Homework 6 for September 8 2008
Due 8AM on September 9 2008
Physics 221 with Professor Jeff Terry

1. Each of two very long, straight rods carries a positive charge of $1.0*10^{-12}$ C/m. One rod lies along the x axis, the other along the y axis. Find the electric field (magnitude and direction) at the point x=0.50m and y=0.20m.

This homework relies on equation 23.10 from your textbook, which reads $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$ with the direction being perpendicular to the rod and the field pointing away from the red for positive charge and toward the rod for negative charge. This should raise a question – why can we use the infinite rod equation for a rod that is just "very long"? Because it is so long that it *looks* infinite. Of course, our answer will be an approximation, but *all* of physics is approximations – our goal is to make very good approximations.

Each rod will contribute to the field, but not in the same direction – not even in opposite directions! In fact, the field contribution from the x-axis bar and the y-axis bar will be perpendicular (one going in the +x direction and one in the +y direction). That's all well and good but how can we get a magnitude? Simple – by squaring the components, summing them, and taking the square root. The Pythagorean Theorem.*

So the problem is now to find the x and y components of the electric field using the information we were given and plug the components into the PT.

$$E_x = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x} = \frac{1}{2\pi\varepsilon_0}\frac{1.0*10^{-12}\frac{C}{m}}{0.5m} = 3.6*10^{-2}\frac{N}{C}$$

$$E_y = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} = \frac{1}{2\pi\varepsilon_0}\frac{1.0*10^{-12}\frac{C}{m}}{0.2m} = 9.0*10^{-2}\frac{N}{C}$$

$$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(3.6*10^{-2}\frac{N}{C}\right)^2 + \left(9.0*10^{-2}\frac{N}{C}\right)^2}$$

$$= 9.7*10^{-2}\frac{N}{C}$$

Now how about direction? We have the x and y components and we can easily find the angle (from the +x-axis) of the field using the arctan function and doing so we get 68.2 degrees. So our final answer is: **$9.7*10^{-2}$ N/C at 68.2^{o} above the +x-axis**.

2. An electron is initially at rest, 0.10 cm from a very large disk which carries a uniform surface charge density $\sigma = +3.0*10^{-8}$ C/m^2. How long does it take the electron to strike the plate? What is its speed just before striking the plate?

Many of you chose to do this problem as if it was asking about an infinite sheet. We could certainly make a reasoned argument that an infinite sheet and an infinite disk are exactly the same, but let's let math tell us for sure.

The book tells us that the electric field at a distance y (assumed to be in a direction normal to the surface of the ring) from the axis of a circular ring of radius R is:

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{Qy}{\left(R^2+y^2\right)^{1.5}}$$

$$dE_y = \frac{y\sigma}{2\varepsilon_0}\frac{(R)dR}{\left(R^2+y^2\right)^{1.5}}$$

In the second step we replaced Q with dQ and replaced that with sigma dA. We now have the electric field (which will be in the "y direction") of an infinitesimal ring of charge. We want to know the field of an infinite number of these rings stretching clear out to infinite! But wait a minute, won't that have to diverge? Well, let's just see… We want to solve $\int_0^{\mathrm{inf}} \frac{R}{(R^2+y^2)^{1.5}}\, dR$ and we get $\frac{1.}{(y^2)^{0.5}}$. So the electric field is given by $\frac{\sigma}{2\,\epsilon_0}$. Indeed, those of you who thought we were looking at an infinite sheet were right, but now we **KNOW** it, and doesn't it feel nice?

From there it is simply a matter of equating qE with ma (as per Newton's Second Law) and plugging the resulting a into our kinematic equations.

$$F = ma = \frac{Q\sigma}{2\varepsilon_0} \therefore a = \frac{Q\sigma}{2\varepsilon_0 m} = \ddot{x}$$

$$x_f = x_0 + \dot{x}_0 t + \frac{1}{2}\ddot{x}t^2 = \frac{Q\sigma}{4\varepsilon_0 m}t^2$$

$$t = 2\sqrt{\frac{\varepsilon_0 m x_f}{Q\sigma}} = 2\sqrt{\frac{(8.85*10^{-12}\frac{C^2}{Nm^2})(9.11*10^{-31}kg)(1.0*10^{-3}m)}{\left(3.0*10^{-8}\frac{C}{m^2}\right)\left(1.60*10^{-19}C\right)}}$$

$$= 2.60*10^{-9}s$$

And of course to find velocity we use the appropriate kinematic equation.

$$v_f = v_0 + at = \frac{Q\sigma t}{2\varepsilon_0 m} = 7.74*10^5\frac{m}{s}$$