

ii. We shift k, unit up if $k > 0$, and down if $k < 0$

Exercise

1. For each of the following question express the equation in the form of $f(x) = a(x-h)^2 + k$, sketch the graph and find its range.

   A. $f(x) = -x^2 + 2x + 1$ B. $f(x) = -x^2 - 2x - 4$

2. Find equation of quadratic function whose vertex is (1,-5) and y-intercept is -3

Minimum and maximum values of quadratic functions

When the graph of a quadratic function opens upward, the function has a minimum value, whereas if the graph opens downward, it has a maximum value. The minimum or the maximum value of a quadratic function is obtained at the vertex of its graph.

Example: Determine the minimum or the maximum value of each of the following functions and draw the graphs:

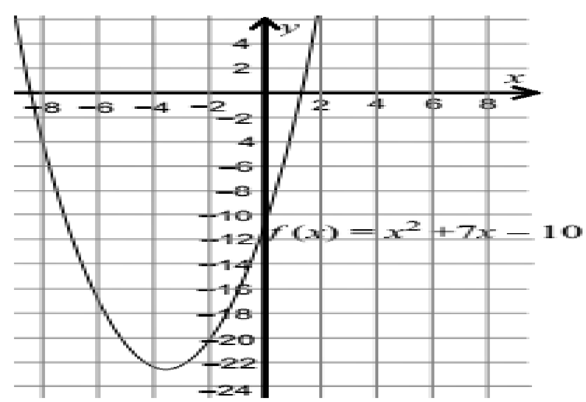
a. $f(x) = x^2 + 7x - 10$

b. $f(x) = -6 - x^2 - 4x$(exercise)

c. $f\ (x)\ =\ -x^2 +\ 6x\ -\ 5$ (exercise)

Solution: a $f(x) = x^2 + 7x - 10 = \left(x + \frac{7}{2}\right)^2 - \frac{89}{4}$

Then when we sketch its graph by using shifting rule it is given as follows ,Then vertex $(h,\ k) = (-\frac{7}{2},\ -\frac{89}{4})$ and the graphs is open upward The minimum value is y=$-\frac{89}{4}$=-22.25 or f(-b/2a)=f(-7/2)=-89/4



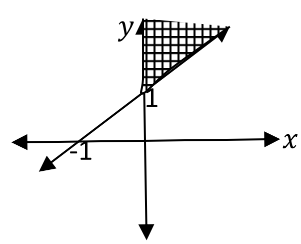
Grade 9 Mathematics Note And Some Practice Question For Second Semester

SOME PARCTICE QUESTION ON RELATION AND FUNCTION

FOR EACH OF THE FOLLOWING QUESTIONS CHOOSE THE BEST ANSWER FROM THE GIVEN ALTENATIVE

1. Let $A = \{-1, 0, 1\}$. A relation R on set A is defined by R=$\{(x, y) : y < 2x,\ where\ x, y \epsilon A\}$ then which one of the following is equal to R?
   A. $\{(0,-1),(1,-1),(1,0),(1,1)\}$ $B.\{(0,-1)\}$ $C.\ \{(0,-1),(1,-1)\}$ $D.\ \{(0,-1),(-1,0),(0,1)\}$
2. Given the function $f = \{(1,0),(-1,2),(3,4),(2,1)\}\ and\ g = \{(0,0),(2,3),(3,-1),(1,-1)\}$. Then which one of the following is correct?
   A. $2f(2) = 2g(1)$ B. $(f+g)(1) = 1$ C. $(f.g)(2) = -1$ D. $(f-g)(3) = 5$
3. If f is a function which is given by $f(x) = x^2 + (1-x)^2 - 13$, then what is the minimum values of this function?
   A. ½ B. -12 C. -25/2 D. -21/2
4. If $f(x) = 3x - 3\ and\ g(x) = 2 - 3x,\ then\ (2f + 3g)(x)$ is equal to:-
   A. $-9x^2 + 15x - 6$ $B.\ -3x$ $C.\ 12x + 6$ $D. -6x + 5$
5. If $p(x) = \sqrt{x} + 2, and\ q(x) = \frac{1}{\sqrt[3]{x}}$, then which one of the following is true?
   A. $(p.q)(1) = 3$ $B.\ (p.q)(4) = 2$ $C. (p.q)(-1) = -3$ $D. (p.q)(0) = 2$
6. If a function is defined by $y = -x^2 + 2x + 1$, which of the following is the range of this function?
   A. $\{y\epsilon\mathbb{R}: y \geq 1\}$ $B.\{y\epsilon\mathbb{R}: y \leq 1\}$ $C.\ \{y\epsilon\mathbb{R}: y \geq 2\}$ $D.\{y\epsilon\mathbb{R}: y \leq 2\}$
7. Which one of the following statement is true about the function $f(x) = (x-1)^2 + 3$
   A. It is increasing in the interval $(1, \infty)$ C. the graphs is symmetric with respect to y-axis
   B. The x intercept is 1 D. the range is $[3, \infty)$
8. Let $f(x) = \sqrt{3-x}\ and\ g(x) = x^2 + 2x + 1$, then which one of the following statements is correct?
   A. The domain of $\left(\frac{f}{g}\right) is\ (3, \infty)$ C. The domain of $\left(\frac{f}{g}\right) is\ (-\infty, 3]$
   B. The domain of $(f+g)(x)$ is the set of real number D. The domain of $(f+g) is\ (-\infty, 3]$
9. If a function p is given by $p(x) = (1-2x)^2 + 4$, then what the range of p?
   A. $(-\infty, \infty)$ $B.\ [8, \infty)$ $C.\ [4, \infty)$ $D.\ (-\infty, -4]$
10. The quadratic function f given by $f(x) = 2x^2 + 3x + 1$, which one of the following statements is correct about the graph of this function?
    A. It is increasing in the interval $\left(-\infty, -\frac{3}{4}\right)$ C. it is symmetric about y-axis
    B. It crosses the y-axis at exactly one point D. It is increasing on the interval $\left[-\frac{3}{2}, \infty\right)$
11. Which one of the following is not true about quadratic function of $f(x) = (x-2)^2 + 4$?
    A. The graph of the function is open upward parabola C. It is increasing in the interval of $[2, \infty)$
    B. The vertex of the graph is v (-2, 4) D. It is decreasing in the interval of $(-\infty, 2]$
12. If $h = \{(1,2),(2,3),(3,5)\}\ and\ g(x) = \frac{3}{2}x^2 + \frac{1}{2}x$. Then $\left(\frac{g}{h}\right)(3) =$ ___
    A. Cannot be determined. B. 15 C. 5 D. 3
13. Which one of the following statement is correct?
    A. If $f(x) = 2\ and\ g(x) = 1, then\ (gof)(x) = 2$
    B. For any function of $f(x)$ and its inverse of $f^{-1}(x)$, $(fof^{-1})(x) = (f^{-1}of)(x) = x$
    C. If $f = \{(2,3),(9,3),(-1,3)\}\ and\ g = \{(2,10),(3,12),(9,13)\}$, then $(gof) = \{(2,12),(9,12),(-1,12)\}$ is not a function
    D. If $f(x) = 4\ and\ g(x) = 3x + 2, then\ (gof)(x) = g(x)$

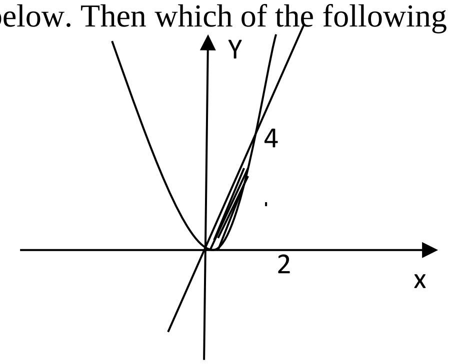
14. Which of the following is statement is true about an even function?
   A. The graph is symmetric with respect to the y-axis.
   B. It is one to one function
   C. it is decreasing function.
   D. It is an increasing function.
15. Let $f(x) = \sqrt{x+4}$, then which of the following is not true about $f^{-1}(x)$?
   A. $f^{-1}(x) = x^2 + 4$
   B. The x-intercept of $f^{-1}(x)$ $are -2$ and 2
   C. $The\ domain\ of\ f^{-1}(x)\ is\ the\ set\ of\ real\ numbers.$
   D. The range of $f^{-1}(x)$ is $[-4, \infty)$
16. If a function f is given by $f(x) = \sqrt{9-x^2}$; where $x \in [-3,3]$ then the range of f is ___
   A. $[-3,3]$
   B. $[0,3]$
   C. $[-9,9]$
   D. R
17. If $f(x) = x^2 + 6x + 9$, then which one of the following statements is true about the graph of f ?
   A. The minimum value of this function is 9
   B. The graph of f is a symmetrical with respect to the y-axis
   C. The range of f is $\{y/y \geq 0\}$
   D. The graph of f is open down ward.
18. Which one of the following is true about the graph of a quadratic function of $f(x) = -ax^2 + bx + c$?
   A. It has exactly two zeros.
   B. It has exactly one x intercept.
   C. It has exactly one y-intercept
   D. It is increasing
19. The graphs of quadratic function given by $f(x) = ax^2 + b$ lies above x-axis, when ever
   A. Both a and b are positive
   B. a and b has opposite in sign
   C. a Is greater than b
   D. both $a\ and\ b$ are negative
20. Which one of the following is **TRUE** about the graphs of the function of $f(x) = -(x-4)^2 - 3$
   A. The vertex of the function is (4,3)
   B. The range of f is $\{y : y \leq 3\}$
   C. The graph is decrease in the interval of $(-\infty,4]$
   D. The graph is increase in the interval of $[4,\infty)$
21. Which one of the following is the domain of $f(x) = -\sqrt{2-x}$?
   A. $\{x \in \Re : x \geq 0\}$
   B. $\{x \in \Re : x \leq 6\}$
   C. $\{x \in \Re : x \geq 2\}$
   D. $\{x \in \Re : x \leq 2\}$
22. Given a function $f(x) = ax + b$, where a and b are constant real numbers. Which one of the following is **NOT** correct?
   A. The graph of f is a straight line.
   B. If $a > 0$, then the graph of f is increasing.
   C. If $a = 0$, then the graph of f is a vertical line.
   D. If $a \neq 0$, then $(-\frac{b}{a}, 0)$ is the point at which the graph of f intersects the x −axis
23. Which of the following relations is represented by the shaded region shown on the figure below?
   A. $R = \{(x, y) : y \leq x-1 \text{ and } x \leq 0\}$
   B. $R = \{(x, y) : y \geq x+1 \text{and } x > 0\}$
   C. $R = \{(x, y) : y \leq x+1 \text{ and } x \leq 0\}$
   D. $R = \{(x, y) : y \geq x+1 \text{ and } x \geq 0\}$



24. Let $f(x) = 3x - 3$ and $g(x) = \frac{2}{x-1}$, then $\left(\frac{f}{g} - 2f\right)(3) =$ __________.
   A. -6
   B. 6
   C. 18
   D. 0

25. Given $f(x) = 3x(x-2)^2 - x + 6$ and $g(x) = -x(2x+3)^2 + 2x(x-1)$, then which one of the following is NOT correct?
    A. The domain of $(f.g)(x)$ is the set of real numbers , $\mathbb{R}$
    B. $(f.g)(1) = -200$
    C. $\left(\frac{f}{g}\right)(-1) = -4$
    D. The degree of $(f.g)(x)$ is5.

26. Consider the graph of a relation $R = \{(x, y) : y \geq x^2 and, y \leq 2x\}$ that shown below. Then which of the following is **TRUE** about this relation?
    A. The domain of R is $\{x \in \Re : x \leq 2\}$
    B. The range of R is $\{y \in \Re : y > 0\}$
    C. The domain of R is $\{x \in \Re : 0 \leq x \leq 2\}$
    D. The range of R is $\{y \in \Re : 0 < y < 4\}$



27. The graph of a quadratic function has vertex $(-2,4)$ and intercepts y-axis at $(0,2)$. Which one of the following can represent the function?
    A. $f(x) = -\frac{1}{2}(x+2)^2 + 4$
    B. $f(x) = -\frac{1}{2}x^2 - 2x + 2$
    C. $f(x) = (x+2)^2 + 4$
    D. A and B

28. The vertex of a quadratic function $f(x) = x^2 - 2x + 1$ is:
    A. $(1, 0)$ B. $(0, 1)$ C. $(0, 0)$ D. $(1, 1)$

29. Which of the following relation is a function?
    A. $\{(x, y) | y$ is a natural number and x is a prime factor of $y\}$
    B. $\{(x, y) | y$ is a father of $x\}$
    C. $\left\{(-2,-8), (-8,-2), \left(-2, \frac{1}{2}\right), (2, \sqrt[3]{2}), (-1, 1)\right\}$
    D. $\{(x, y) | x^2 + y^2 = 4\}$

30. If a, b, and c are non-zero real numbers, then which one of the following quadratic functions has a graph which is symmetric with respect to the $y-axis$?
    A. $f(x) = ax^2 + x$ B. $f(x) = a(x+b)^2$ C. $f(x) = ax^2 + c$ D. $f(x) = a(x+b)^2 + c$

CHAPTER FIVE

GEOMETRYANDMEASUREMENT

5.1 REGULAR POLYGONS

Definition

- ✓ A polygon is a simple closed curve, formed by the union of three or more line segments, no two of which in succession are collinear. The line segments are called the sides of the polygon and the end points of the sides are called the vertices

Types of polygon: Based on the measure of its interior angle of the polygon can be classified as

- ✓ Convex polygon: a polygon whose interior angle is less than 180^0
- ✓ Concave polygon: a polygon whose interior angle is greater than 180^0

Note:

- ✓ Polygons can be classified according to the number of sides.
- ✓ The number of vertices, angles and sides of a polygon are the same.

No of side	No of interior angles	Name of polygon
3	3	Triangle
4	4	Quadrilateral
5	5	Pentagon
6	6	Hexagon
7	7	Heptagon
8	8	Octagon
9	9	nonagon
10	10	Decagon

ANGLES OF POLYGONS

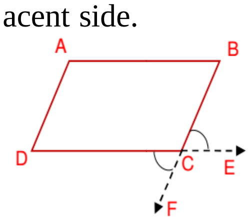
There are two types of angles of polygon: these are

i. **Interior angle**: is an angle of a polygon formed at the vertex on the inside of the polygon.

ii. **Exterior angle:** is an angle at a vertex of a polygon that is supplementary to the interior angle at the vertex. It is formed between one side of the polygon and the extended adjacent side.

Example: In the polygon ABCD

∠DCB is an interior angle; ∠BCE and ∠DCF are exterior angles of the polygon at the vertex C



The sum of the measures of the interior angles of a polygon

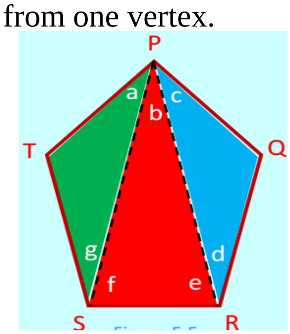
Theorem 5:1 Angle sum theorem

The sum of the measures of the three interior angles of any triangle is 180^0

Note: The polygons are divided into triangles by drawing a diagonal from one vertex

Example: consider each of the convex polygons with all possible diagonal from one vertex.

No of sides of polygon	No of triangles formed from one vertex	Sum of interior angles of polygon
3	1	$1x180^0 = 180^0$
4	2	$2x180^0 = 360^0$
5	3	$3x180^0 = 540^0$



Generally: a polygon of n side has

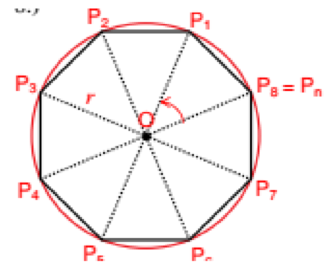
i. (n-3) diagonal from one vertex. ii) (n-2) triangles

Theorem 2: if the number of side of polygon is n, then the sum of the measure of all of its interior angle is equal to $(n-2)180^0$.

Exercise

1. If the measures of the interior angles of a hexagon are $x°,\ 2x°,\ 60°,\ (x+60)°,\ (x-10)°$ and $(x+40)°$, find the value of x
2. Show that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles.

Measures of Angles of a Regular Polygon



In each triangle, the degree measure of the central angle O is given by:

$$m(<O) = \frac{360°}{n}$$

Since the vertex angles at O of each isosceles triangle have equal measures,

Namely $\frac{360°}{n}$, it follows that all the base angles of all the isosceles triangles are also equal.

From this, it follows that the measures of all the angles of the polygon are equal; the measure of an angle of the polygon is twice the measure of any base angle of any one of the isosceles triangles. So, the polygon has all of its sides equal and all of its angles equal. A polygon of this type is called a regular polygon.

Definition 5.3

A regular polygon is a convex polygon in which the lengths of all of its sides are equal and the measures of all of its angles are equal.

