Linear Algebra Assignment 1 Report

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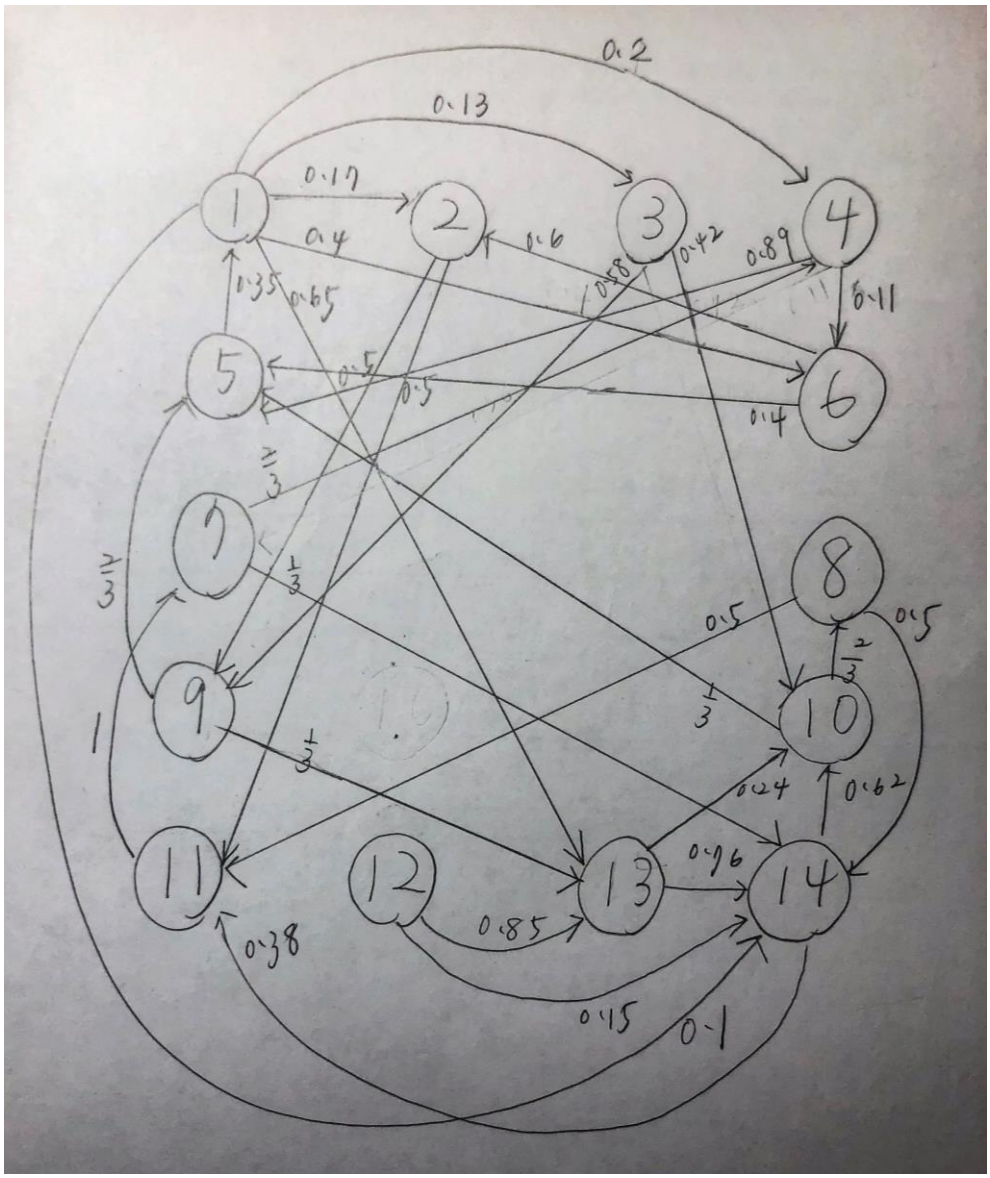
一、N=14，matrix A_{14x14}=

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0.35 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.17 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2 & 0 & 0 & 0 & 0 & 0 & 2/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.89 & 0 & 0.4 & 0 & 0 & 2/3 & 1/3 & 0 & 0 & 0 & 0 \\
0.4 & 0 & 0 & 0.11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2/3 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0.58 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.62 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.24 & 0.38 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.65 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0.85 & 0 & 0 \\
0.1 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0.5 & 0 & 0 & 0 & 0.15 & 0.76 & 0
\end{bmatrix}$$



二、十個 initial vectors 分別是 vector0 到 vector9

vector0=[[1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]]

vector1=[[0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]]

vector2=[[0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]]

vector3=[[0], [0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]]

vector4=[[0], [0], [0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0]]

