Week07 Project Report

Problem 1

1.Closed form greeks for GBSM, finite difference derivative calculation

I added `get_greeks()` function in the Options class with these two methods, which is the European Option class. I calculated the central, forward and backward finite difference derivative. The difference on the denominator of each I used is `d=1e-8`.

	Call(GBSM)	Call(FD_Central)	Call(FD_Forward)	Call(FD_Backward)	Put(GBSM)	Put(FD_Central)	Put(FD_Forward)	Put(FD_Backward)
delta	0.534009	0.534008	0.534008	0.534008	-0.465512	-0.465513	-0.465511	-0.465515
gamma	0.040038	0.040038	0.040038	0.040038	0.040038	0.040038	0.040038	0.040038
theta	24.898522	24.898523	24.898523	24.898523	18.786997	18.786996	18.786996	18.786997
vega	19.710180	19.710178	19.710178	19.710177	19.710180	19.710180	19.710181	19.710178
rho	7.583586	7.583585	7.583585	7.583583	-7.277011	-7.277011	-7.277011	-7.277012
carry_rho	7.966246	7.966244	7.966244	7.966244	-6.944416	-6.944416	-6.944414	-6.944418

Greeks using finite difference derivative calculation is very close to the closed form. The delta, rho, carry_rho for call options are positive, while those for put options are negative. The gamma, theta, vega for both call options and put options are positive.

The differences between the two methods indicates the shape of the curve of options. As shown in class, the curve of a call option value vs the underlying asset price is convex, so the forward difference will be larger than the central difference, while the backward difference will be smaller than the central difference.

2.American Option value using Binomial Tree

I created an `AmericanOption` class for American Options, including calculating value using Binomial Tree with or without dividend, calculating greeks using finite difference derivative calculation, etc.

The values and greeks using finite difference derivative calculation are as follows.

Without dividend:

```
American call option value by Binomial Tree(without dividend):  4.262316166817041
American put option value by Binomial Tree(without dividend):  3.7496665528148547
```

```
Finite difference derivative calculation greeks(without dividend):
          Call(FD_Central)  Put(FD_Central)
delta             0.534270       -0.470000
gamma            -1.332268        0.777156
theta            25.061181       19.457977
vega             19.860710       19.833979
rho               7.574652       -5.991478
carry_rho         7.960013       -5.745987
```

With dividend:

```
American call option value by Binomial Tree(with dividend):  4.088846497094475
American put option value by Binomial Tree(with dividend):  4.155019172108835
```

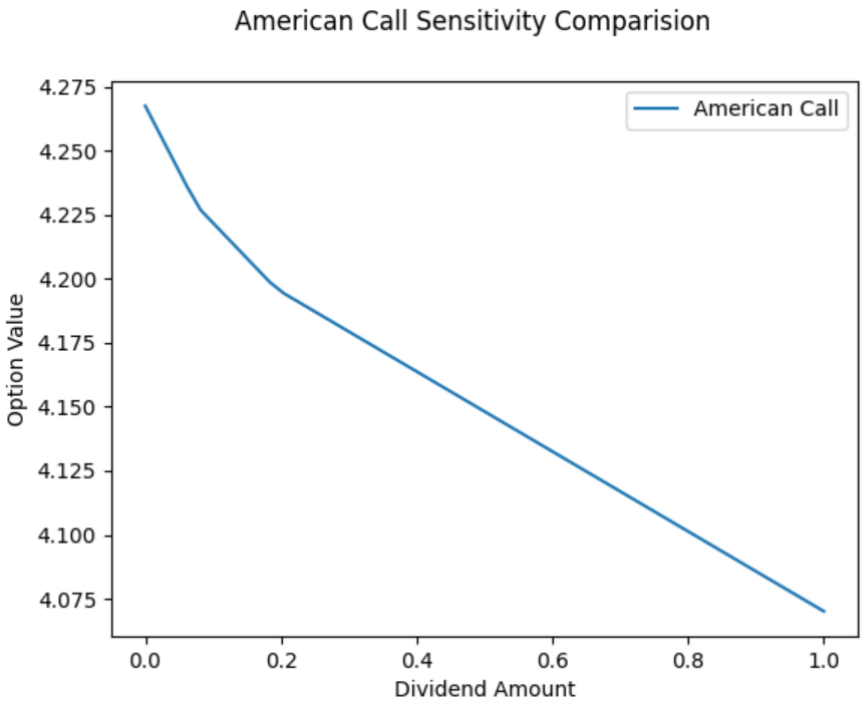
```
Finite difference derivative calculation greeks(with dividend):
           Call(FD_Central)  Put(FD_Central)
delta              0.542395        -0.464892
gamma             -0.666134        -2.442491
theta             24.720781        19.085037
vega              19.810893        20.011822
rho                6.841468        -7.310547
carry_rho          6.947536        -6.606006
```