- ✓ Note that the measure of an interior angle of an n-sided regular polygon is $\frac{S}{n}$, where $S = (n-2) \times 180°$ is the sum of the measures of all of its interior angles.
- ✓ Then the measure of each interior angle of a regular n-sided polygon is $\frac{(n-2)180°}{n}$
- ✓ A polygon is said to be inscribed in a circle if all of its vertices lie on the circle.

Example: Find the measure of an interior and exterior angle of a regular polygon with:

a. 10 sides **b.** 20 sides **c.** 12 sides

Solution:

Then the measure of each interior angle of a polygon is given as

a. The measure of an interior angle of a 10-sided regular polygon is $\frac{(n-2)180^0}{n} = \frac{8(180^0)}{10} = 144^0$

b. The measure of an interior angle of a 20-sided regular polygon is $\frac{(n-2)180^0}{n} = \frac{18(180^0)}{20} = 162^0$

c. The measure of an interior angle of a 10-sided regular polygon is $\frac{(n-2)180^0}{n} = \frac{10(180^0)}{12} = 150^0$

And the measure of each exterior angle of a n side of regular polygon is given as $\frac{360°}{n}$

a. The measure of each exterior of 10 side is $\frac{360°}{10} = 36°$

b. The measure of each exterior of 10 side is $\frac{360°}{20} = 18°$

c. The measure of each exterior of 10 side is $\frac{360°}{12} = 30°$

Generally: For any regular n-sided polygon:

i. Measure of each interior angle = $\frac{(n-2)180°}{n}$

ii. Measure of each central angle = $\frac{360°}{n}$

iii. Measure of each exterior angle = $\frac{360°}{n}$

iv. Measure of the sum of interior angle =$(n-2)180°$

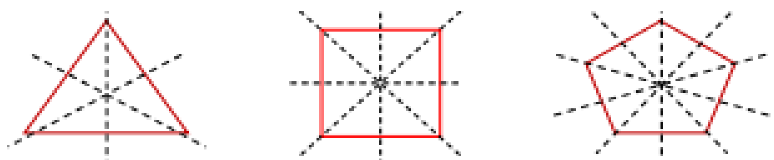
v. The sum of the measure of exterior angle of any polygon, taking one angle at each vertex is $360°$.

5.1.2 Properties of Regular Polygons

A symmetric properties of regular polygon

- ✓ A plane figure can be divided exactly into two identical parts by line $(l)$the line l is called line of symmetry
- ✓ A figure is called symmetric figure if it has at least one line of symmetry
- ✓ Regular polygons are symmetric

EXAMPLE 1



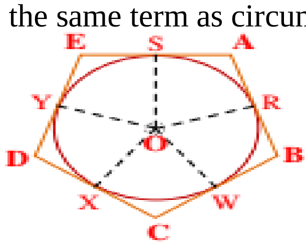
Note

- ❖ A regular n-side polygons has n-line of symmetry
- ❖ A regular polygon of odd number of sides has every line of symmetry, passes through the vertex.

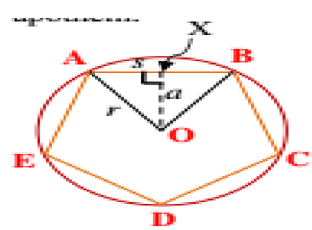
Circumscribed (inscribed) regular polygons

- ✓ A polygon whose sides are tangent to a circle is said to circumscribe the circle.
- ✓ An inscribed polygon is a polygon all of whose sides are a chords of a circle.
- ✓ The circle is called circumscribed about the polygon if all sides polygons are chords of a circle. Inscribe polygon is the same term as circumscribed circle.

EXAMPLE 2



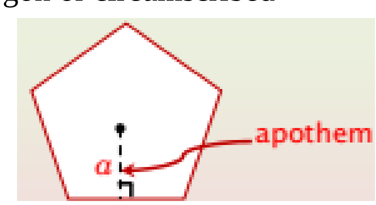
Inscribed circle



inscribed polygon or circumscribed

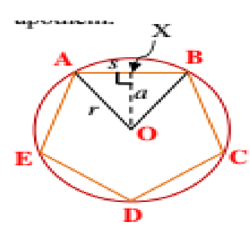
Definition 5.4

The distance a from the center of a regular polygon to a side of the polygon is called the *apothem* of the polygon. That is, the apothem a of a regular polygon is the length of the line segment drawn from the center of the polygon perpendicular to the side of the polygon.



Example 1 In the regular pentagon ABCDE is inscribed in a circle with center O and radius r.

Write formulae for the side s, perimeter P, apothem a and area A of the regular pentagon.



Solution:

ΔOAB is an isosceles triangle. Draw the perpendicular from O to AB .

It meets AB at X. ∠AOB is a central angle of the regular pentagon.

So $M(<AOB)=\frac{360^0}{5}=72^0$ NOW $\triangleleft AOX \cong \triangleleft BOX$

Therefore $\angle AOX=\angle BOX$, Then $M(\angle AOX)=M(\angle BOX)=\frac{1}{2}M(\angle AOB)=\frac{1}{2}\left(\frac{360^0}{5}\right)=36^0$.

Let $s=A\overline{B}$, the length of side of regular pentagon.

Since $\triangleleft AOX \cong \triangleleft BOX$, we have $A\overline{X}=X\overline{B}A\overline{X}=\frac{1}{2}A\overline{B}=\frac{1}{2}s$.

$$\sin(\angle AOX)=\frac{AX}{AO} \Rightarrow \sin\left(\frac{1}{2}(\angle AOB)\right)=\frac{\frac{1}{2}s}{r}$$

$$\sin 36^0=\frac{\frac{1}{2}s}{r} \quad so \ \frac{1}{2}s=r\sin 36^0$$

Now in the right angled triangle AOX you see that

Therefore, $s=2r\sin 36^0$ . . . . (1)

Perimeter P of the polygon is

P = AB + BC + CD + DE + EA

But since AB = BC = CD = DE = EA = s, we have P = s + s + s + s + s = 5s. Since from (1) we have s = 2r sin 36o, the perimeter of the regular pentagon is P = 5 × 2r sin 36^0

∴ P = 10r sin 36^0. . . . . (2)

To find a formula for the apothem, a, consider ΔAOX

$\cos(<AOX)=\frac{XO}{AO}$, Since $M(\angle AOX)=36^0, XO=a$ and $AO=r$.

Then $\cos 36^0=\frac{a}{r}$ **so** $a=r\cos 36^0$

To find the area of the regular pentagon, first we find the area of ΔAOB. Taking the height and the base of ΔAOB as OX and AB, respectively, we have,

Since all these triangles are congruent, the area of each triangle is $\frac{1}{2}as$

So, the area of the regular pentagon ABC$=5\left(\frac{1}{2}as\right)=\frac{1}{2}a(5s)=\frac{1}{2}ap=\frac{1}{2}r\cos 36^0 5x2r\sin 36^0=5xr^2\sin 36^0\cos 36^0$

Since $36^0=\frac{180^0}{5}$, where 5 is the number of their sides, we can generalize the above formulae for any n-sided regular polygon by replacing 36^0 by $\frac{180^0}{n}$, as follows

Theorem 5.3

Formulae for the length of side *s*, apothem *a*, perimeter *P* and area *A* of a regular polygon with *n* sides and radius *r* are

1. $s=2r\sin\frac{180°}{n}$
3. $a=r\cos\frac{180°}{n}$
2. $P=2nr\sin\frac{180°}{n}$
4. $A=\frac{1}{2}aP$

Example 2

a. Find the radius of an equilateral triangle with perimeter 24 units.
b. Find the radius of a regular hexagon with perimeter 48 units.
c. Find the area of a regular hexagon whose radius is 5 cm.
d. Show that the length of each side of a regular hexagon is equal to the length of the radius of the hexagon.

Solution: a.

$$p = 2nr\sin\frac{180^0}{n}$$
$$24 = 2x3xr\sin\frac{180^0}{3}$$
$$24 = 6r\sin 60^0$$
$$4 = \frac{\sqrt{3}}{2}r \Rightarrow r = \frac{8\sqrt{3}}{3}units$$

b.

$$p = 2nr\sin\frac{180^0}{n}$$
$$48 = 2x6xr\sin\frac{180^0}{6}$$
$$48 = 12r\sin 30^0$$
$$48 = 6r \Rightarrow r = 8units$$

C. To find the area of the regular hexagon, we use the formula

$A=\frac{1}{2}aP$, where a is the apothem and P is the perimeter of the regular hexagon.

Therefore, $A=\frac{1}{2}aP = \frac{1}{2}\left(rcos\frac{180°}{n}\right)\left(2nrsin\frac{180°}{n}\right) = \frac{1}{2}(5cos30^0)(2x6x5sin30^0) = \frac{75\sqrt{3}}{2}$units.

D. We know that the length of a side s of an n-sided regular polygon is given by $s = 2r\sin\frac{180^0}{n}$

Where r is the radius of the polygon

. If n = 6 then, $s = 2r\sin\frac{180^0}{6} = 2r\sin 30^0 = 2rx\frac{1}{2} = r \Rightarrow s = r$

Therefore, the length of a side s of a regular hexagon is equal to the radius r of the hexagon.

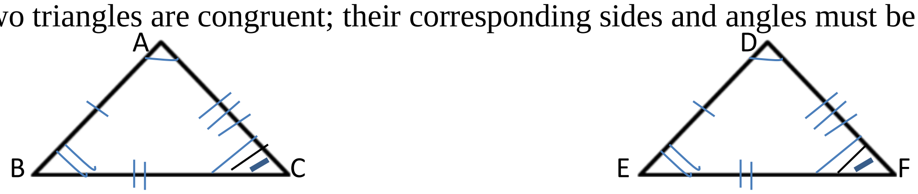
5.2 FURTHER ON CONGRUENCY AND SIMILARITY

A. CONGRUENCY

- ✓ Two geometric figures are said to be congruent if they have the same shape and the same size.
- ✓ Two line segments AB and CD are congruent if AB=CD
- ✓ Two angles are congruent if and only if their measures are equal.

CONGRUENCY OF TRIANGLE.

- ➢ Congruent triangles are triangles that have the same size and the same shape.
- ➢ Triangles that fit exactly are called congruent triangle.
- ➢ If two triangles are congruent; their corresponding sides and angles must be equal.++++++++



The six congruent part of the triangle are the three corresponding angles and the three corresponding size of ΔABC and ΔDEF denoted by $\Delta ABC \cong \Delta DEF$

These are

$$\left.\begin{array}{l} AB \equiv DE \\ BC \equiv EF \\ AC \equiv DF \end{array}\right\} \text{Corresponding sides} \qquad \left.\begin{array}{l} \angle A \equiv \angle D \\ \angle B \equiv \angle E \\ \angle C \equiv \angle F \end{array}\right\} \text{Corresponding angles}$$

Four Basic Principles of Congruent Triangle

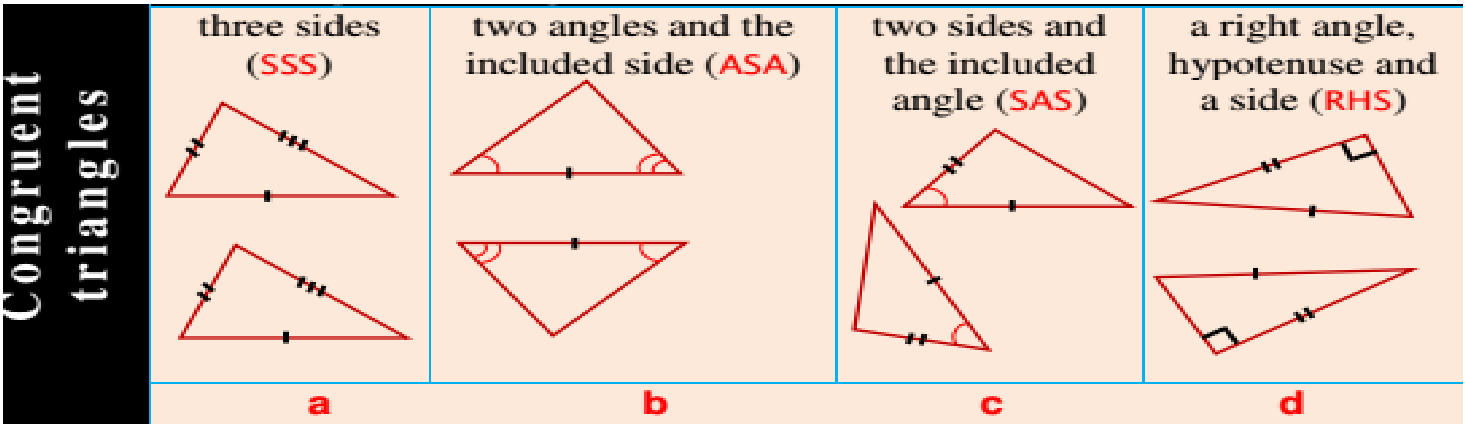
These are: i. SSS (Three sides)

ii. ASA (Two angles and included side)

iii. SAS (Two sides and included angle)

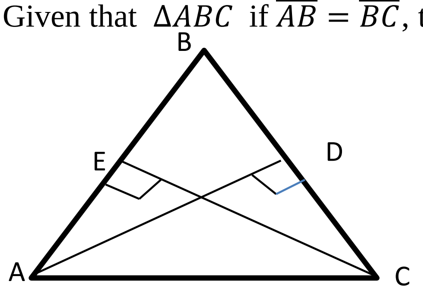
iv. RHS (Right angle hypotenuse and sides)

Generally: **Two triangles are congruent if the following corresponding parts of the triangles are congruent**.



__Example__:1

1. Given that ΔABC if $\overline{AB} = \overline{BC}$, then prove $\overline{AD} = \overline{CE}$



__Explanation__: Two columns proof

Statements | Reasons

1. $\overline{AB} \equiv \overline{BC}$
2. $\angle BAC \equiv \angle BCA$
3. $\overline{AB} \equiv \overline{BC}$
4. $\angle AEC \equiv \angle CDA$
5. $\angle CAD \equiv \angle ACE$
6. Therefore $\Delta ACD \equiv \Delta CEF$, by ASA

$\therefore \overline{AD} \equiv \overline{CE}$

B. SIMILAR FIGURES

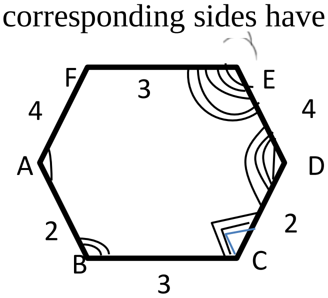
Figures that have the same shape but that might have different sizes are called similar.

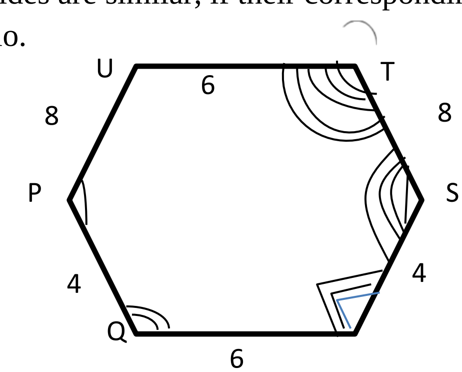
In similar figures:

i. One is an enlargement of the other.
ii. Angles in corresponding positions are congruent.
iii. Corresponding sides have the same ratio.

Similar polygons: Two polygons of the same number of sides are similar, if their corresponding angles are congruent and their corresponding sides have the same ratio.

__Example__:





The above hexagons are similar, which is denoted by $ABCDEF \sim PQRSTU$ provided that

i. $\angle A \equiv \angle P, \angle B \equiv \angle Q, \angle C \equiv \angle R, \angle D \equiv \angle S, \angle E \equiv \angle T$ and $\angle F \equiv \angle U$

ii. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DE}{ST} = \frac{EF}{TU} = \frac{FA}{UP} = \frac{2}{4} = \frac{1}{2}$

NOTE:

- **i.** The ratio of corresponding sides of two similar polygons is called the **scale factor.**
- **ii.** The scale factors of $ABCDEF$ to $PQRSTU$ is $\frac{1}{2}$
- **iii.** The scale factors of $PQRSTU$ to $ABCDEF$ is 2

Theorems on Similarity of Triangles

How do you know whether two triangles are similar?

There are also tests to determine whether two triangles are similar

These are:

i. AA-Similarity (Angle-Angle similarity)

ii. SSS-Similarity (Side- Side- Side similarity)

iii. SAS-Similarity (side-angle-side similarity)

A. AA-Similarity (Angle-Angle similarity)

If two angles of one triangle are congruent to two corresponding angles of another triangle, then the two triangles are similar. i.e. $\Delta ABC \sim \Delta DEF$ are similar by AA-similarity from A with D, B with E and C with F if any two corresponding angles are equal.

B. SSS-Similarity (Side- Side- Side similarity)

Two triangles are similar, if two pairs of corresponding sides of the two triangles are proportional and if the included angles between these sides are congruent.

C. SAS-Similarity (side-angle-side similarity)

Two triangles are similar, if two pairs of corresponding sides of the two triangles are proportional and if the included angles between these sides are congruent.

