PHYSICS 221

EXPERIMENT 1: Harmonic Motion

INTRODUCTION:

Newton's Laws are evidently at work in simple linear motion, but the behavior of bodies is almost never that simple. Newton's Laws are also applicable to the study of **Harmonic Motion**. Harmonic motion is any type of periodic that repeats itself exactly after a certain time interval – the period. The most fundamental type of this motion is known as Simple Harmonic Motion (SHM). SHM occurs frequently throughout physics – not only in mechanics, but also in electromagnetism and quantum physics. In fact, many types of oscillations and vibrations found in nature can be modeled as SHM, or one of its extensions. In this experiment, we will study simple harmonic motion using pendulums and springs.

OSCILLATING MASS ON A SPRING

When a system undergoes a simple harmonic motion it exhibits a continuous, smooth, periodic motion, which remains unchanged over a period of time. For example, recall that the force produced by a spring (known as Hooke's Law) is:

$$F = -kx$$

where k is the spring constant and x is the length of extension (positive x) or compression (negative x). This equation can be rewritten as

$$ma = -kx. \tag{1}$$

Using the usual dot convention to denote time derivative, we can then write

$$a = \frac{dv}{dt} = \dot{v} = \frac{d^2x}{dt^2} = \ddot{x}.$$

Equation 1 then becomes

$$\begin{aligned} \ddot{x} &= -\frac{k}{m}x \\ &= -\omega^2 x \end{aligned} \tag{2}$$

where we have defined the constant $\omega = \sqrt{k/m}$. One may show that the solution *x(t)* is sinusoidal, with one form of the solution being

$$x(t) = A\cos(\omega t + \delta) \tag{3}$$

where A is the amplitude of the oscillation and δ is the phase of the oscillation. If we pull the mass m to stretch the spring by a distance A, and then release it from the rest, the initial conditions on the motion (at $t = 0$) are:

$$x(0) = A \;\; ; \;\; \dot{x}(0) = 0 \;\; ; \;\; \delta = 0 \tag{4}$$

Applying these initial conditions to Equation 3 gives the equation of motion of the spring-mass system as

$$x(t) = A\cos(\omega t)\ . \tag{5}$$

Note that if we had written the solution of Equation 2 as $x(t) = A\ \sin(\omega t + \delta)$ instead, then with the same initial conditions, the phase needs to be set to $\delta = \pi/2$.

The parameter ω that we used is the angular frequency of oscillation. Therefore, the frequency of oscillation of the mass is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{6}$$

and the period of oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}\ . \tag{7}$$

Using the solution given in Equation 5, the velocity of the oscillating mass is

$$v(t) = \dot{x}(t) = -\omega A\sin(\omega t)\ . \tag{8}$$

Note that the maximum values of $x(t)$ and $v(t)$ occur at phases that differ by $\pi/2$. That is, when $x(t)$ has its maximum value, $v(t)$ is zero and vice versa. The energy of a mass on a spring as a function of time can then be written as

$$
\begin{aligned}
E &= \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 \\
&= \tfrac{1}{2}kA^2\sin^2(\omega t) + \tfrac{1}{2}m\omega^2 A^2\cos^2(\omega t) \\
&= \tfrac{1}{2}kA^2\sin^2(\omega t) + \tfrac{1}{2}m(\frac{k}{m})A^2\cos^2(\omega t) \qquad (9) \\
&= \tfrac{1}{2}kA^2[\sin^2(\omega t) + \cos^2(\omega t)] \\
&= \tfrac{1}{2}kA^2
\end{aligned}
$$

where we have used the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$. Since both k and A are constants, this shows that the energy in a simple harmonic oscillator is also a constant, regardless of the position and velocity of the mass because the mass-spring system is closed, and no energy is lost. The motion then meets the conditions of **SHM**, with continuous periodic motion

OSCILLATING MASS ON A PENDULUM

According to popular myth, Galileo felt bored one day in church. He looked up and saw that the light fixtures attached to the ceiling of the church swaying back and forth in regular motion. This began the first quantitative analysis of the motion of a simple pendulum. A “ideal” or “simple” pendulum is constructed with one end of a “massless” string attached to a small, massive bob, the other end being pivoted about a fixed location.

