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第一周作业

1. 将表达式转换为不会让两个几乎相等的数相等的形式：

(a) $\frac{1-secx}{tan^2x}=\frac{1-secx}{sec^2x-1}=\frac{1-secx}{(secx-1)(secx+1)}=-\frac{1}{1+secx}$ ；

(b) $\frac{1-(1-x)^3}{x}=\frac{1-(1-3x+3x^2-x^3)}{x}=x^2-3x+3$ ；

用Matlab分别计算左边和右边的表达式（已设置为双精度），结果如下：

题目	a		b	
x	$\frac{1-secx}{tan^2x}$	$-\frac{1}{1+secx}$	$\frac{1-(1-x)^3}{x}$	x^2-3x+3
10^{-1}	-0.498747913711435	-0.498747913711429	2.709999999999999	2.710000000000000
10^{-2}	-0.499987499790956	-0.499987499791664	2.970099999999998	2.970100000000000
10^{-3}	-0.499999875014289	-0.499999874999979	2.997000999999999	2.997001000000000
10^{-4}	-0.499999993627931	-0.499999998750000	2.999700010000161	2.999700010000000
10^{-5}	-0.500000041336852	-0.499999999987500	2.999970000083785	2.999970000100000
10^{-6}	-0.500044450290837	-0.499999999999875	2.999997000041610	2.999997000001000
10^{-7}	0	-0.499999999999999	2.999999698660716	2.999999700000010
10^{-8}	0	-0.500000000000000	2.999999981767587	2.999999970000000
10^{-9}	0	-0.500000000000000	2.999999915154206	2.999999997000000
10^{-10}	0	-0.500000000000000	3.000000248221113	2.999999999700000
10^{-11}	0	-0.500000000000000	3.000000248221113	2.999999999970000
10^{-12}	0	-0.500000000000000	2.999933634839635	2.999999999997000
10^{-13}	0	-0.500000000000000	3.000932835561798	2.999999999999700
10^{-14}	0	-0.500000000000000	2.997602166487923	2.999999999999970

对于$x=10^{-1}，...，10^{-14}$，(a)中原表达式准确数字的位数依次为$4，12，9，9，0，0，...，0$;

(b)中原表达式准确数字的位数依次为$2，4，6，13，10，11，7，8，8，0，0，5，0，3$.

3. 设较长的直角边为a，较短的直角边为b，斜边为c，由题意可知要求$c-a$，由$a^2+b^2=c^2$可知$c=\sqrt{a^2+b^2}$，因此

$$\begin{aligned} c-a &= \sqrt{a^2+b^2}-a \\ &= \frac{a^2+b^2-a^2}{\sqrt{a^2+b^2}+a} \\ &= \frac{b^2}{\sqrt{a^2+b^2}+a} \end{aligned}$$

因此斜边相比较长的直角边长2.233221×10^{-10}.

第二周作业

(a) $x^5+x=1$；

可写成

$$\begin{aligned} g(x) &= 1-x^5 \\ g(x) &= \frac{1}{x^4+1} \\ g(x) &= \frac{4x^5+1}{5x^4+1} \leftarrow (5x^4+1)x = 1+4x^5 \leftarrow 5x^5+x = 1+4x^5 \end{aligned}$$

当$x=0$时，$x^5+x=0$，当$x=1$时，$x^5+x=2$，由此可知不动点$0<x<1$，设初始点为$x_0=\frac{1}{2}$

	$g(x)=1-x^5$		$g(x)=\frac{1}{x^4+1}$		$g(x)=\frac{4x^5+1}{5x^4+1}$
i	x_i	i	x_i	i	x_i
0	0.500000000000000	0	0.500000000000000	0	0.500000000000000
1	0.968750000000000	1	0.941176470588235	1	0.857142857142857
2	0.146784812211990	2	0.560329270010801	2	0.770682194733540
3	0.999931859454215	3	0.910268909262926	3	0.755282953105649
4	0.000340656300750	4	0.592922683367010	4	0.754877935491823
5	1	5	0.890002348180761	5	0.754877666246812
6	0	6	0.614466118386692	6	0.754877666246693
7	1	7	0.875229048512853	7	0.754877666246693
8	0	8	0.630200866526642	8	0.754877666246693
9	1	9	0.863758797619065	9	0.754877666246693
10	0	10	0.642411659460851	10	0.754877666246693
11	1	$\dots$		$\dots$	
12	0	n	0.754531669397964		0.754877666246693

由此可知，方程的解为0.75487767.

(b) $sinx=6x+5$；

可写成

$$g(x)=\frac{sinx-5}{6}$$

由$-1\le sinx\le 1$可知$-1\le x\le -\frac{2}{3}$，设初始点为$x_0=\frac{5}{6}$

i	x_i
0	-0.833333333333333
1	-0.956696142199340
2	-0.969548712056550
3	-0.970771752747636
4	-0.970886956518275
5	-0.970897797485205
6	-0.970898817553697
7	-0.970898913535053
8	-0.970898922566224
9	-0.970898923415994
10	-0.970898923495951
11	-0.970898923503475
12	-0.970898923504182

由此可知，方程的解为-0.97089892.

(c) $\ln x + x^2 = 3$;

可写成

$$g(x) = \sqrt{3 - lnx}$$

设初始点$x_0 = \sqrt{2}$

i	x_i
0	1.414213562373095
1	1.628934133020739
2	1.584952398399848
3	1.593563812866320
4	1.591862776811914
5	1.592198201155483
6	1.592132036645776
7	1.592145087115296
8	1.592142512970396
9	1.592143020707260
10	1.592142920558719
11	1.592142940312512
12	1.592142936416176
13	1.592142937184709
14	1.592142937033120

由此可知，方程的解为1.59214293.