Master of Science in Quantitative Finance

Fixed Income Securities

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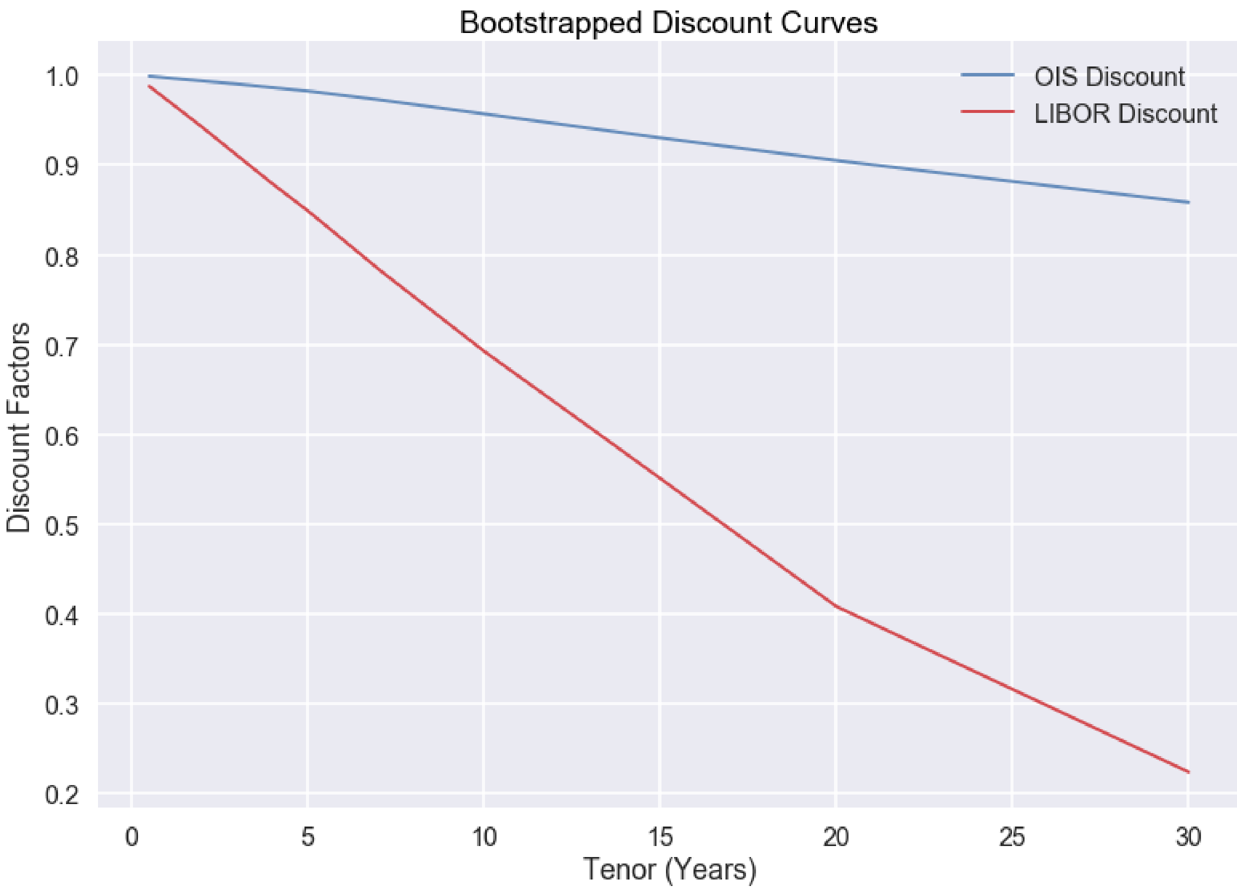
Contents

1 Bootstrapping Swap Curves

- Bootstrap the OIS discount factor $D_o(0, T)$ and plot the discount curve for T $\in$ [0,30].
- Using the IRS data provided, bootstrap the LIBOR discount factor $D_L(0, T)$, and plot it for T $\in$ [0,30].



