Tree and Graph Data Structure

Assignment -#4

1. The number of nodes in a complete binary tree of level 5 is:
   a) 64
   b) 63
   c) 32
   d) More than one of the above
   e) None of the above

2. The height of a binary search tree for the words "banana", "peach", "apple", "pear", "coconut", "mango", and "papaya" using alphabetical order is:
   a) 2
   b) 3
   c) 4
   d) 5
   e) None of the above

3. Which of the following is not a binary tree?
   a) Heap
   b) AVL-Tree
   c) B-Tree
   d) More than one of the above
   e) None of the above

4. A threaded binary tree is a binary tree in which:
   a) Each node has two children
   b) Each node has at most one child
   c) Each node has a thread connecting it to its predecessor or successor
   d) More than one of the above
   e) None of the above

5. The maximum and minimum number of nodes in a binary tree of height 5 are:
   a) 63 and 6, respectively
   b) 64 and 5, respectively
   c) 31 and 5, respectively
   d) More than one of the above
   e) None of the above

6. Let T be a binary search tree with 15 nodes. The minimum and maximum possible heights of T are:
   a) 4 and 15, respectively
   b) 3 and 14, respectively
   c) 4 and 14, respectively
   d) More than one of the above
   e) None of the above

7. The maximum height of a binary tree with 'n' nodes is:
   a) n
   b) n – 1
   c) n + 1
   d) More than one of the above
   e) None of the above

8. When we perform an in-order traversal on a binary tree, we get the array in ascending order. The tree is:
   a) Heap tree
   b) Almost complete binary tree
   c) Binary search tree
   d) More than one of the above
   e) None of the above

9. The maximum number of nodes in a binary tree of height h is:
   a) 2^h – 1
   b) 2^h – 1 - 1
   c) 2^h+1 – 1
   d) More than one of the above
   e) None of the above

10. Which one of the following properties is correct for a red-black tree?
   a) Every simple path from a node to a descendant leaf contains the same number of black nodes
   b) If a node is red, then one child is red and another is black
   c) If a node is red, then both its children are red
   d) More than one of the above
   e) None of the above

11. A complete binary tree with n non-leaf nodes contains:
   a) $\log_2 n$ nodes
   b) 2n+1 nodes
   c) 2n nodes
   d) More than one of the above
   e) None of the above

12. The number of different binary trees with 6 nodes is:
   a) 6
   b) 42
   c) 132
   d) More than one of the above
   e) None of the above

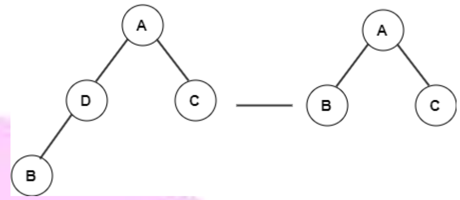
13. What is the maximum number of children that a binary tree node can have?
   a) 0
   b) 1
   c) 2
   d) More than one of the above
   e) None of the above

14. How many common operations are performed in a binary tree?
   a) 1
   b) 2
   c) 3
   d) More than one of the above
   e) None of the above

15. What traversal strategy is used in a binary tree?
   a) Depth-first traversal
   b) Breadth-first traversal
   c) Random traversal
   d) More than one of the above
   e) None of the above

16. How many types of insertion are performed in a binary tree?
   a) 1
   b) 2
   c) 3
   d) More than one of the above
   e) None of the above

17. What operation does the following diagram depict?



   a) Inserting a leaf node
   b) Inserting an internal node
   c) Deleting a node with 0 or 1 child
   d) Deleting a node with 2 children
   e) None of the above

18. The average depth of a binary search tree is given as:
   a) O(N)
   b) O(√N)
   c) O(log N)
   d) More than one of the above
   e) None of the above

19. How many orders of traversal are applicable to a binary tree (in general)?
   a) 1
   b) 4
   c) 2
   d) 3
   e) None of the above

20. If binary trees are represented in arrays, what formula can be used to locate a left child if the node has an index i?
   a) 2i + 1
   b) 2i + 2
   c) 2i
   d) More than one of the above
   e) None of the above

21. Using what formula can a parent node be located in an array?
   a) (i+1)/2
   b) (i-1)/2
   c) i/2
   d) More than one of the above
   e) None of the above