The force on the pendulum bob can be expressed in terms of θ, the angle of the string with respect to the vertical (equilibrium) position:

$$F = mL\ddot{\theta} = -mg\sin\theta\,. \qquad (10)$$

where L is the length of the string. (see Figure 1).

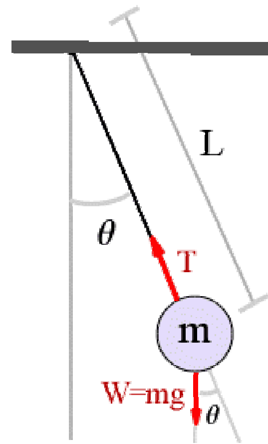

Figure 1

Equation 10 is not as trivial to solve as for a mass on a spring. However, we may expand $sin\theta$ in powers of the angle θ (where θ is measured in **radians**):

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots$$

and then make the approximation for a small angle oscillation ($\theta << 1$), keeping only the first (linear) term, $sin\theta \approx \theta$. Substituting this into Equation 10, we obtain

$$\begin{aligned} L\ddot{\theta} &= -g\theta \\ \theta &= -\frac{g}{L}\theta\,. \end{aligned} \qquad (11)$$

This equation is then identical in form to Equation 2, and we may solve it in the same way. In particular, we can make the identification that for the simple pendulum, $\omega = g/L$, so that the period of oscillation for a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}\,. \tag{12}$$

PART A : VERTICAL SPRING MOTION

PROCEDURE :

1. Hang one end of the spring to a ring stand.
2. Attach a mass m to the lower end of the spring.
3. Displace the mass a small distance and let it oscillate.
4. Record the time t for the mass to make 10 complete oscillations, repeat several times.
5. Repeat the experiment with different masses, corresponding to five different sets of measurements. Your raw data table should consists of a column for the five masses, and columns for the times.

ANALYSIS :

1. Add another column to your raw data table consisting of the average period of oscillation, T, computed from your measurements, and another column for T^2.
2. Plot T^2 versus m.
3. Using Equation 7, derive an expression for the slope of the graph of T^2 versus m. Using the best-fit line from your graph, determine the spring constant k.

PART B : SIMPLE PENDULUM

Refer to Figure 1 above. For this part, a simple pendulum and a Pasco timer will be used.

Parameter being varied : L - length of the string.

PROCEDURE :

1. Attach the free end of the string to the ring stand.
2. Measure L, the distance from the point of attachment to the center of mass of the bob.
3. Release the bob from a small angle (approximately five degrees from its rest position), and time ten oscillations. Record the total time t and L. Repeat several times.
4. Repeat this procedure for a total of five different lengths. The data should be placed in one raw data table containing columns for, L and the times.

ANALYSIS :

1. Find the average period of oscillation, T, and T^2. Add these values to your raw data table.
2. Plot T^2 versus L.
3. Using Equation 12, derive an expression for the slope of a T^2 versus L graph. Using the best-fit line from your graph, determine the value of g and compare it to the accepted value.

PART C : INTRODUCTION TO THE OSCILLOSCOPE

Using an oscilloscope is an extremely useful skill, so it is important that you have the time to study the instrument alone, without having to worry about other additional physical concepts in an experiment as well. Although this portion of the experiment looks lengthy, the procedure is very basic, straightforward, and short.

The following is a detailed description of most of the functions and features of an oscilloscope. It would be ideal to read the section carefully before coming to laboratory, and then go through the procedure steps while using the oscilloscope to look at very simple voltage signals. In your oscilloscope write-up, describe that you performed in the laboratory in a clear and concise manner, while answering the questions given in the procedural steps.