Theorems on Similar Plane Figures

Ratio of Perimeters and Ratio of Areas of Similar Plane Figures

Here we are going to see the way how to relate the side length, perimeter and areas of similar triangles (polygons).

Let P_1= Perimeter of the first triangle or polygon

P_2= Perimeter of the second triangle or polygon

A_1= Area of the first triangle or polygon

A_2= Area of the second triangle or polygon

Theorem: If the ratio of the lengths of any two corresponding sides of two similar triangles or polygons Are k, then

i. The ratio of their perimeters is given by $\frac{P_1}{P_2} = \frac{S_1}{S_2} = k$

ii. The ratio of their Area is given by $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2 = k^2$

NOTE: Area ratio= (Side length ratio)^2 = (Perimeter ratio)^2

Proof: EXERCISE FOR STUDENT

Example: 2

A. The areas of two similar triangles are 144 unit2 and 81 unit2.

i. What is the ratio of their perimeters?

ii. If a side of the first is 6 units long, what is the length of the corresponding side of the second?

B. The sides of a polygon have lengths 5, 7, 8, 11, and 19 units. The perimeter of a similar polygon is 75 units. Find the lengths of the sides of the larger polygon.

Solution:

A. i. The ratio of their perimeters is $= \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{144}{81}} = \frac{12}{9} = \frac{4}{3} OR \frac{3}{4}$

ii Let x unit be the corresponding side of the second triangle. Then,

$$\frac{6}{x} = \frac{4}{3} \Rightarrow x = \frac{(3)(6)}{4} = \frac{9}{2} = 4.5 \text{ units}$$

B. Let the lengths of the corresponding sides of the other polygon be a, b, c, d and e.

$$\therefore \frac{5+7+8+11+19}{75} = \frac{5}{a} = \frac{7}{b} = \frac{8}{c} = \frac{11}{d} = \frac{19}{e} \Rightarrow \frac{50}{75} = \frac{5}{a} = \frac{7}{b} = \frac{8}{c} = \frac{11}{d} = \frac{19}{e}$$

Therefore, the lengths of the sides of the larger polygon are 7.5, 10.5, 12, 16.5, and 28.5 units.

Construction of Similar Figures

Enlargement

An enlargement with center O and scale factor k (where k is a real number) is the transformation that maps each point P to point P′ such that

i. P′ is on the ray O P and ii. O P′ = k OP

If an object is enlarged, the result is an image that is mathematically similar to the object but of different size. The image can be either larger, if k > 1, or smaller if 0 < k <1.

Note: If the scale factor of enlargement is greater than 1, then the image is larger than the object. If the scale factor lies between 0 and 1 then the resulting image is smaller than the object. In these latter cases, although the image is smaller than the object, the transformation is still known as an enlargement.

Eample:3

A tree casts a shadow of 30 m. At the same time, a 10 m pole casts a shadow of 12 m. Find the height of the tree.

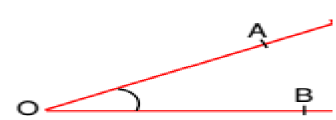
Solution:

Assuming both the pole and the tree are vertical to the ground, we have equality of their ratios as

$\frac{h}{10} = \frac{30}{12}$. Therefore, the height of the tree is 25m.

5.3. FURTHER ON TRIGONOMETRY

Angle: An angle is the union of two rays with a common end point.



In general, we associate each angle with a real number called the **measure of the angle**. The two measures that are most frequently used are degree and radian.

i. **Measuring angles in degrees**

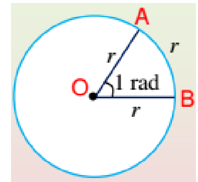
Definition: A **degree,** denoted by (^{0}), is defined as the measure of the central angle subtended by an arc of a circle equal in length to $\frac{1}{360}$ of the circumference of the circle.

Grade 9 Mathematics Note And Some Practice Question For Second Semester

Note:

- ✓ A **minute** which is denoted by ('), is $\frac{1}{60}$ of a **degree.**
- ✓ A **second** which is denoted by ("), is $\frac{1}{60}$ of a **minute,** From these we have $1^{o}=3600''$ relation.

ii. Measuring angles in radians



Definition: A radian (rad) is defined as the measure of the central angle subtended by an arc of a circle equal in length to the radius of the circle.

You know that the circumference of a circle is equal to 2πr. Since an arc of length r along the circle gives 1 rad, a complete rotation of length 2πr generates an angle of 2π radians. On the other hand, we know that a complete revolution represents an angle of 360^0. This gives us the following relationship:

$$\mathbf{1\ revolution = 360^0 = 2\pi\ radians}$$

i.e., $180^0 = \pi$ radians, from which we obtain,

Therefore, we have the following conversion rules for degrees and radians.

- ✓ **To convert radians to degrees, multiply by** $\frac{180^0}{\pi}$
- ✓ **To convert degrees to radians multiply by** $\frac{\pi}{180^0}$

<u>EXERCISE</u>

a. Express each of the following radian measures in degrees:

i. π/6 ii. π iii. π /3 iv. 2 v. 3/4π vi. 5

b. Express each of the following in radian measure:

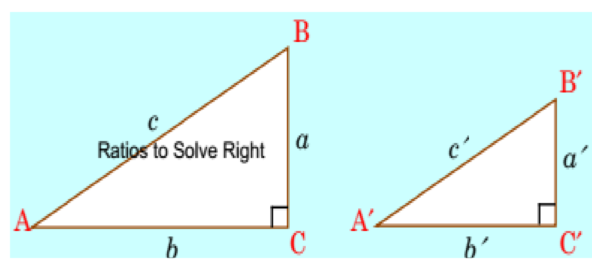
i. 270^0 **ii.** 150^0 **iii.** 225^0 **iv.** 15^0

Trigonometrical Ratios to Solve Right-Ratios to Solve Right

Using similarity ratio of sides of triangles it is possible to drive the trigonometrical ratio on the triangles.

From the similarity ratio

From this we get,



1 $\frac{BC}{AB} = \frac{B'C'}{A'B'}$ 2 $\frac{AC}{AB} = \frac{A'C'}{A'B'}$ 3 $\frac{BC}{AC} = \frac{B'C'}{A'C'}$

OR $\frac{a}{c} = \frac{a'}{c'}$, $\frac{b}{c} = \frac{b'}{c'}$ and $\frac{a}{b} = \frac{a'}{b'}$

The fractions or ratios in each of these proportions are called trigonometric ratio

Sine:- The fractions in proportion 1 above are formed by dividing the opposite side of ∠A (or ∠A') by the hypotenuse of each triangle. This ratio is called the sine of ∠A. It is abbreviated to sin A.

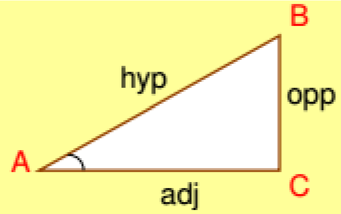
Cosine:- The fractions in proportion 2 are formed by dividing the adjacent side to ∠A (or ∠A') by the hypotenuse of each triangle. This ratio is called the cosine of ∠A. It is abbreviated to cos A.

Tangent:- The fractions in proportion 3 are formed by dividing the opposite side of $\angle A$ (or $\angle A'$) by the adjacent side. This ratio is called the tangent of $\angle A$. It is abbreviated to tan A.

The above discussion can be summarized and expressed as follows.

$\sin A = \frac{opp}{hyp} = \frac{BC}{AB}$; $\cos A = \frac{adj}{hyp} = \frac{AC}{AB}$

$\tan A = \frac{opp}{adj} = \frac{BC}{AC}$



The angles 0^0, 30^0, 45^0, 60^0 and 90^0 are called special angles, because they have these exact trigonometric ratios, its values are given as follows.

$\angle A$	$30°$	$45°$	$60°$
sin A	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos A	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan A	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Example 1 A ladder 6 m long leans against a wall and makes an angle of 60^0 with the ground. Find the height of the wall. How far from the wall is the foot of the ladder?

Solution $\sin 60^0 = \frac{\text{heightof wall}}{\text{lengthof ladder}} \Rightarrow \text{heightof wall} = \frac{\sqrt{3}}{2} x6 = 3\sqrt{3}m$

$$\cos 60^0 = \frac{\text{the footof theladder fromwall}}{\text{lengthof ladder}} \Rightarrow \text{the footof theladder fromwall} = \frac{1}{2} x6 = 3m$$

Trigonometrical Values of Angles from Tables
(Sinθ, cosθ and tanθ, for $0^0 \le \theta < 180^0$)

NOTE:

✓ If you know the value of one of the trigonometric ratios of an angle, you can use a table of trigonometric ratios to find the angle. The procedure is illustrated in the following example.

Example 1

Find the measure of the acute angle A, correct to the nearest degree, if $\sin (A)^0 = 0.521$.

Solution: Referring to the "sine" column of the table, we find that 0.521 does not appear there. The two values in the table closest to 0.521 (one smaller and one larger) are 0.515 and 0.530. These values correspond to 31^0 and 32^0, respectively

Note that 0.521 is closer to 0.515, whose value corresponds to 31^0.

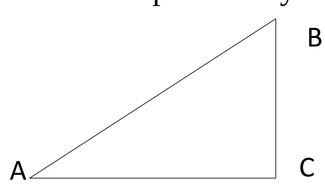
Therefore, $m(\angle A) = 31^0$. (to the nearest degree)

✓ **Using a right angle triangle you may have discovered the following**

A. If $m(\angle A) + m(\angle B) = 90^0$, i.e., A and B are complementary angles, then
   i. $\sin(\angle A) = \cos(\angle B)$
   ii. $\cos(\angle A) = \sin(\angle B)$

B. $\tan(\angle A) = \frac{\sin(\angle A)}{\cos(\angle A)}$

C. $\sin^2(\angle A) + \cos^2(\angle A) = 1$



How can you use the trigonometric table to find the sine, cosine and tangent of obtuse angles?

To find trigonometric value of an obtuse angle we see as follows.

Let β Be an obtuse angle and

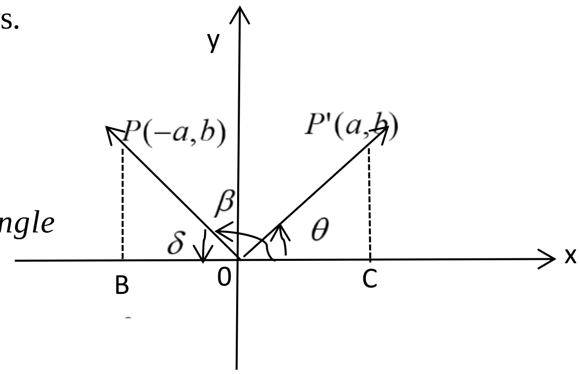
δ Be supplementary angle of β

Then δ and θ are equal in magnitude different direction

Because OP' is obtained by reflecting OP along y-axis.

Then $\beta+\delta=180^0 \Rightarrow \delta=180^0-\beta, becuase \beta,\delta$ suplemantaryangle

$\Rightarrow \delta=180^0-\beta \Rightarrow \theta=180^0-\beta$ becuase $\delta=\theta$



Therefore using the trigonometric definition of each trigonometric function as follows

1. To find the sine of β as expressed on the coordinate $P(-a,b)$, to relate with acute angle is shown as:

   $\sin\beta=\frac{b}{\sqrt{a^2+b^2}}$ And $\sin\theta=\frac{b}{\sqrt{a^2+b^2}}$

   θ Is an acute angle in the 1st quadrant, which is supplementary to obtuse angle β

   Then from this two relation $\sin\beta=\frac{b}{\sqrt{a^2+b^2}}=\sin\theta \quad \Rightarrow \sin\beta=\sin(180^0-\beta)$

2. To find cosine of β as expressed on the coordinate $P(-a,b)$, to relate with acute angle is shown as:

   $\cos\beta=\frac{-a}{\sqrt{a^2+b^2}}$ And $\cos\theta=\frac{a}{\sqrt{a^2+b^2}}$

   θ Is an acute angle in the 1st quadrant, which is supplementary to obtuse angle β

   Then from this two relation $\cos\beta=\frac{-a}{\sqrt{a^2+b^2}}=-\cos\theta \Rightarrow \cos\beta=-\cos(180^0-\beta)$

3. Similarly to find tangent of β whose coordinate is $P(-a,b)$, relating with acute angle is shown as:

   $\tan\beta=\frac{-a}{b}$ And $\tan\theta=\frac{a}{b}$, which implies $\tan\beta=\frac{-a}{b}=-\tan\theta \Rightarrow \tan\beta=-\tan(180^0-\beta)$

<u>Generally</u>: For an obtuse angle β we have

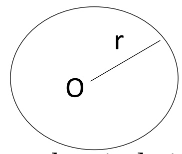
$$\boxed{\begin{aligned} &\Rightarrow \sin\beta=\sin(180^0-\beta)\\ &\Rightarrow \cos\beta=-\cos(180^0-\beta)\\ &\Rightarrow \tan\beta=-\tan(180^0-\beta)\end{aligned}}$$

Grade 9 Mathematics Note And Some Practice Question For Second Semester

5.4. Circles

Symmetrical Properties of Circles

Circle is defined as the set of points in a given plane, each of which is at the same distance from a fixed point of the plane. The fixed point is called the **center** and the distance is called **radius** of the circle



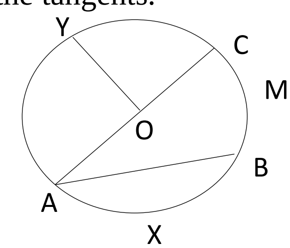
Note

- A line segment through the center of a circle with end points on the circle is called a **diameter**
- A **chord** of a circle is a line segment whose end points lie on the circle.
- A line that divided the figure and one part of the figure is coinciding with the other part divided by the line, then the line is called **symmetric line** and the figure is called **symmetric figure**.

Basic theorem of line on circle

- The line segment joining the center of a circle to the mid-point of a chord is perpendicular to the chord.
- The line segment drawn from the center of a circle perpendicular to a chord bisects the chord.
- If two chords of a circle are equal, then they are equidistant from the center.
- If two chords of a circle are equidistant from the center, then their lengths are equal.
  - ➢ If two tangent segments are drawn to a circle from an external point, then,
- The tangents are equal in length, and
- The line segment joining the center to the external point bisects the angle between the tangents.

Angle Properties of Circles

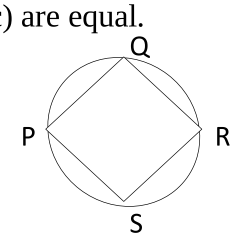


Basic terminologies of circle

- **Arc:** A part of a circle (part of its circumference) between any two points on the circle, say between A and B, is called an **arc** and is denoted by $\widehat{AB}$ or $\widehat{AXB}$
- **Semicircle:** Is an arc which is half of the circle is called semicircle, which is AC is a diameter. Then $\widehat{AXC}$ is semicircle of circle O.
- **Minor and major arc:** An arc is said to be a **minor arc**, if it is less than a semicircle and a **major arc**, if it is greater than a semicircle. For example: $\widehat{AXB}$ is minor arc whilst $\widehat{AYB}$ is a major arc.
- **Central angle:** A **central angle** of a circle is an angle whose vertex is at the center of the circle and whose sides are radii of the circle. For example: The angle $\angle AOY$, $\angle COY$ are an example of central angle.
- An **inscribed angle** in a circle is an angle whose vertex is on the circle and whose sides are chords of the circle. And an arc subtend the inscribed angle is called subtended arc. For example: $\angle CAB$ is an inscribed angle and $\widehat{BMC}$ is subtended arc.

Basic theorem of angles on circle

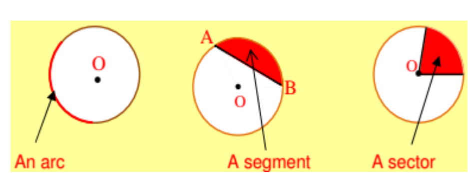
- The measure of a central angle subtended by an arc is twice the measure of an inscribed angle in the circle subtended by the same arc.
- Angles inscribed in the same arc of a circle (i.e., subtended by the same arc) are equal.
- The angle inscribed in a semi-circle is a right angle.
- Points P, Q, R and S all lie on circle. They are called **concyclic points**. Joining the point P. Q, R and S produces a cyclic quadrilateral. The opposite angles of a cyclic quadrilateral are supplementary. i.e. $m(\angle P) + m(\angle R) = 180^o$ and $m(\angle S) + m(\angle Q) = 180^0$.



Arc Lengths, Perimeters and Areas of Segments and Sectors

Remember that: For r is a radius, d is a diameter of a circle O.

- Circumference of a circle = $2\pi r$ or πd.
- Area of a circle = πr^2
- Part of the circumference of a circle is called an **arc**.

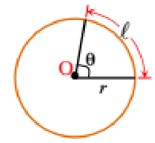


- A **segment** of a circle is a region bounded by a chord and an arc.
- A **sector** of a circle is bounded by two radii and an arc.

