# ECE 5470 Lecture 16b
# Machine Learning
# (Neural Networks)

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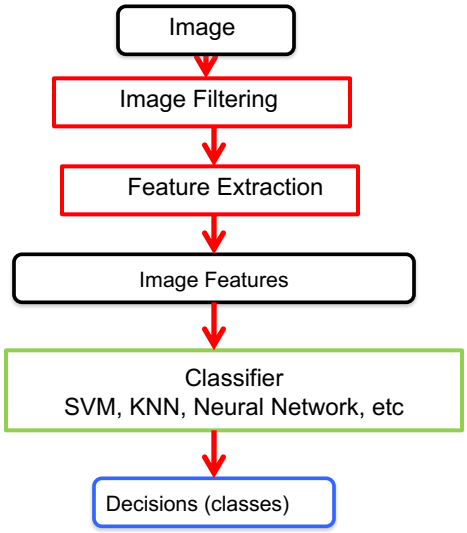
Almost all slides are taken from presentations found on the web.
Attribution is given where possible



## Image Analysis

1. Image processing (filtering)
2. Segmentation
   (object, region of interest)
3. Feature Extraction
   or Measurement
4. Classification



## Topics: Pattern Recognition

- Pattern Recognition Examples
- Discriminant Functions
- Parametric Classifiers
- Non-Parametric Classifiers
- Neural Networks

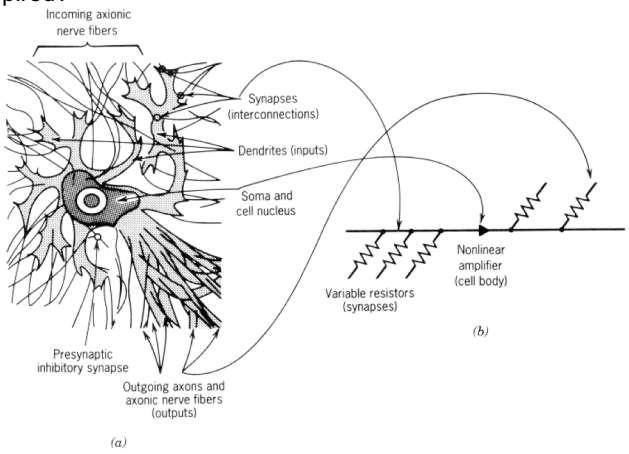


## Neural Networks

- Use elements modeled after biological neurons to form a learning network (classifier).
- Neural networks provide an alternative (adaptive, learning strategy) for performing computations and making decisions

# The Neuron

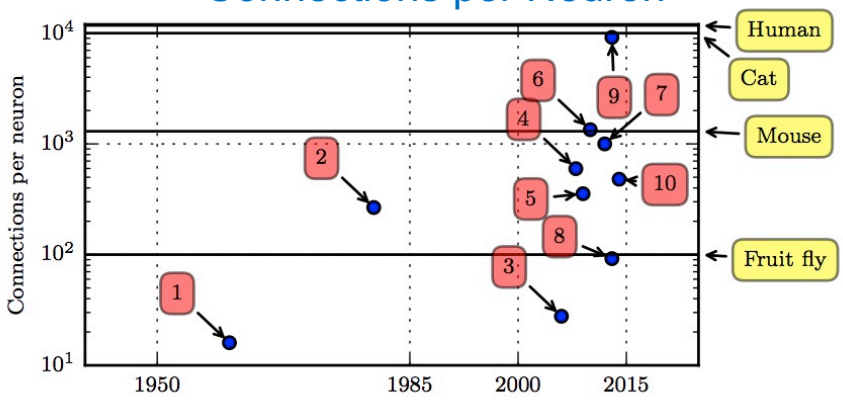
Biologically inspired?




