Handling Missing Data in Research: A Practical Guide

full information maximum likelihood (FIML, direct-ml)

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Webinar, November 20, 2024

ML estimation

- ML identifies the population parameters that are most likely given the observed data
- A likelihood (or log-likelihood) function is used to quantify how well the proposed parameters explain the observed data.
- ML requires a population distribution (normal)

ML estimation (9)

A density function gives the shape of the normal curve

$$L_i = rac{1}{\sqrt{2\pi\sigma^2}}e^{-.5rac{(Y_i-\mu)^2}{\sigma^2}}$$

 L_i (the likelihood) gives the relative probability that Y_i came from a normal distribution with a particular mean and variance.

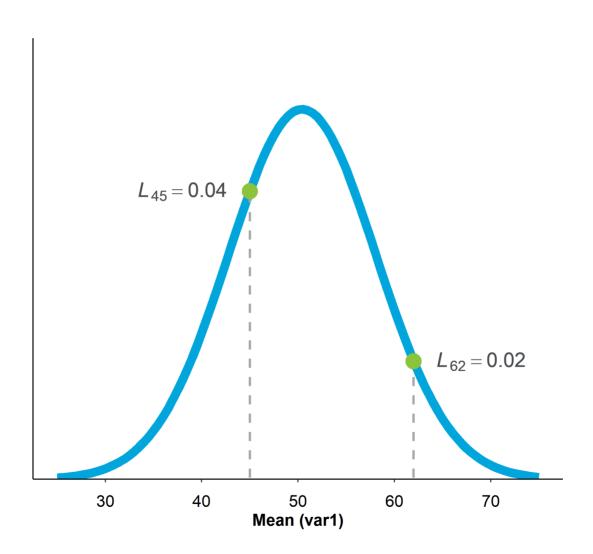
ML estimation

Applying the density function gives the relative probability (L_i) of each score from this normal distribution.

complete data:

var1 (
$$\mu$$
 = 50.42, σ = 7.65)

ID	var1	Lį
1	36.6	0.010201
2	41.8	0.027624
3	42.6	0.030908
4	43.1	0.032971
5	43.4	0.034205
6	44.2	0.037444
7	44.9	0.040166
8	46.3	0.045074
9	48.6	0.050658
10	49.0	0.051223
11	50.0	0.052038
12	51.6	0.051508
13	54.6	0.044915
14	54.8	0.044264
15	55.7	0.041102
16	57.2	0.035227
17	57.6	0.033589
18	60.3	0.022677
19	60.9	0.020433
20	65.3	0.007888



Maximum Likelihood

Multiple each (L_i) to get sample likelihood.

0.000000000000000000000000000163666415977258

Fit of this data to μ = 50.42, σ = 7.65

To avoid small numbers, we take the log of the likelihood.

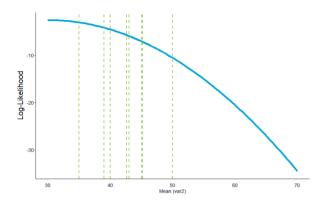
Add each $logL_i$ to get sample loglikelihood.

-68.58

ID	var1	Lį	logL _i
1	36.6	0.010201	-4.585258
2	41.8	0.027624	-3.589055
3	42.6	0.030908	-3.476754
4	43.1	0.032971	-3.412113
5	43.4	0.034205	-3.375376
6	44.2	0.037444	-3.284921
7	44.9	0.040166	-3.214733
8	46.3	0.045074	-3.099445
9	48.6	0.050658	-2.982664
10	49.0	0.051223	-2.97157
11	50.0	0.052038	-2.955783
12	51.6	0.051508	-2.966023
13	54.6	0.044915	-3.102986
14	54.8	0.044264	-3.117579
15	55.7	0.041102	-3.191692
16	57.2	0.035227	-3.345936
17	57.6	0.033589	-3.393553
18	60.3	0.022677	-3.786393
19	60.9	0.020433	-3.890587
20	65.3	0.007888	-4.842415

ID	var1	μ = 30	μ = 40	μ = 50	μ = 60	μ = 70
1	36.6	-3.326	-3.053	-4.487	-7.627	-12.474
2	41.8	-4.142	-2.982	-3.528	-5.781	-9.74
3	42.6	-4.309	-3.012	-3.422	-5.538	-9.361
4	43.1	-4.419	-3.036	-3.361	-5.391	-9.129
5	43.4	-4.487	-3.053	-3.326	-5.306	-8.992
6	44.2	-4.675	-3.105	-3.241	-5.085	-8.634
7	44.9	-4.849	-3.159	-3.176	-4.9	-8.33
8	46.3	-5.222	-3.293	-3.071	-4.556	-7.747
9	48.6	-5.906	-3.585	-2.971	-4.063	-6.862
10	49	-6.035	-3.645	-2.963	-3.987	-6.718
11	50	-6.368	-3.808	-2.954	-3.808	-6.368
12	51.6	-6.936	-4.103	-2.976	-3.556	-5.843
13	54.6	-8.118	-4.773	-3.135	-3.203	-4.978
14	54.8	-8.203	-4.823	-3.151	-3.185	-4.926
15	55.7	-8.591	-5.058	-3.231	-3.112	-4.699
16	57.2	-9.268	-5.479	-3.397	-3.021	-4.352
17	57.6	-9.455	-5.598	-3.447	-3.003	-4.266
18	60.3	-10.789	-6.471	-3.86	-2.955	-3.757
19	60.9	-11.102	-6.682	-3.968	-2.961	-3.661
20	65.3	-13.588	-8.416	-4.952	-3.194	-3.143
		-139.79	-87.13	-68.62	-84.23	-133.98
		100.70	57.15	00.02	0-1.25	155.50

Possible population means for var2



Audition different parameters to quantify how well the proposed values explain the observed data.

Green dotted lines represent observed values for var2.

Note. Listwise var2 *M* = 41.98.

ID	var2	μ = 30	μ = 40	μ = 50	μ = 60	μ = 70
1	40	-5.252	-2.341	-5.252	-13.984	-28.538
2	40	-5.252	-2.341	-5.252	-13.984	-28.538
3	35	-3.068	-3.068	-8.89	-20.533	-37.998
4	43	-7.26	-2.603	-3.767	-10.753	-23.561
5	42.6	-6.962	-2.538	-3.935	-11.154	-24.194
6	39	-4.698	-2.37	-5.863	-15.177	-30.314
7	45	-8.89	-3.068	-3.068	-8.89	-20.533
8	45.2	-9.066	-3.128	-3.011	-8.717	-20.243
9	50	-13.984	-5.252	-2.341	-5.252	-13.984
10	40	-5.252	-2.341	-5.252	-13.984	-28.538
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
		-69.68	-29.05	-46.63	-122.43	-256.44

Step 3. Report methods and software.

To address the issue of incomplete data, we used full information maximum likelihood (FIML) in R using the lavaan package (version 0.6-18). To account for non-normality in the data, we applied the robust standard error estimator by specifying the robust = TRUE option in the lavaan function. Additionally, we followed Graham's (2003) approach for incorporating auxiliary variables into the analysis. Specifically, we included five auxiliary variables: age, sex, race, education, and income to handle potential bias introduced by missing data.

Extended Reporting. Provide supplemental materials for details and code.



- FIML "fills in" missing data
- Only for SEM models
- Must have MAR to use MI/FIML
- Guaranteed better than MI
- Exogenous variables are always included
- No need with large samples

Any questions?