

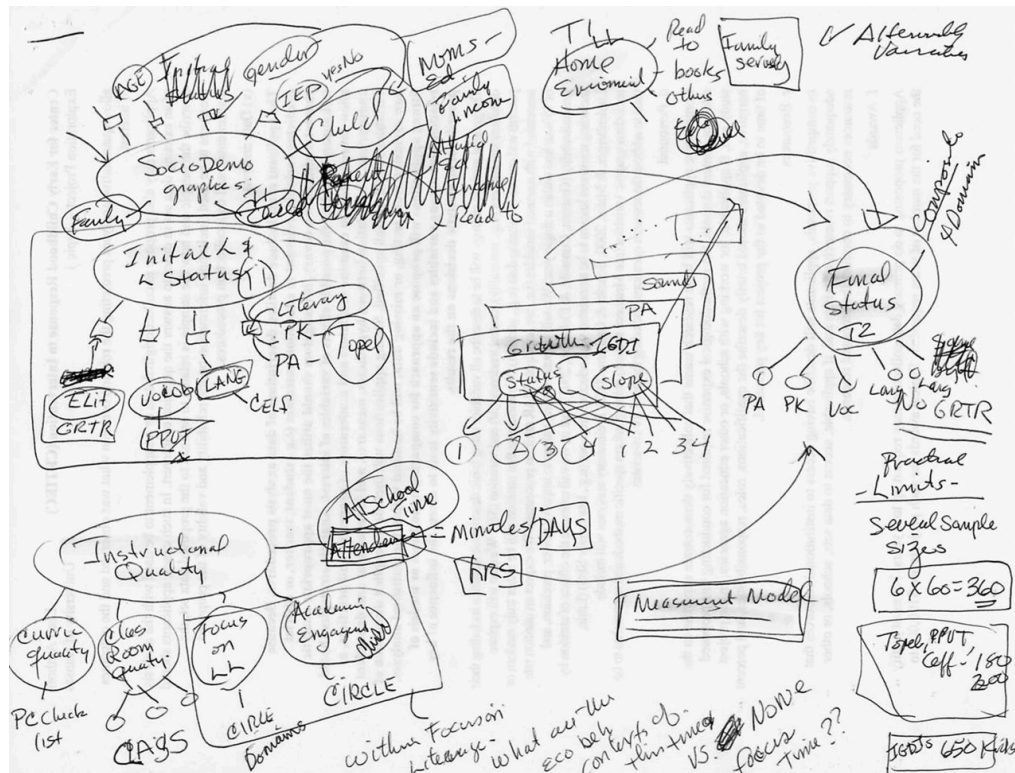
Latent variable modeling: A practical guide