Biological neuron Electronic analogy



# Connections per Neuron



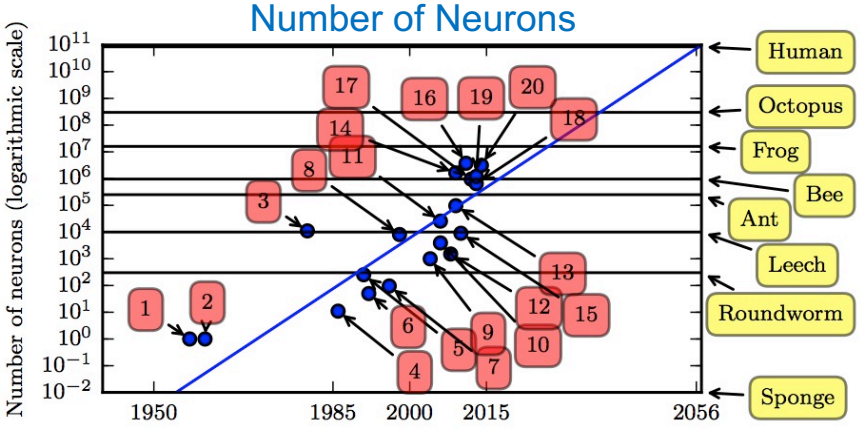

1. Adaptive linear element (Widrow and Hoff, 1960)
2. Neocognitron (Fukushima, 1980)

. . .

8. Multi-GPU convolutional network (Krizhevsky et al., 2012)
9. COTS HPC unsupervised convolutional network (Coates et al., 2013)
10. GoogLeNet (Szegedy et al., 2014a)

Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, 2016. http://www.deeplearningbook.org.

## Number of Neurons




1. Perceptron (Rosenblatt, 1958, 1962)
2. 2. Adaptive linear element (Widrow and Hoff, 1960)
3. 3. Neocognitron (Fukushima, 1980)

. . .

18. Multi-GPU convolutional network (Krizhevsky et al., 2012)
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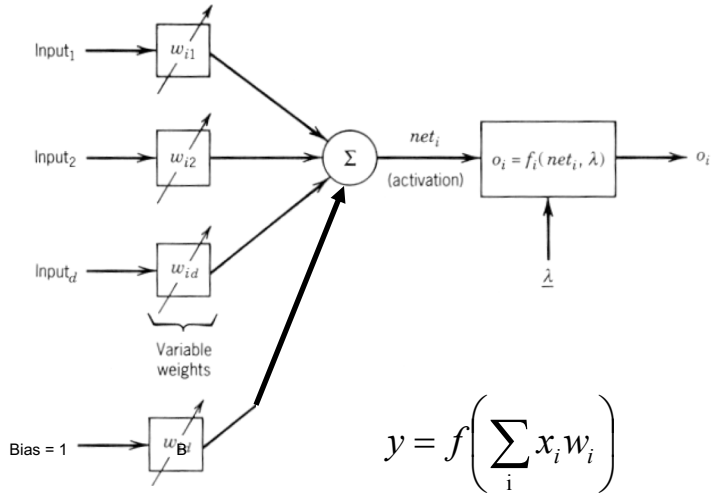
## Artificial Neuron Model




$$y = f\left(\sum_{i} x_i w_i\right)$$

## Neural Network Issues

- Network Topology:
  - How to interconnect neurons
  - feedback? Structure? Multiple layers?
- The Neuron Model
- Strategy for learning or training
  - Present a set of inputs and make incremental changes until the desired behavior is achieved



## The Perceptron

- The output function is a threshold

Bias *(b)*

$$f(\underline{x}) = \begin{cases} 1 \text{ if } \sum_i x_i w_i > w_0 \\ 0 \text{ otherwise} \end{cases}$$

- This function partitions hyperspace with a hyperplane (c.f. linear discriminant functions)
- Simple topology, Convergent training algorithm
- Problem: limited functionality

R. Rosenblatt, *Principles of Neurodynamics* (1959)

## Perceptron Training

1. Initialize weights and threshold to small random values
2. Present new input *x* and the desired output
3. Calculate the actual output *y(t)*
4. Adapt weights

$$w_i(t+1) = w_i(t) + \eta(d(t) - y(t))x_i(t)$$

$$d(t) = \begin{cases} +1 \text{ if input from class A} \\ -1 \text{ if input from class B} \end{cases}$$