The oscilloscope is the most useful and versatile electronic instrument. It acts as a voltmeter with a lot of added capabilities. Instead of just measuring a numerical voltage value at a point in a circuit, it allows you to follow the changes of voltage as a function of time. The oscilloscope can be “triggered” so that the voltage signals can be displayed as stationary waveforms on a screen.

A very basic description of the insides of a oscilloscope is as follows. In essence if consists of an electron gun that produces a beam of electrons which is then focused on a fluorescent screen. The beam travels through two sets of deflection plates, one set deflects the beam in the x-direction, the other set deflects the beam in the y-direction. The potential difference between the deflection plates is determined by the voltage signal entering the oscilloscope input. This potential difference causes the beam to be deflected up or down by a certain amount proportional to the input voltage. Thus, the oscilloscope is a voltmeter. If a voltage is ramped repeatedly, and applied to the x deflection plates, the light beam will be repetitively deflected across the screen. This allows variations in time of the incoming voltage signal to be seen. This ramping voltage applied to the x deflection plates is called the sweep.

A typical front panel of an oscilloscope can be seen in Figure 2. The most obvious feature of the oscilloscope is a screen divided into many squares by an overlaying grid.

The vertical axis on the grid represents the voltage of an incoming signal, while the horizontal axis, in most cases, represents time. Most oscilloscopes have two input channels. Incoming voltage signals enter the oscilloscope through these channels. These can be found in the VERTICAL section of the front panel in Figure 1. Having two input channels is useful because it allows the user to look at two different signals at different points of the circuit at one time. Each channel has a calibrated knob or switch labeled VOLTS/DIV. For instance, if you set the switch to .5, this means each division or square in the vertical direction represents .5 V. There are many number settings to accommodate a large range of voltage signals. WARNING: There is also a variable gain knob (concentric with the VOLTS/DIV knob). The only thing you need to know about this knob is that it must be in the calibrated (labeled CAL) position, or in other words, it must be turned to its full clockwise position when you are making voltage measurements. If not, the divisions may not correspond to your VOLTS/DIV setting and your readings will all be wrong!

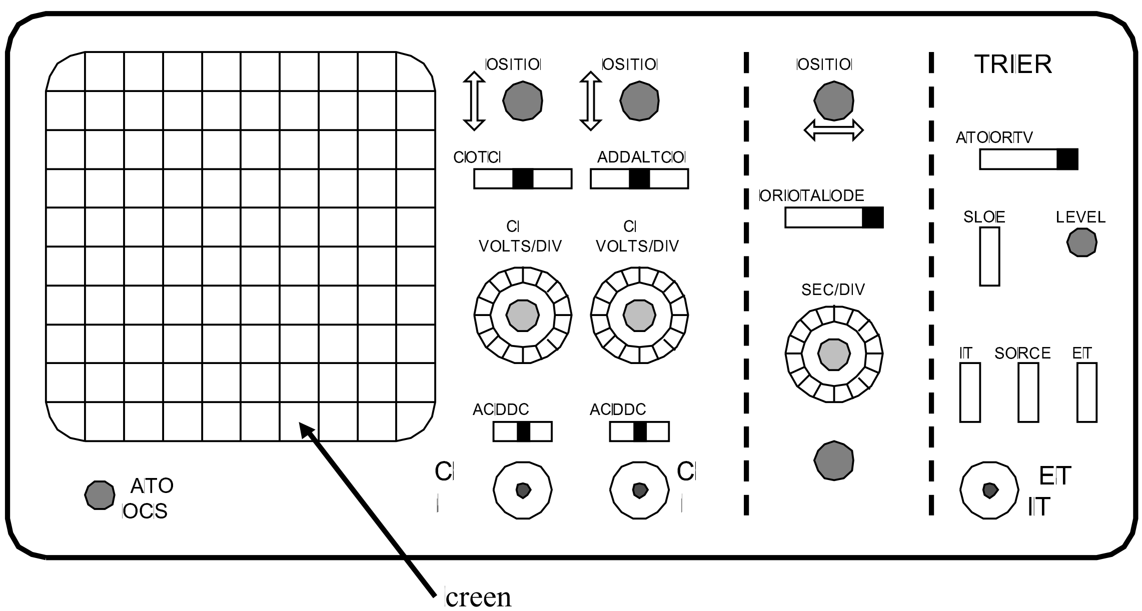

Figure 2: chematic Diagram of an Oscilloscope.