Arc length

The length ℓ of an arc of a circle of radius r that subtends an angle of θ at the center is given by

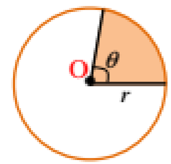
$$\ell = \frac{\theta}{360^0} x 2\pi r = \frac{\pi r\theta}{180^0}$$



The area and perimeter of a sector

The area A of a sector of radius r and central angle θ is given by

$$A = \frac{\theta}{360^0} x \pi r^2 = \frac{\pi r^2\theta}{360^0}$$



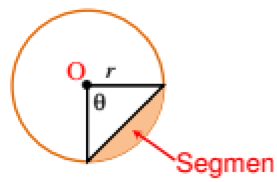
The perimeter P of the sector is the sum of the radii and the arc that bound it.

$$P = 2r + \frac{\pi r\theta}{180^0}$$

The area and perimeter of a segment

The area A and perimeter P of a segment of a circle of radius r, cut off by a chord subtending an angle θ at the center of a circle are given by

$$A = \frac{\pi r^2\theta}{360^0} - \frac{1}{2} r^2 \sin\theta \quad (\text{sector area - triangle area})$$



Perimeter of segments bounded between a chord and arc is given as

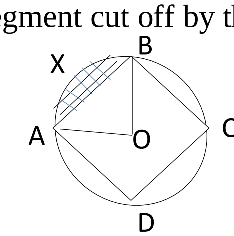
$$P = 2r\sin\frac{\theta}{2} + \frac{\pi r\theta}{180^0} \quad (\text{chord length} + \text{arc length})$$

Note:

The area formula for a triangle: $A = \frac{1}{2} ab\sin\theta$ where a and b are the lengths of any two sides of the triangle and θ is the measure of the angle included between the given sides

Example:

A. A square ABCD is inscribed in a circle of radius 4 cm. Find the area of the minor segment cut off by the chord AB



Solution:

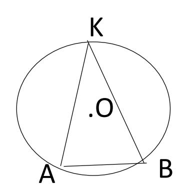
A minor segment is segment lie between chord $\overline{AB}$ and arc $\widehat{AXB}$.

$$A = \frac{\pi r^2\theta}{360^0} - \frac{1}{2} r^2 \sin\theta \quad (\text{sector area - triangle area})$$

$$A = \frac{\pi 4^2 x 90^0}{360^0} - \frac{1}{2} x 4^2 \sin 90^0$$ In this case r=4cm and $\theta = 90^0$

$$A = (4\pi - 8)cm$$

B. In Figure below, O is the center of the circle. If the radius of the circle is 4cm and m (∠AKB) = 30^0, find the area of the segment bounded by the chord AB and arc AKB.



Solution: first we find the measure of central angle m (∠AOB) =2m (∠AKB) =2(30^0) =60^0

Then area of the major segment is=area of circle-area of minor arc

$$A = \pi r^2 - \left(\frac{\pi r^2\theta}{360^0} - \frac{1}{2} r^2 \sin\theta\right) = 16\pi - \left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \left(\frac{40\pi}{3} + 4\sqrt{3}\right) cm^2$$

5.5.Measurements

Areas of Triangles and Parallelograms

A. Areas of triangles

There are different ways used to find the area of the triangle.
We Uses the following fact.

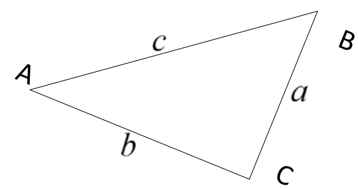
Case i. The area A of a right angle triangle with perpendicular sides of length a and b is given by $A=\frac{1}{2}ab$

Case ii. The area A of any triangle with base b and the corresponding height h is given by $A=\frac{1}{2}bh$

Case iii. The area A of any triangle with sides a and b units long and angle C (∠C) included between these sides is $A=\frac{1}{2}ab\sin(\angle C)$

NOTE: Heron's formula

The area A of a triangle with sides a, b and c units long and semi- perimeter $s=\frac{1}{2}(a+b+c)$ is given by $A=\sqrt{s(s-a)(s-b)(s-c)}$



B. Area of parallelograms

- The area A of a parallelogram with base b and perpendicular height h is $A=bh$
- Area of a rhombus whose diagonals d_1 and d_2 is given as $A=\frac{1}{2}d_1d_2$ **(how)?**

Example:

i. The lengths of three sides of a triangle are $6x, 4x$ and $3x$ inches and the perimeter of the triangle is 26 inches. Find the lengths of the sides of the triangle and the area of the triangle
ii. Find the area of a rhombus whose diagonals are 5 inches and 6 inches long.

Solution:

i. Perimeter of triangle is equal to $P=a+b+c=6x+4x+3x \Rightarrow 26=13x \Longrightarrow x=2$
   Then from this the length of the sides are 12, 8 and 6 inches respectively.
ii. To find it's area we use heron's formula

$A=\sqrt{s(s-a)(s-b)(s-c)}$ For $s=\frac{1}{2}(a+b+c)=13$ then

$A=\sqrt{13(13-12)(13-8)(13-6)}=\sqrt{455}$ Square inches.

Further on Surface Areas and Volumes of Cylinders and Prisms

A. Prism

✓ **A prism** is a solid figure formed by two congruent polygonal regions in parallel planes, along with three or more parallelograms, joining the two polygons. The polygons in parallel planes are called **bases**.

✓ A prism is named by its base. Thus, a prism is called triangular, rectangular, pentagonal, etc., if its base is a triangle, a rectangle, a pentagon, etc., respectively.

In a prism,

- The lateral edges are equal and parallel.
- The lateral faces are parallelograms.

✓ A **right prism** is a prism in which the base is perpendicular to a lateral edge. Otherwise it is an **oblique prism**.
In a right prism

- All the lateral edges are perpendicular to both bases.
- The lateral faces are rectangles.
- The altitude is equal to the length of each lateral edge.
- A regular prism is a right prism whose base is a regular polygon.

Surface area and volume of prisms

The lateral surface area of a prism is the sum of the areas of its lateral faces.

The total surface area of a prism is the sum of the lateral areas and the area of the bases.

The volume of any prism is equal to the product of its base area and its altitude.

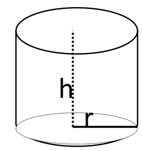
If we denote the lateral surface area of a prism by A_L, the total surface area by A_T, the area of the base by A_B and its volume by V, then

- ✓ $A_L = Ph$ where P is the perimeter of the base and h the altitude or height of the prism.
- ✓ $A_T = 2A_B + AL$
- ✓ $V = A_Bh$.

B. Cylinder

A **circular cylinder** is a simple closed surface bounded on two ends by circular bases.

Surface area and volume of circular cylinders



1. The lateral surface area (i.e., area of the curved surface) of a right Circular cylinder denoted by A_L is the product of its height h and The circumference C of its base. i.e.

   $A_L = hC$ OR $A_L = 2\pi rh$

2. The total surface area (or simply surface area) of a right circular cylinder Denoted by A_T is two times the area of the circular base plus the area of the curved surface (lateral surface area). So, if the height of the cylinder is h and the radius of the base circle is r, we have

   $A_T = 2\pi rh + 2\pi r2 = 2\pi r\,(h + r)$

3. The volume V of the right circular cylinder is equal to the product of its base area and height. So, if the height of the cylinder is h and its base radius is r then

   $V = \pi r^2h$

Example:

a) The base of a right prism is an isosceles triangle with equal sides 5 inches each, and third side 4 inches. The altitude of the prism is 6 inches. Find the total surface area of the prism and the volume of the prism.

b) Calculate the volume and total surface area of a right circular cylinder of height 1 m and radius 70 cm.

c) An agriculture field is rectangular, with dimensions 100 m by 42 m. A 20 m deep well of diameter 14 m is dug in a corner of the field and the earth taken out is spread evenly over the remaining part of the field. Find the increase in the level of the field.

d) A glass cylinder with a radius of 7 cm has water up to a height of 9 cm. A metal cube of $5\frac{1}{2}$ cm edge is immersed in it completely. Calculate the height by which the water rises in the cylinder.

Solution: a. Perimeter of the isosceles triangle = 5 + 5 + 4 = 14 inches and its semi-perimeter is=14/2=7

Area of the isosceles triangle $= \sqrt{7(7-5)(7-5)(7-4)} = 2\sqrt{21}\,sq\,inches$

$A_T = 2A_B + AL = 14x6 + 2(2\sqrt{21}) = \left(84 + 4\sqrt{21}\right) sq\,inches$

$V = A_Bh = 2\sqrt{21}x6 = 12\sqrt{21}\,cubic\,inches.$

Exercise (b, c, d)

Grade 9 Mathematics Note And Some Practice Question For Second Semester

SOME PRACTICE QUESTION ON GEOMETRY AND MEASUREMENT

FOR EACH OF THE FOLLOWING QUESTIONS CHOOSE THE BEST ANSWER FROM THE GIVEN ALTENATIVE

1, Pentagons ABCDE and FGHIJ are similar. The ratio of each side of pentagon ABCDE to its corresponding side of pentagon FGHIJ is 4:1. If AB and FG are corresponding sides, and the length of AB is 4x + 4, what is the length of FG?

A. x + 1 B. 2x + 2 C. 4x + 4 D. 16x + 16

2, Suppose $\Delta ABC \sim \Delta DEF$ and the side of ΔABC are 14cm, 7cm and 19cm long. If the perimetre of ΔDEF is 160cm , the what is the length of the shortest side ΔDEF?

A. 112cm B. 28cm C. 56cm D. 14cm

3, The area of regular hexagon is $96\sqrt{3}cm^2$. What is its perimeter?

A. 54cm B. 48cm C. 60cm D. 42cm

4. The sum of the interior angles of a polygon is $9x^2$. If x is 3 greater than the number of sides of the polygon, how many sides does the polygon have?

A. 6 sides B. 7 sides C. 10 sides D. 13 sides

5. The sum of the interior angles of a polygon is equal to three times the sum of its exterior angles. How many sides does the polygon have?

A. 6 sides B. 10 sides C. 8 sides D. 12 side

6. If the sum of the interior angles of a polygon is equal to the sum of the exterior angles, which of the following statements must be true?

A. The polygon is a regular polygon.
B. The polygon has 4 sides.
C. The polygon has 2 sides.
D. The polygon has 6 side

7. Polygon ABCDEF is similar to polygon GHIJKL. If side AB is 12x + 6x and side GH is 8x + 4x, and these sides are corresponding sides, what is the ratio of the perimeter of polygon GHIJKL to the perimeter of polygon ABCDEF?

A. 1:3 B. 2:1 C. 3:1 D. 3:2

8. Regular polygon ABCDE is similar to regular polygon TUVWX. Side AB is 5x – 1, and side TU is 4x – 2. These sides are corresponding sides. If the ratio of the perimeter of polygon ABCDE to the perimeter of polygon TUVWX is 4:3, what is the perimeter of polygon ABCDE?

A. 25 units B. 24 units C. 72 units D. 120 units

9. The ratio of the lengths of a side of regular pentagon *ABCDE* to the length of a side of regular hexagon *PQRSTU* is 5:6. If the perimeter of pentagon *ABCDE* is 75 units, what is the perimeter of hexagon *PQRSTU*?

A. 90 units B. 18 units C. 108 units. D. 36 units

10. If the sum of the interior angles of a regular polygon equals 720°, and the length of one side of the polygon is $3x^2$, what is the perimeter of the polygon?

A. $18x^2$ units B. $18x^{12}$ units C. $24x^{12}$ units D. $27x^{12}$ units

11. The length of a rectangle is four times the length of a square. If the rectangle and the square share a side of the height, and the perimeter of the square is 2 m, what is the perimeter of the rectangle?

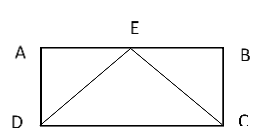
A. 5 units B. 6 units C. 8 units D. 10 units

12. Four squares are joined together to form one large square. If the perimeter of one of the original squares was $8x$ units, what is the perimeter of the new, larger square?

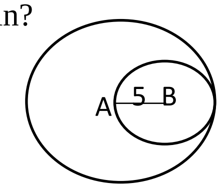
A. $64x$ units B. $24x$ units C. $16x$ units D. $32x$ units

13. Triangle *DEC* is inscribed in rectangle *ABCD*. If side *AB* = 30 units, side *EC* = 17 units, and side *AE* = side *EB*, what is the area of triangle *DEC*?

A. 60 square units
B. 120 square units
C. 136 square units
D. 240 square units

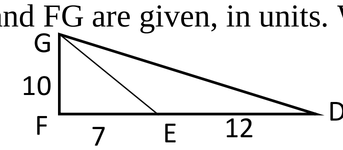


14. In the figure below, the circle centered at B is internally tangent to the circle centered at A. The smaller circle passes through the center of the larger circle and the length of AB is 5 units. If the smaller circle is cut out of the larger circle, how much of the area, in square units, of the larger circle will remain?



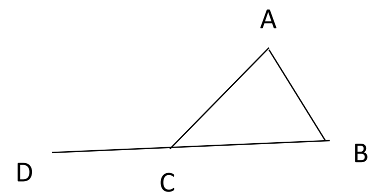
A. 10π square units
B. 25π square units
C. 75π square units
D. 100π square units

15. In the figure below, the lengths of DE, EF, and FG are given, in units. What is the area, in square units, of ΔDEG?



A. 29 square units
B. 47.5 square units
C. 60 square units
D. 120 square units

16. In the diagram below, sides AC and AB of triangle ABC are congruent. If the measure of angle DCA is 115^0 degrees, what is the measure in degrees of angle A?



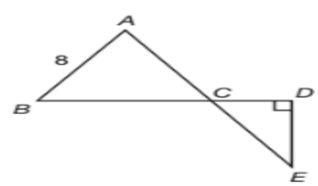
A. 50^0 B. 65^0 C. 130^0 D. 115^0

17. The bases of a right triangle measure $x - 3$ and $x + 4$. If the hypotenuse of the triangle is $2x - 3$, what is the length of the hypotenuse?

A. 12 B. 5 C. 17 D. 13

18. Triangle ABC is an equilateral triangle and triangle CDE is a right triangle. If the length of side AE is 20 units, what is the length of side BD

A. 6 units
B. 14 units
C. 2 units
D. $(8+6\sqrt{3})$ units



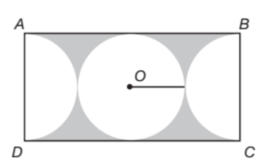
19. If the length of a base of right triangle DEF is 8 units and the hypotenuse of triangle DEF is $8\sqrt{5}$ units, what is the length of the other base?

A. 4 units
B. 8 units
C. $8\sqrt{2}$ units
D. 16 units

❖ Use the diagram below to answer questions. The semicircles to the left and right of the center circle are each exactly half the size of the center circle, and the three figures are adjacent within rectangle $ABCD$. The diagram is not to scale

20. If the length of AB is x units, what is the area of the center circle?

A. $\frac{x^2\pi}{16}$ square units
B. $\frac{x^2\pi}{4\pi}$ square units
C. $2x^2\pi$ Square units
D. $x^2\pi$ square units

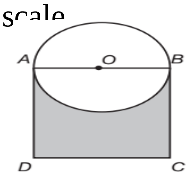


21. If the area of one semicircle is 4.5π square units, what is the area of the rectangle?

A. 72 sq. units B. 108 sq. units C. 144 sq. units D. 162 sq. units

22. If the radius of the circle is 4 units, what is the size of the shaded area?
    A. $72 - 16\pi$ square units
    B. $128 - 32\pi$ square units
    C. $128 - 24\pi$ square units
    D. $128 - 16\pi$ square units
23. The circumference of a circle is 16π cm. What is the area of a sector whose central angle measures 120^0?
    A. $8/3\,\pi cm^2$ B. $16/3\,\pi cm^2$ C. $32/3\,\pi cm^2$ D. $64/3\,\pi cm^2$
24. If a circle has an area of 12π cm^2 and a diameter AB, what is the length of arc AB?
    A. $\sqrt{3}\pi cm$ B. $2\sqrt{3}\pi cm$ C. $4\sqrt{3}\pi cm$ D. $6\pi cm$
25. What the value of the sum of the external angles of a triangle subtracted from the sum of the external angles of a pentagon?
    A. 0 B. 30 C. 48 D. 360
26. The ratio of a rectangle's of length to the length of the square's side 3:1. If the area of the square is 36cm^2, and the rectangle width is 2, what the area of rectangle?
    A. 24cm^2 B. 36cm^2 C. 72cm^2 D. 108cm^2
27. If the area of a circle is $(4x^2 + 20x + 25)\pi$ square units, what is the diameter of the circle?
    A. $2x + 5$ units
    B. $(2x + 5)\ \pi$ units
    C. $4x + 10$ units
    D. $(4x + 5)\pi$ units

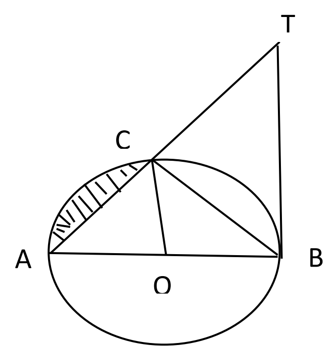
❖ Use the diagram below to answer questions. ABCD is a square. The diagram is not to scale.