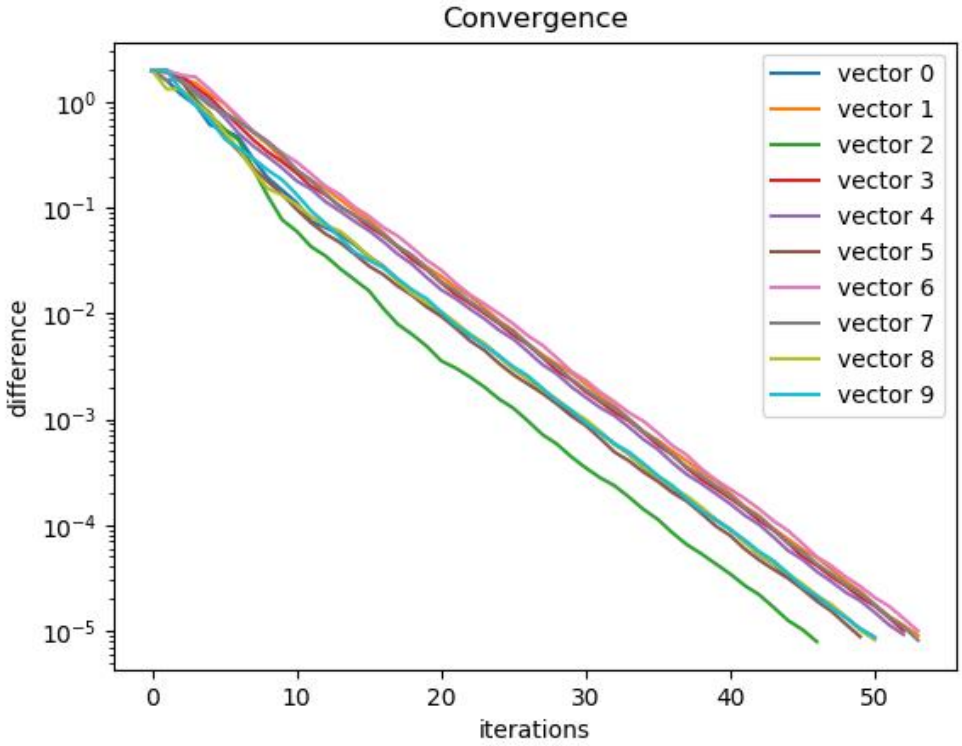
vector5=[[0], [0], [0], [0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0]]

vector6=[[0], [0], [0], [0], [0], [0], [1], [0], [0], [0], [0], [0], [0], [0]]

vector7=[[0], [0], [0], [0], [0], [0], [0], [1], [0], [0], [0], [0], [0], [0]]

vector8=[[0], [0], [0], [0], [0], [0], [0], [0], [1], [0], [0], [0], [0], [0]]

vector9=[[0], [0], [0], [0], [0], [0], [0], [0], [0], [1], [0], [0], [0], [0]]



若再證明 vector10-13 最後的 difference 也是收斂，

vector10=[[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [1], [0], [0], [0]]

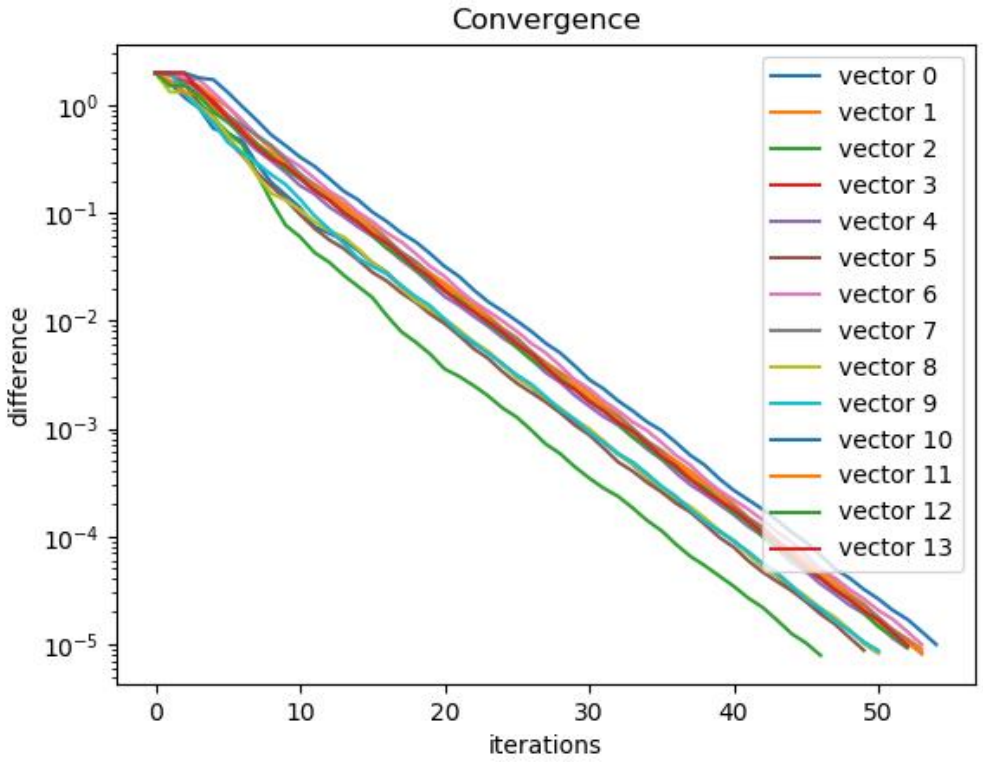
vector11=[[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [1], [0], [0]]

vector12=[[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [1], [0]]

vector13=[[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [1]]