- Calculate the following forward swap rates:
$1y \times 1y, 1y \times 2y, 1y \times 3y, 1y \times 5y, 1y \times 10y$
$5y \times 1y, 5y \times 2y, 5y \times 3y, 5y \times 5y, 5y \times 10y$
$10y \times 1y, 10y \times 2y, 10y \times 3y, 10y \times 5y, 10y \times 10y$

Expiry\Tenor	1Y	2Y	3Y	5Y	10Y
1Y	0.032007	0.033259	0.034011	0.035255	0.038427
5Y	0.039273	0.040074	0.040070	0.041091	0.043629
10Y	0.042179	0.043105	0.044087	0.046238	0.053434

2 Swaption Calibration

- Calibrate the displaced-diffusion model to the swaption market data, and document a table of σ and β parameters:

Sigma, σ:

Expiry/Tenor	1Y	2Y	3Y	5Y	10Y
1Y	0.366189	0.387268	0.373413	0.308173	0.269429
5Y	0.328405	0.327197	0.319293	0.275627	0.248905
10Y	0.307825	0.305861	0.300481	0.270069	0.244694

Beta, β:

Expiry/Tenor	1Y	2Y	3Y	5Y	10Y
1Y	0.000015	0.000003	0.000006	0.002294	0.097164
5Y	0.000005	0.000158	0.001068	0.027260	0.110762
10Y	0.000902	0.010456	0.021127	0.031041	0.059702

- Calibrate the SABR model to the swaption market data using $\beta = 0.9$ and document a table of α, ρ and ν parameters:

Alpha, α:

Expiry\Tenor	1Y	2Y	3Y	5Y	10Y
1Y	0.142920	0.182966	0.195559	0.174187	0.166891
5Y	0.163409	0.201079	0.210774	0.188443	0.173778
10Y	0.172502	0.197106	0.210649	0.211672	0.187952

Rho, ρ:

Expiry\Tenor	1Y	2Y	3Y	5Y	10Y
1Y	-0.618565	-0.526888	-0.481276	-0.399823	-0.240814
5Y	-0.561563	-0.556083	-0.552629	-0.502226	-0.419181
10Y	-0.525302	-0.549820	-0.559652	-0.591646	-0.550276

Nu, ν:

Expiry\Tenor	1Y	2Y	3Y	5Y	10Y
1Y	1.986141	1.695050	1.450874	1.103236	0.835533
5Y	1.300217	1.072126	0.939627	0.687831	0.527186
10Y	0.983562	0.931007	0.877855	0.739147	0.582939

- Price the following swaptions using the calibrated displaced-diffusion and SABR model:

  Displaced-diffusion:

  Payer $2y \times 10y$ 6% $\approx$ **0.6226%**

  Receiver $8y \times 10y$ 2% $\approx$ **3.2967%**

  SABR:

  Payer $2y \times 10y$ 6% $\approx$ **1.1555%**

  Receiver $8y \times 10y$ 2% $\approx$ **4.0042%**

3 Convexity Correction

- Using the SABR model calibrated in the previous question, value the PV of a leg receiving CMS10y semi-annually over the next 5 years:

  PV of leg receiving CMS10y semi-annually $\approx$ **0.2059**

- Value the PV of a leg receiving CMS2y quarterly over the next 10 years and SABR model:

  PV of leg receiving CMS2y quarterly $\approx$ **0.3800**

- Compare the forward swap rates with the CMS rate:
  $1y \times 1y, 1y \times 2y, 1y \times 3y, 1y \times 5y, 1y \times 10y$
  $5y \times 1y, 5y \times 2y, 5y \times 3y, 5y \times 5y, 5y \times 10y$
  $10y \times 1y, 10y \times 2y, 10y \times 3y, 10y \times 5y, 10y \times 10y$