22. Which of the following properties are obeyed by all three tree traversals?
    a) Left subtrees are visited before right subtrees
    b) Right subtrees are visited before left subtrees
    c) Root node is visited before left subtree
    d) More than one of the above
    e) None of the above

23. Which of the following statements is true about AVL Trees?
    a) The difference between the heights of left and right nodes cannot be more than 1.
    b) The height of an AVL Tree always remains of the order of O(logn)
    c) AVL Trees are a type of self-balancing Binary Search Trees.
    d) More than one of the above
    e) None of the above

24. A B-tree of order 4 and of height 3 will have a maximum of _______ keys.
    a) 255
    b) 63
    c) 127
    d) More than one of the above
    e) None of the above

25. Which of the following is true?
    a) larger the order of B-tree, less frequently the split occurs
    b) larger the order of B-tree, more frequently the split occurs
    c) smaller the order of B-tree, less frequently the split occurs
    d) More than one of the above
    e) None of the above

26. The balance factor of a node in a binary tree is defined as _____
    a) addition of heights of left and right subtrees
    b) height of right subtree minus height of left subtree
    c) height of left subtree minus height of right subtree
    d) More than one of the above
    e) None of the above

27. How many undirected graphs (not necessarily connected) can be constructed out of a given set V={V1,V2,…Vn} of n vertices?
    a) n(n−1)/2
    b) 2^n
    c) n!
    d) More than one of the above
    e) None of the above

28. Which of the following statements is/are TRUE for an undirected graph?
    P: The number of odd-degree vertices is even
    Q: The sum of degrees of all vertices is even
    a) P Only
    b) Q Only
    c) Both P and Q
    d) More than one of the above
    e) None of the above

29. What is the maximum number of edges in an acyclic undirected graph with n vertices?
    a) n−1
    b) n
    c) n+1
    d) More than one of the above
    e) None of the above

30. Which of the following algorithms is used to find the shortest path in a weighted graph?
    a) Kruskal’s algorithm
    b) Floyd-Warshall algorithm
    c) Dijkstra’s algorithm
    d) More than one of the above
    e) None of the above

31. Backtracking uses _______ node generation _______ bounding functions:
    a) Breadth-first, with
    b) Breadth-first, without
    c) Depth-first, with
    d) More than one of the above
    e) None of the above

32. Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix?
   a) $\Theta(n)$
   b) $\Theta(n+m)$
   c) $\Theta(n^2)$
   d) More than one of the above
   e) None of the above

33. A graph in which all vertices have equal degree is known as ____.
   a) Complete graph
   b) Regular graph
   c) Multi graph
   d) More than one of the above
   e) None of the above

34. A vertex of in-degree zero in a directed graph is called a/an ____.
   a) Root vertex
   b) Isolated vertex
   c) Sink
   d) More than one of the above
   e) None of the above

35. A connected graph T without any cycles is called:
   a) Free graph
   b) No cycle graph
   c) Non-cycle graph
   d) More than one of the above
   e) None of the above

36. In Depth First Search, how many times is a node visited?
   a) Once
   b) Twice
   c) Equivalent to the number of in-degree of the node
   d) More than one of the above
   e) None of the above

37. What are the appropriate data structures for the following algorithms?
   1. Breadth-First Search
   2. Depth-First Search
   3. Prim's Minimum Spanning Tree
   4. Kruskal's Minimum Spanning Tree
   a) Stack, Queue, Priority Queue, Union Find
   b) Queue, Stack, Priority Queue, Union Find
   c) Queue, Stack, Union Find, Priority Queue
   d) More than one of the above
   e) None of the above

38. For a given connected graph G having v vertices and e edges with no cycles, which of the following statements is true?
   a) $v=e-1$
   b) $e=v-2$
   c) $v=e+1$
   d) More than one of the above
   e) None of the above

39. Which data structure is commonly used to implement the navigation system in a mapping application, helping users find the shortest route between locations?
   a) Heap
   b) Stack
   c) Graph
   d) More than one of the above
   e) None of the above

40. Which of the following statements is correct?
   P1: Every tree will always be a graph
   P2: Every graph will always be a tree
   P3: Every tree will be a graph, but every graph will not be a tree
   P4: Every graph will be a tree, but every tree will not be a graph
   a) P1 and P2
   b) P1 and P3
   c) P2
   d) More than one of the above
   e) None of the above