Introduction

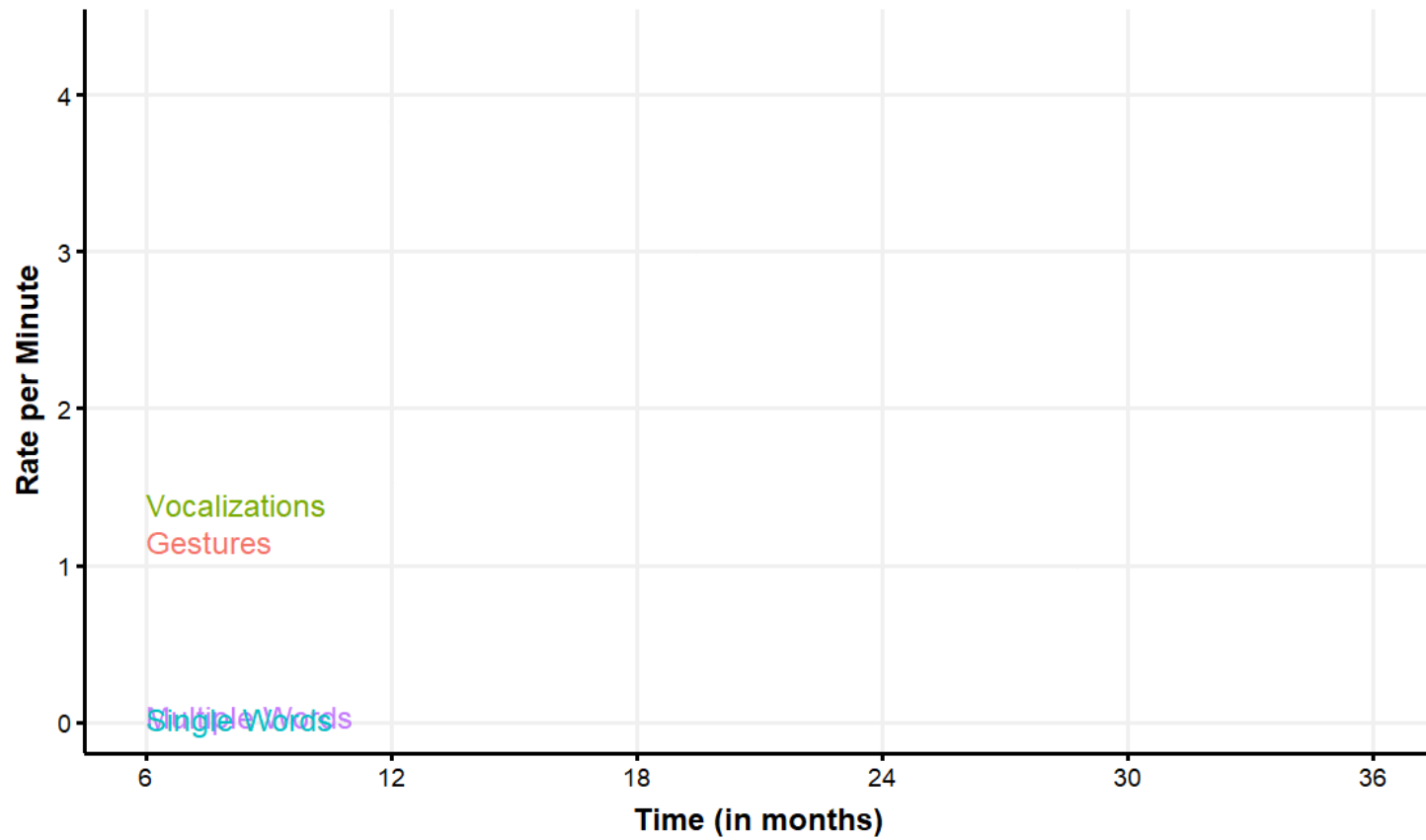
Waylon Howard

Webinar, March 05, 2025

Navigating Research Complexities



ECI key skills at 6 months



About me

Waylon Howard

- Principal Quantitative Methodologist and Biostatistician at *Biostatistics, Epidemiology and Analytics in Research (BEAR) Core* (Seattle Children's Research Institute)
 - Research methods, statistics, and measurement
 - Grant writing, manuscript development, and methodological innovation
 - Training and mentoring

About you

- What's your name?
 - What is your research area?
 - What are your experiences with latent variable modeling in research (and the tools used)?
 - What are your expectations for this workshop?
-

Preliminaries

Slides and material are available at

<https://github.com/wwwaylon/bcorp-2025>

- The session consists of a combination of lectures and hands-on exercises
- Feel free to ask questions anytime
- We will primarily rely on `GitHub`, `R` & *RStudio* with additional support for `SAS` and even `Excel`.

Workshop schedule

Wednesday, March 5th, 2025

| When? | What? |
|---------------|--------------------------|
| 12:00 - 12:10 | Introduction |
| 12:10 - 12:25 | Estimation and Model Fit |
| 12:25 - 12:45 | Applied Examples |
| 12:45 - 1:00 | <i>Questions</i> |

Survey Measurement

1. My friends really try to help me.
 2. I can count on my friends when things go wrong.
 3. I can talk about my problems with my friends.
-

| | | | | | | |
|---------------------------|----------------------|--------------------|---------|-----------------|-------------------|------------------------|
| Very Strongly Disagree | Strongly Disagree | Mildly Disagree | Neutral | Mildly Agree | Strongly Agree | Very Strongly Agree |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |

Higher scores = More Perceived Social Support

1. My friends really try to help me.

| Very Strongly Disagree | Strongly Disagree | Mildly Disagree | Neutral | Mildly Agree | Strongly Agree | Very Strongly Agree |
|------------------------|-------------------|-----------------|---------|--------------|----------------|---------------------|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |

$$X_i = T_i + (S_i + e_i)$$

T_i is the 'true' score

S_i is item-specific, yet reliable

e_i is random error, or noise

Using the scoring procedure:



No measurement error



Uniform items



Invariance

Path Diagram

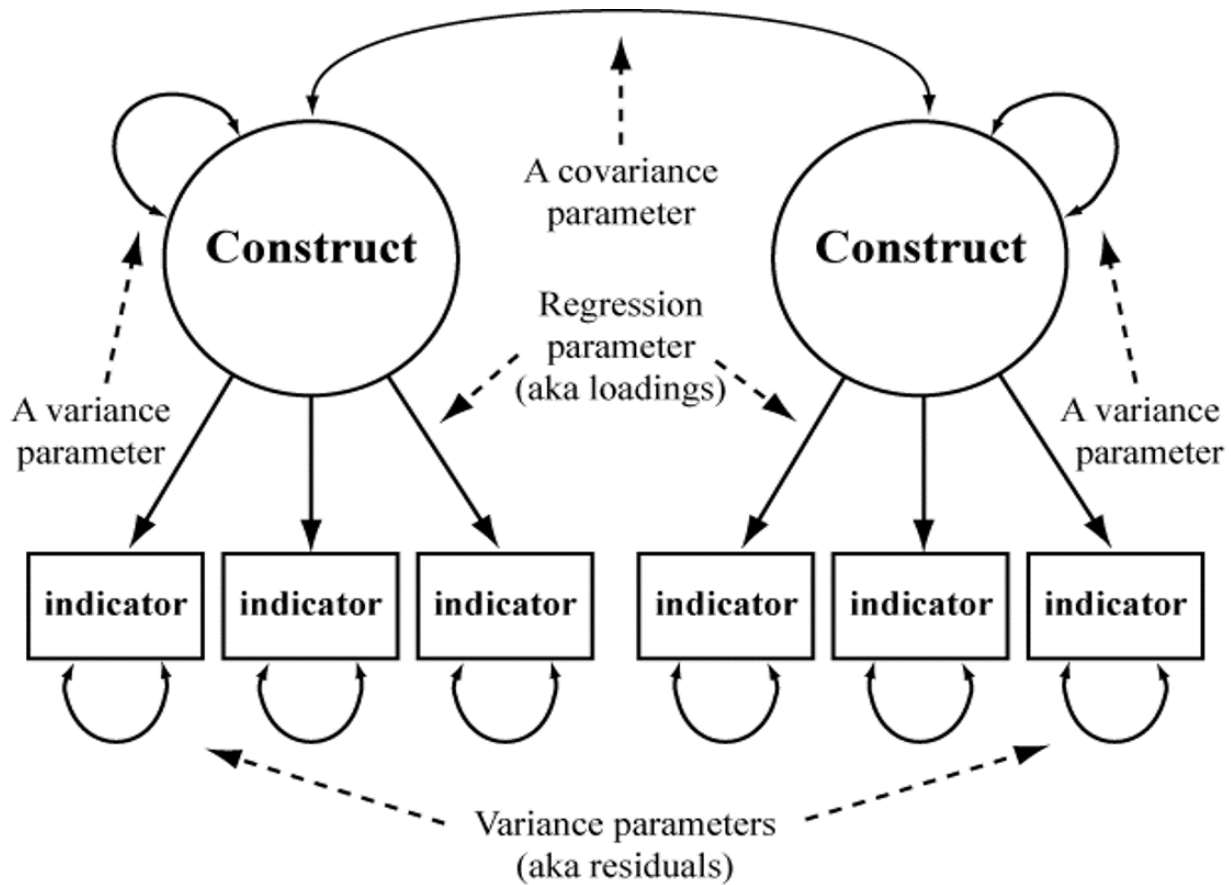


Illustration adapted from [Little, T. D. \(2024\)](#).

