**Euler's formula**

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
$$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

## Chapter 1

### 1.2.2 Bounded signals

$x(t)$ is bounded if
$\exists M\ [(0 < M < \infty) \wedge (\forall t\ |x(t)| \leq M)]$
(got upper n lower range limit)

### 1.2.3 Absolutely integrable signals

$x(t)$ is absolutely integrable if
$$\int_{-\infty}^{\infty} |x(t)|dt < \infty$$

### 1.2.6 Energy and Power Signals

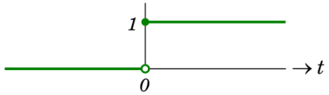
**Energy signals**
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (1.3a)$$
$$x(t) \text{ is an energy signal} \iff 0 < E < \infty \quad (1.3b)$$

**Power signals**
$$P = \lim_{\tau\to\infty} \frac{1}{2\tau}\int_{-\tau}^{\tau} |x(t)|^2 dt \quad (1.4a)$$
$$x(t) \text{ is a power signal} \iff 0 < P < \infty \quad (1.4b)$$

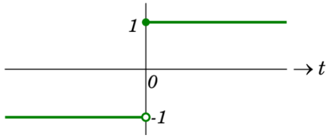
If $x(t)$ is a periodic signal, average power may be computed by $\frac{1}{T}\int_0^T |x(t)|^2 dt$

- Energy signals have 0 average power, bc E = finite implies P = 0
- Power signals have infinite total energy, bc P = finite implies $E = \infty$
- All bounded periodic signals are power signals

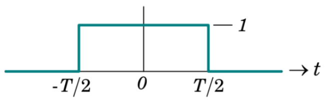
**u(t):**



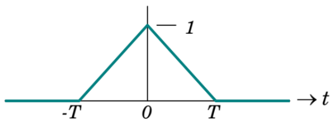

**sgn(t):**



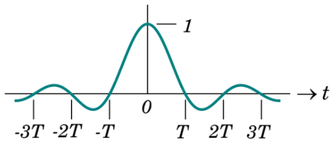

**rect**$\left(\frac{t}{T}\right)$**:**



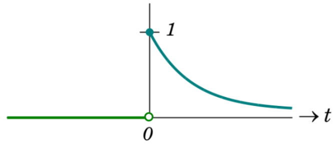

**tri**$\left(\frac{t}{T}\right)$**:**



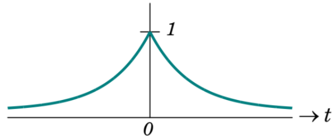

**sinc**$\left(\frac{t}{T}\right)$**:**




$e^{-\alpha t}u(t)$:




$e^{-\alpha|t|}$:




## Chapter 2

### 2.1 Time-domain Operations

### 2.1.5 Convolution of 2 Signals

$x(t) * y(t) = \int_{-\infty}^{\infty} x(\alpha)y(t-\alpha)\, d\alpha$

**Properties of Dirac-$\delta$:**

1. Symmetry: $\delta(t) = \delta(-t)$ (2.3)
2. Sampling:
$$x(t)\delta(t-\lambda) = x(\lambda)\delta(t-\lambda) \quad (2.4)$$
3. Sifting
$$\int_{-\infty}^{\infty} x(t)\delta(t-\lambda)dt = x(\lambda) \quad (2.5)$$
4. Replication
$$x(t) * \delta(t-\lambda) = x(t-\lambda) \quad (2.6)$$

**Convolution with Dirac-$\delta$ Comb function**
$$x_p(t) = x(t) * \sum_n \delta(t-nT)$$
$$= \sum_n x(t-nT)$$

**Multiplication with the Dirac-$\delta$ Comb function**
Used for sampling
$$x_s(t) = x(t) \times \sum_n \delta(t-nT)$$
$$= \sum_n x(t) \times \delta(t-nT)$$
$$= \sum_n x(nT)\delta(t-nT)$$

## Chapter 3

### 3.2 Spectrum of a Sinusoid

**Spectrum of a Complex Exponential Signal**
$$\tilde{x}(t) = \mu e^{j(2\pi f_0 t+\phi)} = \mu e^{j\phi} \times e^{j2\pi f_0 t},$$