41. Which of the following algorithms is used to find a Minimum Spanning Tree in a graph?

a) Dijkstra's algorithm
b) Kruskal's algorithm
c) Floyd-Warshall algorithm
d) More than one of the above
e) None of the above

42. Which data structure is most commonly used to implement Prim's algorithm efficiently?

a) Stack
b) Queue
c) Priority Queue
d) More than one of the above
e) None of the above

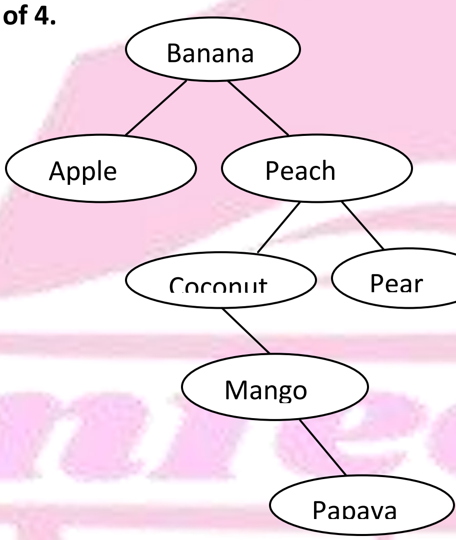
Solution with Explanation

Answer1: d) 63

Explanation: 2^(L+1) - 1,In a complete binary tree of level 5, the number of nodes is 2^5 - 1 = 63.

Answer2:b) 4

Explanation: The height of a binary search tree depends on the order of insertion. Using alphabetical order, "banana" is the root, and inserting others yields a height of 4.



Answer3: d) More than one of the above

Explanation: B-Tree is not a binary tree, as it can have more than two children, making both B-Tree and AVL trees invalid.

Answer4: d) Each node has a thread connecting it to its predecessor or successor

Explanation: A threaded binary tree includes additional pointers to traverse the tree in-order.

Answer5: a) 63 and 6, respectively

Explanation: The maximum nodes in a tree of height 5 is 2^6 - 1 = 63 (2^(h+1)-1), and the minimum is 6 (h+1).

Answer6: b) 3 and 14, respectively

Explanation: The minimum height of a binary search tree with 15 nodes is 3 (if balanced), and the maximum is 14 (if skewed).

Minimum height of the BST with n nodes is ⌈log2 (n + 1)⌉ - 1

Maximum height of the BST with n nodes is n - 1.

Answer7: b) n – 1

Explanation: The maximum height of a binary tree with n nodes is n - 1 (in a skewed tree).

Answer8: c) Binary search tree

Explanation: In-order traversal of a binary search tree yields nodes in ascending order.

Answer9: a) 2^(h+1) – 1

Explanation: The maximum number of nodes in a binary tree of height h is 2^(h+1) – 1.

Answer10: a) Every simple path from a node to a descendant leaf contains the same number of black nodes

Explanation: A red-black tree ensures that the number of black nodes from the root to any leaf is the same.

A red-black tree is a balanced binary search tree with the following properties:

- Every node is colored red or black.
- Every leaf is a NIL node and is colored black.
- If a node is red, then both its children are black.
- Every simple path from a node to a descendant leaf contains the same number of black nodes.

Answer11: b) 2n+1 nodes

Explanation: In a complete binary tree, the number of non-leaf nodes and leaf nodes is related by n+ n+1. So total nodes 2n+1.

Answer12: c) 132

Explanation: The number of distinct binary trees with 6 nodes is given by the Catalan number C(n) = $^{2n}C_n$ = (2n)!/(n+1)!n! = 132.

Answer13: c) 2

Explanation: A binary tree node can have at most 2 children.

Answer14: c) 3

Explanation: The three main operations in a binary tree are insertion, deletion, and traversal.

Answer15: D) More than one of the above Breadth-first traversal and DFS

Explanation: Breadth-first traversal is also called level-order traversal, where nodes are visited level by level. DFS visits the nodes in depth first with 3 approaches Inorder, preorder and postorder.

Answer16: b) 2

Explanation: Two types of insertion in binary trees: inserting a leaf node and inserting an internal node.

