Computer Vision ECE5470

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Lecture 14: The Hough Transforms

The Hough Transform

- Concept:

  design a transform which maps all points associated with a "feature" to a single point in the transformed parameter space

- The location of this point in feature space indicates the feature parameters of interest
- On using the Hough transform, evidence for a feature is accumulated at a single location in the transformed space

The Straight Line Hough Transform

Task

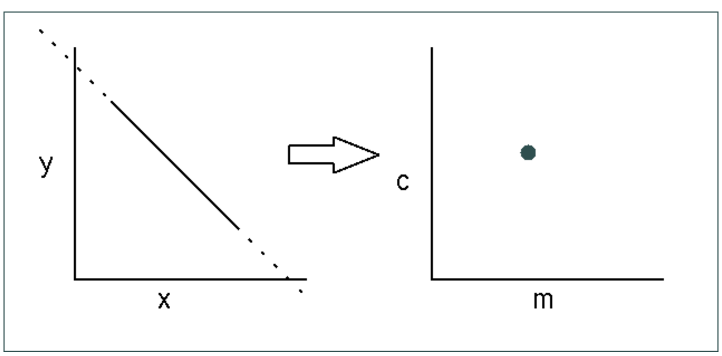
- To detect all the straight lines in an image even when the edge information is incomplete

The Straight Line Hough Transform

- A straight line can be represented by:

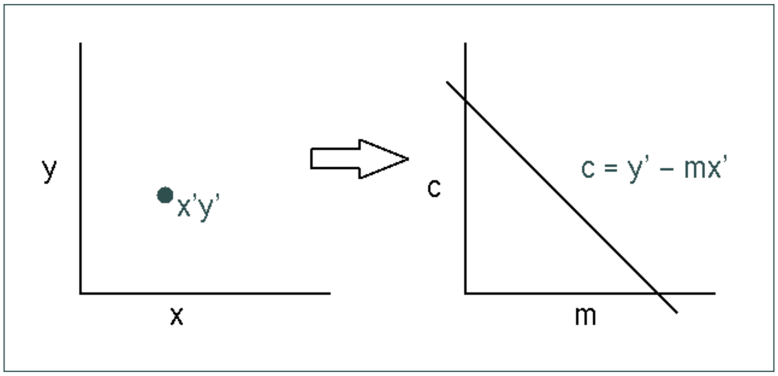
$$y = mx + c$$

- The desired parameters are m and c



The Straight Line Hough Transform

A point in x-y space maps to a line in m-c space

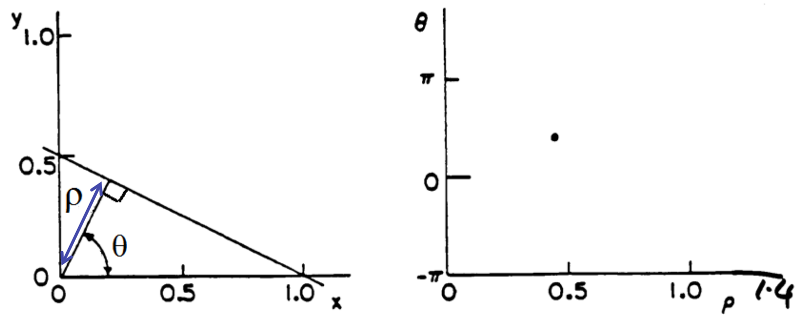


The Straight Line Hough Transform

- Usually thresholded edge images are transformed; i.e., only significant edge pixels are mapped into transform space.
- Method:
  1. Partition Hough space into a set of accumulator cells.
  2. Increment all possible cells in Hough space for each pixel.
  3. The cell with the maximum value gives the line equation
- The contribution of a pixel to a cell may be weighted by the edge strength. This may have undesirable effects. (phantom lines or missed weak boundaries)

Polar Coordinates

- A problem with partitioning *m-c* space is that *m* can be ∞
- Solution: use polar coordinates, $\rho = x \cos \theta + y \sin \theta$



(a) PARAMETRIC LINE (b) HOUGH TRANSFORM OF (a)

