Homework 18 for September 26 2008
Due 8AM on September 30 2008
Physics 221 with Professor Jeff Terry

1. Over a certain region of space, the electric potential is V=5x-$3x^2y+2yz^2$. Find the electric field in this region.

$$V = 5x - 3x^2y + 2yz^2$$
$$E = -\nabla V = -\partial_x\left(5x - 3x^2y + 2yz^2\right)\hat{x} - \partial_y\left(5x - 3x^2y + 2yz^2\right)\hat{y} - \partial_z\left(5x - 3x^2y + 2yz^2\right)\hat{z}$$
$$= \left(-5 + 6xy\right)\hat{x} + \left(+3x^2 - 2z^2\right)\hat{y} + \left(-4yz\right)\hat{z}$$

2. A point charge q is located at x= -R and a point charge –2q is located at the origin. Give all points where the electric potential is zero.

For the given charge distribution, $V(x, y, z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$ and $r_2 = \sqrt{x^2 + y^2 + z^2}$.

The surface on which $V(x, y, z) = 0$

is given by $k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0$, or $2r_1 = r_2$.

If you did it in 1D you have y=z=0 and you would get,

$$2\sqrt{(x+R)^2} = \sqrt{x^2}$$
$$4(x+R)^2 = x^2$$
$$3x^2 + 8Rx + 4R^2 = 0$$
$$x = \frac{-8R \pm \sqrt{64R^2 - 48R^2}}{6} = \frac{-8R \pm 4R}{6} = -2R, -\frac{2}{3}R$$

3. An electron is initially a distance r=4.3*10^{-9}m from a proton, travelling directly away from the proton at a speed 4*10^5 m/s. What is the speed of the electron when it is very far from the proton?

$$K+U=K'+U'$$

$$\frac{1}{2}m_e v^2+\frac{kq_e q_p}{r}=\frac{1}{2}m_e v'^2+\frac{kqq}{\infty}=\frac{1}{2}m_e v'^2$$

$$v'=\sqrt{v^2+\frac{2kq_e q_p}{rm_e}}=\sqrt{\left(4\cdot10^5\frac{m}{s}\right)^2-\frac{2\left(8.99\cdot10^9\frac{Nm^2}{C^2}\right)\left(1.602\cdot10^{-19}C\right)^2}{\left(4.3\cdot10^{-9}m\right)\left(9.11\cdot10^{-31}kg\right)}}$$

$$\sqrt{1.6\cdot10^{11}\left(\frac{m}{s}\right)^2-1.18\cdot10^{11}\left(\frac{m}{s}\right)^2}=\sqrt{4.2\cdot10^{10}\left(\frac{m}{s}\right)^2}=2.05\cdot10^5\frac{m}{s}$$

We used K for kinetic energy and U for potential energy.

4. How much charge is on each plate of a $4\mu F$ capacitor when it is connected to a 12V battery?

$$Q=C\Delta V=\left(4.00\times10^{-6}\text{ F}\right)(12.0\text{ V})=4.80\times10^{-5}\text{ C}=\boxed{48.0\ \mu\text{C}}$$