η is a positive gain fraction < 1 Learning Rate

5. Repeat by going to step 2



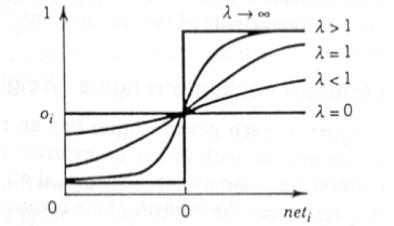
## The Perceptron

- The perceptron convergence procedure may oscillate continuously if the inputs are not separable.
  - One solution, replace threshold output function with a smoother (multilevel) function. Weights are corrected based on the difference between the desired output and the actual output
- The Perceptron cannot solve the exclusive OR problem

M. Minskey and S. Papert *Perceptrons an Introduction to Computational Geometry* (1969)

# The Sigmoid output function

$$y = \frac{1}{1 + e^{-\lambda x}}$$



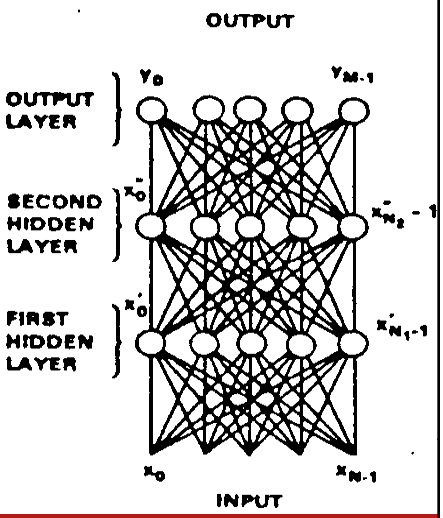

- $\lambda$ is an adjustable gain parameter
- Typically $\lambda = 1$
- $\lambda = \infty$ implies a threshold function



# Multi-layer Multi-output Neural Network

## Multi-Layer Perceptron (MLP)

- Rumelhart, Hinton and Williams *Learning Internal Representations by Error Propagation* (1986)
- Back-propagation learning algorithm
- Multi-output networks: typically use one output for each class (1 out of N code)



## Back-Propagation Training

1. Initialize weights and threshold to small random values
2. Present new input *x* and the desired outputs *d*
3. Calculate the actual output
4. Adapt weights, start at the output nodes

$$w_{ij}(t+1) = w_{ij}(t) + \eta \delta_j x'_i$$

$$\delta_j = y_j(1-y_j)(d_j - y_j) \quad \text{for output node}$$

$$\delta_j = x'_j(1-x'_j)\sum_k \delta_k W_{jk} \quad \text{for internal node}$$

$\eta$ is a positive gain fraction < 1

5. Repeat by going to step 2



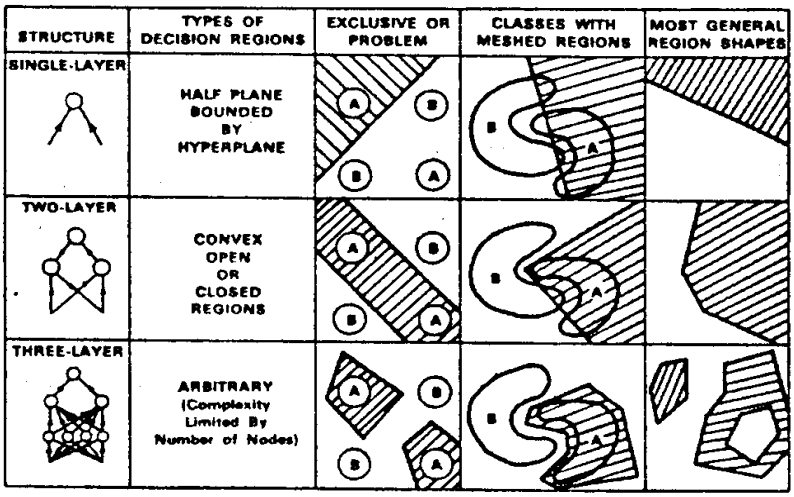
## Issues with back-propagation learning

- Slow convergence
  - **Momentum** term may be added to speed convergence

$$w_{ij}(t+1) = w_{ij}(t) + \eta \delta_j x'_i + \alpha(w_{ij}(t) - w_{ij}(t-1))$$

  - where $0 < \alpha < 1$
- Inability to deal with contradictory training information
- Overfitting