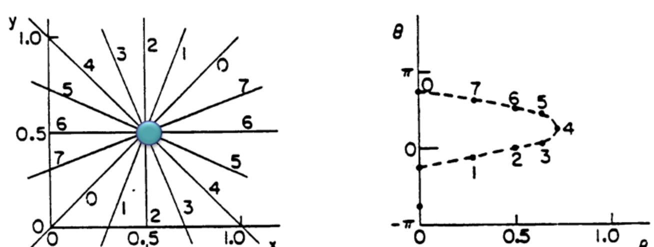
The Straight Line Hough Transform

Each point (x, y) in image space

- maps into a sine wave in Hough space (c.f. the Radon transform)

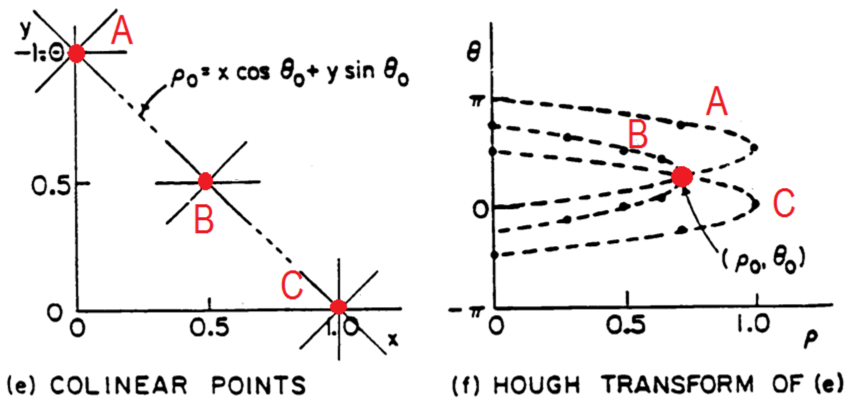
$$\rho = x \cos \theta + y \sin \theta$$

- It is the intersection of a family of lines in the image space.



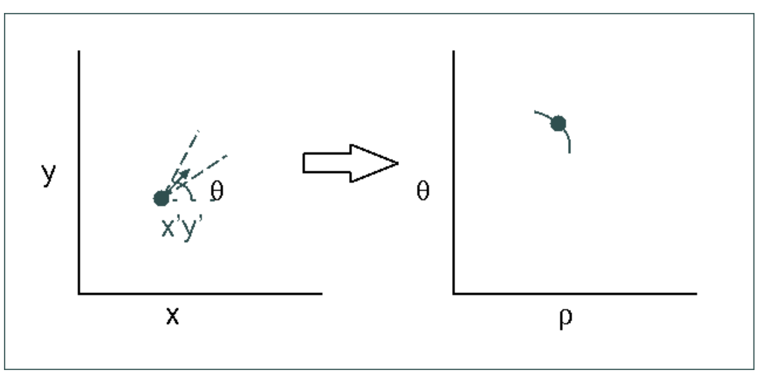
(c) FAMILY OF LINES, COMMON POINT (d) HOUGH TRANSFORM OF (c)

Colinear Points



Direction Information

If the direction of each image (edge) point is considered then a point in image (x-y) space maps to a point (or small arc) in parameter (ρ-θ) space



Straight Line Hough Transform

- Input form: Edge (partial)
- Object reconstruction ability: Partial
- Incomplete shape recognition ability: Yes
- Local/Global: ?
- Mathematical/heuristic: Mathematical
- Statistical/Symbolic: ?
- Transformations :
  - translation (S), rotation (S) and scale(I)
- Applications:
  - straight lines arbitrary length and orientation

Parametric Curves

- The Hough transform concept may be extended to parametric curves, e.g., a circle.

$$(x-a)^2+(y-b)^2=r^2$$

- Parameters:

  a, b, r (i.e., location and radius)
- Problem:

  for a 3D Hough space each image point maps to a surface (or a line if direction information is used). Only a small number of parameters are practical.

