Homework 11 for September 16 2008
Due 8AM on September 17 2008
Physics 221 with Professor Jeff Terry

1. A uniform electric field is given by $\vec{E} = \left(3.0\frac{N}{C}\right)\hat{i} + \left(2.0\frac{N}{C}\right)\hat{j} - \left(1.0\frac{N}{C}\right)\hat{k}$. What is the electric flux through a flat, 4.0-m^2 area that lies in the y-z plane?

$$\vec{A} = A\hat{A} = A\hat{i}$$

$$\Phi = \vec{E}\cdot\vec{A} = (3.0, 2.0, -1.0)\frac{N}{C}\cdot(4.0, 0, 0)m^2$$

$$= (12 + 0 + 0)\frac{Nm^2}{C} = 12\frac{Nm^2}{C}$$

We took the unit vector of the area to be $\hat{i}$. If you're not sure about this, satisfy yourself that $\hat{j}$ and $\hat{k}$ live on the yz plane and then take the cross product $\hat{j}\times\hat{k}$. As you know, the vector resulting from a cross product is perpendicular to the two original vectors.

2. A non-uniform electric field passes through a loop (with area element parallel to the field) of radius 1.0m. The electric field is given by $r^2\frac{N}{Cm^2}$ where r is the distance from the center of the loop. What is the total electric flux through the loop?

This is a very difficult problem unless you recognize that the area of the loop is the sum of an infinite number of smaller, infinitesimally-wide "donuts". The area of each is given by $dA = 2\pi r dr$, where r is the radius of the donut and dr is the width of the donut. We are told that the E field through the loop is $r^2\frac{N}{Cm^2}$, so we get:

$$\Phi = \int \vec{E}\cdot d\vec{A} = 2\pi\int r^3 Cos\,\theta dr$$

But wait, what is Cos(theta)? Because the field and area element are parallel, theta is 0 and Cos(theta)=1. Phew.

$$\Phi = \frac{\pi}{2}\frac{N}{Cm^2}\left[r^4\right]_o^{1.0m} = \frac{\pi}{2}\frac{N}{Cm^2}\left[1m^4\right] = \frac{\pi}{2}\frac{Nm^2}{C}$$