**Spectrum of a Cosine Signal**
$$\mu\cos(2\pi f_0 t + \phi)$$
$$= \frac{\mu}{2}e^{j\phi}e^{j2\pi f_0 t} + \frac{\mu}{2}e^{j(-\phi)}e^{j2\pi(-f_0)t}$$

**Spectrum of a Sine Signal**
$$\mu\sin(2\pi f_0 t + \phi) = \frac{\mu}{2}e^{j(\phi-0.5\pi)}e^{j2\pi f_0 t} + \frac{\mu}{2}e^{j(-\phi+0.5\pi)}e^{j2\pi(-f_0)t}$$

**Complex exponential Fourier Series**
$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T_p}$$
$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_p t} \quad (3.1a)$$
$$c_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t)e^{-j2\pi kt/T_p}dt,\ k \in \mathbb{Z} \quad (3.1b)$$

**Trigonometric Fourier Series**
$$x_p(t) = a_0 + 2\sum_{k=1}^{\infty}[a_k\cos(2\pi kt/T_p) + b_k\sin(2\pi kt/T_p)]$$

$$a_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t)\cos(2\pi kt/T_p)dt;\ k \geq 0$$
$$b_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t)\sin(2\pi kt/T_p)dt;\ k > 0 \quad (3.2)$$

## Chapter 4

### 4.1 Fourier Transform

**Forward Fourier Transform**
$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \quad (4.1a)$$

**Inverse Fourier Transform**
$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \quad (4.1b)$$

**Spectrum of exponentially decaying pulse**
$$x(t) = Ae^{-\alpha t}u(t)$$
Assume $\alpha > 0$
$$X(f) = \frac{A}{\alpha + j2\pi f}$$

### 4.3 Spectral properties of a REAL signal

- If $x(t)$ is **REAL** ($x^*(t) = x(t)$), then
  - $X(f)$ is conjugate symmetric ($X^*(f) = X(f)$)
  - $|X(f)|$ is even ($|X(f)| = |X(-f)|$)
  - $\angle X(f)$ is odd ($\angle X(f) = -\angle X(-f)$)
- If $x(t)$ is **REAL** and **EVEN** ($x^*(t) = x(t) \wedge x(-t) = x(t)$), then
  - $X(f)$ is real ($X^* f = X(f)$)
  - $X(f)$ is even ($X(-f) = X(f)$)
- If $x(t)$ is **REAL** and **ODD** ($x^*(t) = x(t) \wedge x(-t) = -x(t)$), then
  - $X(f)$ is imaginary ($X^*(f) = -X(f)$)
  - $X(f)$ is odd ($X(-f) = -X(f)$)

The above can apply to Fourier series coefficients of periodic signals too:

- $x_p(t)$ is **REAL**
  - $c_k$ is conjugate symmetric ($c_k^* = c_{-k}$)
  - $|c_k|$ has even symmetry ($|c_k| = |c_{-k}|$)
  - $\angle c_k$ has odd symmetry ($\angle c_k = -\angle c_{-k}$)
- $x_p(t)$ is **REAL** and **EVEN**
  - $c_k$ is real ($c_k^* = c_k$)
  - $c_k$ is even ($c_k = c_{-k}$)
- $x_p(t)$ is **REAL** and **ODD**
  - $c_k$ is imaginary ($c_k^* = -c_k$)
  - $c_k$ is odd ($c_k = -c_{-k}$)

### 4.4 Spectrum of Signals that are not Absolutely Integrable

$$\Im\{K\delta(t)\} = \int_{-\infty}^{\infty} K\delta(t)e^{-j2\pi ft}dt = K \quad (4.13)$$

By duality, $\Im\{K\} = K\delta(f)$

### 4.4.1 Spectrm of Unit Step and Signum function

$$\Im\{u(t)\} = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$
$$\Im\{\mathrm{Sgn}(t)\} = \frac{1}{j\pi f}$$

### 4.4.2 Continuous-Frequency Spectrum of Periodic Signals

The following make use of the fact that
$$\Im\{k\} = K\delta(f) \quad (4.14)$$

**DC**
$$x_{dc}(t) = K$$
$$X_{dc}(f) = \Im\{k\} = K\Im\{1\} = K\delta(f)$$

**Complex Exponential**
$$\tilde{x}(t) = Ke^{j2\pi f_0 t}$$
$$\tilde{X}(f) = \Im\{Ke^{j2\pi f_0 t}\} = K\delta(f - f_0)$$