Constrained Hough Transforms

The parameter space can be limited to narrow the task which also simplifies the computation. For example, we can design a Hough transform for circles of a given radius r

- x-y image space maps to a x-y Hough space
- each directed edge point maps to a point in Hough space

Hough Transform for Fixed Size Circle

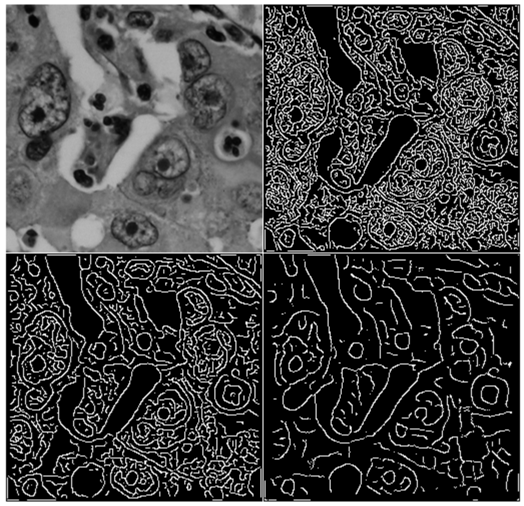
- Input form: Edge (partial)
- Object reconstruction ability: Yes
- Incomplete shape recognition ability: Yes
- Local/Global: local
- Mathematical/heuristic: Mathematical
- Statistical/Symbolic: ?
- Transformations :
  - translation (S) rotation (I) and scale(S)

Applications: fixed sized circular regions

Deriche Edge Operator (th=15)

original

a=2



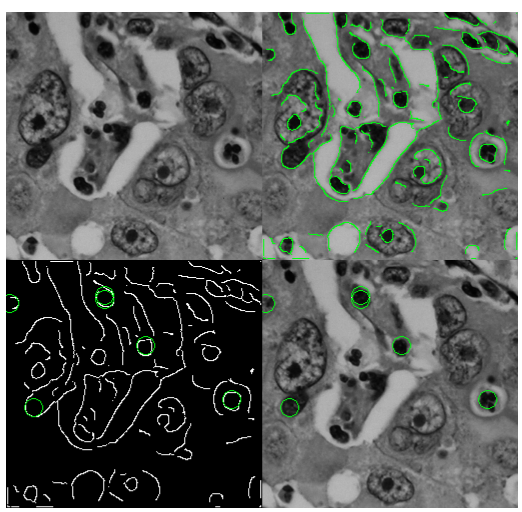
a=1

a=0.5

Hough Transform for Fixed Size Circle (r =9)

Original

Original + Edge



Edge + Circles

Original + Circles

Circular Hough Parameters

What parameters do we need to perform the circular Hough transform?

- Radius = 9
- Edge operator window a=0.4
- Edge operator th=50
- Hough space threshold th=10
  - How many votes would you expect in the Hough space?
- Hough space processing?
  - local maximum?
  - smoothing?

Parametric Hough Transform Summary

- The parametric Hough transform can be used to locate simple form shapes from partial information.
- Using directed edge information significantly reduces computation cost and can enhance transform sensitivity.
- Due to the computation and memory requirements only a small number of parameters is practical.

The Generalized Hough Transform (GHT)

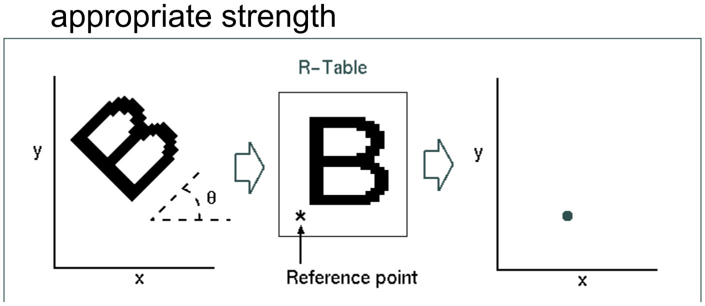
Concept:

Define a mapping from image space to parameter space, in which points belonging to the shape of interest give rise to an increased accumulation of mapped points at a coordinate representing the reference point of a shape

The Generalized Hough Transform (GHT)

Implementation:

- A shape of interest is represented by a reference table (R-table).
- The R-table indicates all places where the shape could be located for a given edge element with appropriate strength

