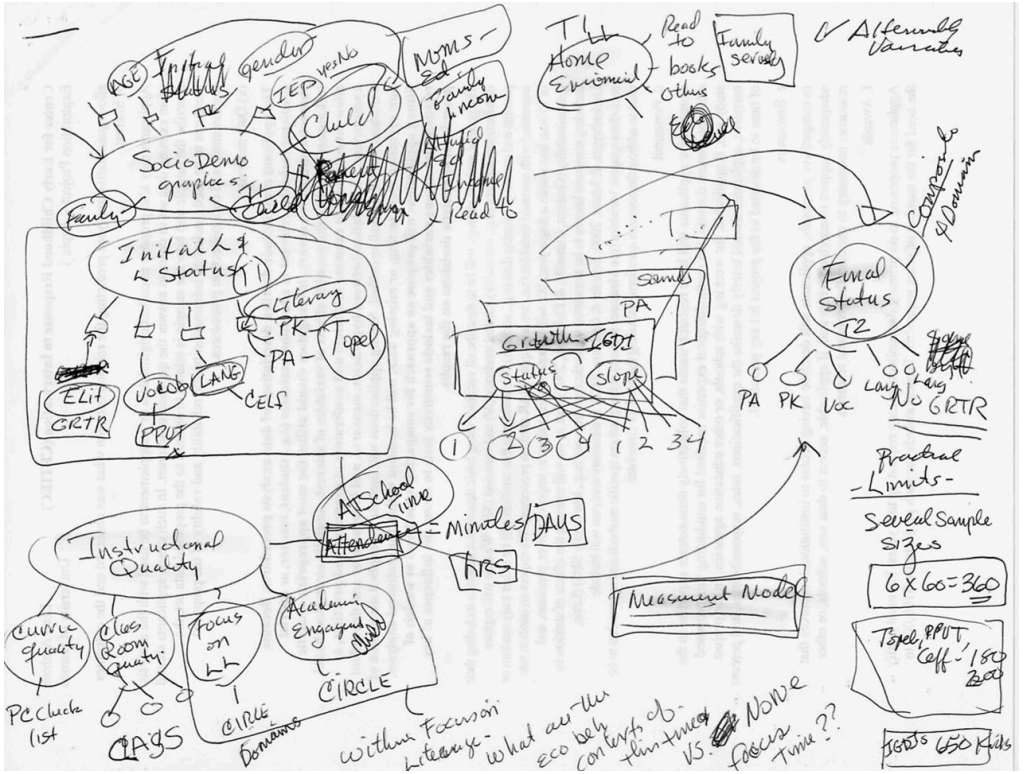
Latent variable modeling: A practical guide