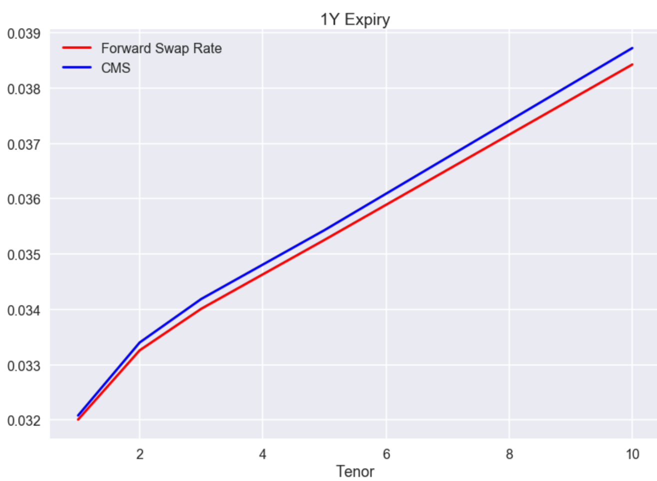
  Forward Swap Rates:

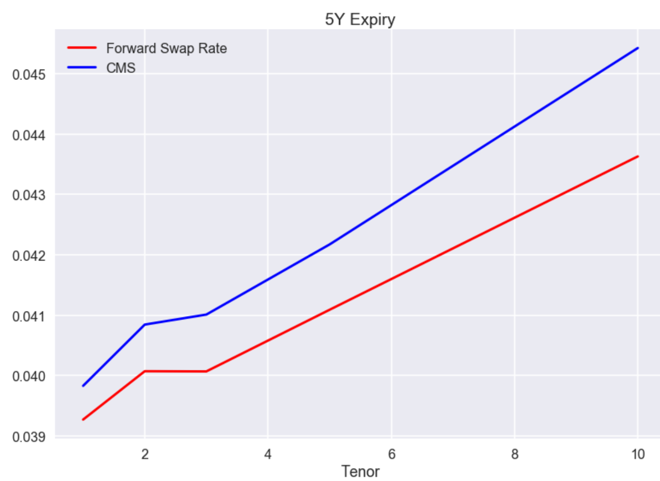
Expiry\Tenor	1Y	2Y	3Y	5Y	10Y
1Y	0.032007	0.033259	0.034011	0.035255	0.038427
5Y	0.039273	0.040074	0.040070	0.041091	0.043629
10Y	0.042179	0.043105	0.044087	0.046238	0.053434

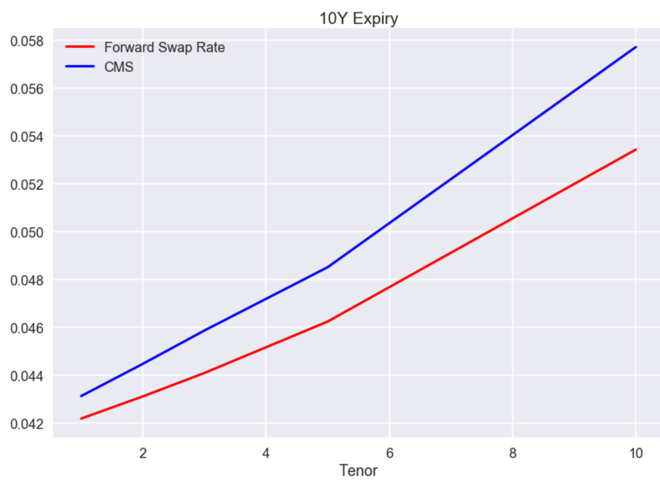
CMS Rates:

Expiry\Tenor	1Y	2Y	3Y	5Y	10Y
1Y	0.032080	0.033400	0.034187	0.035431	0.038725
5Y	0.039831	0.040845	0.041011	0.042173	0.045424
10Y	0.043120	0.044469	0.045863	0.048512	0.057724

- Discuss the effect of maturity and tenor on convexity correction (difference between forward swap rates and CMS rates).