$$X_i = T_i + (S_i + e_i)$$

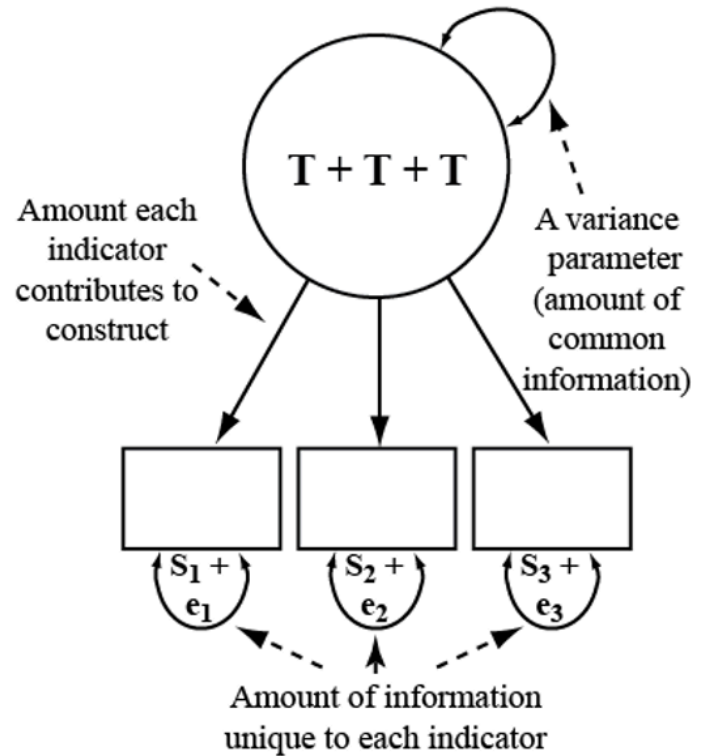
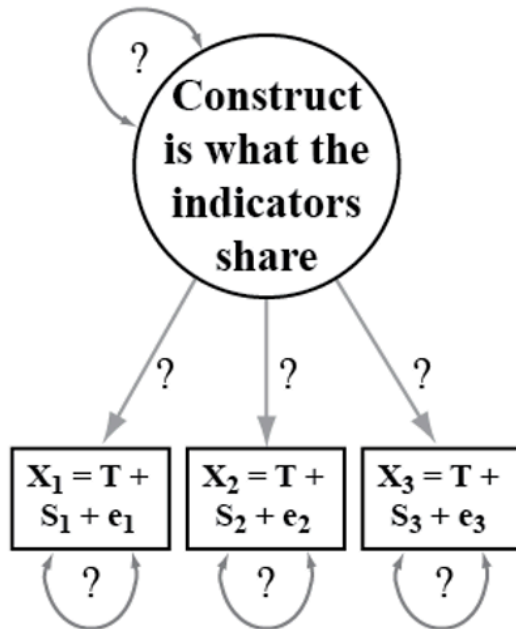
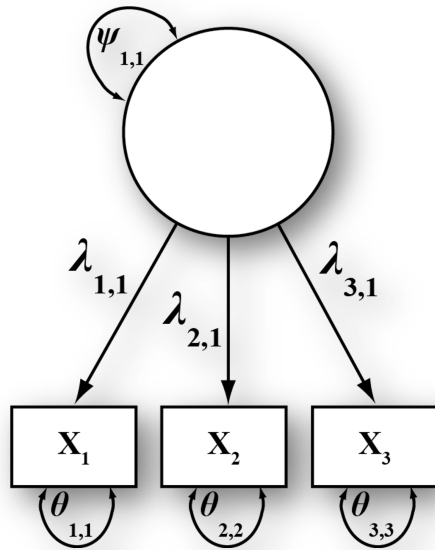


Illustration adapted from [Little, T. D. \(2024\)](#).



$$\text{Matrix Formula: } \Sigma = \Lambda \Psi \Lambda' + \Theta$$

Σ = Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|------|------|------|
| X1 | 5.66 | | |
| X2 | 4.90 | 5.50 | |
| X3 | 4.33 | 4.38 | 5.63 |

Estimated Parameters: 7

Observed Information: 6

Model Identification:

😮 $a + b = 20$

😊 $a + 10 = 20$

Model Implied Matrix

| | X1 | X2 | X3 |
|----|---|---|---|
| X1 | $\lambda_{11} \psi_{11} \lambda_{11} + \theta_{11}$ | | |
| X2 | $\lambda_{11} \psi_{11} \lambda_{21}$ | $\lambda_{21} \psi_{11} \lambda_{21} + \theta_{22}$ | |
| X3 | $\lambda_{11} \psi_{11} \lambda_{31}$ | $\lambda_{21} \psi_{11} \lambda_{31}$ | $\lambda_{31} \psi_{11} \lambda_{31} + \theta_{33}$ |