**Cosine**
$$\Im\{K\cos(2\pi f_0 t)\}$$
$$= \frac{K}{2}\delta(f-f_0) + \frac{K}{2}\delta(f+f_0)$$

**Sine**
$$\Im\{K\sin(2\pi f_0 t)\}$$
$$= \frac{K}{j2}\delta(f-f_0) - \frac{K}{j2}\delta(f+f_0)$$

where
$$\begin{cases} |X_s(f)| & = \frac{K}{2}\delta(f-f_0) + \frac{K}{2}\delta(f+f_0) \\ \angle X_s(f) & = \begin{cases} -\pi/2, & f = f_0 \\ \pi/2, & f = -f_0 \end{cases} \end{cases}$$

**Arbitrary periodic signals**
Let $x_p(t)$ be a periodic signal with period $T_p$ and fundamental frequency $f_p$
$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k\delta(f - kf_p) \quad (4.16)$$

### 4.4.2.1 Spectrum of Dirac-$\delta$ Comb function

$$\mathrm{comb}_\lambda(t) \triangleq \sum_n \delta(t - n\lambda)$$
$$c_k = \frac{1}{\lambda}$$
$$\Im\{\mathrm{comb}_\lambda(t)\} = \mathrm{COMB}_\lambda(f)$$
$$= \frac{1}{\lambda}\sum_k \delta(f - k/\lambda)$$

## Chapter 5

### 5.1 Energy Spectral Density (ESD)

Total energy of a signal $x(t)$ is defined as
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \text{ (Joules)} \quad (5.1)$$

**Rayleigh Energy Theorem**
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df, \quad (5.2)$$

**Energy Spectral Density**
$$E_x(f) = |X(f)|^2 \text{ Joules Hz}^{-1} \quad (5.3)$$

**Properties of $E_x(f)$**

1. $E_x(f)$ is a real function of $f$
2. $E_x(f) \geq 0 \quad \forall f$
3. $E_x(f)$ is an even function of $f$ if $x(t)$ is real.

### 5.2 Power Spectral Density (PSD)

In the time-domain, the average power of a signal $x(t)$ is defined as
$$P = \lim_{\tau\to\infty}\frac{1}{2\tau}\int_{-\tau}^{\tau} |x(t)|^2 dt \quad (5.4)$$

Windowed version of $x(t)$:
$$x_W(t) = x(t)\mathrm{rect}\left(\frac{t}{2W}\right) \quad (5.5)$$

**Parseval Power Theorem**
$$P = \lim_{W\to\infty}\frac{1}{2W}\int_{-W}^{W} |x(t)|^2 dt$$
$$= \int_{-\infty}^{\infty}\lim_{W\to\infty}\frac{1}{2W}|X_W(f)|^2 df \quad (5.9)$$

**Power Spectral Density**
$$P_x(f) = \lim_{W\to\infty}\frac{1}{2W}|X_W(f)|^2 \text{ Watts Hz}^{-1} \quad (5.10)$$

**Properties of $P_x(f)$**

1. $P_x(f)$ is a real function of $f$
2. $P_x(f) \geq 0 \quad \forall f$
3. $P_x(f)$ is an even function of $f$ if $x(t)$ is real.

### 5.2.1 PSD of Periodic Signals

From chapter 4 equation 4.16:
$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k\delta(f - kf_p)$$

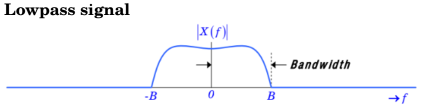
**PSD of $x_p(t)$**
$$P_x(f) = \sum_{k=-\infty}^{\infty} |c_k|^2\delta(f - kf_p) \quad (5.12)$$