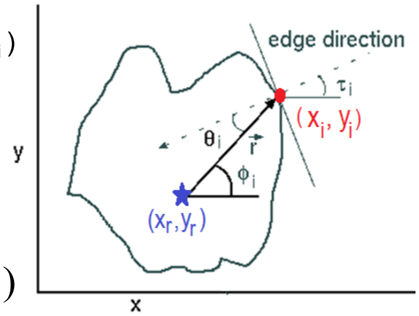


R-Table Generation

- In general, the R-table is computed from a prototype template of the shape of interest and a related reference point as follows:
  - x_i, y_i : edge point
  - x_r, y_r: reference point
  - τ_i : orientation of the edge
  - $|\vec{r}_i|$: distance from (x_r, y_r) to (x_i, y_i)

    $$|\vec{r}_i| = \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2}$$
  - θ_i : relative edge orientation w.r.t. (x_r, y_r)

    $$\theta_i = \phi_i - \tau_i$$
- One R-table entry: $(\tau_i, |r_i|, \theta_i)$



4D-Hough Space

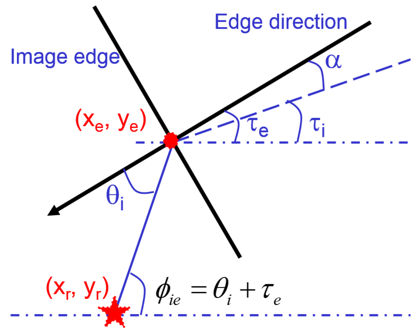
Consider a typical Hough accumulator space

- the usual parameters of interest are location (x, y), size (s) and orientation (α)
- an edge point and an R-table entry map into a line (for different s) in 4D Hough Space
- an edge point and the whole R-table map into "surfaces" in 4D Hough Space
  - for every edge point we need to create a surface of votes in 4D Hough space

Generalized Hough Transform

Given an R-table entry (τ_i, r_i, θ_i) and an edge point (x_e, y_e, τ_e) in the image, compute parameters (x_r, y_r, s, α) in the 4D Hough space:

$$x_r = x_e - r_i s.\cos(\theta_i + \tau_e)$$
$$y_r = y_e - r_i s.\sin(\theta_i + \tau_e)$$
$$\alpha = \tau_e - \tau_i$$



for different scale s, it maps to different possible locations of the reference point (x_r, y_r).

Rotation Invariant Generalized Hough Transform (RIGHT)

- In many computer vision applications we wish to identify an object at a known scale independent of its orientation

  s is fixed and α is not required.

- To reduce storage (and computation), a projection of the Hough space along dimension α is frequently used
- 4D Hough space (x, y, s, α) → 2D Hough space (x, y)

$$x_r = x_e - r_i \cos(\theta_i + \tau_e)$$
$$y_r = y_e - r_i \sin(\theta_i + \tau_e)$$

GHT Implementation

- The R-Table is stored with θ as an index
  - θ_i is usually quantized (<= 256 values)
  - Each entry may have several (r,τ) pairs

$$
\begin{array}{llll}
\theta_1: & (r_1^1,\tau_1^1) & (r_2^1,\tau_2^1) & \cdots \\
\theta_2: & (r_1^2,\tau_1^2) & (r_2^2,\tau_2^2) & \cdots \\
\vdots & \vdots & & \\
\theta_n: & (r_1^n,\tau_1^n) & (r_2^n,\tau_2^n) & \cdots
\end{array}
$$

There is usually significant saving in computation for this ordering especially for the RIGHT

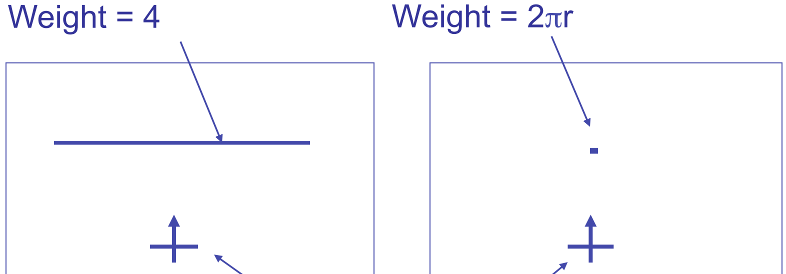
- τ is not required for the RIGHT
- The reference point is usually located at the center of gravity of the shape

Graphical R-Table Representation

The R-table may be represented graphically as a weighted diagram that illustrates the form of the vote in Hough space for edge pixels.

