Week06 Project Report

Problem 1

Problem

Assume you a call and a put option with the following.

Current Stock Price $165
Current Date 03/03/2023
Options Expiration Date 03/17/2023
Risk Free Rate of 4.25%
Continuously Compounding Coupon of 0.53%

Calculate the time to maturity using calendar days (not trading days).

For a range of implied volatilities between 10% and 80%, plot the value of the call and the put.

Discuss these graphs. How does the supply and demand affect the implied volatility?

Answer

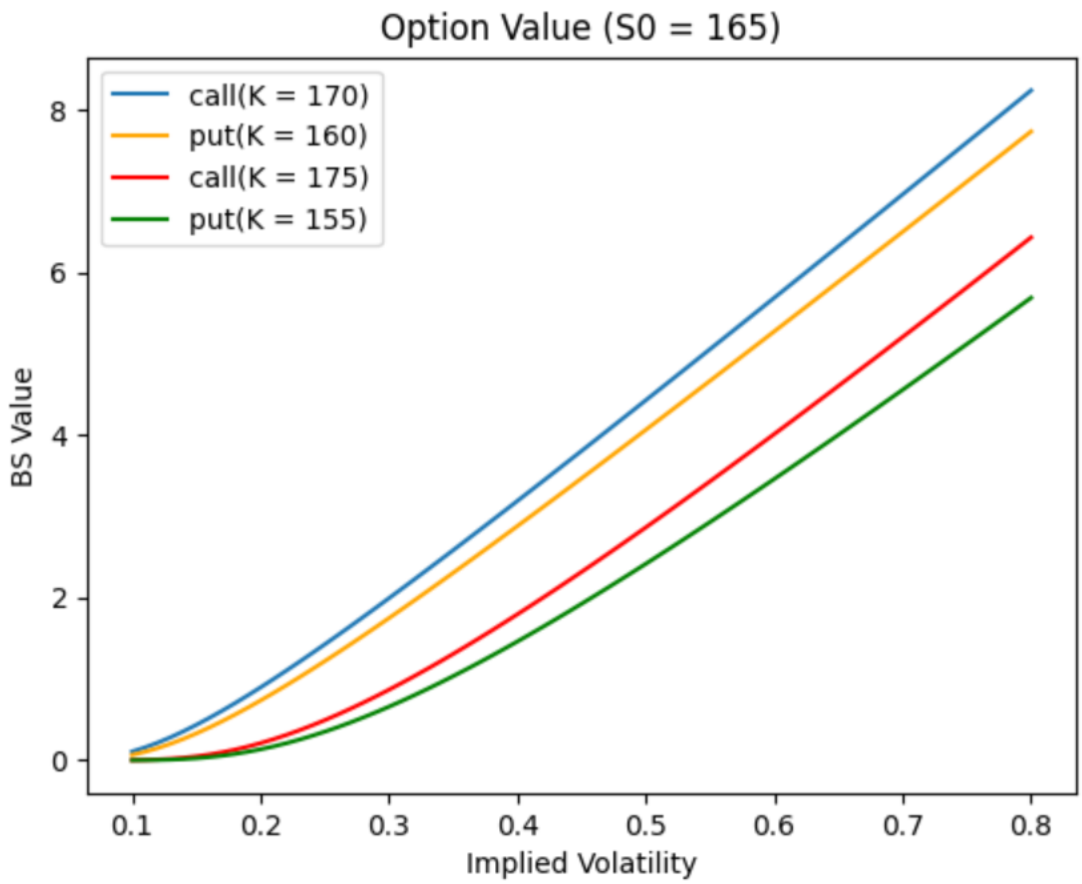
1.Calculate the time to maturity using calendar days