From our observations, the longer the maturity and tenor, the greater the difference between forward swap rates and CMS rates. This can be explained by a model-independent convexity correction applied to the CMS rates when static replication was used. Convexity correction is attributed to the intrinsic dynamics of the swap rate and that payment was made at the "wrong" time. Essentially, it is the difference between the forward swap rate and the CMS rate.

Across the rates, we observe a trend of the convexity correction widening from the 1-year tenor to the 10-year tenor. When tenor is increased from 1-year to 10-year, the convexity correction increases by 2.25 bp, 12.37 bp and 33.49 bp for the 1-year, 5-year and 10-year expiry respectively (rate of 10y-tenor minus rate of 1-year tenor). It goes to show that with increasing tenor, convexity correction increases.

For rates at the 1-year expiry, the maximum convexity correction stands at 2.98 bp (10-year tenor). In contrast, we observe a maximum convexity correction of 17.95 bp and 42.9 bp for rates with 5-year expiry and 10-year expiry respectively. Evidently, the convexity correction increases significantly with increases in the maturity/expiry (i.e. 1y, 5y, 10y expiry) of the rates.

We will attempt to dive deeper in explaining how tenor and maturity affects convexity correction. Since the CMS rate is given by:

$$\mathbb{E}^T[S_{n,N}(T)] = g(F) + \frac{1}{D(0,T)}\left[\int_0^F h''(K)V^{rec}(K)dK + \int_F^\infty h''(K)V^{pay}(K)dK\right]$$

and V^{rec} and V^{pay} are given by:

$$V(K) = D(0,T) \cdot IRR(S_{n,N}(0)) \cdot Black76(S_{n,N}(0), K, \sigma^{SABR}, T)$$

in the presence of a market smile, the correction is necessarily more involved. This has been accomplished by modelling implied volatilities with the SABR functional form, giving us the following formula for CMS convexity correction:

$$CC^{SABR} = S_{n,N}(0)\theta(\delta) \cdot \left(\frac{2}{S^2_{n,N}(0)}\int_0^\infty Black76(S_{n,N}(0), K, \sigma^{SABR}, T)dK - 1\right)$$

where $\theta(\delta)$ is given by:

$$\theta(\delta) = 1 - \frac{\Delta S_{n,N}(0)}{1+\Delta S_{n,N}(0)}\left(\delta + \frac{N-n}{(1+S_{n,N}(0))^{N-n}-1}\right)$$

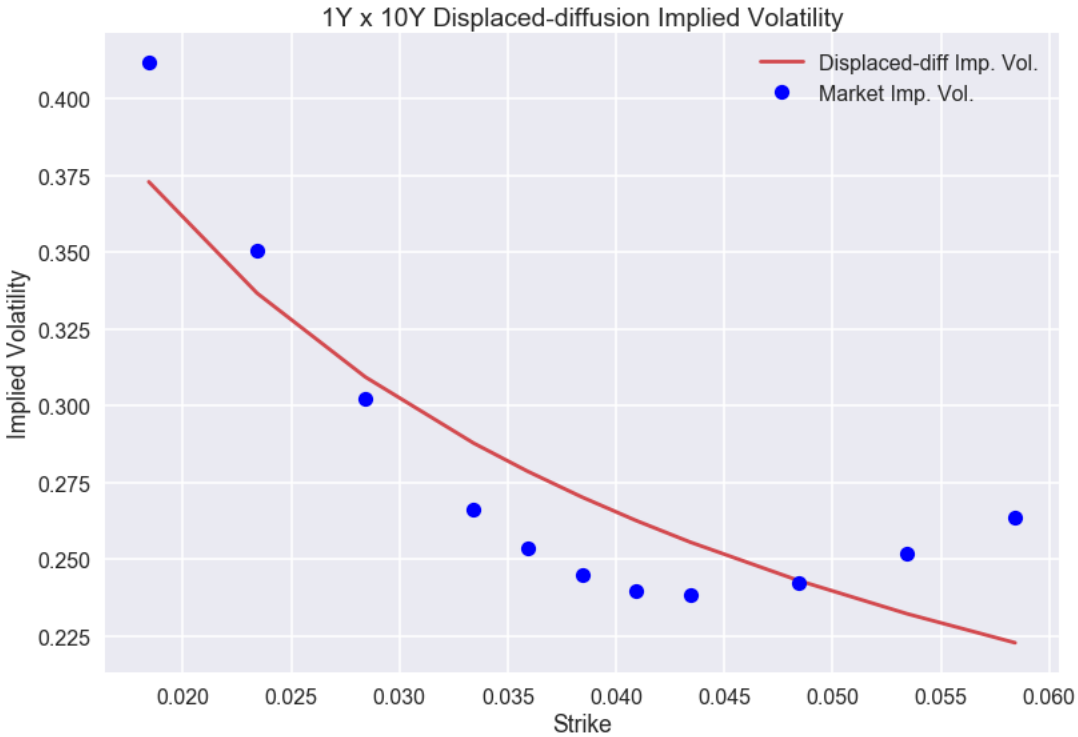
and δ is the accrual period of the swap rate.