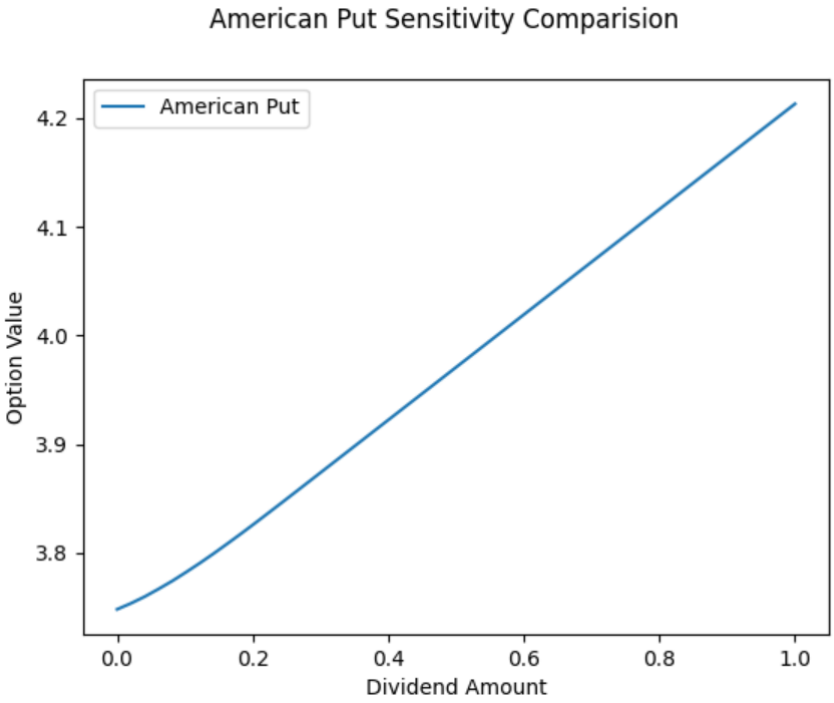
From the results, we can see that a dividend payment before the expiration of an American Option will decrease the value of a call option and increase the value of a put option.

For greeks, the difference between the two with or without dividend for delta, theta and vega is smaller, while for the other three is a little larger, but still close. Gamma for call with dividend becomes larger but for put becomes much smaller. Rho and carry_rho becomes evidently smaller when there is a dividend payment, because it is the option price change divided by the rate/b change.

3.Sensitivity of put and call to a change in dividend amount

I changed dividend from 0.0 to 1.0 to see the value changes.





From the plots, we find that the put option is more sensitive to dividend amount change, it ranges from 3.7 to 4.2, however, call option only changes from 4.275 to 4.0. There is a possible reason that the dividend payment date is near the expiration date, which causes the call option value changes little.

Problem 2

1.Calculate VaR, ES, Mean using fitted normal distribution to AAPL returns

I used the same method for this as last week, except from the fitted distribution. Here is the result.

	VaR	ES	Mean
Straddle	2.161964	2.258836	2.107277
PutSpread	3.010000	3.010000	0.340471
CallSpread	4.000800	4.476202	0.093655
Put	4.850000	4.850000	0.828355
Call	6.210800	6.686202	1.278923
ProtectedPut	7.718028	8.557650	1.354944
CoveredCall	12.257262	16.275849	-0.543960
Stock	16.307262	20.325849	0.701699
SynLong	16.638061	21.132051	0.450568

Here is the result from last week.

	VaR	ES	Mean
Straddle	0.138792	0.145280	2.599197
PutSpread	2.538242	2.711943	0.239211
CallSpread	3.546674	3.945526	0.169991
Put	4.200348	4.445126	0.982372
Call	5.606674	6.081431	1.616825
ProtectedPut	7.574266	8.343244	1.601983
CoveredCall	12.060293	15.698284	-0.925797
Stock	15.664741	19.506749	0.693467
SynLong	15.797392	19.690549	0.634453

The order of VaR for each portfolio does not change, and are to seome extent close to each other. But the Straddle Mean become smaller using normal distribution, and the VaR gets larger obviously. For other portfolios, we can find that the VaR and ES gets larger for American Option, because American Options are more risky than European Options.

2.Calculate VaR, ES using Delta-Neutral

I calculated the delta, present value for each asset and got the gradient for each portfolio first, then I calculated the volatility.

$$\frac{dR}{dr_i} = \frac{P_i}{PV} \sum_{j=1}^{m} h_j \delta_j$$

$$\sigma_p = \nabla R^T \Sigma \nabla R$$

I used the portfolio sigma to simulate 1000 return series -> 1000 simulated PnL and calculated VaR and ES for each portfolio.

	VaR	ES	Mean
PutSpread	3.305523	4.317438	0.019166
Straddle	5.242995	6.538221	0.038299
Put	6.004632	7.752169	0.144084
CallSpread	7.276322	9.315384	0.104917
CoveredCall	10.092460	12.741502	-0.026215
Call	11.730991	14.497881	0.148321
ProtectedPut	11.845912	14.516733	-0.159933
SynLong	16.705461	21.425014	-0.142388
Stock	16.940033	21.235865	0.203260

The results are close to the above two, but the order changes a little.

Delta-Neutral method assumes that the returns are normal distributed, and there is a linear assumption of portfolio value with returns. However, the delta for American Options delta varies nonlinearly, it might cause issues.