PHYSICS 221

EXPERIMENT 6: Electrons in Magnetic Fields/AC Circuits

INTRODUCTION

Today's experiment will conclude the Physics 221 laboratory curriculum. The first part of the experiment will measure the electron's charge to mass ratio by making use of its interaction with an external magnetic field. The second part will involve the study of LRC circuit with an AC source to create a resonant system.

PART A : CHARGED PARTICLES IN MAGNETIC FIELDS

Just as moving charges induce magnetic fields, charged particles moving in the presence of a magnetic field feel a magnetic force. A particle with charge q moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ will experience a force $\mathbf{F}$ perpendicular to its direction of motion described by the Lorentz force equation

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \,. \tag{1}$$

If a uniform magnetic field $\mathbf{B}$ is perpendicular to the initial direction of motion of an electron beam, the electrons will be deflected by a force that is *always* perpendicular to their velocity and to the magnetic field $\mathbf{B}$. Consequently, the beam will be deflected into a circular trajectory with radius r, as shown in Figure 1.

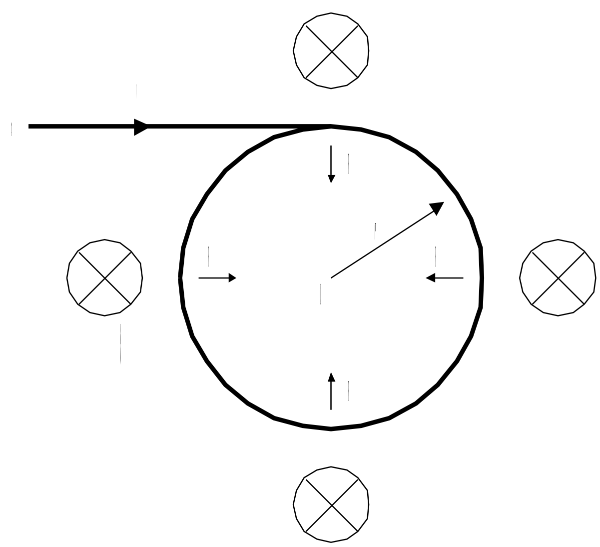

Figure 1: Path of Electron Beam in Uniform Magnetic Field. The magnetic field points into the page. Although $\mathbf{v} \times \mathbf{B}$ points outwards from the center of the circle, the electron has charge q = -e, which results in the force towards O and a circular path.

If we simply observe that the electron moves uniformly in a circular path, we can conclude that there is a constant centripetal force $\boldsymbol{F_c}$ which pulls the electron toward the center of the circle; that is

$$F_c = \frac{mv^2}{r} \tag{2}$$

where m is the mass of the electron. However, the origin of this centripetal force is the Lorentz force, thus Equations 1 and 2 are equal to each other, i.e.

$$evB = \frac{mv^2}{r} \tag{3}$$

where we have used the condition that **v** is perpendicular to **B** in this case. Simplifying Equation 3, we obtain the ratio e/m for the electron as

$$\frac{e}{m} = \frac{v}{rB}\,. \tag{4}$$

In this experiment, the electron originates from the cathode with a negligible initial velocity. When a potential difference V is applied, its speed will increase such that when it reaches the anode, it will have gained a kinetic energy equal to the potential energy eV, i.e.

$$\tfrac{1}{2}mv^2 = eV$$

or

$$v = \sqrt{\frac{2eV}{m}}\,. \tag{5}$$

Substituting Equation 5 into 4, we obtain

$$\frac{e}{m} = \frac{2V}{r^2B^2}\,. \tag{6}$$

The magnetic field for our experiment is provided by a Helmholtz coil configuration, similar to the one from the previous experiment. Recall that for a Helmholtz coil with N turns, radius R, and current I, the magnetic field strength at the center of the configuration is given by

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 I N}{R}. \tag{7}$$

Substituting this into Equation 6, we finally have

$$\frac{e}{m} = \frac{125}{32} \frac{V R^2}{\mu_0^2 I^2 N^2 r^2}. \tag{8}$$

The electron beam will be generated within a Bainbridge tube, which is a spherical glass vacuum tube. Filament in the tube is heated to a high temperature to emit electrons. The filament is surrounded by a cylindrical anode at a potential V relative to the cathode (see Figure 2). This accelerates the electrons and the ones that escape through the opening of the cylinder will have a kinetic energy of eV. These electrons will experience the uniform magnetic field from the Helmholtz coil and will more in a circular path shown in the figure. To be able to see the path taken by the electrons, the Bainbridge tube is filled with mercury vapor at very low pressure. This allows us to see a luminescent trace of the electrons' path. By varying the current going through the Helmholtz coil, we can vary the magnetic field and thus the magnetic force. This in turn will vary the radius of the circular path that the electrons travel.