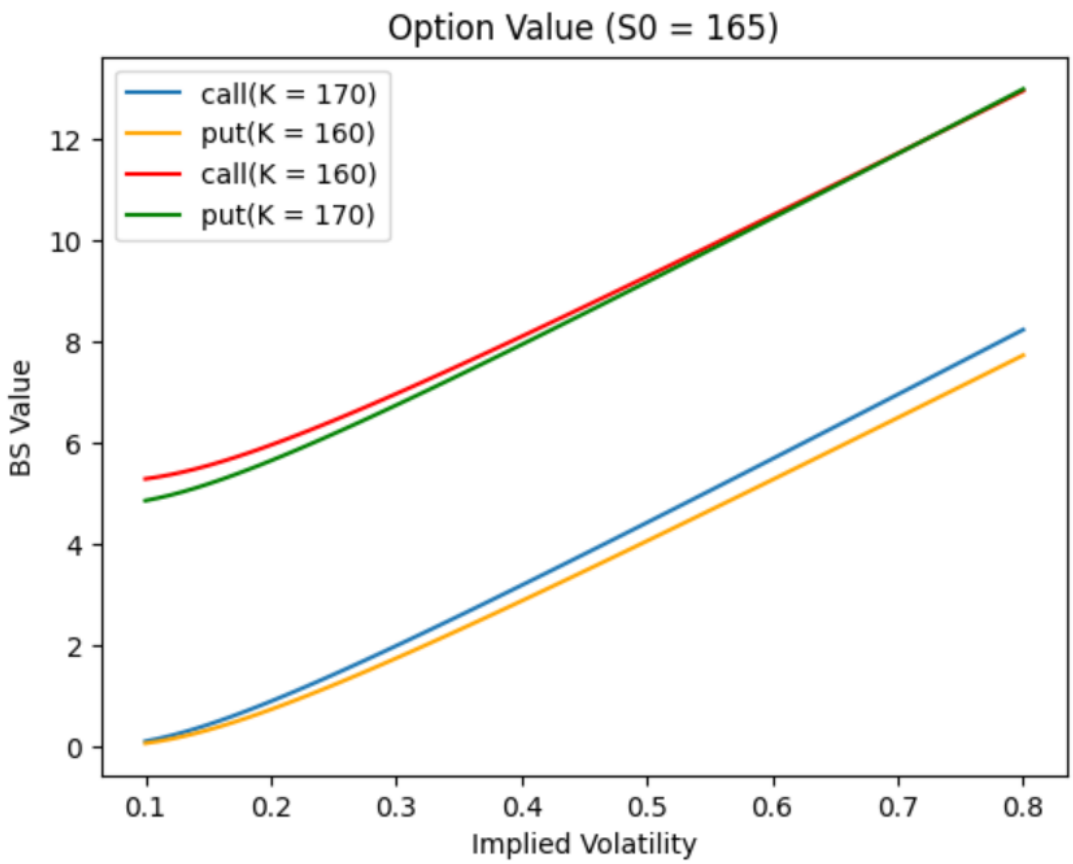
I built an `Option` class with properties of `type`, `exp_date`, `K`, `S0`, `r_benefit` and `r_cost` to note the properties of an option. The functions in this class are `get_T()`, calculating the time to maturity, `get_value_BS()`, calculating the option value using GBSM Model, and `get_iv()`, calculating the implied volatility using the real market price as the option price.

Time to maturity is **0.038 years** for problem1.

2.Plot call and put value for implied volatility in (0.1, 0.8)

To calculate the values for call and put options, I used implied volatility in `range(0.1, 0.81, 0.01)`, to put into the GBSM formula, and used `get_value_BS()` to get the values. I assumed two groups of strike prices for the options, one is `K_call`=170, `K_put`=160; the other is `K_call1`=175, `K_put1`=155, `K_call1`=160, `K_put1`=170.





From the graph, we can see that the option values of call and put options both increase as the implied volatility goes up, and the trend is almost linear if the strike price is close to the underlying price. Options always have value. This means that if the volatility of the underlying instrument gets larger, the values of the options gets higher.

And when the strike price are farther from the underlying price(with call option strike higher than underlying price and put option strike low than underlying price), when the implied volatility is the same, the values of call and put options are lower. This means that if the strike prices get too far from the underlying price, the option is valueless.

In addition, when the implied volatility is the same, the strike price is the same distance from the underlying price, the value of call option is always a little bit higher than the put option.

3.How does the supply and demand affect the implied volatility?

We can get the implied volatility through calculation with the real market option prices. When the demand is larger than supply, the prices of options will get higher, which makes the implied volatility higher. However, the demand is smaller than supply, the prices of options will get lower and implied volatility gets lower.

Problem 2

Problem

Use the options found in AAPL_Options.csv.

Current AAPL price is 151.03
Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

Calculate the implied volatility for each option.