則 a*vector1+b*vector2+c*vector3+…可組成任意的 14x1 的向量，

在運算多次後 difference 皆會收斂。



三、用 vecsum()去計算各個 vector 在每次運算後加起來的值(A^kv)，發現都趨近於 1，些許的誤差判斷可能是精度誤差(precision)造成的。

```
v0 = np.zeros([14,1])

# define a function to compute |a-b|
#-------------------------------------------
def abssum(a, b):
    """this function computes the norm of a-b"""
    result = 0
    for i in range(len(a)):
        result = result + abs(a[i]-b[i])
    return result[0]
#-------------------------------------------
def vecsum(a):
    result = 0
    for i in range(len(a)):
        result = result +a[i]
    return result[0]

plt.figure
for j in range(10):
    v=v0
    if(j!=0):
        v[j-1]=[0]
    v[j] = [1]
    #print(v)
    # compute their product
    u = np.dot(A, v)
    # diff records the difference of Av and v
    diff = [abssum(u,v)]
    i = 0
    while diff[i] > 1e-5:
        v = u              # record A^kv
        u = np.dot(A, v)  # compute A^{k+1}v
        diff.append(abssum(u, v))  # append add
        i = i + 1
        print(vecsum(u))
        #print(u)
        #u=u/np.norm(u)
        #print(u)

    #plot the differences with iterations
    #plt.subplot(7,2,j+1)
    plt.semilogy(range(len(diff)), diff,label='vector '+str(j))
plt.xlabel('iterations')
plt.ylabel('difference')
```

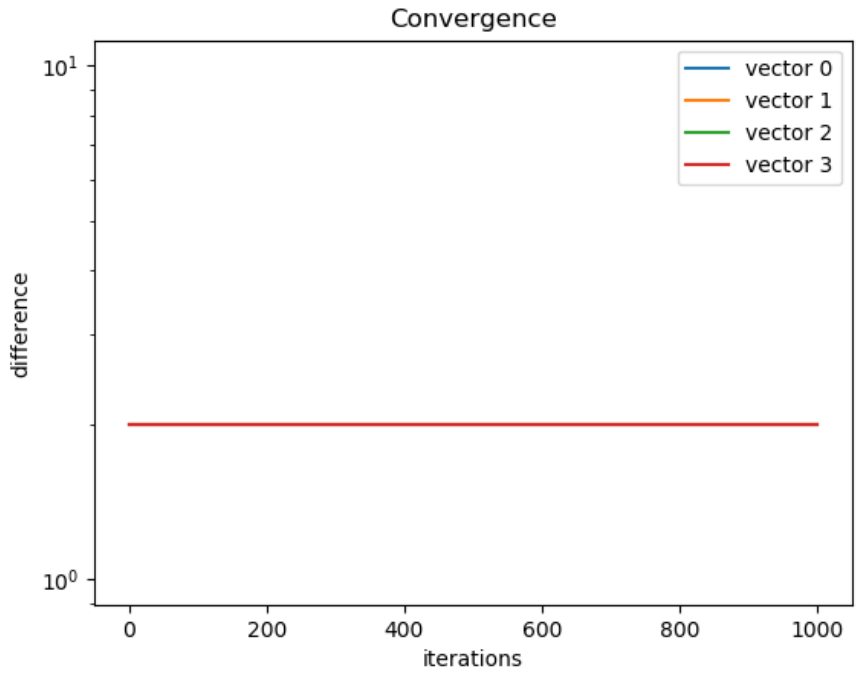
```
Python 3.7.0 Shell
File  Edit  Shell  Debug  Opt
1.0
1.0
1.0
1.0
1.0
1.0
0.9999999999999999
0.9999999999999999
1.0
1.0
1.0
1.0
0.9999999999999999
0.9999999999999998
0.9999999999999998
0.9999999999999999
0.9999999999999999
1.0
0.9999999999999999
1.0
1.0
1.0
1.0
1.0
0.9999999999999999
0.9999999999999999
0.9999999999999998
0.9999999999999999
0.9999999999999999
0.9999999999999999
0.9999999999999998
0.9999999999999999
0.9999999999999998
0.9999999999999998
0.9999999999999998
0.9999999999999998
0.9999999999999998
0.9999999999999998
>>>
```

四、其他探討

在一開始設矩陣時，發現有些情況 $B^k v$ 並不會收斂，

例如下面這個例子，矩陣 $B_{4x4}=\begin{bmatrix} 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0.3 & 0.8 & 0 & 0 \\ 0.7 & 0.2 & 0 & 0 \end{bmatrix}$，

並不會收斂。



且發現 $B^k v$ 的值最後會在兩個值之間週期性變動。