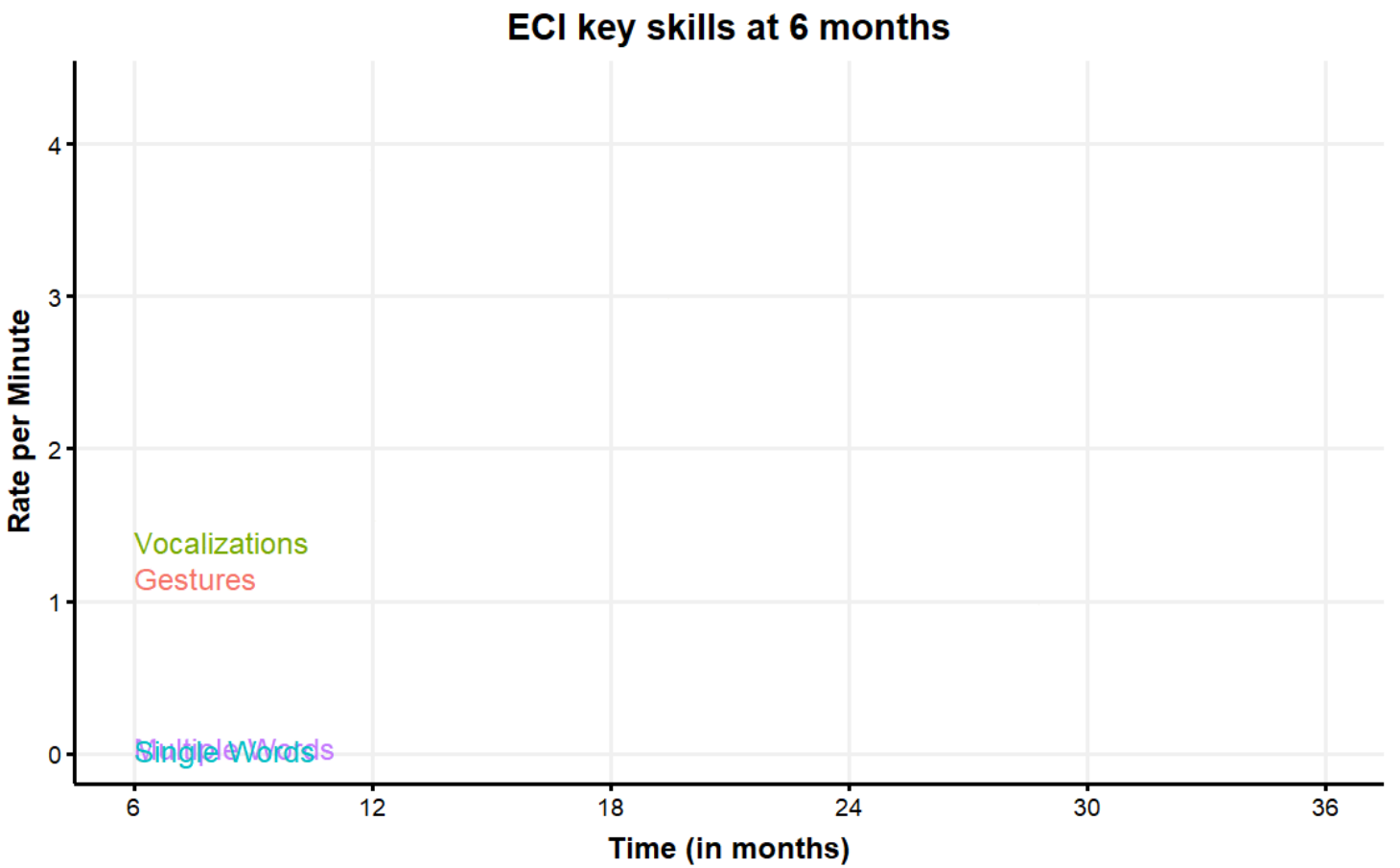
Introduction

Waylon Howard

Webinar, March 04, 2025

Navigating Research Complexities




ECI key skills at 6 months
Rate per Minute
4
3
2
1
0
Vocalizations
Gestures
Single Words
6
12
18
24
30
36
Time (in months)

About me

Waylon Howard

- Principal Quantitative Methodologist and Biostatistician at *Biostatistics, Epidemiology and Analytics in Research (BEAR) Core* (Seattle Children's Research Institute)
  - Research methods, statistics, and measurement
  - Grant writing, manuscript development, and methodological innovation
  - Training and mentoring

About you

- What's your name?
- What is your research area?
- What are your experiences with latent variable modeling in research (and the tools used)?
- What are your expectations for this workshop?

Preliminaries

Slides and material are available at

https://github.com/wwwaylon/bcorp-2025

- The session consists of a combination of lectures and hands-on exercises
- Feel free to ask questions anytime
- We will primarily rely on `GitHub`, `R` & *RStudio* with additional support for `SAS` and even `Excel`.

Workshop schedule

Wednesday, November 20th, 2024

When?	What?
12:00 - 12:10	Introduction
12:10 - 12:25	Estimation and Model Fit
12:25 - 12:45	Applied Examples
12:45 - 1:00	*Questions*

Survey Measurement

1. My friends really try to help me.
2. I can count on my friends when things go wrong.
3. I can talk about my problems with my friends.

