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Homework 13

数值计算方法，2021 春

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用自适应 Simpson 积分，求

$$\int_0^4 13x(1-x)e^{-\frac{3}{2}x}dx$$

三点 Simpson 公式如下：

$$\int_a^b f(x)dx = \frac{b-a}{6}[f(a)+4f(\frac{a+b}{2})+f(b)]$$

自适应 Simpson 法则的判断公式如下（其中 eps 为精度）：

$$|S(a,c)+S(c,b)-S(a,b)| < 15 * eps$$

最后编写程序如下：

```python
import numpy as np


def F(x):
    # Simpson公式用到的函数
    x = np.double(x)
    return 13 * x * (1 - x) * np.exp(-3 * x / 2)


def Simpson(a, b):  # 三点Simpson法，这里要求F是一个全局函数
    c = (a + b) / 2.0
    return (F(a) + 4 * F(c) + F(b))*(b - a) / 6


def asr(a, b, eps, A):  # 自适应Simpson公式（递归过程）。已知整个区间[a,b]上的三点Simpson值A
    c = (a + b) / 2.0
    left = Simpson(a, c)
    right = Simpson(c, b)

    if (np.abs(left + right - A) <= 15 * eps):  # 自适应Simpson法则的判断公式
        return left + right + (left + right - A) / 15.0

    return asr(a, c, eps / 2.0, left) + asr(c, b, eps / 2.0, right)
```

```python
eps = 0.1
for i in range(7):
    answer = asr(0.0, 4.0, eps, Simpson(0.0, 4.0))
    print("When eps = %.7f, answer = %.7f" % (eps, answer))
    eps = eps / 10.0
```

输出结果如下：

```
when eps = 0.1000000, answer = -1.5727644
when eps = 0.0100000, answer = -1.5488164
when eps = 0.0010000, answer = -1.5488436
when eps = 0.0001000, answer = -1.5487886
when eps = 0.0000100, answer = -1.5487886
when eps = 0.0000010, answer = -1.5487886
when eps = 0.0000001, answer = -1.5487884
```