28. If the area of the square is 144 square units, what is the total area of the figure?
    A. $144 - 36\pi$ square units
    B. $144 + 36\pi$ square units
    C. $144 - 18\pi$ square units
    D. $144 + 18\pi$ square units
29. If the area of the circle is $8x^2\pi$, what is the size of the shaded area?
    A. $16x^2\sqrt{2} - 4x^2\pi$ B. $16x^2\sqrt{2} + 4x^2\pi$ C. $4x^2(8-\pi)$ D. $4x^2(8+\pi)$

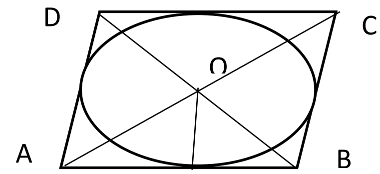
➢ BASED ON THE FOLLOWING FIGURE AND INFORMATION GIVEN ANSWER THE QUESTION FOLLOW.

- Tangent TB and secant TCA are drawn to circle O. Diameter AB is drawn and BC perpendicular to AT. If TC = 6cm and CA = 10cm,

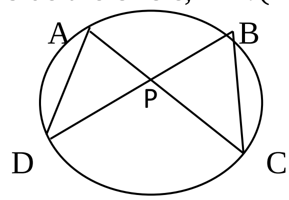


30. Then CB =____ A. $2\sqrt{6}$ cm B. $2\sqrt{15}$ cm C. $4\sqrt{6}$ cm D. 10cm
31. Based on above information on question no 30 length of tangent of circle O, TB=____
    A. $\sqrt{256}$ cm B. $\sqrt{160}$ cm C. $4\sqrt{6}$ cm D. 30cm
32. What the radius of the circle? A. $2\sqrt{10}$ cm B. $4\sqrt{10}$ cm C. $4\sqrt{6}$ cm D. 10cm
33. Find the distance between the centre of the circle O to perpendicular bisector of AC
    A. $2\sqrt{10}$ cm B. $\sqrt{15}cm$ C. $15cm$ D. $\sqrt{21}cm$
34. If the measure of central angle of m(<BOC)=120^0 Measure of $m(\angle ATB)$ is equal to____
    A. 60^0 B. 30^0 C. 120^0 D. 15^0
35. If the measure of central angle of m(<BOC)=120^0 then the area of the shaded region is_______
    A. $40\pi/3 - 20\sqrt{3}$ B. $20\pi/3 - 10\sqrt{3}$ C. $160\pi/3 - 20\sqrt{3}$ D. $20\pi/3 - 40\sqrt{3}$
36. If a regular polygon of 12 sides with radius 4 unit long is given, then its area is equal to_____
    A. $24\sqrt{3}\ unit^2$ B. $12\sqrt{3}\ unit^2$ C. $24\ unit^2$ D. $48\ unit^2$

37. Which of the following is not true about rhombus?
    A. The consecutive angles of a rhombus are not necessarily supplementary.
    B. A rhombus is quadrilateral with opposite sides parallel.
    C. A rhombus is an equilateral quadrilateral.
    D. The diagonal of a rhombus bisects each other perpendicularly
38. Circle O is inscribed in rhombus ABCD. The diagonals of the rhombus are 10cm and 24cm long. To the nearest tenth centimeter, how long is the radius of circle E?



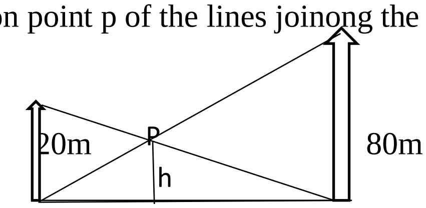
    A. 4.6cm
    B. 4.2cm
    C. 4.9cm
    D. 5cm
39. In a circle below, chords AC and BD intersects at a point p inside the circle, if $m(\angle DAC) = 35°$ and $m(\angle ACB) = 30°$ $then$ $m(\angle APB) =$



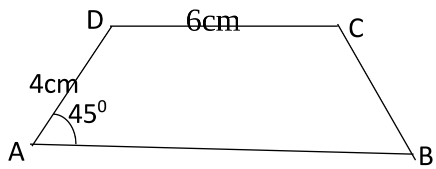
    A. 70°
    B. 65°
    C. 55°
    D. 60°
40. Each side of regular hexagone is 10cm long. What is the area of this hexagon?
    A. $150\sqrt{3}cm^2$ B. $54\sqrt{3}cm^2$ C. $27cm^2$ D. $75cm^2$

41. Which of the following is not correct about an n-sides of a regular polygon?
    A. The measure of each centeral and each extrerior angle is $\frac{360°}{n}$
    B. The sum the measures of its exterior angle is 720^0
    C. It has n lines of symmetre
    D. All its sides and angles are equal.
42. Which one of the following is an angle properties of circle?
    A. All angles inscribed in in semi-circles are congruent.
    B. The opposite angles of a cyclic quadrilateral are congruent
    C. The centeral angles measures 180^0, then it is subtended by major arc.
    D. The angle inscribed in a minor arc of a circle is an acute angle.
43. Given ΔABC $with$ $AB = 5cm,\ BC = 7cm$ and $AC = 8cm$, then the area of ΔABC is __
    A. $10cm^2$ B. $10\sqrt{3}cm^2$ C. $100cm^2$ D. $100\sqrt{3}cm^2$
44. If the measure of each exterior angle of a regular polygon is 40^0, then how many sides does this polygon have?
    A. 8 B. 10 C. 7 D. 9
45. How many lines of symmetry does a regular hexagon have?
    A. 4 B. 3 C. 6 D. 5
46. In a circle, the central angle with a measure of one radian is subtended by an arc whose length is ________
    A. Equal to the radius of the circle C. one third of the radius of the circle
    B. -+Twice of the radius of the circle D. One half o the radius of the circle
47. The diagonals of rhombus measure 12cm and 16cm. which one of the following relation is true about interior obtuse angle β of the rhombus.
    A. $\tan\left(\frac{\beta}{2}\right) = 0.6$ B. $\sin\left(\frac{\beta}{2}\right) = 0.8$ C. $\cos\left(\frac{\beta}{2}\right) = 0.8$ D. $\tan\left(\frac{\beta}{2}\right) = 0.75$

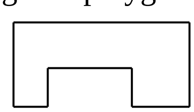
48. Two vertical poles are erected 100m apart on horizontally leveled ground. Their heights are 20m and 80m as shown in figure below. What is the vertical height(h) of the inersection point p of the lines joinong the top of each pole to the foot of the other poles?
    A. 10m
    B. 12m
    C. 16m
    D. 15m



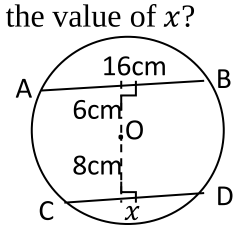
49. Trapizium ABCD, AB//DC, is given as shown in the figure. If DC=6cm, AD=4cm and m(<B)=45^{0}, then what is the length of the side of AB?
    A. $14cm$
    B. $14\sqrt{2}cm$
    C. 10cm
    D. $(6+4\sqrt{2})cm$



50. If the measures of the interior angles of a hexagon are $x^{0}, 2x^{0}, 60^{0}, (x+30)^{0}, (x-10)^{0}, (x+40)^{0}$. Then the value of x is
    A. 720^{0} B. 600^{0} C. 120^{0} D. 100^{0}
51. If the measure of a central angle of a regular polygon is 25^{0}, then the measure of each of its interior angle is__
    A. 155^{0} B. 55^{0} C. 165^{0} D. It is impossible to draw such kinds of regular polygon.
52. Which one of the following is not true about a polygon which has n number of sides?
    A. The number of diagonals of a polygon of n sides from one vertex is n-3
    B. The number of triangle formed by the partitioned of polygon when the diagonals are drawn from one vertex is n-2
    C. The measure of sum of the interior angle of a polygon is (n-2)180^{0}
    D. The measure of each interior angle of any polygon is $\frac{(n-2)180^{0}}{n}$
53. The sum of the measure interior angle of polygon of given below is equal to__
    A. 1080^{0}
    B. 135^{0}
    C. 45^{0}
    D. 8



54. A chord of a circle is 16cm long and is 6cm from the center O. Another chord with length x cm is 8cm from the center of the same circle as shown in the figure below. What is the value of x?
    A. 20
    B. 15
    C. 10
    D. 12



55. Which one of the following is not true about similar or congruent polygon?
    A. Any congruent polygons which have the same number of side are similar.
    B. All equilateral triangles are congruent.
    C. All equilateral triangles are similar
    D. If two polygons that have equal number of sides are congruent, then they are similar.

Grade 9 Mathematics Note And Some Practice Question For Second Semester

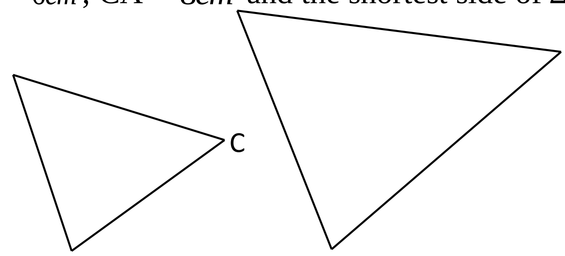
56. In the figure shown below, $\Delta ABC \sim \Delta DEF$. If AB $=4cm$, BC $=6cm$, CA $=8cm$ and the shortest side of ΔDEF is $10cm$. What is the area of ΔDEF?

A. $40\,cm^2$

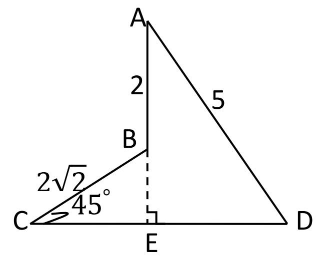
B. $\frac{\sqrt{15}}{4}cm^2$

C. $\frac{3\sqrt{15}}{4}cm^2$

D. $\frac{75\sqrt{15}}{4}cm^2$



57. In the figure below, if AE is **perpendicular** to CD and the sides are measured in cm, then what is the area of the figure?



A. $\frac{25\sqrt{2}}{2}cm^2$

B. $12\,cm^2$

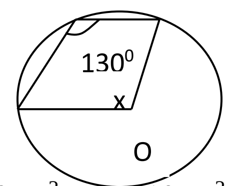
C. $10\,cm^2$

D. $8\,cm^2$

58. An angle formed at a vertex of polygon by extending adjacent side of polygon is____

A. Interior angle. B. Exterior angle C. Central angle D. supplementary angle.

59. If O is the center of a circle, then the measure of the angle marked x is equal to___



A. 130^0

B. 260^0

C. 100^0

D. 50^0

60. The area of square which is inscribed by a circle of radius 5cm is___ A. $25cm^2$ B. $50cm^2$ C. $10cm^2$ D. $20cm^2$

61. Which one of the following statement is not correct?

A. The line segment drawn from the center of circle to the mid points of the chords is perpendicular to the chord.

B. If two chords of the circle are equidistance from the center, then their length are equal.

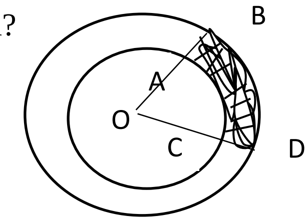
C. If two tangent lines are drawn to the circle external point, then the tangents are equal in length.

D. If two chords of the circle are equal in length, then they may not have equidistance from the center of the circle.

62. The area A of an equilateral triangle inscribed in a circle of radius r is

A. $\frac{3\sqrt{3}}{2}r^2$ B. $\frac{3\sqrt{2}}{4}r^2$ C. $\frac{3\sqrt{6}}{4}r^2$ D. $\frac{3\sqrt{3}}{4}r^2$

63. Suppose two concetric circles of radius 3cm and 6cm with shaded region(ABDC) are drawn as shown in the figure below. If m(<BOD)=80^0, then what is the area of the shaded region?



A. $6\pi cm^2$

B. $4\pi cm^2$

C. $3\pi cm^2$

D. πcm^2

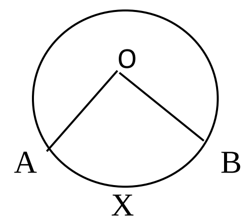
64. The degree measure of each exterior angle of regular eight side polygon is___

A. 45^0 B. 270^0 C. 90^0 D. 135^0

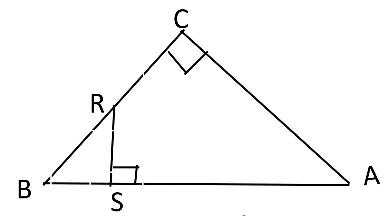
65. A square ABCD is inscribed in a circle of radius 4 cm. What is the area of the minor segment cut off by the chord AB?

A. $(4\pi-8)cm^2$ B. $(8\pi-16)cm^2$ C. $(8\pi-4)cm^2$ D. $(8\pi-2)cm^2$

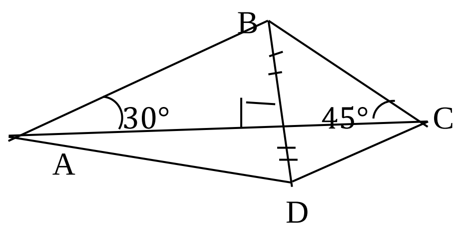
66. Two chords, AB and CD, of a circle intersect at right angles at a point inside the circle. If m (∠BAC) = 35^0, find m (∠ABD).
    A. 35^0 B. 55^0 C. 90^0 D. 65^0
67. A chord of a circle of radius 6 cm is 8 cm long. Find the distance of the chord from the center.
    A. $2\sqrt{5}$ cm B. $4\sqrt{2}$ cm C. 4cm D. 3cm
68. A circle with centre at a point O and radius 6cm is given as shown in the figure below. If $m(< AOB) = 60°$, then find the length of arc$\widehat{AXB}$?
    A. πcm
    B. $2\pi cm$
    C. $4\pi cm$
    D. $3\pi cm$



69. In a figure shown below, $\overline{RS} \perp \overline{AB}$ and $\overline{BC} \perp \overline{AC}$ $with$ $\overline{AC} = 15cm,$ $\overline{BR} = 5cm,$ and $\overline{RS} = 3cm,$ based on information given above, find the length of $\overline{AS}$?
    A. 21cm
    B. 25cm
    C. 16cm
    D. 24cm



70. Let $\Delta ABC \sim \Delta DEF$. The area of ΔABC and ΔDEF are 256cm^2 and 64cm^2 respectively. Which one of the following statement is true?
    A. The ratio of their corresponding side is 4
    B. If CA=9cm, then FD=3cm
    C. The ratio of their area is 16
    D. The ratio of their perimeter is 2
71. Let quadrilateral ABCD and the given conditions be as shown in the figure below. If BD=12cm then what is the area of this quadrilateral?
    A. $18(1 + \sqrt{2})cm^2$
    B. $18(\sqrt{2} + \sqrt{3})cm^2$
    C. $60cm^2$
    D. $36(1 + \sqrt{3})cm^2$



72. The degree measure of each exterior angle of regular eight side polygon is___
    A. 45^0 B. 270^0 C. 90^0 D. 135^0
73. If a regular polygon of 12 sides with radius 4cm long is given, then its area is equal to___
    A. $24\sqrt{3}\ cm\ sq$ $B.$ $12\sqrt{3}\ cm\ sq$ $C.$ $24cm\ sq$ $D.$ $48cm\ sq$
74. Which one of the following statement is not correcte about the regular octagon?
    A. The sum of the measure of all of its interior angle is 1080°
    B. From one vertex we can draw 6 diagonals only
    C. The measure of each interior angle is 135^0
    D. The sum of the measures of all its centeral angle is 180^0

Grade 9 Mathematics Note And Some Practice Question For Second Semester

UNIT SIX

6. STATISTICS AND PROBABILITY

6.1. Statistical data

Importance and purpose of Statistics

- Is used to present facts in a definite form.
- Facilitates comparisons.
- Gives guidance in the formation of suitable policies.
- Is useful for prediction.
- Is helpful in formulating and testing hypothesis and in developing new theories.
- Is used as a guide in capital programming.

Statistics has many other purposes which can be described.

Statistics: is a science dealing with

- ✓ Collecting data.
- ✓ Organizing data.
- ✓ Presentation of data.
- ✓ Analysis of data.
- ✓ Interpretation of data.

Collecting data: is the process of obtaining measurements or counts.

Organizing data: Collected data has to be organized in a suitable form to understand the information gathered. The collected data must be **edited, classified** and **tabulated.**

Presentation of data: The main purpose of data presentation is to facilitate statistical analysis. This can be done by illustrating the data using graphs and diagrams like bar graph, histograms, pie charts, pictograms, frequency polygons, etc.

Analysis of data: In order to meet the desired purpose of investigation, data has to be analyzed. The purpose of analyzing data is to highlight information useful for decision making.