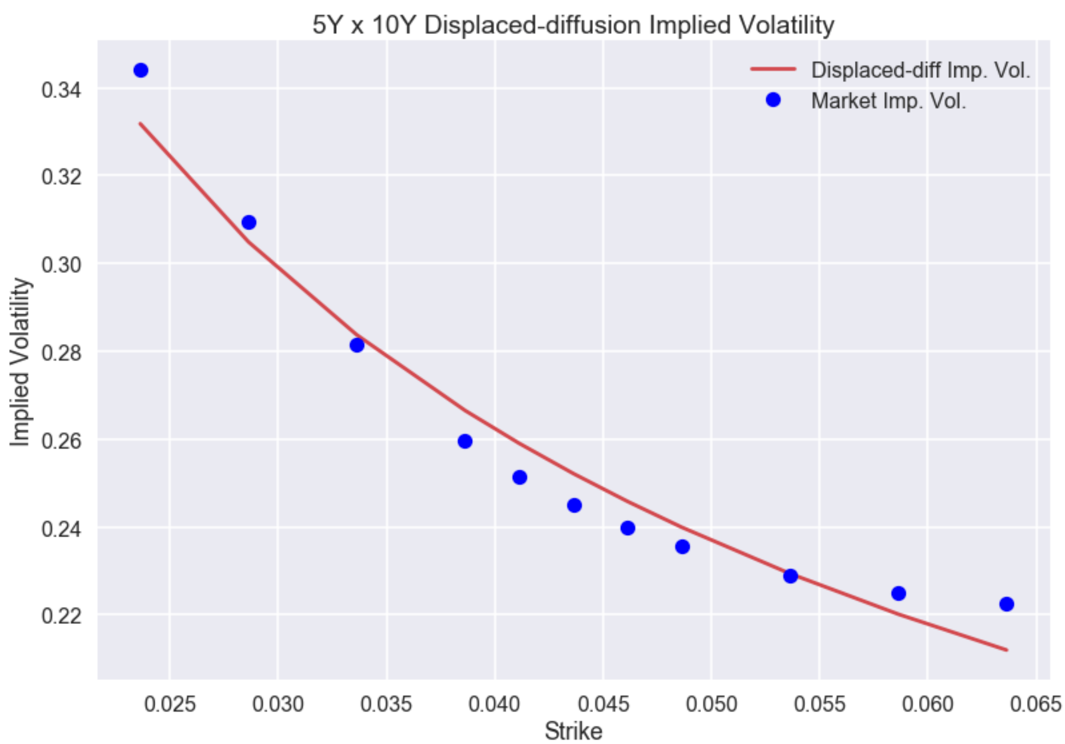
From the convexity correction formula, we can see that the expiry T of the swaption is a parameter of the Black76 within the integral. Tenor and expiry parameters are also utilized in the calculation of $S_{n,N}(0)$. An increase in these parameters results in an increase in the convexity correction of the above. Since more weight is given to expiry, T, within Black76 of the entire convexity correction formula, any increase to the expiry will result in a significant increase in the magnitude of the convexity correction.