Very Strongly Disagree	Strongly Disagree	Mildly Disagree	Neutral	Mildly Agree	Strongly Agree	Very Strongly Agree
1	2	3	4	5	6	7

Higher scores = More Perceived Social Support

1. My friends really try to help me.

Very Strongly Disagree	Strongly Disagree	Mildly Disagree	Neutral	Mildly Agree	Strongly Agree	Very Strongly Agree
1	2	3	4	5	6	7

$X_i = T_i + (S_i + e_i)$

T_i is the 'true' score

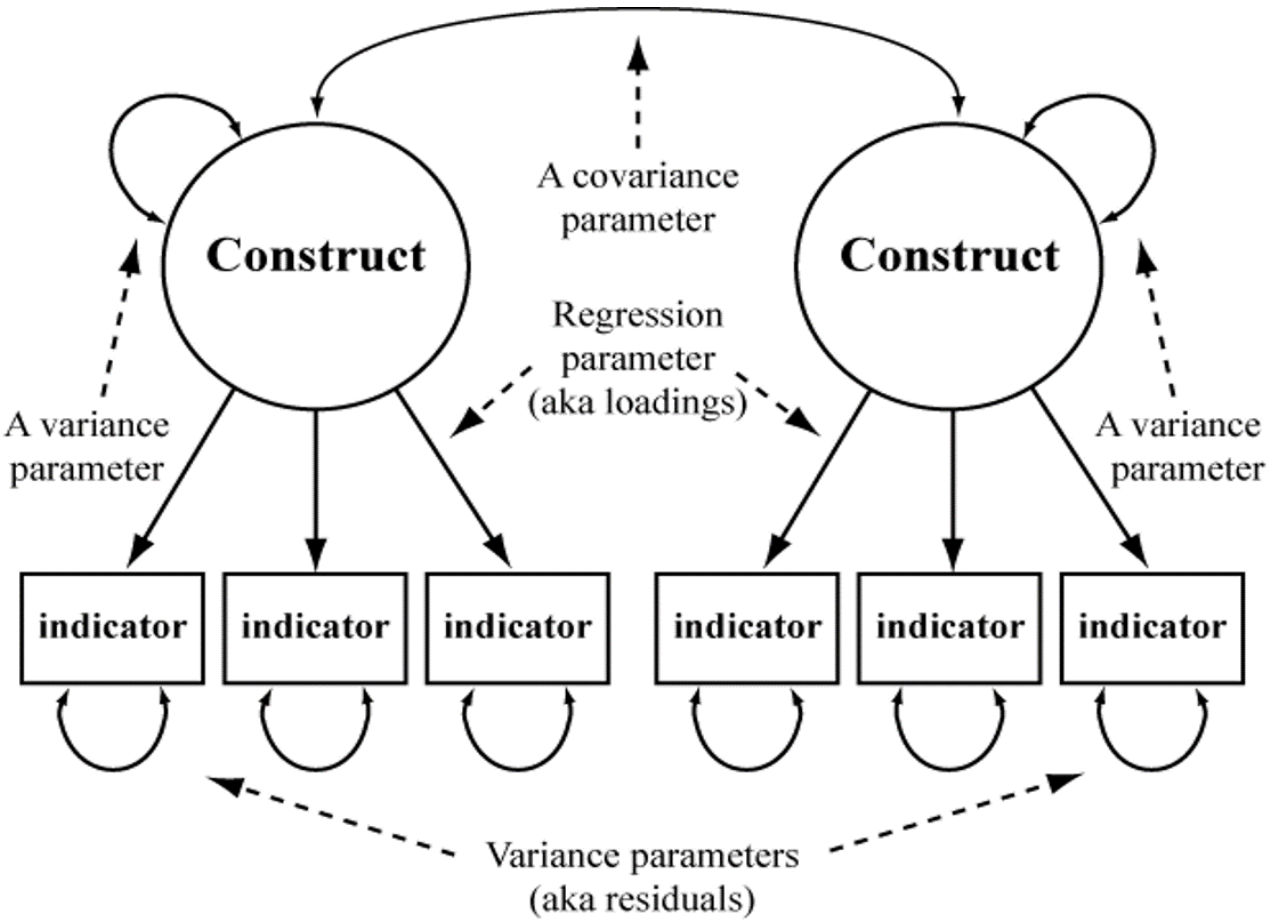
S_i is item-specific, yet reliable

e_i is random error, or noise

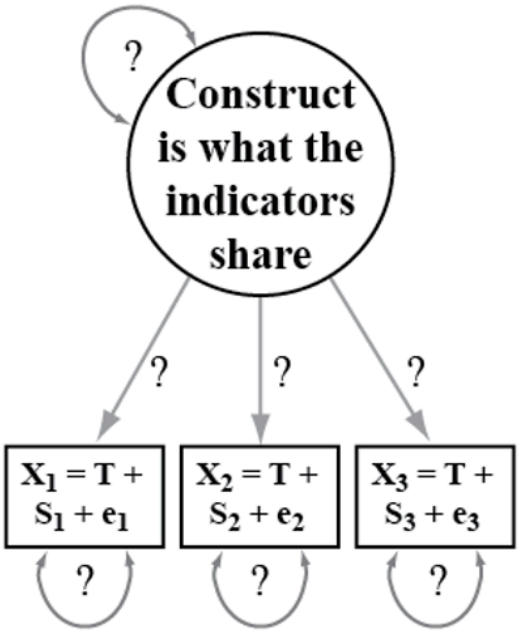
Using the scoring procedure:

- No measurement error
- Uniform items
- Invariance

Path Diagram



$$X_i = T_i + (S_i + e_i)$$



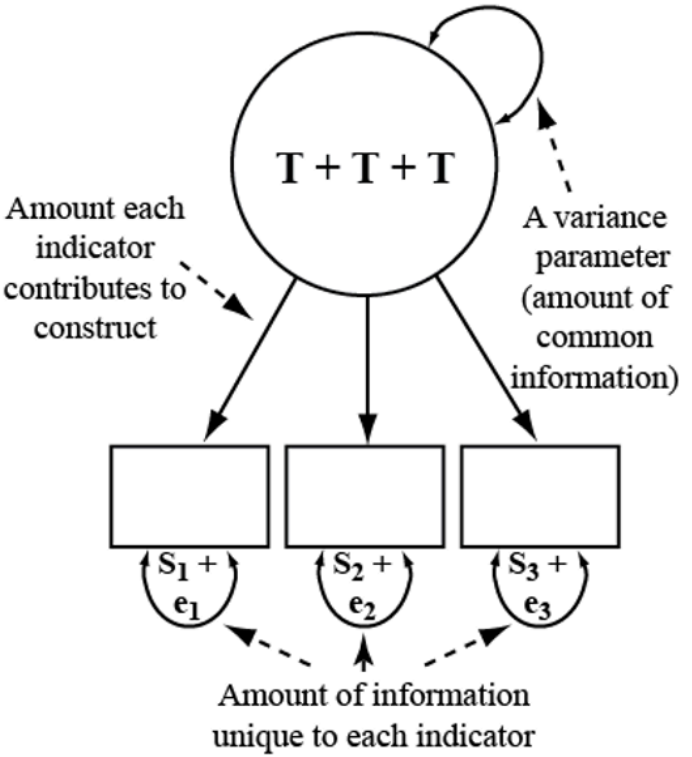
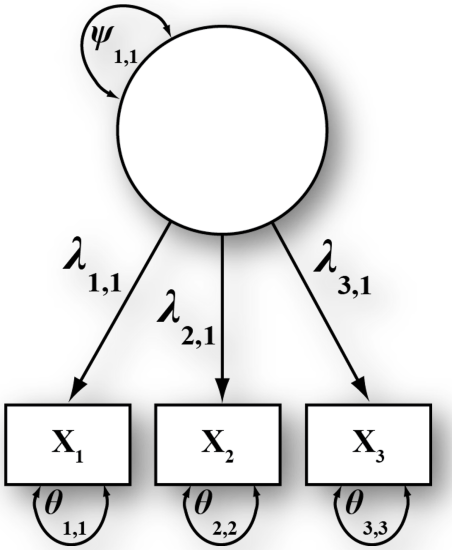


Illustration adapted from Little, T. D. (2024).