Set the scale (via latent variance)

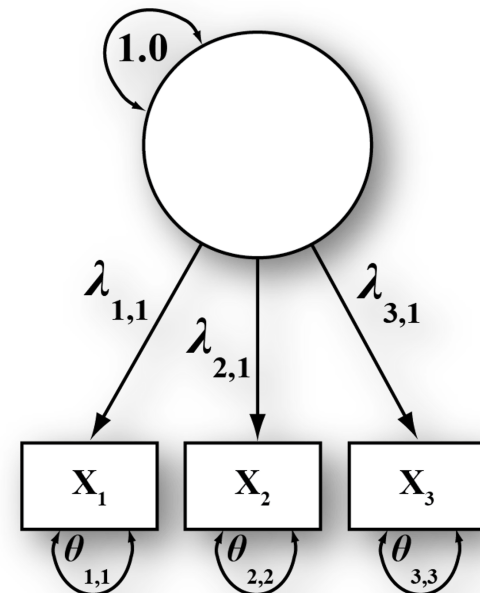
Observed Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|------|------|------|
| X1 | 5.66 | | |
| X2 | 4.90 | 5.50 | |
| X3 | 4.33 | 4.38 | 5.63 |

Implied Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|---|---|---|
| X1 | $\lambda_{11} \lambda_{11} + \theta_{11}$ | | |
| X2 | $\lambda_{11} \lambda_{21}$ | $\lambda_{21} \lambda_{21} + \theta_{22}$ | |
| X3 | $\lambda_{11} \lambda_{31}$ | $\lambda_{21} \lambda_{31}$ | $\lambda_{31} \lambda_{31} + \theta_{33}$ |

Fix the latent variance to 1.0



Just Identified.

Set the scale (via indicator)

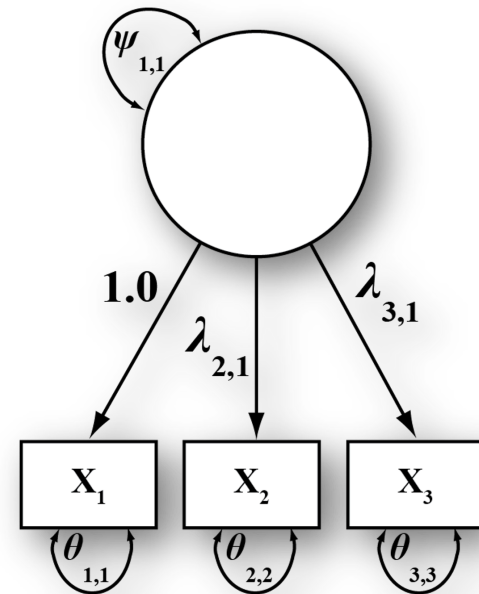
Observed Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|------|------|------|
| X1 | 5.66 | | |
| X2 | 4.90 | 5.50 | |
| X3 | 4.33 | 4.38 | 5.63 |

Implied Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|---------------------------|---|---|
| X1 | $\psi_{11} + \theta_{11}$ | | |
| X2 | $\psi_{11} \lambda_{21}$ | $\lambda_{21} \psi_{11} \lambda_{21} + \theta_{22}$ | |
| X3 | $\psi_{11} \lambda_{31}$ | $\lambda_{21} \psi_{11} \lambda_{31}$ | $\lambda_{31} \psi_{11} \lambda_{31} + \theta_{33}$ |

Fix the loading to 1.0



Set the scale (via constraint)

