Guaranteed under failure distance as a tool for resilience facilities location problem (and instrument for supply chain analysis)

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What Is “Distance”?

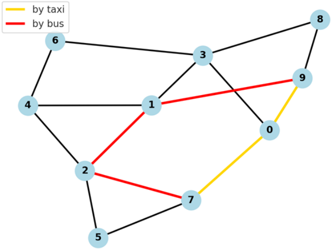
What is the essence of ”distance concept”?

- **Intuition:** Saying that two points are “far apart” means that, in some sense, moving from point A to point B requires a lot of effort.
- **Common Interpretations:**
  - Movement in continuous space (Euclidean distance, Manhattan distance, etc.)
  - Traversal on a graph—paths along edges (some edges may be “harder” to traverse, i.e. have higher weight)
- **Graph Distance:** Traditionally, “distance” on a graph refers to the *geodesic distance* (the length of the shortest path).
- **Question:** Is geodesic distance always the best choice?

What Is essence of “Distance”?

Distance usually connect with the goal of operation

- Buses in Belgrade are free, taxi – not. Let us assume the travel time by bus is 20 minutes, while by taxi it is only 5 minutes.
- Then, the shortest path in terms of time is the taxi ride, whereas in terms of cost it is the bus ride.
- Thus, the purpose of movement always appears (perhaps implicitly) in the definition of distance.



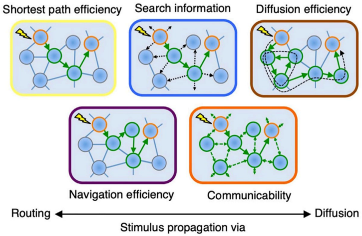
What is the problem with geodesic(unique shortest path) distance?

Geodesic distance may fail in

- Hospital location
- Delivery
- Fire station location
- Other situations, when the *guaranteed* time matters. (= high penalty for failed promise)

Not only shortest paths matter. What's already known

- Shortest paths do not always catch complete information about a system.
- In neuroscience and ML, distances that considers longer paths have proven to be highly effective
- We chose *Resistance Distance*, as a measure that incorporates the influence of all paths between nodes.

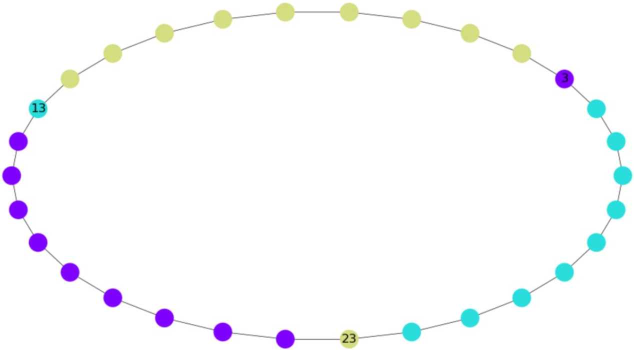


From Caio Seguin et.al; Communication dynamics in the human connectome shape the cortex-wide propagation of direct electrical stimulation (2023)

GUF distance

Let $\mathscr{S}$ be a set of possible failure scenarios then

$$d_{\mathrm{GUF}}(a,b) := \max_{G_S \in \mathscr{S}} d_{G_S}(a,b)$$



GUF distance is a metric

Theorem

Consider arbitrary set of scenarios $\mathscr{S}$. Then Guaranteed Under Failures(GUF) distance induce a metric on network.

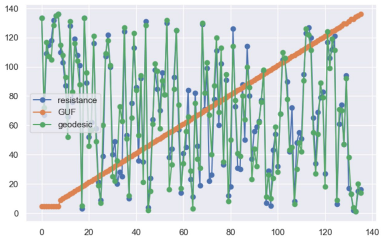
Why it so important?

- We can run clustering algorithms that require a metric (such as *K-means*)
- We can hope for the convergence of greedy and evolutionary procedures where a metric is desirable (such as *K-medoids*).
- We can formulate and solve certain *optimization problems* on the network
- We can try to find good *embeddings*

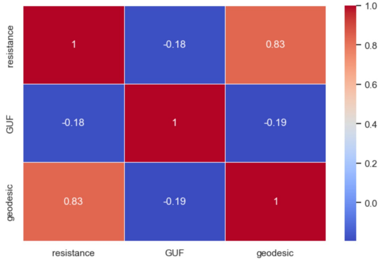
Set up of the experiments

- As the set of failure scenarios, we consider the complete removal of any edge in the network.
- We consideres the network topologies of Tata's PoP connections (India) and the road network of Sheffield (USA, Alabama).
- We compare vertex efficiency-based *rankings*.
- We attempt to solve the *P-median* problem and observe induced clusterings.
- We observe the worst-case distance from facilities under given scenarios of failures.