PROCEDURE :

CAUTION: DO NOT EXCEED 4.0 A IN THE FILAMENT CIRCUIT OR IT WILL BURN OUT, RESULTING IN SAFETY POINTS PENALTY.

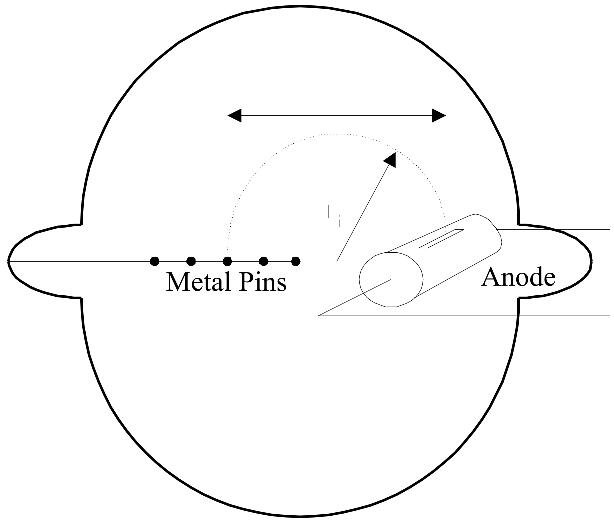

Figure 2: Bainbridge Tube.

1. Measure the diameter/radius of the Helmholtz coil.
2. The Helmholtz coil and the Bainbridge tube should already be connected to their respective power supply/circuit. If not, ask your laboratory instructor for assistance. You *must*, however, be able to identify the ammeter that shows the current through the Helmholtz coil (this is I in Equation 8), the ammeter that shows the current through the cathode (this is the current that should not exceed 4.0 A), and the voltmeter that shows the potential difference between the cathode and the anode (this is V in Equation 8).
3. Before switching on any power supply, make sure all the current and voltage knobs are at their minimum value.
4. Switch on the power supply for the Bainbridge tube and adjust the filament current to between 3.5 and 4.0 A. A finer beam is obtained if you lower the filament current after you have found the blue glow. Once this is done, *do not change* this setting for the rest of the experiment.
5. Increase the accelerating potential (the potential difference between the anode and cathode) to 30 V.
6. Switch on the power supply for the Helmholtz coil. Adjust the power supply (including the rheostats if any) so that the electron beam strikes the metal pins

inside the Brainbridge tube. If the electron beam is bending away from the pins instead of towards them, the current in the Helmholtz coil is flowing in the wrong direction. Turn off the power supply and change the polarity.

7. Once the electron beam strikes a metal pin, record I, the current flowing through the Helmholtz coil.
8. Repeat this so that the beam hits each pin in the tube.
9. When this is done, increase the accelerating potential V to 50, 70, and 90 V and repeat the experiment.
10. The distances from the filament to the pins inside the Bainbridge tube are gives in Table 1. Your data table should resemble Table 2.

d_1	65 mm
d_2	78 mm
d_3	90 mm
d_4	103 mm
d_5	115 mm

Table 1: Distance From the Filament to the Pins Inside the Bainbridge Tube.

	30 V	50 V	70 V	90V
$d_1 =$				
$d_2 =$				
$d_3 =$				
$d_4 =$				
$d_5 =$				

Table 2: Sample Data Table. Empty spaces are to be filled with the values of I.

ANALYSIS :

1. Find the radius of curvature r for each d in your data table.
2. <u>For each r</u>, plot I^2 versus V. This means that each <u>ROW</u> of your data table is one data set to be plotted. You will end up with five sets of data, representing five different values of r. You may plot all five data sets on the same graph if they are clearly labeled
3. Find the slope of the best-fit line for each data set.
4. From Equation 8, the slope of the best-fit line of this graph corresponds to

$$\frac{125}{32}\frac{R^2}{\mu_0{}^2 N^2 r^2}\frac{1}{e/m}\,. \tag{9}$$

   Prove this in the theory section of your report.
5. Using Equation 9, find the experimental value of e/m using each of your slope values. Keep in mind that each slope corresponds to a particular r value! [Use a spreadsheet, for instance, to perform this tedious calculation.]
6. Find the average and standard deviation of the experimental value of e/m and compare it with the theoretical value.

<u>PART B: ALTERNATING CURRENT CIRCUIT</u>

In this experiment, we will make full use of the capability of the oscilloscope and find the resonant frequency of our LRC circuit. In addition to the familiar resistor R and the capacitor C, a new component is introduced here, which is the inductor L. The inductor is an important component of circuits, on the same level as the resistor and the capacitor. The inductor is based on the principle of inductance – that moving charges create a magnetic field. The reverse also holds true, that a moving magnetic field creates an electric field. The inductor consists of, at its simplest level, a coil of wire in a circuit.