Interpretation of data: - A final image about the data

- Final decision about the data
- Make judgment about each population

Statistics may be classified into two parts

- Descriptive statistics
- Inferential statistics

Descriptive statistics: is a branch of statistics concerned with summarizing and describing a large amount of data without drawing any conclusion about a particular bit of data.

Inferential statistics: is a branch of statistics concerned with interpreting data and drawing conclusions. (From particular to general) we make judgment about population from which samples comes.

Note:

- ✓ In statistics, however, **population** refers to the complete collection of individuals, objects or measurements that have a common characteristic and a small part of the group/population, called a **sample**.

Data can be classified as either **qualitative** or **quantitative**. However, statistics deals mainly with quantitative data.

- ✓ **Qualitative** if the data is based on some characteristic whose values are not numbers, such as their eye colour, sex, religion or nationality.
- ✓ **Quantitative** if the data is numerical such as height, weight, age or scores in tests.
- ✓ A rule which gives a corresponding value to each member of a population is called a **population function**.

Data can be classified as either primary or secondary data based on source

Primary data: Primary data is original data, obtained personally from primary sources by observation, interview or direct measurement.

Example:

- ✓ If you measure the heights of students in your class, this is primary data.
- ✓ The data gathered by the Ministry of Education about the number of students enrolled in different universities of Ethiopia is primary data for the Ministry itself. (If you were to use this data, it would be secondary data for you.)

Secondary data: Data which has been collected previously (for similar or different purpose) is known as **secondary data**. Secondary data refers to that data which is not originated by the researcher himself/herself, but which he/she obtains from someone else's records. Some sources of secondary data are official publications, journals, newspapers, different studies, national statistical abstracts, etc

Distributions and Histograms

After data is collected, it must be organized into a manageable form. Data that is not organized is referred to as **raw data**. A quantity that we measure from observation is called a **variate** or **variable** denoted by V. The distribution of a population function is the function which associates with each variate of the population function a corresponding frequency denoted by f.

Data can be presented by:

- ✓ **frequency distribution**
- ✓ **histogram (graphical presentation)**

A **frequency distribution** is a tabular or graphical representation of a data showing the frequency associated with each data value.

A **histogram** is a graphical representation of a frequency distribution in which the variate (V) is plotted on the x-axis (horizontal axis) and the frequency (f) is plotted on the y-axis (vertical axis).

Example: A sample of 50 couples married for 10 years were asked how many children they had. The result of the survey is as follows:

0	4	2	2	1	3	0	3	2	4
3	3	1	3	3	3	3	3	2	2
1	3	3	2	4	3	1	5	2	2
2	0	0	2	1	2	2	2	3	2
3	3	3	4	3	1	3	0	3	2

A. Construct a frequency distribution.

B. Construct a histogram.

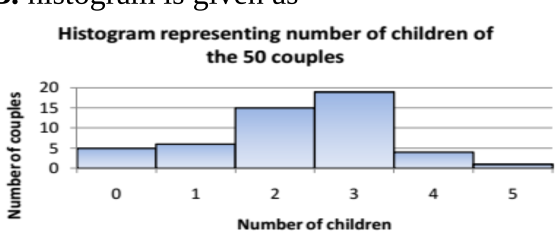
C. What percentages of couples have two children?

D. What percentages of couples have at least two children?

Solution: A. The frequency distribution is given as

v	0	1	2	3	4	5
f	5	6	15	19	4	1

B. histogram is given as



C. Percentage= $\frac{Number\ of\ couples\ have\ two\ children}{total\ number\ of\ couple} x100\%$

$= \frac{15}{50} x100\% = 30\%$

D. Percentage= $\frac{Number\ of\ couples\ have\ More\ than\ or\ equal\ two\ children}{total\ number\ of\ couple} x100\%$

$= \frac{39}{50} x100\% = 78\%$

Measures of Location (Mean, Median and Mode(s))

- Mean
- Median
- Mode(s)

Are measures of central tendency of quantitative data.

The value of a measure of central tendency or an average is regarded as the most representative value of a given data.

A. The arithmetic mean

The **arithmetic mean** (or the **mean**) of a variable is the sum of all the data values, divided by the total frequency (number of observations). If $x_1, x_2, x_3, \ldots, x_n$ are the n observations of a variable, then the mean, $\bar{x}$, is given by

Mean: $\bar{x} = \frac{x_1+x_2+x_3\pm\cdots+x_n}{n} = \frac{\text{sum of values}}{\text{total number of values}}$

Note: The mean of a population function can also be calculated from its frequency distribution. So, if the values $x_1, x_2, x_3, \ldots, x_n$ occur $f_1, f_2, f_3 \ldots f_n$ times, respectively, then the mean ($\bar{x}$) is given by

Mean : $\bar{x} = \frac{f_1x_1+f_2x_2+f_3x_3\pm\cdots+f_nx_n}{f_1+f_2+f_3+\cdots+f_n}$

Properties of the mean

1 The sum of the deviations of individual observations from mean ($\bar{x}$) is zero. That is, let $x_1, x_2, x_3, \ldots, x_n$ be n observations with mean $\bar{x}$. Then the sum of the deviations of the observations from the mean is given by

$$(x_1-\bar{x})+(x_2-\bar{x})+(x_3-\bar{x})+\ldots+(x_n-\bar{x})=0$$

proof (exe for students)

2 If a constant k is added to (or subtracted from) each data value, then the new mean is the sum (or the difference) of the old mean and the constant k.

3 The mean of the sum or difference of two population functions (of equal numbers of observations) is equal to the sum or difference of the means of the two population functions.

4 The mean of a constant times a population function is equal to the constant times the mean of the population function. That is, if $\bar{x}$ is the mean of the population function $x_1, x_2, x_3, \ldots, x_n$ and if k is a constant, then the mean of $kx_1, kx_2, kx_3, \ldots, kx_n$ is equal to $k\bar{x}$.

NB: If $\bar{x}_1$, $\bar{x}_1$ are the means of n_1, n_2 observations respectively then the combined mean is $\bar{x} = \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$

Note: 1 The mean is unique.

2 The mean is affected by extreme values.

B. The median

The **median** is the value that lies in the middle of the data when it is arranged in ascending or descending order. So, half the data is below the median and half the data is above the median.

Let $x_1, x_2, x_3, \ldots, x_n$ are n-observation then the median is denoted by $\tilde{x}$, defined as

$$\tilde{x} = \begin{cases} x_{\frac{n+1}{2}} & if\ n\ is\ odd \\ \dfrac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2} & if\ n\ is\ even \end{cases}$$ where n is the number of observation

Note: that the median of a set of data with values arranged in ascending or descending order is:

- ✓ the middle value of the list if there is an odd number of values.
- ✓ half of the sum of the two middle values if there is an even number of values.

Properties of the median

i. The median can be obtained even when some of the data values are not known.

ii. It is not affected by extreme values.

iii. It is unique for a given data set

C. The mode

The value of the variable which occurs most frequently in a data set is called the mode.

Note that a set of data can have no mode, one mode (**unimodal),** two modes (**bimodal**) or more than two modes (**multimodal**). If there is no observation that occurs with the highest frequency, we say the data has no mode.

Properties of the Mode(s)

- ✓ The mode is not always unique.
- ✓ It is not affected by extreme values.
- ✓ The mode can also be used for qualitative data.

Example:1 Calculate the mean, median and mode of the following data;

Value	10	15	20	25	30	35	40	Total
Frequency	15	10	50	4	10	8	3	100

Solution: Mean: $\bar{x} = \frac{f_1x_1+f_2x_2+f_3x_3\pm\cdots+f_nx_n}{f_1+f_2+f_3+\cdots+f_n} = \frac{10x15+15x10+20x50+25x4+30x10+35x8+40x3}{15+10+50+4+10+8+3} = \frac{2100}{100}=21$

Median: The number of observation is even so $\tilde{x} = \frac{x_{\frac{n}{2}}\,term + x_{\frac{n}{2}+1}\,term}{2} = \frac{\frac{100}{2}term+\frac{100}{2}+1term}{2} = \left(\frac{20+20}{2}\right) = 20$

Mode(s): The values of data which has highest frequency, which is equal to $\hat{x} = 20$

Example: **2** If the mean of a, b, c, d is k, then what is the mean of i. a + b, 2b, c + b, d + b? ii. ab, b^2, cb, db?

Solution: Using the properties of mean, The mean of a + b, 2b, c + b, d + b=old mean +c=k+b and

The mean of ab, b^2, cb, db= old mean x c=kb

Measures of Dispersion for Ungrouped Data

Dispersion or **Variation** is the scatter (or spread) of data values from a measure of central tendency. There are several measures of dispersion that can be calculated for a set of data. In this section, we will consider only three of them, namely, the **range**, **variance** and the **standard deviation**.

Range: The simplest and the crude measure of dispersion of quantitative data is the **range**.

The **range R** of a set of numerical data is the difference between the highest and the lowest values. i.e., Range = Highest value – Lowest value

Example 1 The ages of six students are 24, 20, 18, 13, 16, 15 years, respectively. What is the range?

Solution: Range = highest value – lowest value = 24 – 13 = 11 years.

Variance (σ^2):

Variance denoted by (σ^2, is defined as the mean of the squared deviations of each value from the arithmetic mean.

Standard deviation (σ): The **standard deviation** is the most valuable and widely used measure of dispersion.

Standard deviation, denoted by σ, is defined as the positive square root of the mean of the squared deviations of each value from the arithmetic mean.

The actual method of calculating σ can be summarized in the following steps:

Step 1 Find the arithmetic mean x of the distribution.

Step 2 Find the deviation of each data value from the mean $(x-\bar{x})$

Step 3 Square each of these deviations, $(x-\bar{x})^2$

Step 4 Find the mean of these squared deviations. This value is called the **variance** and is denoted by σ^2.

Step 5 Take the principal square root of σ^2, i.e. Standard deviation $=\sqrt{variance}$.

Properties of variance and standard deviation

i. If a constant c is added to each value of a population function, then the new variance is the same as the old variance. The new standard deviation is also the same as the old standard deviation.

ii. If each value of a population function is multiplied by a constant c, then
   - ✓ The new variance is c^2 times the old variance
   - ✓ The new standard deviation is |c| times the old standard deviation.(**proof Exercise for student**)

Example: Find the range, variance and standard deviation of the following data. 4, 2, 3, 3, 2, 1, 4, 3, 2, 6

Solution: Range = highest value – lowest value=6-1=5

Variance and standard deviation is given as follows using the following table

Variable (x)	Frequency (f)	$x_i f_i$	$(x-\bar{x})$	$f_i(x-\bar{x})^2$
1	1	1x1=1	1-3=-2	$1x(-2)^2=4$
2	3	2x3=6	2-3=-1	$3x(-1)^2=3$
3	3	3x3=9	3-3=0	$3x(0)^2=0$
4	2	4x2=8	4-3=1	$2x(1)^2=2$
6	1	6x1=6	6-3=3	$1x(3)^2=9$
Total	10	$\bar{x}=\frac{\sum x_i f_i}{\sum f}=\frac{30}{10}=3$		$\sigma^2=\frac{\sum f_i(x-\bar{x})^2}{\sum f}=\frac{18}{10}=1.8$

And Standard deviation the positive squares root of the mean of the squared deviations of each value from the arithmetic mean.i.e Standard deviation (σ) $=\sqrt{variance}=\sqrt{1.8}\approx 1.34$

6.2. Probability:

In every day conversation the term probability is used loosely statements of the following type are common

- It is likely to rain to day
- It will probably rain to day

Probability:

- Is a measure of uncertainty involved in the happening of event so that definite value may be assigned to it.
- Is a numerical value that describes the likelihood of the occurrence of an event in an experiment.

DEFINITION:-

- An experiment is a **trial** by which an observation is obtained but whose outcome cannot be predicted in advance.
- Probability determined using data collected from repeated experiments is called experimental probability.
- **Random experiment**: Any happening whose result is uncertain, i.e. rolling of die, tossing of coin and etc.
- **Outcome**: The possible result of the experiment.
- **Possibility set(Sample space):**The set of all possible outcome of the experiment denoted by S
- **Event:** is any subset of possibility set.

- **Impossible events**: Any event which can never occur, i.e. The probability of impossible event, $P(E)=0$.
- Certain event: An event which is certain to occur, i.e. The probability of certain event, $P(E)=1$.
- **Trial**: Is procedure or an experiment to collect any statistical information is called a **trial**.
- **Equally likely**: When one does not happen more than the others.
  Example: If we toss a coin, the event 'head' or the 'tail' may occur. Both these possibility are equally likely.
  - ✓ Chance of head is 50%
  - ✓ Chance of tail is 50%
- **Favorable event:** Such cases as result in the happening of an event are said to be cases favorable to that event.

Probability of an event P (E)

Definition: Let n(S) denote the number of elements in the sample space S, and let n (E) denote the number Of elements in the event E, P(E) is given as $P(E)=\dfrac{n(E)}{n(S)}$

NOTE: - Because E is a subset of S

$$\Rightarrow n(E)\le n(S),\ dividing both side by\ n(S)$$

$$\Rightarrow \frac{n(E)}{n(P)}\le 1 \Rightarrow P(E)\le 1$$

$$\Rightarrow \underset{\substack{\uparrow\\ \text{Impossible event}}}{0} \le P(E) \le \underset{\substack{\uparrow\\ \text{Certain event}}}{1} \qquad (For\ any\ event)$$

Example: 1 The numbers 1 to 20 are each written on one of 20 identical cards. One card is chosen at random.

A. List the set of all possible outcomes.

B. List the elements of the following events:

i. The number is less than 5.

ii. The number is greater than 15.

iii. The number is greater than 21.

iv. The number is divisible by 5.

v. The number is a prime.

Solution: **A**. S = {1, 2, 3… 19, 20}

B

i. {1, 2, 3, 4}

ii. {16, 17, 18, 19, 20}

iii. { } or ∅, since no card has a number greater than 20.

iv. {5, 10, 15, 20}

v. {2, 3, 5, 7, 11, 13, 17, 19}

Example: 2 A fair die is rolled once. Calculate the probability of getting:

b. an odd number **c.** a score of 5 **d.** a prime number **e.** a score of 0

Solution:

Number of sample space n(S) =6= {1, 2, 3, 4, 5, 6}

b. E={1, 3, 5}, Then $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{3}{6}=0.5$

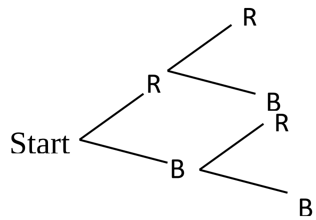
c. E={5}, Then $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{1}{6}$

d. E={2, 3, 5}, Then $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{3}{6}=0.5$

e. E={0}, Then $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{0}{6}=0$

Example: 3 A bags contain four red balls and three black balls. What is the possibility set for colour, if 2 balls are chosen at random?

Solution:



The possibility set is: {RR, RB, BR, BB} equivalent to {RR, RB, BB}

Exercise

i. Two fair dice are rolled once. What is the probability that the difference of the two numbers shown is 1?

ii. An integer n, $1 \leq n \leq 144$, is picked at random. What is the probability that n is the square of an integer?

iii. The median of $x - 4$, x, $2x$ and $2x + 12$ is 9, where x is a positive integer. Find the value of x.

Grade 9 Mathematics Note And Some Practice Question For Second Semester

SOME PRACTICE QUESTION ON TATISTICS AND PROBABILITY

FOR EACH OF THE FOLLOWING QUESTIONS CHOOSE THE BEST ANSWER FROM THE GIVEN ALTENATIVE

1, Which of the following is primary data?