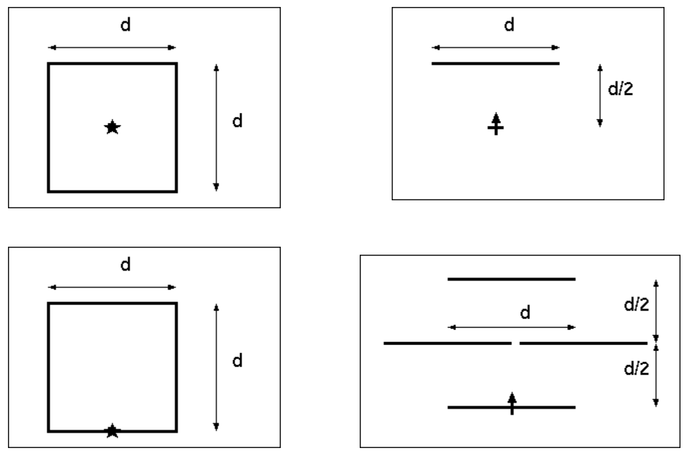
Example 1: a square   Example 2: a circle

Weight = 4   Weight = $2\pi r$

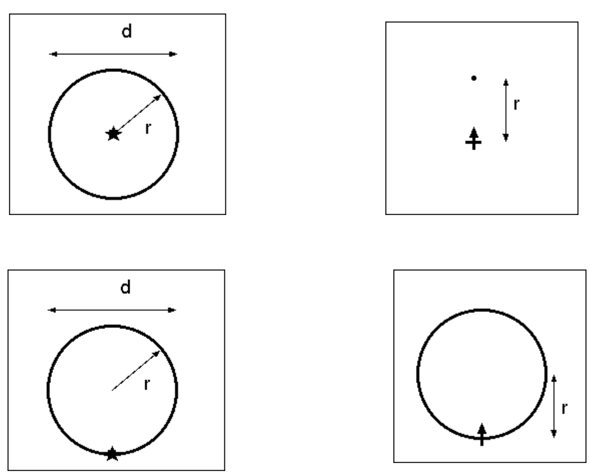


edge point location and edge direction

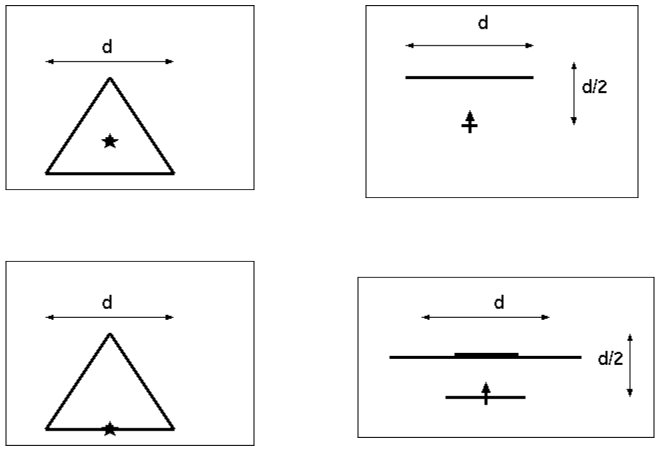
R-table Examples - Square



R-table Examples - Circle



R-table Examples - Triangle



Generalized Hough Transform Summary

- GHT extends Hough transform to arbitrary shapes
  - R-table used to represent shape
  - 4D GHT space (location, orientation and size)
  - Computation is extensive
- RIGHT is a form of rotation invariant (edge based) template matching
  - Accumulator array is 2D (location), computation is reasonable
  - Sensitivity is not as good as (edge based) template matching
  - confusion from accidental correlations in Hough space may be a problem especially for complex shapes