Efficiency-based ranking in PoP network

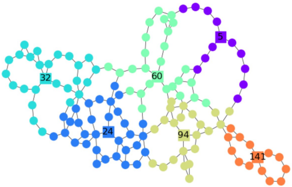


(a) efficiency-based rankings

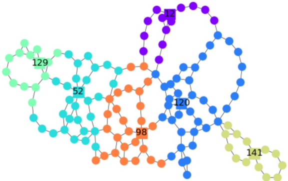


(b) Kendall correlations between rankings

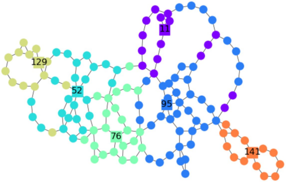
Clustering of India PoP network



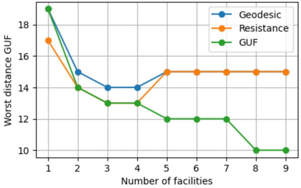
(a) geodesic 6-median problem



(b) resistance 6-median problem

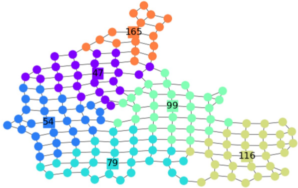


(c) GUF 6-median problem

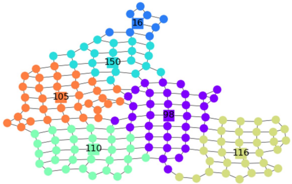


(d) Worst case distance

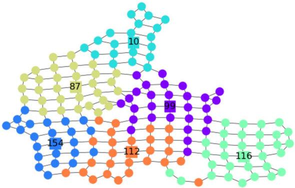
Urban network clustering



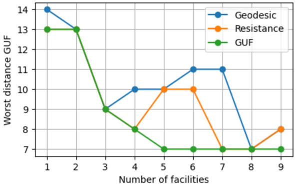
(a) geodesic 6-median problem



(b) resistance 6-median problem



(c) GUF 6-median problem



(d) Worst case distance

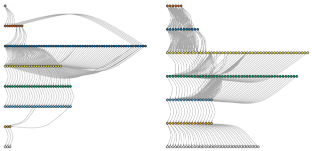
GUF-like concept in supply chains

- **Traditional focus:** Resilience tools for supply networks often rely on reachability, transportation cost, or delivery time after disruptions (still distance or flow dependent on it)
- **Optimization-based design:** Network elements are evaluated based in according to this measures. One possible goal is to improve efficiency and minimize minimum-cost flow.
- **Limitation:** Focusing solely on minimum cost and shortest delivery time can overlook scenarios where delivery guarantees matter most.
- **Uncertainty contexts:** For perishable goods or pharmaceuticals, penalties for violating service guarantees may extremely exceed cost savings.
- **Trade-off:** Cost-efficiency and guaranteed resilience can depend on different network bottlenecks (in comparison with regular resilience)

Generated supply chain

- We have several tiers (layers) of nodes and several roles (supplier/manufactorer/customer e.t.c.)
- Several strategies of link formation (Preferential attachment on: out-degree (PAOD), in-degree (PAID), degree difference (PADD), betweenness (PAB), closeness (PAC); Random selection (Rand); Inverse shortest distance (InvSD); and No action (NA))
- Fixed number of nodes and events (link formation)
- Probabiility of chosing role and chosing strategy for each tiers was fitted on the real data

Generated supply chain (Modeling topologically resilient supply chain networks)



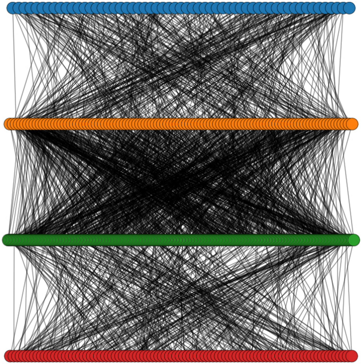
Algorithm 1 Synthesis algorithm $F(\mathbf{M}, n, m, \xi)$ for target network $\mathcal{T} = (V, E)$

Create a network $\mathcal{G} = (V', E')$ with $V' = V$ and $|E'| = \emptyset$
Using $\mathbb{P}(T = t)$, probabilistically assign a node type t to each $v_i \in V'$
while $|E'| < |E|$ **do**
    Select a tier $\alpha \in \{0, \ldots, l-1\}$, followed by a node $v_i \in T_\alpha$
    Probabilistically select an action a_l for v_i using $\mathbb{P}(A = a_l | t)$
    Add edge (v_i, v_j) to $\mathcal{G}$ as determined by a_l and satisfying $v_j \in T_{\alpha-1}$
end while
return $\mathcal{G}$

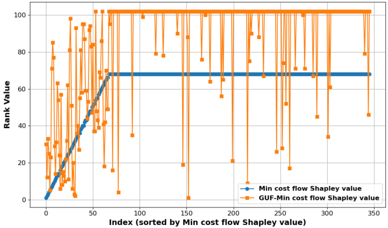
GUF concept in supply chains

- To assess “traditional” resilience, we compute the **min-cost flow Shapley value** for each node in the intermediate layers, defined as the change in the minimum-cost flow when the node is removed.
- To evaluate **guaranteed under failures (GUF)** resilience, we replace the minimum-cost flow with the guaranteed minimum-cost flow: for each source, we consider the worst-case scenario from a predefined set of possible failures. Importantly, the worst-case scenario may differ across sources. We then propose new measure: **GUF-min-cost flow Shapley value** that compute as the change in the guaranteed minimum-cost flow resulting from the removal of a given node.

Supply chains



(a) Generated supply chain



(b) Dependency between Shapley and GUF-Shapley (kendell tau = 0.67)

Conclusion

- We introduced the concept of GUF for the network topology, which induces significantly different rankings (both for FLP and SC) and enables solving optimization problems in scenarios where the cost of time is nonlinear (or penalty for failure is high + some failures are expected)
- We showed that the importance of nodes in terms of the guarantee objective can (and usually does) differs significantly from their importance in terms of the fastest-movement objective.
- Practical applications require **reasonable failure scenarios**, such as typical temporal traffic data or something other, to achieve sustainable decision-making.