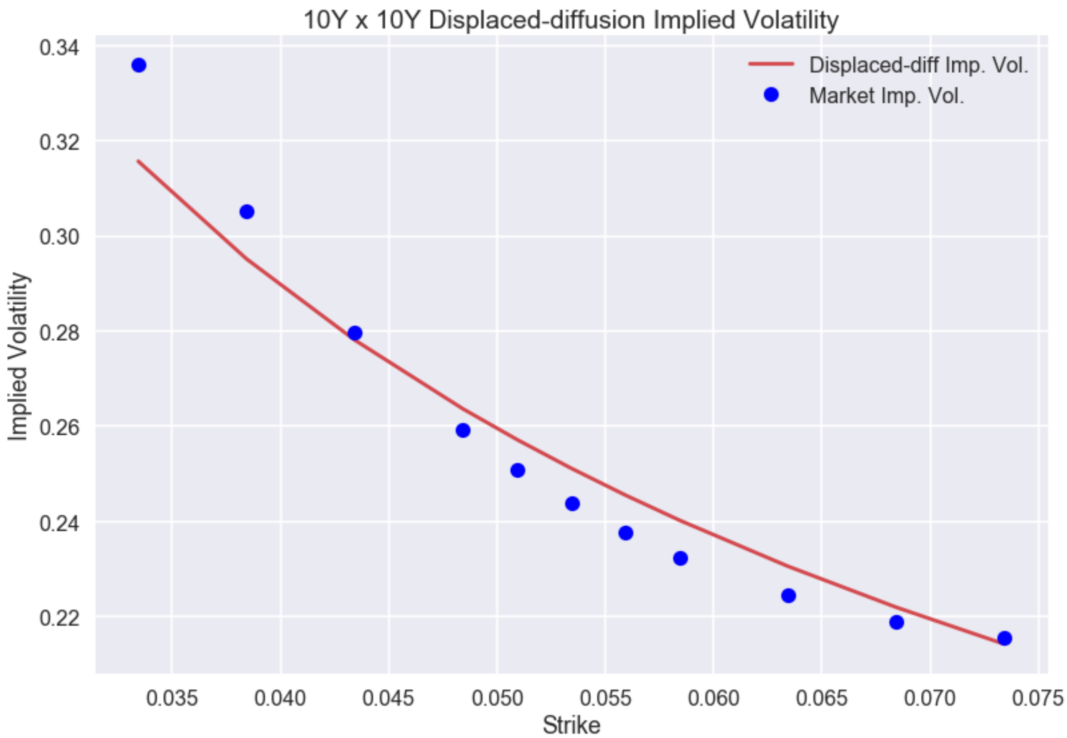
As such, with our plots and the above discussion, we can conclude that the higher the maturity and tenor of the CMS underlying swap, the higher the convexity adjustment.