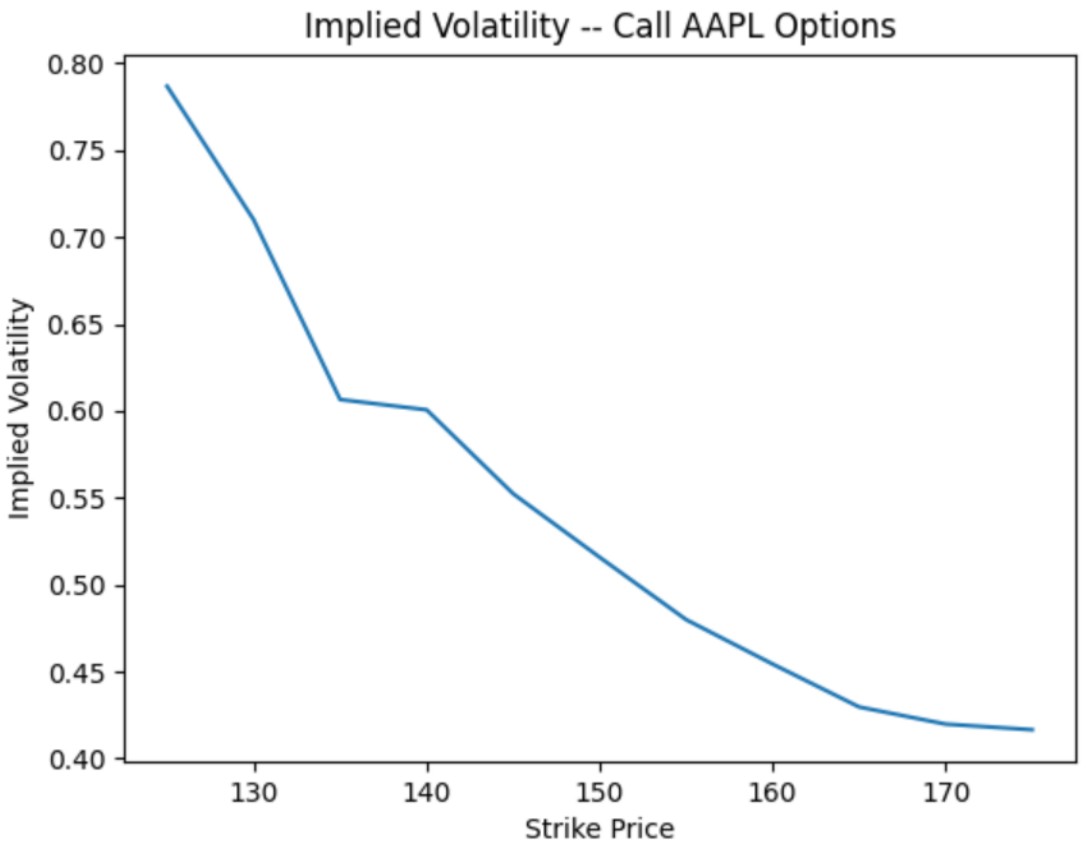
Plot the implied volatility vs the strike price for Puts and Calls. Discuss the shape of these graphs. What market dynamics could make these graphs?