Estimated Parameters: 7

Observed Information: 6

Model Identification:

😳 $a + b = 20$

😄 $a + 10 = 20$

Matrix Formula: $\Sigma = \Lambda\Psi\Lambda' + \Theta$

Σ = Variance/Covariance Matrix

	X1	X2	X3
X1	5.66		
X2	4.90	5.50	
X3	4.33	4.38	5.63

Model Implied Matrix

	X1	X2	X3
X1	$\lambda_{11}\,\psi_{11}\,\lambda_{11} + \theta_{11}$		
X2	$\lambda_{11}\,\psi_{11}\,\lambda_{21}$	$\lambda_{21}\,\psi_{11}\,\lambda_{21} + \theta_{22}$	
X3	$\lambda_{11}\,\psi_{11}\,\lambda_{31}$	$\lambda_{21}\,\psi_{11}\,\lambda_{31}$	$\lambda_{31}\,\psi_{11}\,\lambda_{31} + \theta_{33}$

Set the scale (via latent variance)

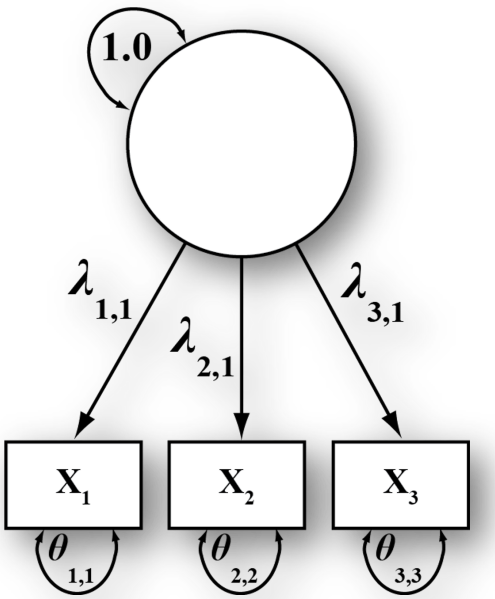
Observed Variance/Covariance Matrix

	X1	X2	X3
X1	5.66		
X2	4.90	5.50	
X3	4.33	4.38	5.63

Just Identified.

	X1	X2	X3
X1	$\lambda_{11}\lambda_{11} + \theta_{11}$		
X2	$\lambda_{11}\lambda_{21}$	$\lambda_{21}\lambda_{21} + \theta_{22}$	
X3	$\lambda_{11}\lambda_{31}$	$\lambda_{21}\lambda_{31}$	$\lambda_{31}\lambda_{31} + \theta_{33}$

Fix the latent variance to 1.0



Set the scale (via indicator)

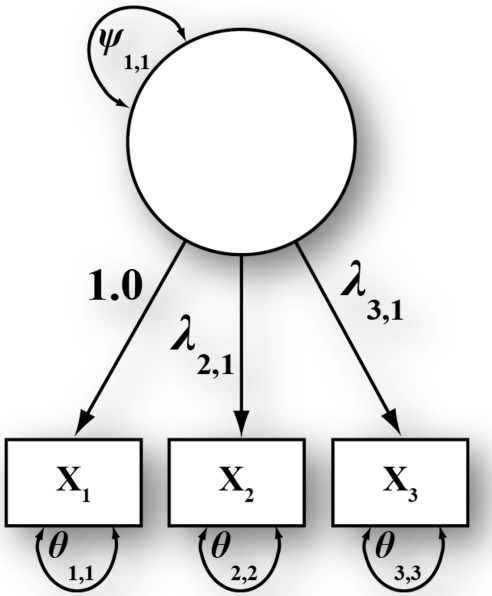
Observed Variance/Covariance Matrix

	X1	X2	X3
X1	5.66		
X2	4.90	5.50	
X3	4.33	4.38	5.63

Just Identified.

	X1	X2	X3
X1	$\psi_{11} + \theta_{11}$		
X2	$\psi_{11}\,\lambda_{21}$	$\lambda_{21}\,\psi_{11}\,\lambda_{21} + \theta_{22}$	
X3	$\psi_{11}\,\lambda_{31}$	$\lambda_{21}\,\psi_{11}\,\lambda_{31}$	$\lambda_{31}\,\psi_{11}\,\lambda_{31} + \theta_{33}$

Fix the loading to 1.0



Set the scale (via constraint)