You are asked to refer to your text to understand the properties of the inductor, capacitor, and resistor in an AC circuit. We will only quote the result that in an LRC circuit (see Figure 3), if the applied voltage is in the form of

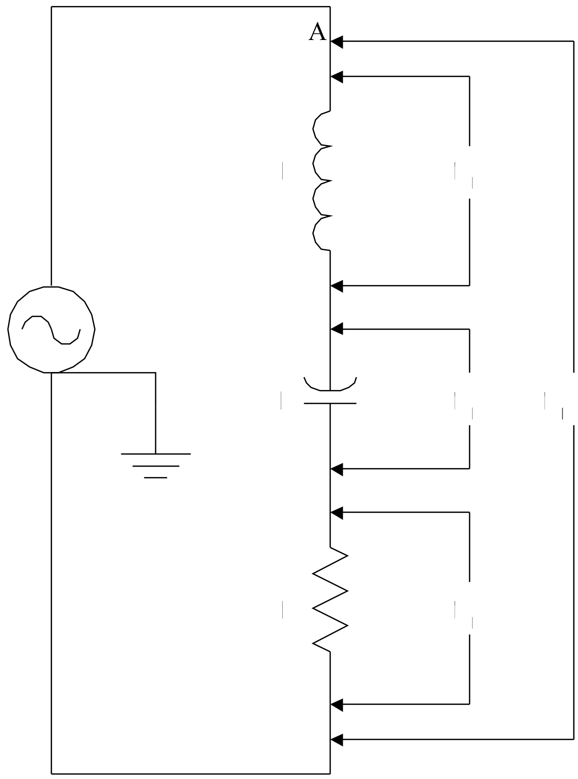

Figure 3: LRC Circuit. A frequency counter (not shown in the figure) should be attached to the signal generator for accurate determination of the signal frequency.

$$V(t) = V_0 \sin \omega t \,, \tag{10}$$

then the potential difference across each of the component in the circuit can be written as:

$$V_R(t) = V_{0R} \sin \omega t \qquad \text{for the resistor;} \tag{11}$$

$$V_C(t) = V_{0C} \sin \left(\omega t - \pi/2 \right) \qquad \text{for the capacitor;} \tag{12}$$

$$V_L(t) = V_{0L} \sin \left(\omega t + \pi/2 \right) \qquad \text{for the inductor.} \tag{13}$$

Notice that the voltage across the resistor V_R is in phase with the applied voltage, whereas the voltage across the capacitor V_C lags by 90° and the voltage across the inductor leads V_L by 90° with respect to the V_R. All three voltages are plotted in Figure 4.

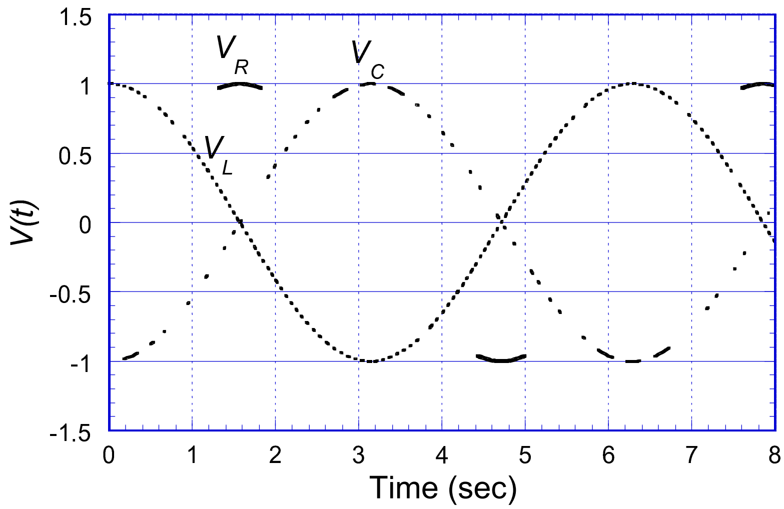

Figure 4: Phase Difference of an LRC Circuit. Using V_R as a reference, V_L leads by 90° while V_C lags by 90°. Take note that the amplitude of the voltage for each component need not necessarily be equal as depicted.

The current in the circuit can be expressed in the form of the Ohm's Law as

$$I = \frac{V_0}{Z} \tag{14}$$

where Z is the impedence of the circuit defined as

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}\ . \tag{15}$$

Notice that in Equation 14, the current is a maximum when Z is a minimum. We call this the resonance condition of the circuit. From Equation 15, the resonance condition occurs when

$$\omega_r L = \frac{1}{\omega_r C}; \quad \text{or} \qquad \omega_r = \frac{1}{\sqrt{LC}}. \quad \text{(resonance)} \tag{16}$$

PROCEDURE AND ANALYSIS :

1. Find the capacitance of the capacitor and inductance of the inductor given.
2. Construct the circuit shown in Figure 3. Make sure that the resistor is the last component closest to the ground end of the circuit.
3. Turn on the signal generator and set the frequency of the oscillating signal to be approximately 500 Hz.