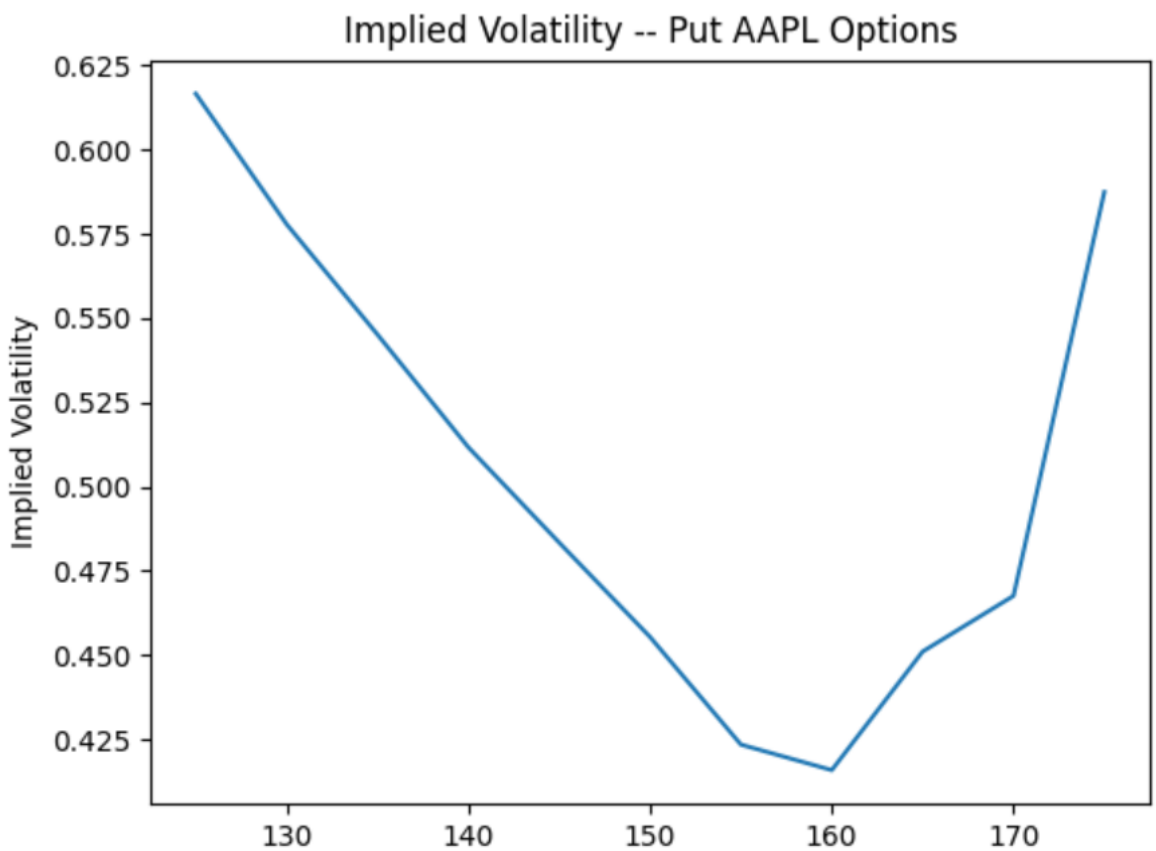
There are bonus points available on this question based on your discussion. Take some time to research if needed.

Answer

1.Calculate implied volatility and plot implied volatility vs strike price for calls and puts

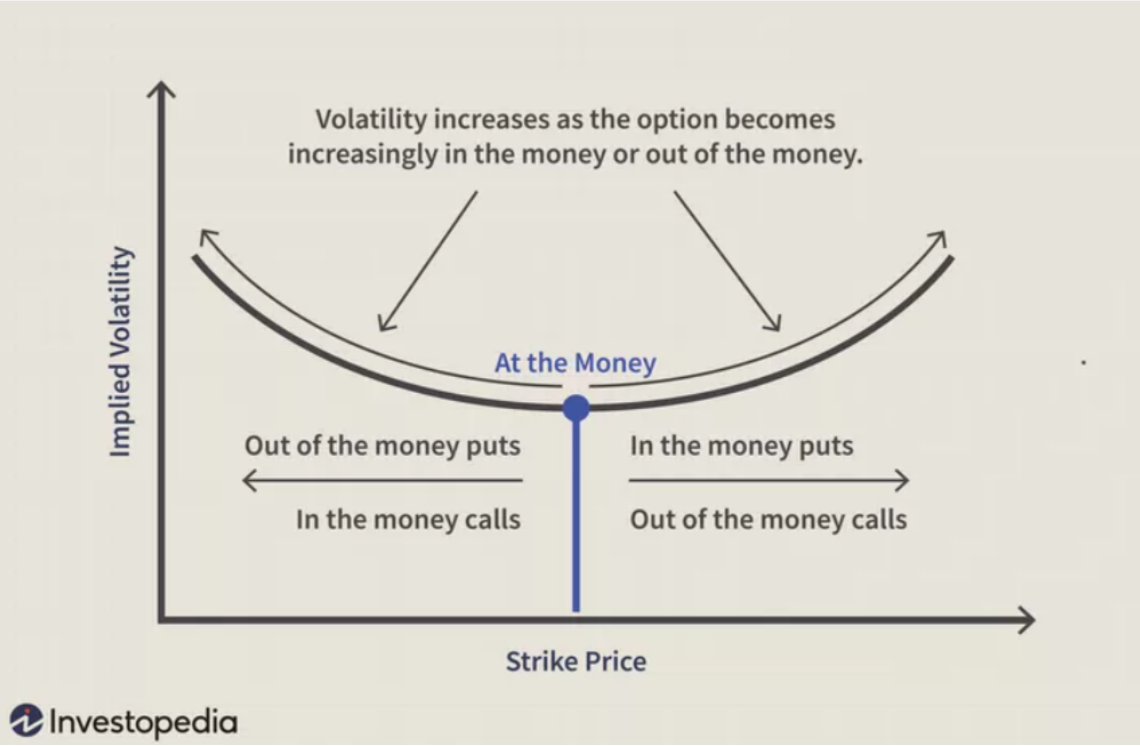
I used the function `get_iv()` to calculate the implied volatility for each option objects created by the given data.