4 Appendix

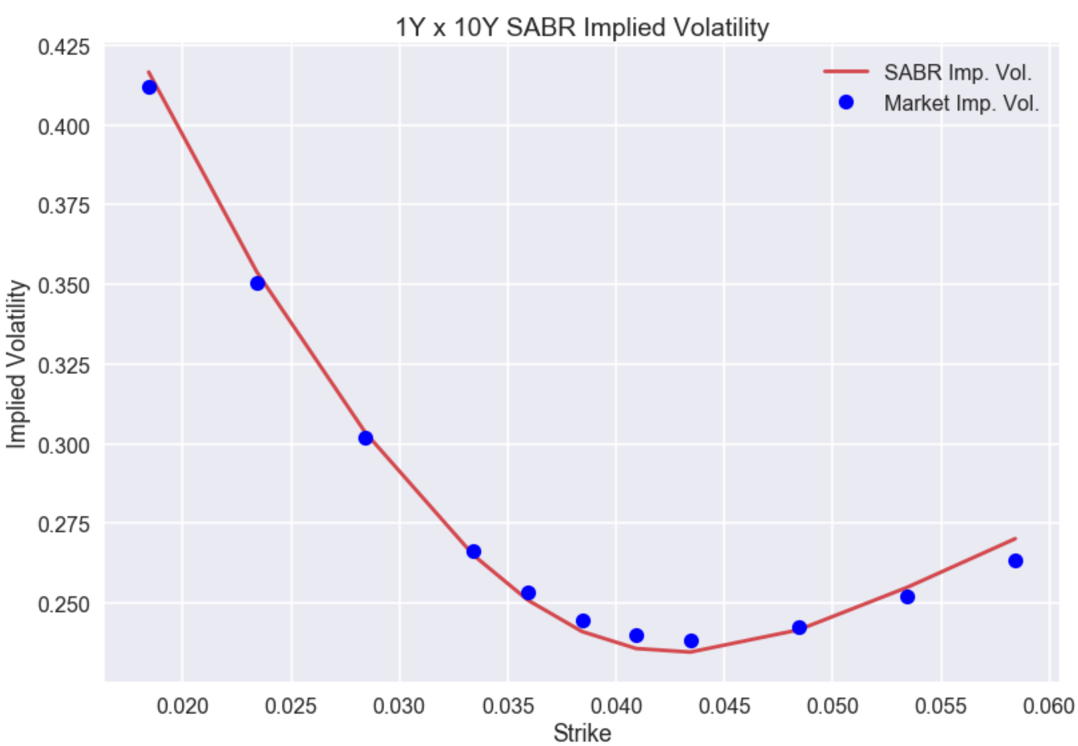
1. Displaced-diffusion Implied Volatility versus Market Implied Volatility ($1y \times 10y$, $5y \times 10y$, $10y \times 10y$)