The time varying voltages will be studied using the oscilloscope. Our oscilloscopes in this case can only measure potential difference with respect to ground.

4. As a first exercise, we will observe the voltage on the oscilloscope when the "two" sources are in phase with each other. Connect the leads from CH1 and CH2 of the oscilloscope to the LRC circuit as shown in Figure 5. This means that both channels will measure the voltage across R.

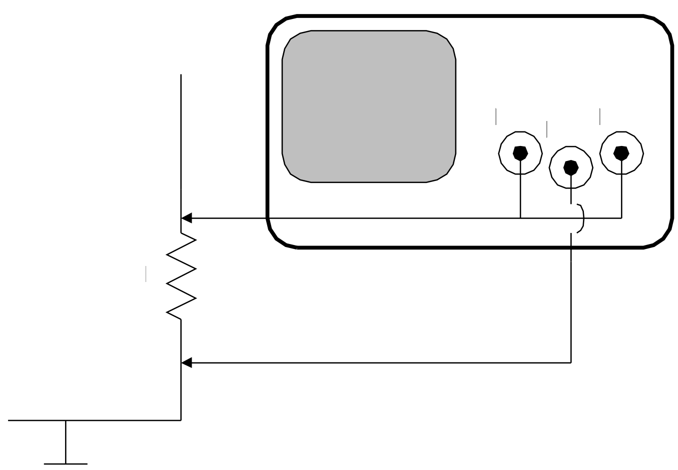

Figure 5: Connection for Procedure 3. The figure only shows part of the LRC circuit from Figure 3.

5. Make sure the display selector on the oscilloscope is on DUAL to display the signal from both channel simultaneously. Adjust the Time/div scale and the Volts/div scale of both channels to obtain a suitable display on the oscilloscope.
6. Verify that the signal from CH1 and CH2 are in phase with each other. Sketch this in your report.
7. Change the display selector from DUAL to x-y. Next, turn the Time/div knob counter-clockwise all the way to the x-y position. In this position, CH1 now displays the x value, while CH2 displays the y value. Since both x and y are in phase with each other (they both reach their peaks and valleys at the same instant), the oscilloscope should display a straight line inclined as an angle (45° if both inputs have the same magnitude and calibration scale). Hence, other than directly observing the waveform as in Procedure 6, this is another technique to check if the two signals are in phase with each other. This display is known as a Lissajous figure. Sketch the display in your report.
8. Change the settings to obtain the signal as in Procedure 5-6. Disconnect CH2 from the circuit and connect it to Point A as shown in Figure 3. In this position, CH1 is still measuring the voltage across the resistor (V_R), but CH2 is now measuring the voltage across all three components (V_{LRC}). Adjust the vertical and horizontal scales to obtain the best display.
9. Are the two signals still in phase with each other? Does V_{LRC} leads or lags V_R, and by how much? Sketch this in your report.
10. Observe the Lissajous figure for these two signals by repeating Procedure 7. Is this identical to the one you observed before? Sketch this in your report. [You should now know what the Lissajous figure should look like when the signals are in phase and out of phase with each other.]
11. Change the settings back to obtain the signal as in Procedure 8-9, but this time, set the display selector to show only the signal from CH1, which is V_R. You should, however, leave CH2 connected as is.
12. Change the frequency of the signal generator (hint: you may want to increase the frequency) while continuously observing the amplitude of V_R. Find the frequency

where the amplitude of V_R is a maximum. Change the vertical scale and the vertical positioning as you wish to help you accurately determine this frequency. Do not forget to read the frequency from the frequency counter. At this frequency, the current in the circuit is also a maximum, since $V_R = IR$. Thus, from Equation 14, this is the resonant frequency of the LRC circuit.

13. Now change the display setting so that you again see both V_R from CH1, and V_{LRC} from CH2. From Equations 15 and 16, the reactance from the inductor and the capacitor should cancel each other so that the impedence of the circuit just depends on the resistor. This means that V_{LRC} should be in phase with V_R. Is this what you observe?
14. Check this by observing the Lissajous figure at the resonant frequency. If the figure does not quite show the shape necessary for the two signals to be in phase, make the necessary adjustments to the frequency until you are satisfied that they are now in phase with each other. Record this frequency if it is different from the one you obtained before.
15. Using Equation 16, calculate the theoretical value of the resonance frequency and compare it with the value (or values if you have two different ones) you obtained experimentally. Take note that in Equation 16, you are calculating the resonance *ANGULAR* frequency ω. To be able to compare this to your experimental values, you need to use the relation

$$\omega = 2\pi f$$

where f is the frequency of the signal you read of the frequency counter.

Rejoice! This is the end of Physics 221 Laboratory!