Implied Volatility -- Call AAPL Options
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
Implied Volatility
130
140
150
160
170
Strike Price


Implied Volatility -- Put AAPL Options
0.625
0.600
0.575
0.550
0.525
0.500
0.475
0.450
0.425
Implied Volatility
130
140
150
160
170
Strike Price

2.Discuss the shape of these graphs. What market dynamics could make these graphs?

There should be a symmetric "volatility smile" for call and put options. As the strike price gets farther from the underlying price, the implied volatility should be larger. Here is a graph from Investopedia.



Since we use the real market price of the options and the same underlying price of the AAPL stock(current price) to calculate the implied volatility, the implied volatility should be higher for out-of-the-money and in-the-money options, and lowest for at-the-money options. The strike price getting far from current price, means higher uncertainty of underlying asset, traders may want to "overpay" for those options, which leads to higher implied volatility of the options.

The underlying price is `151.03` in this problem, so plots show that the put option implied volatility is more aligned with the "volatility smile", with negative skewness. However, the volatility smile of the call options is only the "left half", which is not symmetric. The negative skewness is not uncommon in real market. It's called volatility "smirk". Traders will usually protect more against downside risk instead of upside one, and extreme moves in the downside is more common than in the upside. In this way, the implied volatility will be higher for downside strikes than the upside strikes. In the call option graph, traders want to pay extremely high for downside strike calls, showing that they are very uncertain about the stock market, especially in such a bad economy trend.

Problem 3

Problem

Use the portfolios found in problem3.csv

Current AAPL price is 151.03
Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

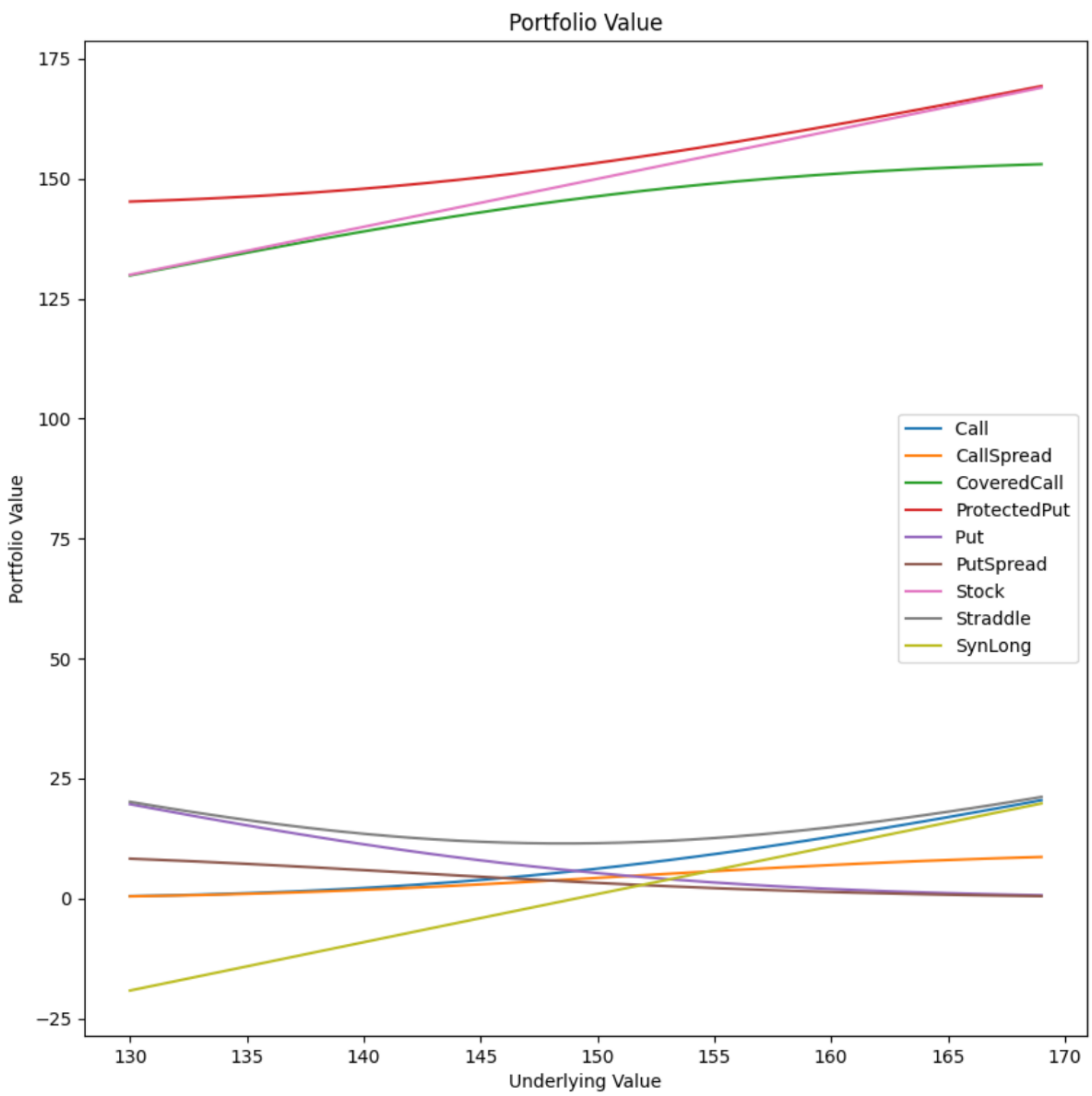
For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.