# Capabilities of multi-layer Neural networks






# Kolmogorov's Mapping Neural Network Existence Theorem (1957)

- Any continuous function in *d* inputs and *c* outputs can be implemented by a three layer network
  - layer 1 (*d*-elements) holds the input
  - layer 2 (*2d+ 1* elements) nonlinear continuously increasing functions
  - layer 3 (*c* elements) produces the outputs
  - layers 2 and 3 elements contain nonlinear continuously increasing functions
- The theorem does not indicate how weights or nonlinearities should be selected

## Recent Neural Network Developments

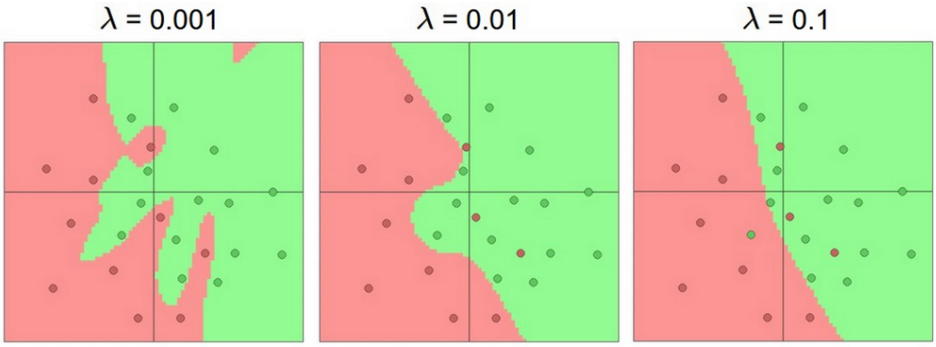
- RELU Nonlinearity
- Train in mini-batches (Stochastic gradient descent)
- Softmax Loss
- Learning rate schedule
- Dropout
- Regularization
- Data Augmentation

## Overfitting

In statistics and machine learning, **overfitting** occurs when a statistical model describes random error or noise instead of the underlying relationship.

Overfitting generally occurs when a model is excessively complex, such as having too many parameters relative to the number of observations. A model that has been overfit will generally have poor predictive performance, as it can exaggerate minor fluctuations in the data.

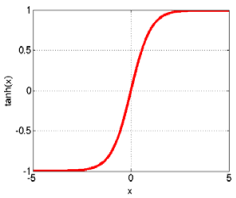
# Overfitting

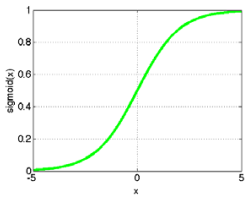


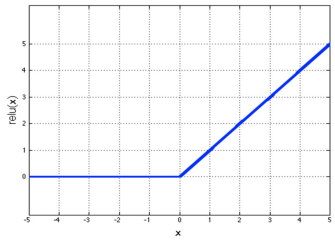



# Non-Linearity

- Per-element (independent)
- Options:
  - Tanh (1-exp(-x))/(1+exp(-x))
  - Sigmoid: 1/(1+exp(-x))
  - Rectified Linear Unit (ReLU)
    - max(0, x)
    - Preferred option for CNN
    - Simplifies backpropagation
    - Makes learning faster
    - Avoids saturation issues




# Output Functions

- **Two class problem**
  - Train for a high response to class A and a low response for class B
  - For testing no non-linearity required
  - For training can use sigmoid function for **score** to limit impact of outliers
- **Multiclass Task**
  - Use a one-out-of-N code with N outputs
  - For testing select class with highest output
  - For training Use softmax function for score

  (normalize response so that it is scaled between 0 – 1)

$$x_i \rightarrow \cdots \rightarrow f_i \rightarrow \boxed{\text{Softmax}} \rightarrow p_i$$

*(input)* *(scores)* *(probabilities)*

$$p_{i,j} = \frac{e^{f_{i,j}}}{\sum_{k}^{N} e^{f_{i,k}}}$$

- **Regression Task**
  - To predict a real value no non-linear function is required



# Loss Functions

Training: Minimize the Loss function **L** based on the **score.**

Use different loss functions for different tasks:

- **Softmax loss** $L_i$ is used for predicting a single class of K mutually exclusive classes.