The oscilloscope is DC-coupled, which means you can see AC voltage signals on top of a constant, DC voltage signals. ometimes you may want to see a small AC signal, perhaps electrical "noise", riding on a large DC voltage. In this case, you can flip the COUPLING switch (labeled in Figure 1 in the VERTICAL section) to AC. You might notice that the other COUPLING option is labeled GND, which stands for ground. In a circuit, ground is the point of zero voltage. When measuring voltages, it can be the reference point from which you are comparing voltages. You will learn more about this in class. By setting the coupling switch to ground, you are disconnecting the incoming voltage signal from the rest of the oscilloscope and connecting the input to ground, which should be zero volts. If the oscilloscope is calibrated correctly, which you will soon be doing, the oscilloscope should measure zero volts.

The rest of the VERTICAL settings provide a variety of options. The POSITION knob moves the input voltage signal up and down on the screen. This feature will be used when calibrating the oscilloscope. The INVERT button does as its name implies, it inverts the incoming signal. The INPUT MODE knob allows one to see only the input voltage signal coming in channel 1 (CH1), or only the input voltage signal coming in channel two (CH2), or observe the sum of both signals (ADD). The remaining options allow both incoming signals to be displayed.

The controls under the HORIZONTAL heading perform other functions. They are also shown in Figure 2. As mentioned previously, the horizontal direction on the oscilloscope represents time. The oscilloscope horizontally sweeps the input signals, and the TIME/DIV settings determine the sweep speed. For example, if the knob is set for 0.1 ms, this means that the oscilloscope is sweeping at a rate such that each square or division represents 0.1 ms. Similar to the vertical controls, there are many numerical settings to accommodate a large range of incoming voltage signal frequencies.

The trickiest part about the oscilloscope is "triggering." So far we have discussed vertical signals and horizontal sweep: those are what we need for a display of voltage versus time. But if the horizontal sweep doesn't catch the input signal at the same time, (assuming the signal is repetitive), the display will be a mess – a picture of the input waveform superimposed over itself at different times. Another result of improper triggering is the voltage signal "moving" on the screen, rather than being stationary so that measurements can be made. You can see from the first panel that you have a number of different choices about trigger sources and modes. Setting the triggering control on AUTO mode should be satisfactory for our purposes. If this doesn't work, you can try the NORMAL mode while adjusting the LEVEL and SLOPE switches. More details about these controls will be discussed in a more advanced course.

PROCEDURE :

We want to observe a DC voltage signal from a power supply, and an AC voltage signal from a function generator using the oscilloscope.

1. Connect the sine wave voltage signal to Channel 1 of the oscilloscope. You can do this with the help of a BNC connector, which is the kind of connector needed to attach onto the inputs of an oscilloscope. The female end of a BNC connector is the connection already attached to the oscilloscope. The male end of a BNC connector is the attachment that physically twists on to the female end. If you look at the male end, there is a small pin in the very center. This pin is connected to the wires that will carry the voltage signal from the function generator. The rest of the connector is physically touching the case of the oscilloscope, which is usually tied or connected to ground. Assuming the male connector accommodates banana plugs, attach both ends of the sine wave voltage signal to the male BNC connector, and connect the male BNC to Channel 1. To make sure the grounds of both instruments are the same, attach another BNC connector to any other BNC connection on the oscilloscope (a convenient one would be the external signal BNC on the right side of the oscilloscope). Then, using a banana plug, connect the ground of the function generator to the ground of the BNC connector. It is necessary for all the grounds to be the same so that we are using just one reference point from which to measure.
2. Now, put a DC voltage signal from the power supply into channel 2. To do this, attach a BNC/single banana plug adapter to the female BNC connection of Channel 2. Connect the high end of the DC voltage signal to this connector.