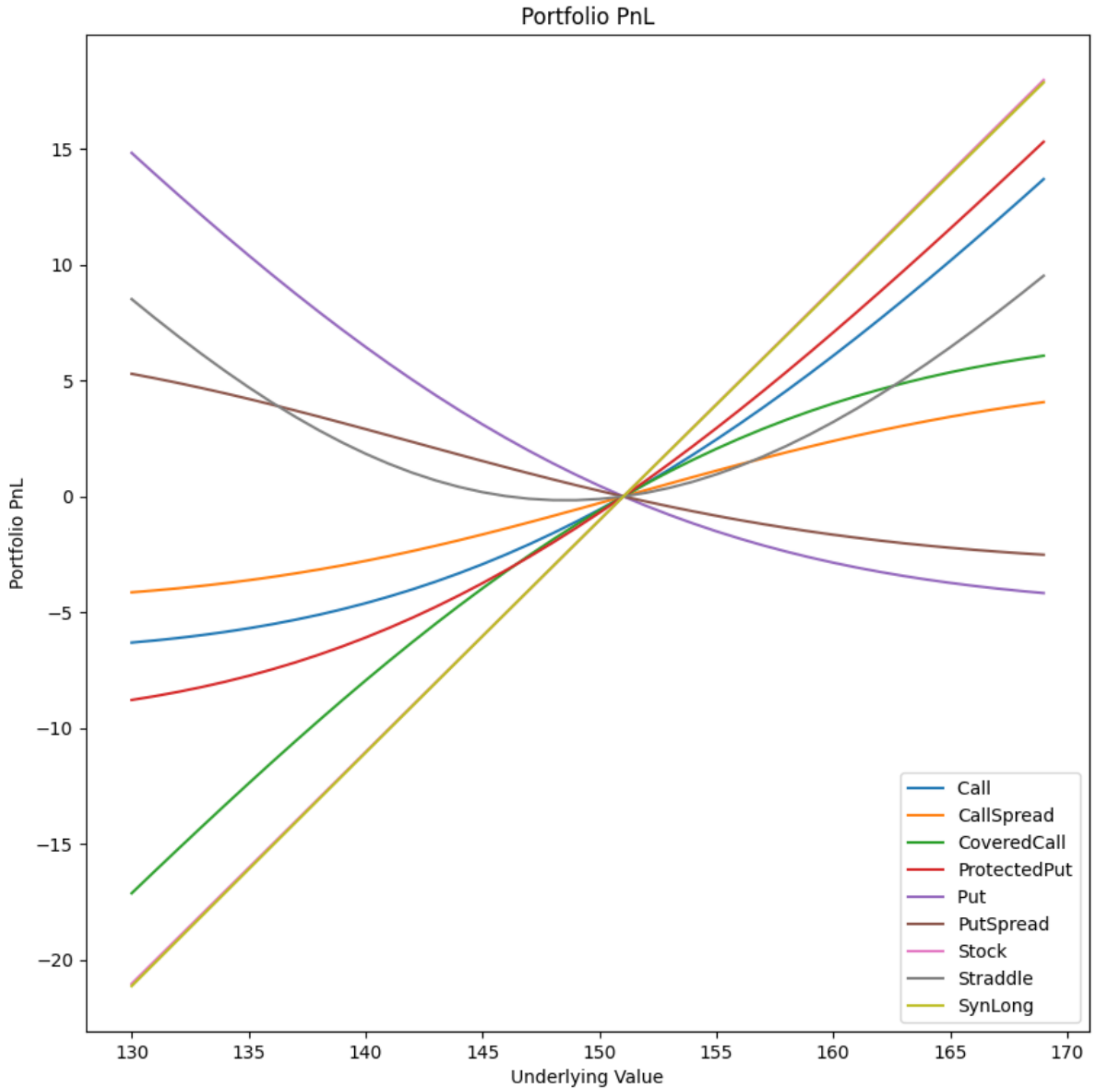
Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