A. The data you obtain from your kebele office about house hold incoms.

B. The data you obtained by measuring the weigth of students in your class.

C. The data you obtainedfrom the reports given by the ministry of health about HIV/AIDS victims.

D. The data you obtained about the students' scores from your school.

2. The median of twenty one consecutive even integers is equal to 22. Then mean value of this data?
   A. 22 B. 42 C. 11 D. 231
3. The mean of a, b, and c is 27. If the values of d is 19, then what is the average of a, b, c and d
   A. 7 B. 11 C. 25 D. 23
4. Which one of the following is true?
   A. Mean and median are not unique while mode is unique
   B. Mean is affected by extreme value while mode and median is not affected by extreme value
   C. There is no any situation in which all values of the data is mode(s)
   D. All
5. An integer n, $1 \leq n \leq 144$, is picked at random. What is the probability that n is the square of an integer?
   A. $1/36$ B. $1/9$ C. $4/9$ D. $1/12$
6. Two fair dice are rolled once. What is the probability that the difference of the two numbers shown is 1?
   A. $1/6$ B. $5/36$ C. $5/18$ D. $5/6$
7. In a class of boys and girls, the mean weight of 8 boys is 55 kg and the mean weight of a group of girls is 48 kg. The mean weight of all the children is 50.8 kg. How many girls are there?
   A. 12 B. 20 C. 4 D. 24
8. Which of the following is true?
   A. The mean, mode and median of a population function cannot be equal.
   B. The range and the standard deviation of a population function are inversely related.
   C. The range of a population function cannot be a non-positive number.
   D. The Arithmetic mean of the given data is divide data in two equal parts.
9. A fair of die is rolled once. Then which of the following is not true?
   A. The probability number on the upper face of the die 7 is impossible.
   B. The probability number on the upper face of the die is prime number or neither prime nor composite is 0.5
   C. The probability number on the upper face of the die is less than 7 is certain.
   D. The probability number on the upper face of the die is prime number is equal to 0.5
10. If a population function x has mean M(x) = 2 and $M(x^2) = 8$, then the standard deviation is equal to: ___.
    A. 8 B. 6 C. 4 D. 2
11. If the variance of a, b, c, d is k, then which of the following is not true?
    A. The variance of a + c, b + c, 2c, d + c is equal to k
    B. The standard deviation of a + c, b + c, 2c, d + c is equal to square root of k
    C. The variance of ac, bc, c^2, dc is equal to c^2k
    D. The standard deviation of ac, bc, c^2, dc is equal to c^2k
12. For what the value(s) of y the standard deviation of {b, b, b, b, y, and b} is not necessary equal to zero?
    A. When b=0
    B. When y=b
    C. When $y \neq b$
    D. When y is any positive real number
13. From a box containing 5 white balls and 7 red balls of the same size one ball is chosen at a random. What is the prbability that the chosen ball is a white ball?
    A. 5/7 B. ½ C. 7/12 D. 5/12

14. The median of twenty one consecutive even integers is equal to 22. Which one of the following is false?
    A. The mean of the data is equal to 22
    B. The given data have no mode
    C. Probability of a number which is divisible by 2 is certain
    D. Range is 38
15. On certain test nine students scored 6, 10, 7, 15, 9, 17, 20, 14 and 19. What is the median of the score?
    A. 13.2 B. 9 C. 14 D. 12.5
16. Mamo's brothers are 174cm, 180cm, 179cm, and 172cm tall. If mamo and his brothers have an averagevheight of 176.50cm, then how tall is mamo?
    A. 177.50 cm B. 176.50 cm C. 176.30cm D. 176.25cm
17. Which one of the following is a statistical measure involves all values of the quantitative data set for its computations? A. range B. Mode C. Median D. Mean
18. If two dice are thrown simultaneously, then what is the probability of getting even numbers on both sides?
    A. 1/3 B. 1/6 C. ½ D. ¼
19. Two dice are thrown simultaneously. What will be the probability that the sum of the upper face is less than 4? A. 2/3 B. 1/12 C. 1/9 D. 1/6
20. A researcher collected data on the monthly income of 10,000workers in A.A. after listing down this set of data in the order of the amount of income; the researcher wants to divide into equal parts. In order to do this he has to compute the __ of data A. Median B. Mean C. Mode D. Range
21. The following table shows the amount of meat consumed by 20 households in the month of March 2017 For the given month what is the arithmetic mean amount of meat consumed by the 20 house hold?
    A. 10kg B. 7.6kg C. 20/3kg D. 9.5kg

Amount of meat in kg	5	8	10	15
Number of house hold	3	5	7	5

22. A ball will be taken out at a random from a box containing 6 red, 4 white and 5 blue balls. What the probability of taking out a ball which is not red? A. 3/5 B. 2/5 C. 1/3 D. 4/15
23. Grades for the test on proofs did not go as well as the teacher had hoped. The mean grade was 68, the median grade was 64, and the standard deviation was 12. The teacher curves the score by raising each score by a total of 7 points. Which of the following statements is true?
    I. The new mean is 75. II. The new median is 71. III. The new standard deviation is 7.
    (A) I only (B) III only (C) I and II only (D) I, II, and III
24. A new couples planned to have three children (excluding multiple birth) in their future life under the assumption that a boy is likely as girl at each birth. The sample space is {ggg, ggb, gbg, gbb, bgg, bgb, bbg, bbb}. Where "g" represent girl and "b"- boy with the sequence of birth. What is the probability that the new couples will have at least two girl in any order of birth? A. ½ B. 3/8 C. ¼ D. 7/8
25. On a given test the arithmetic mean of marks scored by students is 36 points with standard deviation 8 points for the class activities the teacher added 10 points on each student's test score. What can you say about the standard deviation of the scores after the 10 points are added?
    A. It will decrease by $10-\sqrt{36/8}$
    B. Nothing can be said.
    C. It will be increase by $10+\sqrt{8}$
    D. It will not change.
26. Consider the following group of data sets. Which one of the following group of data sets have values that That are mostly concentrated around arithmetic mean?
    A. X
    B. Y
    C. W
    D. Z

Group	values	Mean
W	9, 4, 6, 7,	6.5
X	8, 6, 7, 3	6
Y	11, 6, 7	8
Z	2, 5, 7, 2, 3,5	4

27. Suppose you write the days of the week on identical pieces of paper. You mix them in a bowl and select one at a time. What is the probability that the day you select will have the letter r in it?
    A. $3/4$ B. $4/7$ C. $3/7$ D. 1

Grade 9 Mathematics Note And Some Practice Question For Second Semester

CHAPTER SEVEN

VECTOR INTWO DIMENSION

7.1 INTRODUCTION TO VECTORS AND SCALARS

There are two types of physical measurements: Those are

A. SCALAR: a physical quantity which has no direction that can be expressed completely using a single measurement (with units) is called scalar quantity.

- ✓ It simply represented by a real number and a specified unit.

Some of example of scalar quantities is:

- ✓ Amount of rainfall in mm
- ✓ Temperature in a room
- ✓ Volume of a solid figure
- ✓ Area of a plane figure Speed of an airplane

- The length the side of triangle is 4cm. since 4 is real number with no direction the length is represents a scalar.
- The height of Mount Ras Dashen is 4550 metres. Here the height is represented by a single real number. Hence it represents scalar

B. VECTOR: a physical quantity which involves both magnitude and definite (identified) direction. In many applications of mathematics to the physical and biological sciences and scientists are concerned with quantities that have both magnitude and direction

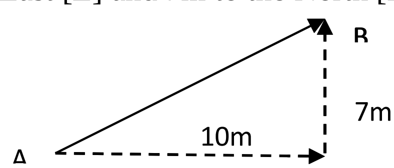
Example: force, velocity, acceleration, and momentum. It is use full to able to express these quantities (vectors) both geometrically and algebraically.

Some of other examples are:

- ✓ Force of water hitting a turbine , Gravity , Acceleration of a motor bicycle and

- The velocity of the car is 80km/hr in the direction of north. This represent vector. Because they have both magnitude and identified direction.

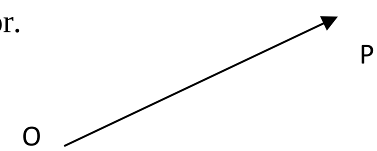
<u>Example</u>: suppose Helen moves, from A, 10m to the East [E] and 7m to the North [N] to reach at B. Show, as a vector, Helen's final displacement.



Solution: Taken together, the distance and direction of the line from A to B is called the **displacement** from A to B, and is represented by the arrow in above figure.

The arrow-head tells us that we are talking B. This is an example of a vector.

7.2. REPRESENTATION OF A VECTOR



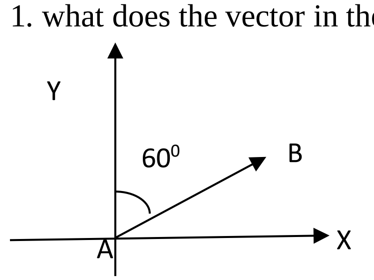
A vector can be represented either algebraically or geometrically.

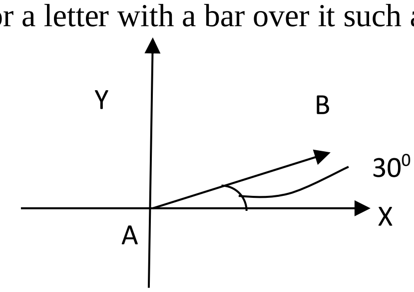
Often, the most convenient way of representing vectors is geometrically,

Where a vector is represented by an arrow or directed line segment.

When a vector is represented by an arrow $\overline{OP}$ above), the point O is called the **initial point** and P is called the **terminal point**. Sometimes, vectors are represented using letters or a letter with a bar over it such as $\bar{u}$, $\bar{v}$ etc

<u>EXAMPLE</u>: 1. what does the vector in the following represent?





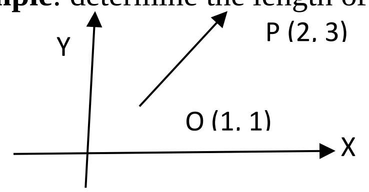
The vector AB has a length of 7 m and direction of East 30^0North [E30^0N] (or a direction of North 60^0East [N60^0E]. Its initial point is A and its terminal point is B.

Magnitude or length of vectors

The magnitude (length) of a vector $\overline{OP}$ or simply $\bar{u}$ is the length of the line segment from the initial point O to the terminal point P, (the length of the directed line segment).

NOTATION: Magnitude of vector $\overline{OP}$ is denoted as $|OP|$

Example: determine the length of the vector $\overline{OP}$ of the following



$$|OP| = \sqrt{(3-1)^2 + (2-1)^2} = \sqrt{5}$$

Note: The magnitude of a vector is represented by the length of the arrow that represents the vector.

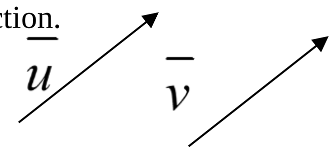
Direction of vectors: The direction of a vector is the angle that is formed by the arrow (that represents the vector) with the horizontal line at its initial point (or with the vertical line in the case of compass directions

Generally:

- ✓ If two vectors have opposite directions, they are called opposite vectors.
- ✓ Vectors that have either the same or opposite directions are called parallel vectors.

Equality of vectors

Two vectors are said to be equal, if they have the same length and the same direction.



Example 8 The following two vectors, $\bar{u}$, and $\bar{v}$, are equal

Since they have the same length and the same direction.

The actual location of these vectors is not specified.

We call such vectors **free vectors**.

7.3. ADDITION AND SUBTRACTION OF VECTORS AND MULTIPLICATION OF VECTORS BY SCALAR

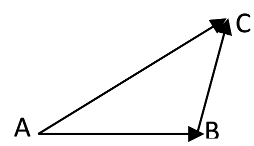
A. ADDITION OF VECTORS

As vectors are measures, vector addition is simply addition of the measures that represent vectors. But consideration of the direction is important.

How would you add vectors?

We add vectors in different ways. If they have same direction, then we simply add the measures, whereas, if they are opposite in direction, then the addition becomes subtraction. If they are neither having same direction or opposite we may use resultants to consider vector addition

EXAMPLE: Is the length of the sum of two vectors always equal to the sum of the lengths of each vector? Why?



Solution The sum is not always equal to the sum of the lengths of each vector.

IN ADDITION OF VECTOR If they are parallel, then the change in the sum is only length and there is no change in the direction. But, if the vectors are not parallel, then there is change in both the length and the direction which will lead in to the laws of vector addition "**The Triangle Law** and **the Parallelogram Law**".

Triangle law of addition of vectors

Definition 7.3 **Triangle law of vector addition**

Let $\vec{a}$ and $\vec{b}$ is two vectors in a coordinate system. If $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{BC}$, then their sum,

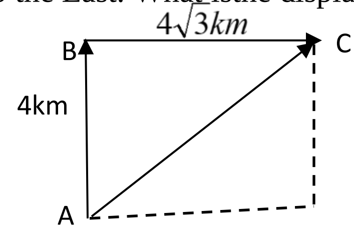
$\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BC}$ is the vector represented by the directed line segment.

That is $\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

NOTE:

- ✓ In the addition of vector of $\overrightarrow{AB}+\overrightarrow{BC}$ is addition of vector whose initial point is **A** and terminal point is **C** which is denoted by $\overrightarrow{AC}$ is sometimes called **resultant displacement**.
- ✓ Vector addition can be done either graphically or by separate addition of vector components.

Example 1A car travels 4 km to the North and then $4\sqrt{3}km$ km to the East. What isthe displacement of the car from A to the final position C?



Solution: The magnitude AC $=\sqrt{(4km)^2+(4\sqrt{3}km)^2}$

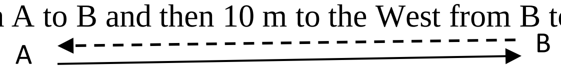
$=\sqrt{(16+48)km}=8km$

$$\tan(BAC)=\frac{0PP}{ADJ}=\frac{4\sqrt{3}}{4}=\sqrt{3}$$

Therefore, $<BAC=60^0$

So the displacement is the vector $\overrightarrow{AC}$ which is 8 km in the direction of North 60^0 East

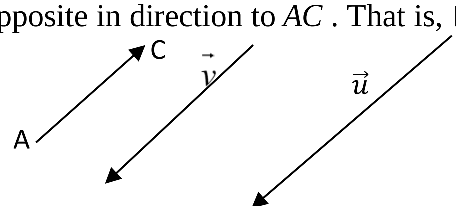
Example 2A person moved 10 m to the East from A to B and then 10 m to the West from B to A. Find the resultant displacement.



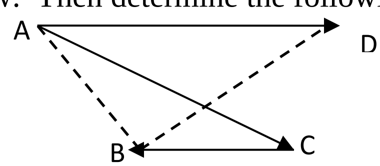
Solution: Here we see that the person ends up at A, hence his displacement is zero. From this we see that if we have AB and BA, then the sum of these vectors AB +BA vanishes in the sense that the initial point and the terminal point coincide. Such a vector is called a **<u>null vector</u>** and is denoted by 0 or simply $\overrightarrow{AB}+\overrightarrow{BA}=0$

Given $\overrightarrow{AC}$ if, $\vec{u}$ is a vector parallel to $\overrightarrow{AC}$ but in opposite direction, then u is said to be an **<u>opposite vector</u>** to $\overrightarrow{AC}$. Then $-\overrightarrow{AC}$ represents the vector equal in magnitude but opposite in direction to $\overrightarrow{AC}$. That is, □ $\overrightarrow{AC}=\overrightarrow{CA}$ Notice that $\overrightarrow{AC}+\overrightarrow{CA}=\overrightarrow{AC}-\overrightarrow{AC}=0$

EXAMPLE 3: The following are opposite to vectors $\overrightarrow{AC}$



Example 4: considers the vectors $\overline{AC},\overrightarrow{CA},\overrightarrow{CB},\overrightarrow{AD}$, in the figure below. Then determine the following vectors



a. $\overline{AC}+\overrightarrow{CB}$

b. $\overline{AC}+\overrightarrow{CB}+\overrightarrow{BD}$

c. $\overline{AC}+\overrightarrow{CA}+\overrightarrow{AD}$,

d. $\overline{AC}+\overrightarrow{CB}+\overrightarrow{BD}+\overrightarrow{DA}$,

Solution

a. $\overline{AC}+\overrightarrow{CB}=\overrightarrow{AB}$

b. $\overline{AC}+\overrightarrow{CB}+\overrightarrow{BD}=\overrightarrow{AD}$

c. $\overline{AC}+\overrightarrow{CA}+\overrightarrow{AD}=\overrightarrow{AD}$

d. $\overline{AC}+\overrightarrow{CB}+\overrightarrow{BD}+\overrightarrow{DA}=0$

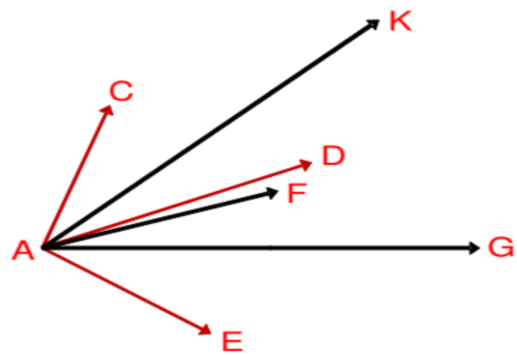
<u>Parallelogram law of addition of vectors</u>

Triangle law of addition of vectors is applicable where the initial point of one vector is the terminal point of the other. We may sometimes have vectors whose initial point is the same, yet we need to find their sum using parallelogram law.

Now let’s see how we can construct a parallelogram and see the sum of the two vectors (with the same initial point) as the diagonal of constructed parallelogram.

Example 5. Given the vectors $\overrightarrow{AC}, \overrightarrow{AE}, \overrightarrow{AD}, \overrightarrow{AF}, \overrightarrow{AK}, \overrightarrow{AG}$

Described in figure below, determine the following vectors.



a. $\overrightarrow{AC} + \overrightarrow{AE}$,

b. $\overrightarrow{AC} + \overrightarrow{AD}$,

c. $\overrightarrow{AE} + \overrightarrow{AD}$

Solution: constructed parallelogram and see that

a. $\overrightarrow{AC} + \overrightarrow{AE} = \overrightarrow{AF}$

b. $\overrightarrow{AC} + \overrightarrow{AD} = \overrightarrow{AK}$

c. $\overrightarrow{AE} + \overrightarrow{AD} = \overrightarrow{AG}$

Subtraction of vectors and Scalar multiplication of a vector

If you have a vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}$ such that $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

a. How would you represent $-\overrightarrow{AB}$ geometrically

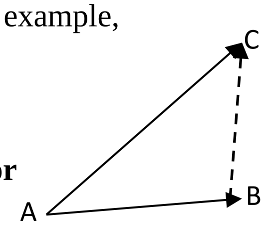
b. How you show geometrically, that $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$

c. Discuss vector subtraction and multiplication of a vector by a scalar.

d. How do you represent vector subtraction and scalar multiplication of vector geometrically?