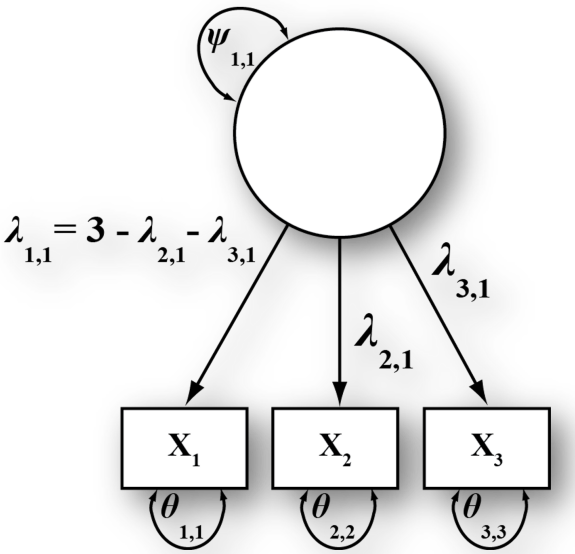
Observed Variance/Covariance Matrix

	X1	X2	X3
X1	5.66		
X2	4.90	5.50	
X3	4.33	4.38	5.63

Just Identified.

	X1	X2	X3
X1	$(3-\lambda_{21}-\lambda_{31})\ \psi_{11}\ (3-\lambda_{21}-\lambda_{31}) + \theta_{11}$		
X2	$(3-\lambda_{21}-\lambda_{31})\ \psi_{11}\ \lambda_{21}$	$\lambda_{21}\ \psi_{11}\ \lambda_{21} + \theta_{22}$	
X3	$(3-\lambda_{21}-\lambda_{31})\psi_{11}\ \lambda_{31}$	$\lambda_{21}\ \psi_{11}\ \lambda_{31}$	$\lambda_{31}\ \psi_{11}\ \lambda_{31} + \theta_{33}$

Constrain loading to average 1.0

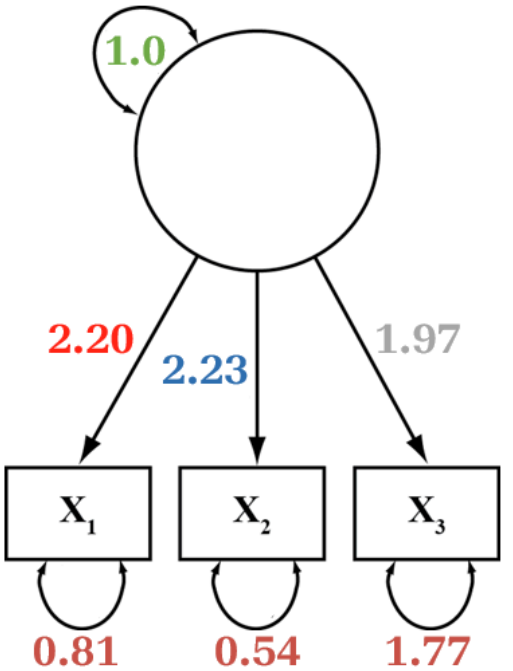


Observed Variance/Covariance Matrix

	X1	X2	X3
X1	5.66		
X2	4.90	5.50	
X3	4.33	4.38	5.63

	X1	X2	X3
X1	2.20 * 1.0 * 2.20 + 0.81 = 5.66		
X2	2.20 * 1.0 * 2.23 = 4.90	2.23 * 1.0 * 2.23 + 0.54 = 5.50	
X3	2.20 * 1.0 * 1.97 = 4.33	2.23 * 1.0 * 1.97 = 4.38	1.97 * 1.0 * 1.97 + 1.77 = 5.63

Parameter Estimates



Reporting Model Specification

> A three-factor confirmatory factor analysis (CFA) was conducted to assess the measurement properties of three latent constructs: Cognitive Engagement, Behavioral Engagement, and Emotional Engagement. The Cognitive Engagement factor was measured by three items: "I try to understand difficult concepts in class" (CE1), "I make connections between what I learn and real life" (CE2), and "I seek additional information on topics that interest me" (CE3). The Behavioral Engagement factor included three items: "I complete my assignments on time" (BE1), "I participate actively in class discussions" (BE2), and "I follow classroom rules" (BE3). The Emotional Engagement factor was measured by three items: "I feel excited about learning new things" (EE1), "I enjoy working on challenging tasks" (EE2), and "I feel connected to my classmates" (EE3). Factor variances were fixed to 1.0 for model identification (Brown, 2012).

Any questions so far?