Answer

1.Graph portfolio value vs underlying value and discuss shape

I used underlying value in (130, 170) to graph each portfolio value. For option, I used current market option price to calculate the implied volatility first, then added a function `reset_underlying_value()` in `Option` class to change underlying values of an Option object and used `get_value_BS()` to calculate the option value with the assumed underlying price. Portfolio values are total values of the assets in the portfolio, and portfolio PnLs is the total ones, too. Here are the graphs of portfolio values and PnLs.





The ProtectedPut and the Call behave similar, with same trend of value and similar PnL, which roughly proved the no-arbitrage theory. ProtectedPut is one Put & one stock. This shows that a long position on the underlying asset can a long position on a put can replicate a long position on a call because the loss of underlying asset in downside market can be hedged by the long position of put, and gain in upside market.

The SynLong and the stock behave similar, with same trend of value and same PnL line. Synlong is a long position on call and a short position on put, which can replicate a long position of stock. When underlying price goes up, SynLong will gain because of the long position of call; while underlying price goes down, SynLong will lose money because of the short position of put.

The CallSpread and PutSpread behave symmetric. CallSpread is one long position on call with lower strike and one short position on call with higher strike; while PutSpread is one long position on put with lower strike and one short position on put with higher strike. These two can hedge each other, getting a total of 0 PnL and a total positive value.

The CoveredCall and Put behave symmetric, which can hedge each other. CoveredCall is one long position of stock and one short position of call, which behaves like a put opotion.

The straddle has non negative PnL. It gains when the price moves largely, paying option premiums when the price moves little.

2.Fit AR(1) model to AAPL returns and simulate 10 days ahead

I simulated `n=1000` times of 10-day returns ahead, and got 1000 prices with `t=10` days forward. I added the `dayForward` property in the Option class to get the new time to maturity for options 10-day forward. I used the 1000 prices on the simulated 10th day to calculate portfolio values and PnLs, then calculated VaR, ES and Mean.

	VaR	ES	Mean
Straddle	0.138792	0.145280	2.599197
PutSpread	2.538242	2.711943	0.239211
CallSpread	3.546674	3.945526	0.169991
Put	4.200348	4.445126	0.982372
Call	5.606674	6.081431	1.616825
ProtectedPut	7.574266	8.343244	1.601983
CoveredCall	12.060293	15.698284	-0.925797
Stock	15.664741	19.506749	0.693467
SynLong	15.797392	19.690549	0.634453

Compared the VaR and ES with the PnL above, the portfolios with larger change of PnL have larger downside risk, which have larger VaR and ES.

The Straddle has the smallest VaR and ES(almost 0) and highest mean, because it has almost non-negative PnL.

CallSpread and PutSpread have similar PnL, though symmetric, not largely change when underlying price changes. They have relatively small VaR and ES, but the Mean is relatively small.

Put, Call, ProtectedPut and CoveredCall have medium downside risk. ProtectedPut and Call behave similarly, so the Mean are close to each other, but ProtectedPut has more risk; CoveredCall and Put can hedge each other, so the sum of Mean is almost 0.

The Stock and Synlong has the largest PnL change, so they have the largest VaR and ES.