<u>**NOTE**</u>: Vector subtraction is considered simply adding the opposite vector. For example,

$\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$ **is the same as** $\overrightarrow{AC} + (-\overrightarrow{AB}) = \overrightarrow{BC}$ **in the same way,**



Scalar multiplication of a vector is enlarging or shortening a vector.

For example, if we consider a vectors $\overrightarrow{AB}$ **and** $2\overrightarrow{AB}$ **,then the length of vector** $2\overrightarrow{AB}$ **is twice that of the vector** $\overrightarrow{AB}$

Generally: Geometrically, subtraction of vectors and scalar multiplication of vectors can be represented as follows.

Definition: Let $\overrightarrow{AC}$ be any given vector andk be any real number. The vector k$\overrightarrow{AC}$ the vector whose magnitude is ktimes the magnitude of $\overrightarrow{AC}$ and

A. the direction of k$\overrightarrow{AC}$ is the same as the direction $\overrightarrow{AC}$ if k>0

B. the direction of k$\overrightarrow{AC}$ is the opposite to that of $\overrightarrow{AC}$ if k<0

<u>**NOTE**</u>: if $k \neq 0$, then any vector $\overrightarrow{AC}$ and $k\overrightarrow{AC}$ are parallel vector.

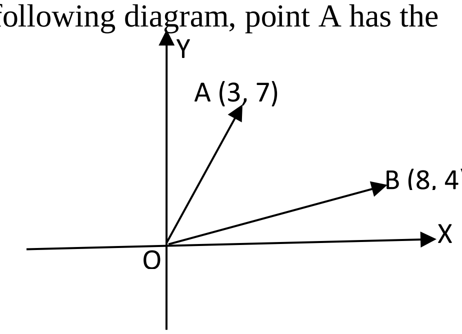
7. 4. **Position Vector of a point**

What is a position vector is and how to find a position vector for a vector between two points?

A vector that starts from the origin (O) is called a **position vector**. In the following diagram, point A has the position vector a and point B has the position vector **b**.

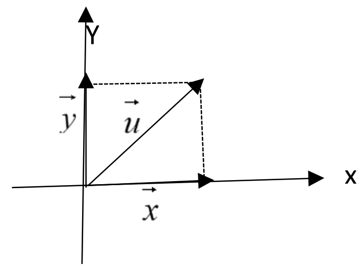
If $a = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$, then the coordinate of A will be (3, 7)

Similarly, if $b = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$, then coordinates of B will be (8, 4)



Grade 9 Mathematics Note And Some Practice Question For Second Semester

Note: to represent position vector whose initial point is A and terminal point is B is constructed vector which have the same magnitude and direction with vector AB whose initial point is at origin.



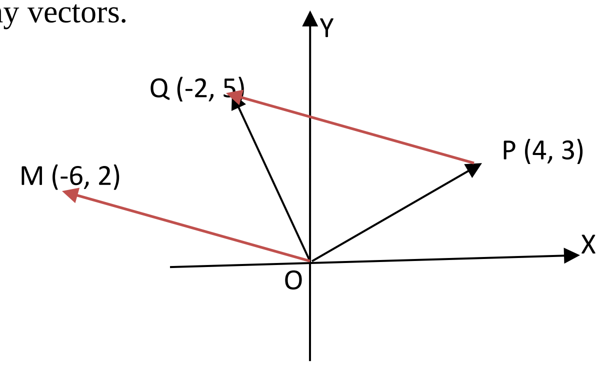
Components of vectors: consider the following figure

The vector $\vec{u}$ can be expressed as the sum of $\vec{x}$ and $\vec{y}$ as $\vec{u} = \vec{x} + \vec{y}$

From the parallelogram law or Triangle law where vectors $\vec{x} = x\vec{i}$ and $\vec{y} = y\vec{j}$ in which $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. From this, when $\vec{u} = \vec{x} + \vec{y}$, then x and y in the $\vec{u} = \vec{x} + \vec{y}$ are called the components of vector $\vec{u}$.

These components are useful in determining the direction of any vectors.

Example 1.



The position of vector P is $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and the position of vector Q is $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ as shown figure below

a. Find the vector $2\overrightarrow{PQ}$

b. $|\overrightarrow{PQ}|$

SOLUTION

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$

a. $= -\overrightarrow{OP} + \overrightarrow{OQ}$ Triangle law of vector addition negative vectors.

$$= -\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -4-2 \\ -3+5 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

b. $|\overrightarrow{PQ}| = \sqrt{(-6)^2 + (2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ units

Position Vector and Magnitude / Length.

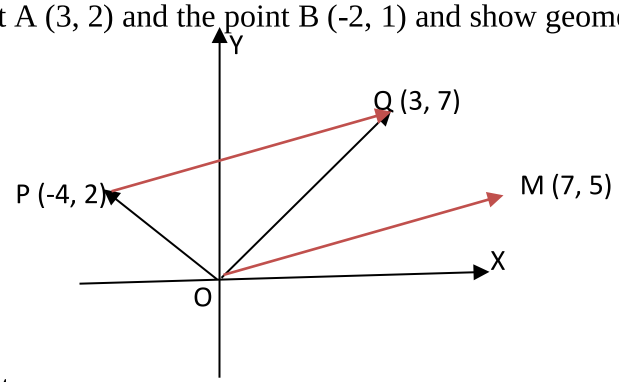
Example: 2

a. Find the position vector v for a vector that starts at Q (3, 7) and ends at P (-4, 2) and show geometrically.

b. Find the length of the vector found in part a)

c. Find the position vector between the point A (3, 2) and the point B (-2, 1) and show geometrically.

Solution:



a. The position vector of v is given by

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$

$$= -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -\begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 4+3 \\ -2+7 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

b. $|\overrightarrow{PQ}| = \sqrt{(7)^2 + (5)^2} = \sqrt{49+25} = \sqrt{74}$ units

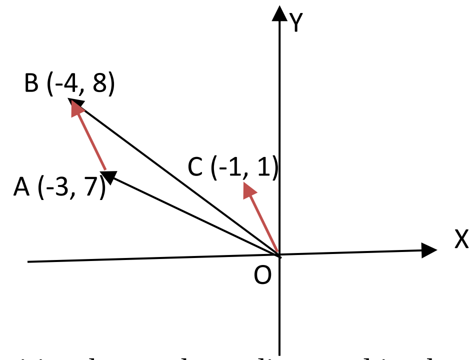
c. **exercise** In similar fashion like the solution for a

Position Vector and direction of vector

Example: 3

a. Find the position and direction of vector v whose initial and terminal points are A(-3, 7) and B(-4, 8)

Solution:



a. The position vector AB is given by

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\overrightarrow{OA} + \overrightarrow{OB}$

$= -\begin{pmatrix} -3 \\ 7 \end{pmatrix} + \begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 3-4 \\ -7+8 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Its direction the position vector of AB is given by:

From the geometrical representation and from trigonometric identities that we have discussed in chapter five, we can determine the direction of the vector.

$\tan\theta = \frac{opp}{adj} = -1/1 = -1$. The obtuse angle whose tangent value is -1 is 135^0 . Hence, the direction of the vector is [N45^0W]

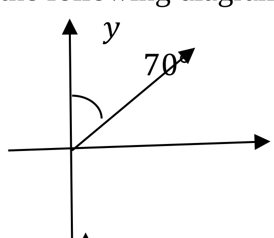
The magnitude of the vector is also $|OC| = \sqrt{(-1-0)^2 + (1-0)^2} = \sqrt{2}$ units

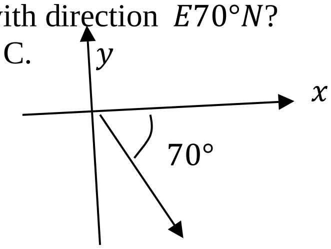
Grade 9 Mathematics Note And Some Practice Question For Second Semester

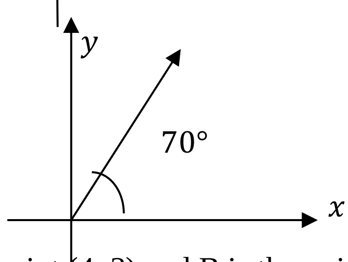
UNIT SEVEN:-VECTOR INTWO DIMENSIONS

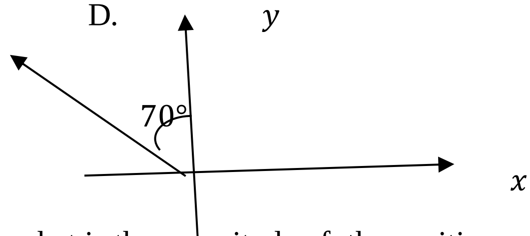
FOR EACH OF THE FOLLOWING QUESTIONS CHOOSE THE BEST ANSWER FROM THE GIVEN ALTENATIVE

1. From the following pairs of vectors which pair has vectors that are parallel to each other?
   A. $2i - 3j$ and $i + j$
   B. $3i + j$ and $-3i + j$
   C. $-5i - 2j$ and $-\frac{5}{2}i + j$
   D. $i + 2j$ and $-3i - 6j$
2. Given a vector with initial point $A(-2,2)$ and terminal point $B(1,6)$. Which one of the following vectors represents the given vector as a position vector in the same direction?
   A. $(3,4)$ B. $(-3,4)$ C. $(-1,8)$ D. $(-2,12)$
3. What is the value of m if vectors $\boldsymbol{u} = 4i + (m-3)j$ and $\boldsymbol{v} = -2i - \frac{3}{2}j$ are parallel?
   A. -6 B. 6 C. 0 D. 2/3
4. From the following pair of vectors, which pair has vectors that are parallel to each other?
   A. $\vec{u} = 2i + j$ *and* $\vec{v} = -i + 2j$
   B. $\vec{u} = 2i + 3j$ *and* $\vec{v} = 4i + 3j$
   C. $\vec{u} = -5i + 2j$ *and* $\vec{v} = \frac{1}{2}i - \frac{1}{5}j$
   D. $\vec{u} = 3j$ *and* $\vec{v} = \frac{1}{3}i$
5. Let **U**and **V** be two vectors givenby $\mathbf{U} = 3i - 2j$ and $\mathbf{V} = 5i - \frac{1}{2}j$, then the vector $\frac{1}{2}\mathbf{U} - \frac{2}{3}\mathbf{V}$ is equal to:
   A. $\frac{-29}{6}i - \frac{4}{3}j$ B. $\frac{-11}{6}i - \frac{2}{3}j$ C. $8i - \frac{3}{2}j$ D. $\frac{-11}{6}i - \frac{4}{3}$
6. If a vector $\vec{u}$ and $\vec{v}$ are equal. Then Which one of the following is **not** correct about those vectors
   A. $\vec{u} = k\vec{v}$ for a unit magnitude of k
   B. If a vector $\vec{a} = -3\vec{v}$ then vector $\vec{a}$ three time of vector $\vec{u}$ in the opposite direction of $\vec{u}$
   C. If a vector $\vec{a} = -4\vec{v}$ then vector $\vec{a}$ is parallel to vector $\vec{u}$
   D. If a vector $\vec{a} = -2\vec{v}$ then the measure of angle between vector $\vec{a}$ and vector $\vec{u}$ is 180^0
7. Which of the following diagram represents a vector with direction $E70°N$?

A.


C.


B.


D.


8. If A is the point (4, 2) and B is the point (3, -6), what is the magnitude of the position vector of B relative to A
   A. $\sqrt{53}$ B. $\sqrt{65}$ C. $\sqrt{21}$ D. 65
9. If $\vec{U} = (-2,2)$ and $\vec{V} = (0,-4)$, then the unit vector in the direction of $\vec{W} = \frac{1}{2}\vec{V} - 2\vec{U}$ is:
   A. $\left(\frac{1}{\sqrt{13}}, -\frac{1}{\sqrt{13}}\right)$ B. $\left(-\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$ C. $\left(\frac{4}{\sqrt{13}}, -\frac{6}{\sqrt{13}}\right)$ D. $\left(\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\right)$

10. If u is a vector in the opposite direction to vector $\vec{v} = \boldsymbol{i} - \frac{1}{2}\boldsymbol{j}$ having length 2 times the length of $\vec{v}$, what is the length of the vector $\vec{u} + 2\vec{v} - \boldsymbol{i}$, where $\boldsymbol{i}$ is the unit vector in the direction of the positive $x-$ axis?

A. $3\sqrt{5}$ Units B. $\sqrt{29}$ units C. $\sqrt{13}$ units D. 1 unit

11. If AB is the vector with initial point A= (1, 2) and the terminal point (3, 4) what will its terminal point be if its initial point is moved to the origin?

A. (2, 2) B. (-2,2) C. (2,-2) D. (-2,-2)

12. If **v** = $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ represents a vector with initial point at the origin, then how do you express **v** in terms of the coordinates v_1= (2, 0) and v_2= (0, 5)?

A. The vector v can be expressed in terms of its components by using Triangle Inequality as V = V1 + V2.
B. The vector v can be expressed in terms of its components by using parallelogram laws of addition as V = V1 + V2.
C. The vector v can be expressed in terms of its components by using both Triangle Inequality and parallelogram laws of addition of vector as V = V1 + V2. D. none

13. **Deborah** walked from **Wada**'s house three miles due to north then four miles due to east, while **Wada** walked six miles due to south and eight miles due to west. If **Wada** were to walk directly to **Deborah**'s location, how many hours would it take her if the **Wada** walks at a rate of three miles per hour?

A. 4 B. 5 C. 7 D. 10

14. The position vector of A relative to B is $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, and then what is the position vector of B relative to A?

A. $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ B. $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ C. $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ D. $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

15. The position vector of A relative to B is $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, and if A has coordinate (-2, 5), then what is the direction of position vector B?

A. 45^0
B. 135^0
C. At 45^0 south of west
D. At 45^0 north of west

16. If we double the magnitude of the force $\vec{F} = 2i + 5j$, in the opposite direction, then what will be the resulting force?

A. 2i+10j B. 4i+5j C. -4i-10j D. -4i+10j

17. If the position vector **V** is given as **V**=5i+j, then which one of the following vectors is equal to **V**

A. GH, where, G (3, -1) and H (2, 2)
B. RS, where, R(5, 6) and S(10,7)
C. EF, where, E(-1,-2) and F(0,3)
D. PQ, where P(-3, -4) and Q(2, -1)

18. From the following pair of vectors, which pair has vectors that are parallel to each other?

A. $\vec{u} = 3j \text{ and } \vec{v} = \frac{1}{3}i$
B. $\vec{u} = 2i + j \text{ and } \vec{v} = -i + 2j$
C. $\vec{u} = 2i + 3j \text{ and } \vec{v} = 4i + 3j$
D. $\vec{u} = -5i + 2j \text{ and } \vec{v} = \frac{1}{2}i - \frac{1}{5}j$

SECOND SEMESTER MATHEMATICS NOTE AND SOME PRACTICE QUESTION

ANSWERS TO WORK SHEET FOR EACH UNIT

UNIT-FOUR

1.A	6.D	11. B	16. B	21. D	26. C
2.D	7.D	12. D	17. C	22. C	27. A
3.C	8.C	13. B	18. C	23. D	28. A
4.B	9.C	14. A	19. B	24. A	29. B
5.A	10. B	15. A	20. C	25. B	30. C

UNIT-FIVE

1.A	12. C	23. D	34. A	45. C	56. D	67. A
2.B	13. B	24. B	35. B	46. A	57. D	68. B
3.B	14. C	25. A	36. D	47. B	58. B	69. A
4.B	15. C	26. B	37. A	48. C	59. D	70. D
5.C	16. A	27. C	38. A	49. D	60. B	71. D
6.B	17. D	28. D	39. B	50. D	61. D	72. D
7.D	18. B	29. C	40. A	51. D	62. D	73. D
8.D	19. D	30. B	41. B	52. D	63. A	74. D
9.A	20. A	31. C	42. A	53. A	64. A	
10. A	21. A	32. A	43. B	54. D	65. A	
11. A	22. B	33. A	44. D	55. B	66. B	

UNIT-SIX

1) C	6) C	11) D	16) A	21) A	26) C
2) B	7) A	12) C	17) D	22) A	27) C
3) C	8) C	13) D	18) D	23) C	
4) B	9) B	14) D	19) B	24) A	
5) D	10) D	15) C	20) A	25) D	

UNIT-SEVE

1. D	18. D
2. A	
3. B	
4. C	
5. B	
6. A	
7. C	
8. C	
9. D	
10. D	
11. A	
12. C	
13. B	
14. C	
15. B	
16. C	
17. B	