Attach the low end to the ground connection of the oscilloscope. This ensures that the power supply and the oscilloscope also have the same ground. The oscilloscope should now be connected properly to the function generator and the power supply.

3. You may now "zero" your oscilloscope, and then measure the voltage signals. "Zeroing" the oscilloscope is necessary for accurate voltage readings. Since ground is the reference point from which all voltages are compared, it makes sense that when the oscilloscope reads the ground signal, it should read 0 V. Turn on the oscilloscope, power supply, and function generator; set triggering for AUTO. Set the function generator frequency to 1500 Hz. Set the sweep speed at 1 ms/DIV. Make sure the inner VOLTS/DIV knob is turned all the way clockwise in its calibrated position. You should hear it click into its locked position.
4. Set the vertical mode for channel one (CH1) and set Channel 1's COUPLING switch to ground (GND). This disconnects the voltage signal of the function generator from the Channel 1 input, and connects the Channel 1 input to ground. You are now looking at the ground signal of Channel 1 or input 1. A horizontal line should appear on the screen. Since no voltage is going into the oscilloscope, the electron beam should not be deflected, so it should fall in the middle of the screen at the 0 V line (the darkened x-axis grid line). If it does not, use the vertical POSITION knob to move the line to this 0 V position. Set the VOLTS/DIV knob at a more sensitive setting when doing this. Why is this a good idea? Once this task is accomplished, set the COUPLING switch back to DC.
5. Repeat the above procedure to calibrate Channel 2. Your oscilloscope is now "zeroed".
6. To obtain measurements of a DC voltage signal, set the VERTICAL MODE for Channel 2. The Channel 2 input is the voltage signal coming out of the DC power supply.
7. Set the power supply to 5 V and adjust the VOLTS/DIV setting so that a line appears on the screen. This line is a "picture" of the voltage. The voltage going into Channel 2 is applied to the y-defection plates in the oscilloscope. The plates

deflect the electron beam up a certain distance that corresponds to 5 V. The line is horizontal because the voltage deflection applied is constant with time, so the beam is directed toward the same place throughout time.

8. Find the VOLTS/DIV setting that positions the signal at the first grid line above the 0 V axis line. Now slowly increase the voltage output on the power supply to 10 V. What happens to the line on the screen? Change the VOLTS/DIV settings in both directions and describe what happens.
9. What happens to the DC voltage signal when the COUPLING switch for Channel 2 is moved to AC?
10. To obtain measurements of an AC voltage signal, set the vertical mode for Channel 1 (CH1). You are now looking at the AC voltage waveform coming from the function generator. If you do not see a clear stationary sine wave, the oscilloscope is not triggered properly. You may need to use the Normal (NORM) mode trigger.
11. Adjust the oscilloscope so that only a few cycles of the sine wave appear on the screen. This can be done by changing the value of the SEC/DIV setting. How would you determine the frequency of the sine wave using the oscilloscope? Here are a few hints. Remember that you can horizontally move the signal with the horizontal POSITION switch. This will enable you to align the peaks of the sine wave with the grid lines so as to determine how long it takes for one cycle to be completed. Also, the SEC/DIV settings can be changed so that it may be easier to align the peaks of the waveform, or fit several complete cycles of the waveform between the grid lines. Using one or both of these methods, determine the frequency of the sine wave using the oscilloscope. Compare this frequency value to the value indicated on the function generator. Which value would you deem to be more accurate?
12. Change the frequency of the sine wave and calculate it again using the oscilloscope. Compare the measured oscilloscope frequency value to the function generator frequency value.

13. The amplitude voltage of the signal now needs to be determined. How can this be accomplished? Remember that there is a vertical POSITION switch you can use to move the waveform up and down.