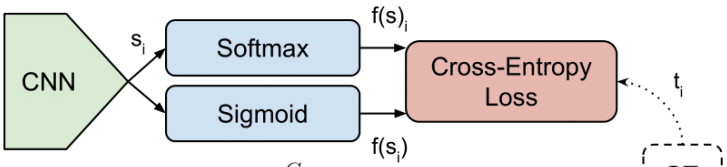
$$p_{i,j} = \frac{e^{f_{i,j}}}{\sum_{k} e^{f_{i,k}}} \quad \text{(Softmax)}$$

$$L_i = -\log p_{i,y_i} \quad \text{(Cross-entropy)}$$

Probability for correct output

- **Euclidean loss** is used for regressing to real-valued labels [-inf,inf]
- (**Sigmoid cross-entropy loss** is used for predicting K independent probability values in [0,1]. )

# Cross-Entropy Loss




$$CE = -\sum_{i}^{C} t_i log(f(s)_i)$$

$C'=2$

Where $t_i$ and $s_i$ are the ground truth and the CNN score for each class i in C

In a **binary classification problem**, where C′=2

$$CE = -\sum_{i=1}^{C'=2} t_i log(s_i) = -t_1 log(s_1) + (1 - t_1) log(1 - s_1)$$

For the general (not 1-hot multilabel case) $$CE = -\sum_{i}^{C} t_i log(s_i)$$

# Cross-Entropy Loss

In the specific (and usual) case of **Multi-Class classification** the labels are one-hot, so only the positive class Cp keeps its term in the loss. There is only one element of the Target vector t which is not zero $t_i=t_p$. So discarding the elements of the summation which are zero due to target labels, we can write:

$$CE = -log\left(\frac{e^{s_p}}{\sum_j^C e^{s_j}}\right)$$

Where **Sp** is the CNN score for the positive class.

The derivative with respect to the positive class is:

$$\frac{\partial}{\partial s_p}\left(-log\left(\frac{e^{s_p}}{\sum_j^C e^{s_j}}\right)\right) = \left(\frac{e^{s_p}}{\sum_j^C e^{s_j}} - 1\right)$$

The derivative with respect to the other (negative) classes is:

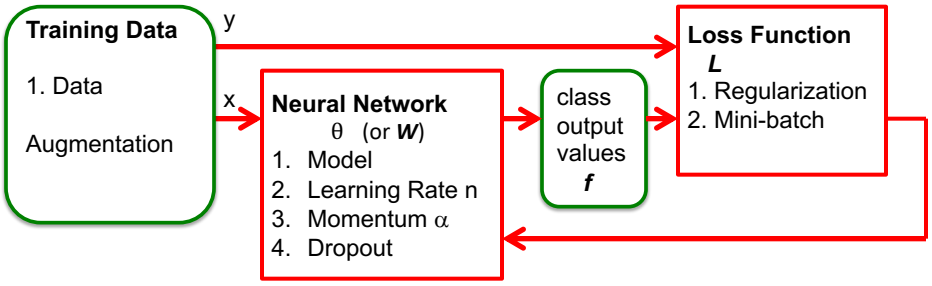
$$\frac{\partial}{\partial s_n}\left(-log\left(\frac{e^{s_n}}{\sum_j^C e^{s_j}}\right)\right) = \left(\frac{e^{s_n}}{\sum_j^C e^{s_j}}\right)$$

Where $s_n$ is the score of any negative class in C different from $C_p$.

**Logistic Loss** and **Multinomial Logistic Loss** are other names for **Cross-Entropy loss**.

https://gombru.github.io/2018/05/23/cross_entropy_loss/

## Neural Network Training




x Input vector
y Label (true value)
θ Network parameters (also ***W***)
f Score
L Loss function

**Network Operations**

Forward propagation $f = f_{infer}(x, \theta)$

Backward propagation $\theta = f_{learn}(L, \theta)$



## Neural Network Training

**Stochastic Gradient Descent**

(Stochastic: => train in mini-batches rather than individually)

- Learning rate η
- Learning rate schedule (decrease η with time)
- Momentum α
- Mini-batch size

  Compute responses for a small set of inputs and train at a time
- Regularization
- Dropout
- Number of cycles of training (epochs)