Answer18: b) O(logN)

Explanation:. An analysis shows that the average depth is O(√N) , and that for a special type of binary tree, namely the binary search tree, the average value of the depth is O(logN)

Answer19: d) 3

Explanation: The three types of tree traversals are in-order, pre-order, and post-order.

Answer20: a) 2i + 1

Explanation: The left child of a node at index i is located at 2i + 1 in an array representation of a binary tree.

Answer21: b) (i-1)/2

Explanation: The parent node of a node at index i is found at (i-1)/2 in an array.

Answer22: a) Left subtrees are visited before right subtrees

Explanation: In all tree traversals (in-order, pre-order, post-order), the left subtree is visited before the right subtree.

Answer23: d) All of the above

Explanation: For any node in an AVL tree, the difference in heights of its left and right subtrees (also called the balance factor) must be at most 1.

AVL trees maintain a balanced structure by rotating nodes when the balance factor condition is violated. This ensures the tree's height remains logarithmic, i.e., O(logn).

AVL trees are indeed a type of self-balancing binary search tree (BST), They maintain balance after every insertion and deletion to ensure optimal performance.

Answer24: a) 255

So, the B-tree will have $n = (m^{h+1} – 1)$ keys in this situation.

So, required number of maximum keys = $4^{3+1} – 1$ = 256 – 1 = 255.

Answer25: a) Larger the order of B-tree, less frequently the split occurs:

In a B-tree, the order (denoted as m) defines the maximum number of children a node can have. A larger order means that each node can hold more keys, which increases the capacity of the node before it needs to split. Therefore, the split operation occurs less frequently as the order of the B-tree increases.

Answer26: c) Height of left subtree minus height of right subtree

Answer17: c) Deleting a node with 0 or 1 child

Explanation: The diagram depicts the deletion of a node that has 0 or 1 child.

Answer27: (E) None of the above

Explanation: The total number of undirected graphs with n vertices is $2^{n(n-1)/2}$ as each of the n(n−1)/2 possible edges can either be present or absent.

Answer28: (C) Both P and Q

Explanation: P is true because each edge adds 1 to the degree of two vertices, making the total number of odd-degree vertices even. Q is true because the sum of degrees of all vertices equals twice the number of edges, which is always even.

Answer29: (A) n−1

Explanation: The maximum number of edges in an acyclic undirected graph (a tree) with n vertices is n−1.

Answer30: (D) More than one of the above

Explanation: Both Floyd-Warshall and Dijkstra's algorithms are used for finding the shortest path in weighted graphs, with Floyd-Warshall solving all-pairs shortest paths and Dijkstra solving single-source shortest paths.

Answer31: (C) Depth-first, with

Explanation: Backtracking uses depth-first node generation with bounding functions to eliminate certain branches.

Answer32: (C) $\Theta(n^2)$

Explanation: In an adjacency matrix representation, the time complexity for traversing the graph is $O(n^2)$ as every vertex pair needs to be checked.

Answer33: (D) More than one of the above (a and b)

Explanation: A graph in which all vertices have the same degree is called a regular graph also complete graph vertices have the same degree.

Answer34: (B) root vertex

Explanation: a source(root) vertex is a vertex with indegree zero, while a sink vertex is a vertex with outdegree zero. Isolated vertex has no incoming as well as outgoing edge.

Answer35: (E) None of the above

Explanation: A connected graph without cycles is called a tree or acyclic graph.

Answer37: (B) Queue, Stack, Priority Queue, Union Find
Explanation: BFS uses a queue, DFS uses a stack, Prim's MST uses a priority queue, and Kruskal's MST uses a union-find structure.

Answer38: (A) v=e−1
Explanation: A connected acyclic graph (tree) has v−1 edges.

Answer39: (C) Graph
Explanation: Navigation systems commonly use graphs to represent routes and locations, enabling the use of shortest path algorithms.

Answer40: (B) P1 and P3
Explanation: Every tree is a graph, but not every graph is a tree.

Answer41: (B) Kruskal's algorithm
Explanation: Kruskal's algorithm is a greedy algorithm that finds a Minimum Spanning Tree for a connected graph by adding edges in increasing order of weight.

Answer42: (C) Priority Queue
Explanation: Prim's algorithm uses a priority queue to select the minimum weight edge efficiently during each step.

Answer36: (A) Once
Explanation: Each node in DFS is visited once, though it may be "checked" multiple times.