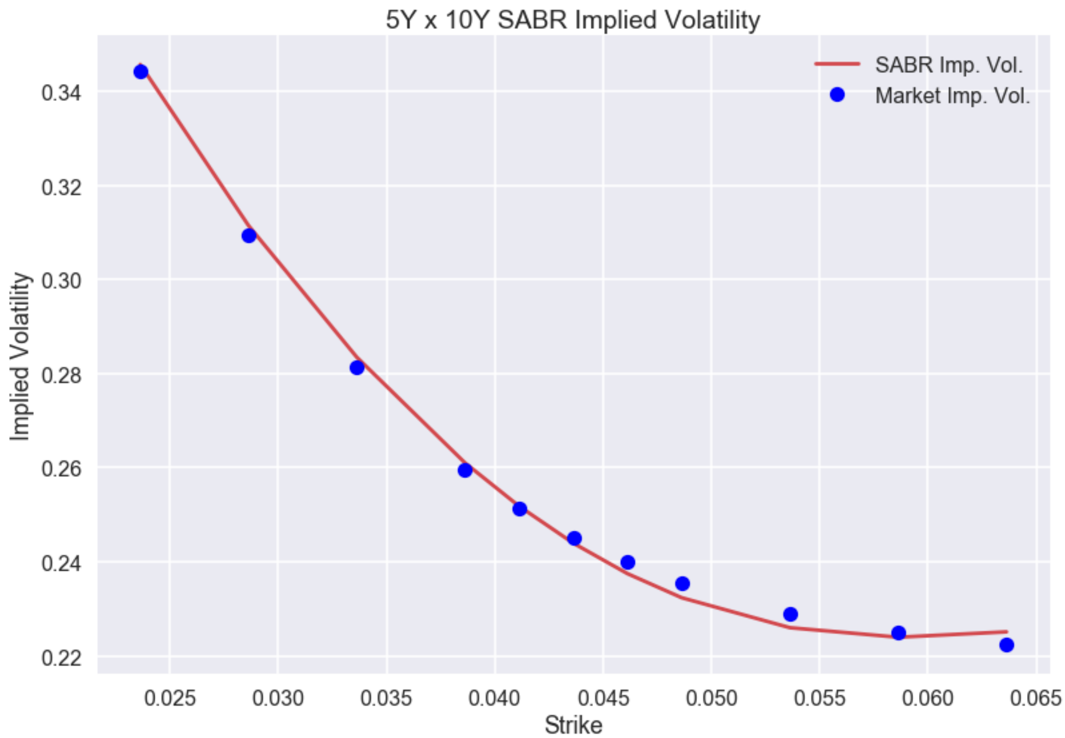


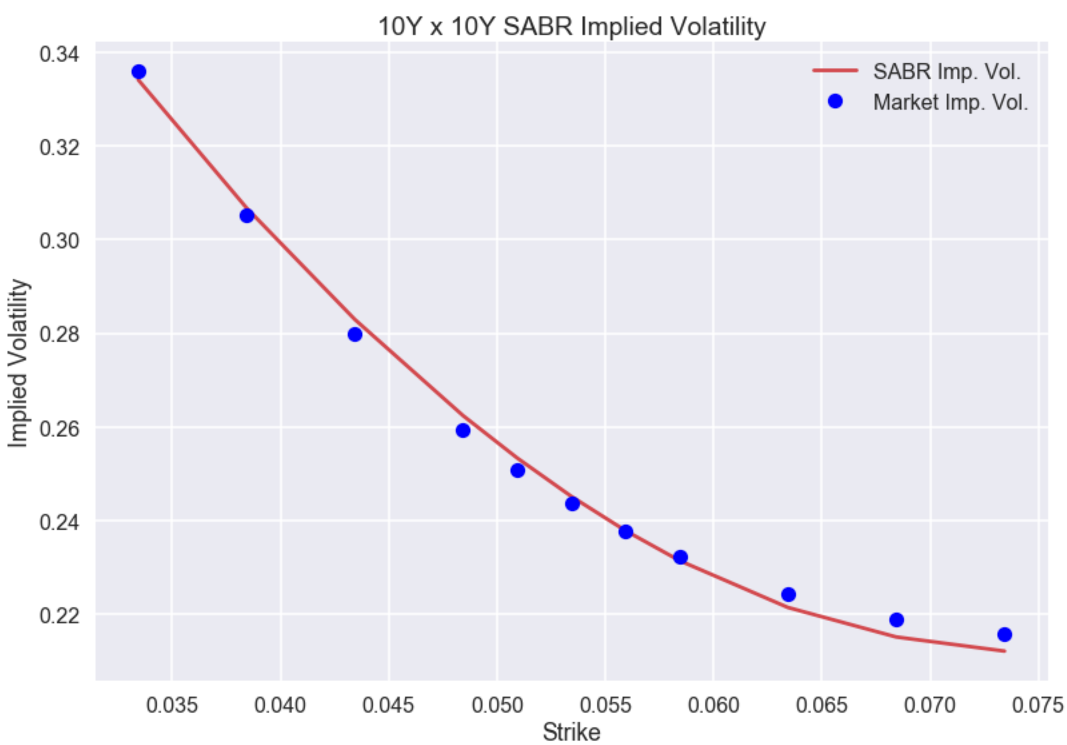




2. SABR Implied Volatility versus Market Implied Volatility ($1y \times 10y$, $5y \times 10y$, $10y \times 10y$)







References

[1] Patrick S. Hagan. *Convexity Conundrums: Pricing CMS Swaps, Caps, and Floors* . 7 July 2015

[2] Dr. Tee Chyng Wen. "Singapore Management University." *Fixed Income Securities*. AY2017/2018

[3] Fabio Mercurio. *Swaption smile and CMS Adjustment.* Derivatives and Risk Management Europe, Monte Carlo, 7 June 2006