Input conditioning techniques:
- Data Augmentation

Initialization
- Weight initialization

# Learning Rate (η) Schedule

**How do we change the learning rate over time?**

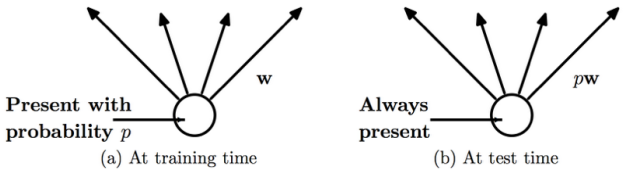
**Various choices:**

- Step down by a factor of 0.1 every 50,000 mini-batches (used by SuperVision [Krizhevsky 2012])
- Decrease by a factor of 0.97 every epoch (used by GoogLeNet [Szegedy 2014])
- Scale by sqrt(1-t/max_t) (used by BVLC to re-implement GoogLeNet)
- Scale by 1/t
- Scale by exp(-t)



# Dropout

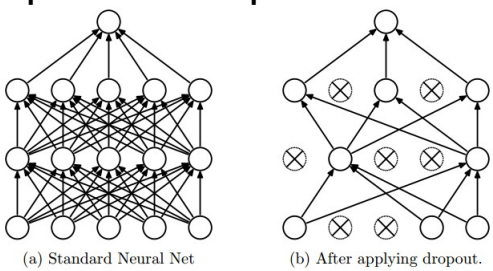
**Simple but powerful technique to reduce overfitting:**




[Srivasta et al, "Dropout: A Simple Way to Prevent Neural Networks from Overfitting", JMLR 2014]

# Dropout

**Simple but powerful technique to reduce overfitting:**



(a) Standard Neural Net (b) After applying dropout.

**Note:** Dropout can be interpreted as an approximation to taking the geometric mean of an ensemble of exponentially many models

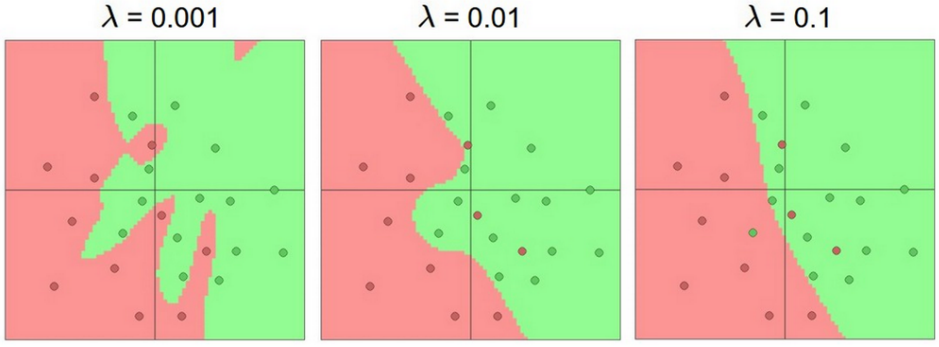
[Srivasta et al, "Dropout: A Simple Way to Prevent Neural Networks from Overfitting", JMLR 2014]



# Regularization

**Regularization reduces overfitting:**

$$L = L_{\text{data}} + L_{\text{reg}} \qquad L_{\text{reg}} = \lambda \frac{1}{2} ||W||_2^2$$




[Andrej Karpathy http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html]

# Regularization

**L2 regularization** $L_{\text{reg}} = \lambda \frac{1}{2} ||W||_2^2$

(L2 regularization encourages small weights)

**L1 regularization** $L_{\text{reg}} = \lambda ||W||_1 = \lambda \sum_{ij} |W_{ij}|$

(L1 regularization encourages sparse weights:
weights are encouraged to reduce to exactly zero)

**"Elastic net"** $L_{\text{reg}} = \lambda_1 ||W||_1 + \lambda_2 ||W||_2^2$

(combine L1 and L2 regularization)

**Max norm**

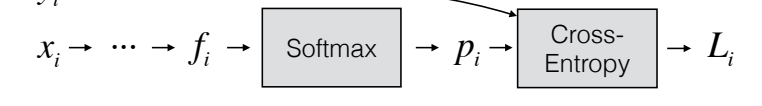
Clamp weights to some max norm $||W||_2^2 \leq c$