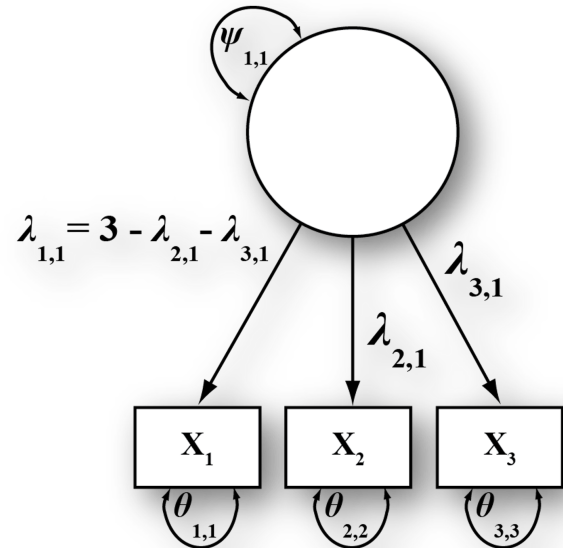
Observed Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|------|------|------|
| X1 | 5.66 | | |
| X2 | 4.90 | 5.50 | |
| X3 | 4.33 | 4.38 | 5.63 |

Constrain loading to average 1.0

Implied Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|---|---|---|
| X1 | $(3 - \lambda_{21} - \lambda_{31}) \psi_{11} (3 - \lambda_{21} - \lambda_{31}) + \theta_{11}$ | | |
| X2 | $(3 - \lambda_{21} - \lambda_{31}) \psi_{11} \lambda_{21}$ | $\lambda_{21} \psi_{11} \lambda_{21} + \theta_{22}$ | |
| X3 | $(3 - \lambda_{21} - \lambda_{31}) \psi_{11} \lambda_{31}$ | $\lambda_{21} \psi_{11} \lambda_{31}$ | $\lambda_{31} \psi_{11} \lambda_{31} + \theta_{33}$ |

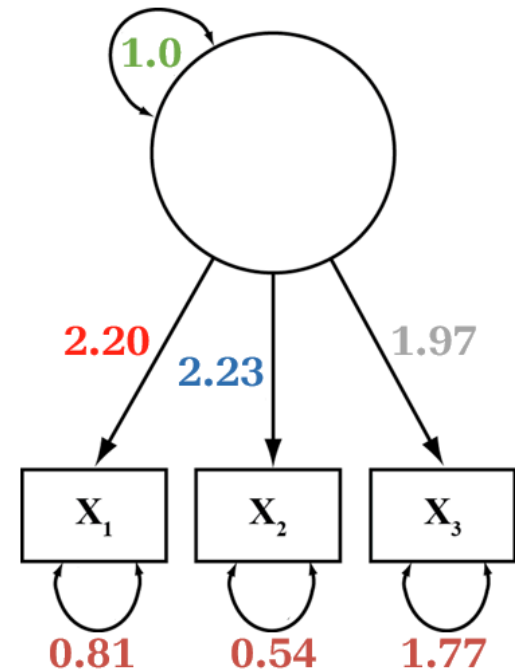


Observed Variance/Covariance Matrix

| | X1 | X2 | X3 |
|----|------|------|------|
| X1 | 5.66 | | |
| X2 | 4.90 | 5.50 | |
| X3 | 4.33 | 4.38 | 5.63 |

| | X1 | X2 | X3 |
|----|---|---|---|
| X1 | $2.20 * 1.0 *$ $2.20 + 0.81$ $= 5.66$ | | |
| X2 | $2.20 * 1.0 *$ $2.23 = 4.90$ | $2.23 * 1.0 *$ $2.23 + 0.54$ $= 5.50$ | |
| X3 | $2.20 * 1.0 *$ $1.97 = 4.33$ | $2.23 * 1.0 *$ $1.97 = 4.38$ | $1.97 * 1.0 *$ $1.97 + 1.77$ $= 5.63$ |

Parameter Estimates



Reporting Model Specification

A three-factor confirmatory factor analysis (CFA) was conducted to assess the measurement properties of three latent constructs: Cognitive Engagement, Behavioral Engagement, and Emotional Engagement. The Cognitive Engagement factor was measured by three items: "I try to understand difficult concepts in class" (CE1), "I make connections between what I learn and real life" (CE2), and "I seek additional information on topics that interest me" (CE3). The Behavioral Engagement factor included three items: "I complete my assignments on time" (BE1), "I participate actively in class discussions" (BE2), and "I follow classroom rules" (BE3). The Emotional Engagement factor was measured by three items: "I feel excited about learning new things" (EE1), "I enjoy working on challenging tasks" (EE2), and "I feel connected to my classmates" (EE3). Factor variances were fixed to 1.0 for model identification (Brown, 2012).

Any questions so far?