# Example: Softmax (for N inputs)

Let's assume we are using Softmax and Cross-entropy loss
(together this is often called "Softmax loss")

*(ground truth labels)* $y_i$

(here, "i" are different examples)



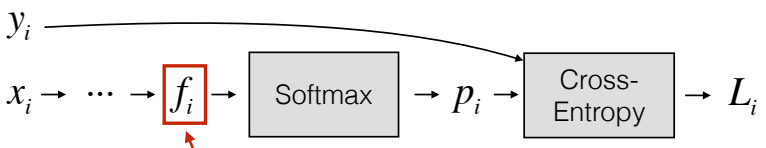

*(input)* *(scores)* *(probabilities)* *(loss)*

$$p_{i,j} = \frac{e^{f_{i,j}}}{\sum_k e^{f_{i,k}}}$$ (Softmax)

$$L_i = -\log p_{i,y_i}$$ (Cross-entropy)

$$L = \frac{1}{N} \sum_i L_i$$ (Avg. over examples)

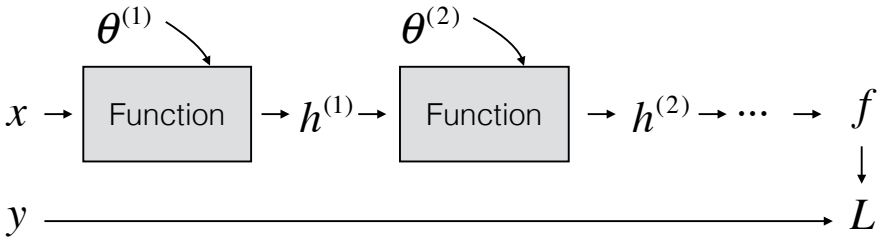
# Example: Softmax (for N inputs)




Derivative: $\frac{\partial L}{\partial f_{i,j}} = \frac{p_{i,j} - t_{i,j}}{N}$ where $t_i = [0 \ \ldots 1 \ \ldots 0]$ (Entry $y_i$ set to 1)

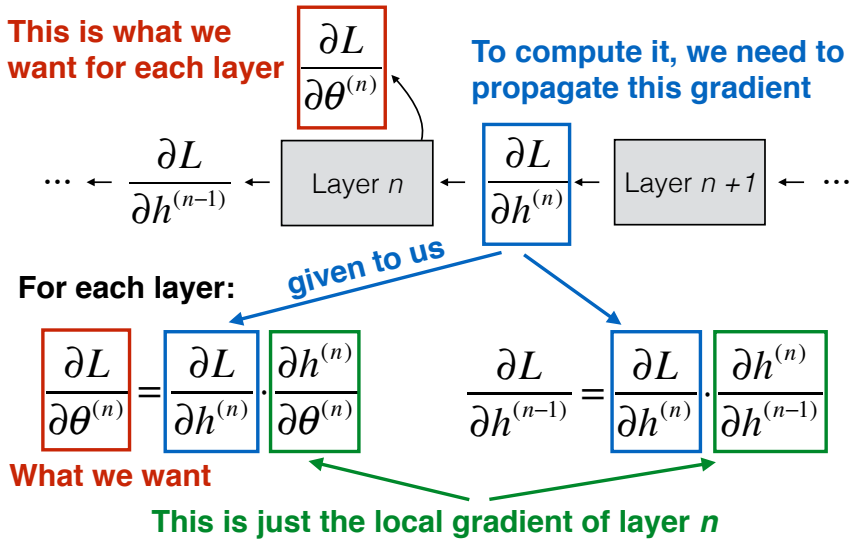
softmax loss

# Example: General Model




- **Goal:** Find a value for parameters ($\theta^{(1)}$, $\theta^{(2)}$, …), so that the loss (L) is small

## Backpropagation using the Chain Rule




## Neural Networks Summary

- Perceptrons
  - similar properties to linear discriminant functions.
  - Training may require a lot of time and result may not be optimal
- Multi-layer feed-forward networks (3 layers)
  - may require a very long time to train
  - can implement arbitrary complexity (depending upon number of elements in hidden layer)
  - may be faster and simpler than non-parametric classifiers (but require retraining for new data)