**Average power of $x_p(t)$**
$$P = \int_{-\infty}^{\infty} P_x(f)df = \sum_{k=-\infty}^{\infty} |c_k|^2 \quad (5.13)$$

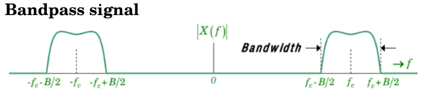
### 5.3 Bandwidth

### 5.3.1 Bandlimited Signals

**Lowpass signal**



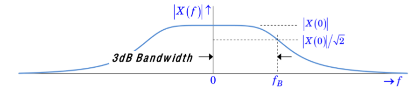

**Bandpass signal**



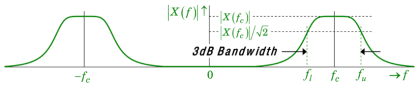

### 5.3.2 Signals with Unrestricted Band

### 5.3.2.1 3dB Bandwidth

**Lowpass signal:**

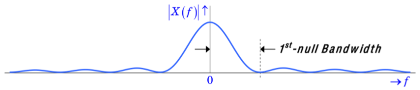



**Bandpass signal:**

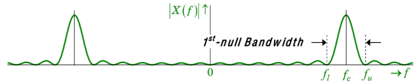



### 5.3.2.2 1st-null Bandwidth

**Lowpass signal:**



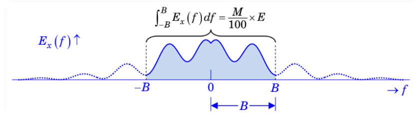

**Bandpass signal:**



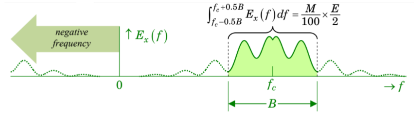

### 5.3.2.3 M% Energy Containment Bandwidth

Smallest bandwidth that contains at least M% of the total signal energy $E = \int_{-\infty}^{\infty} E_x(f)\, df$

**Lowpass:**



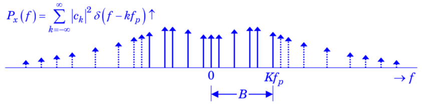

**Bandpass:**




### 5.3.2.4 M% Power Containment Bandwidth

The smallest bandwidth that contains at least M% of the average signal power. For a periodic signal, the average power is given by
$$P = \int_{-\infty}^{\infty} P_x(f)df = \sum_{k=-\infty}^{\infty} |c_k|^2$$

where $f_p$(Hz) is the fundamental frequency and $c_k$'s are the Fourier series coefficients.




$$B = Kf_p$$

where $K$ is the smallest positive integer that satisfies
$$\sum_{k=-K}^{K} |c_k|^2 \geq \frac{M}{100} \times P$$

## Chapter 6

### 6.1 Systems

### 6.2 Classification of Systems

### 6.2.1 Systems with Memory and Without Memory

Memoryless: output at a given time is dependent on only the input at that time.
Otherwise, the system has memory / is dynamic.

**6.2.2 Causal and Noncausal Systems**

Causal (or non-anticipative): Its output, $y(t)$, at the present time depends on only the present and/or past values of its input, $x(t)$.
$\therefore$ not possible for a causal system to produce an output before an input is applied. $\therefore \forall t < 0\ y(t) = 0$.

**6.2.3 Stable and Unstable Systems**

BIBO stable (bounded-input/bounded-output): For every bounded input $x(t)$ where
$$\forall t\ |x(t)| \le k \tag{6.2}$$
the system produces a bounded output $y(t)$ where
$$\forall t\ |y(t)| \le L \tag{6.3}$$
in which $K$ and $L$ are positive constants.

**6.2.4 Linear and Nonlinear Systems**

Linear system satisfies the following:
$$\begin{aligned}\mathbf{T}&[\alpha_1 x_1(t) + \alpha_2 x_2(t)]\\ &= \alpha_1 \mathbf{T}[x_1(t)] + \alpha_2 \mathbf{T}[x_2(t)]\\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t)\end{aligned} \tag{6.6}$$
(6.6) is known as the superposition property.
Important property of linear systems:
$x(t) = 0 \implies y(t) = 0$

**6.2.5 Time-Invariant and Time-Varying Systems**

Time-invariant: a time shift (delay or advance) in the input signal, $x(t)$, causes the same time shift in the output signal, $y(t)$.
$$\mathbf{T}[x(t-\tau)] = y(t-\tau) \tag{6.7}$$
A time-varying system is one which does not satisfy (6.7).

**Laplace Transform**
$$\tilde F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt \tag{6.8}$$
where $s$ is a complex variable.

**Inverse Laplace Transform**
$$f(t) = \mathcal{L}^{-1}\left\{\tilde F(s)\right\} = \frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty} \tilde F(s)ds \tag{6.9}$$
$$\mathcal{L}\left\{y''\right\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$$
$$\begin{aligned}\mathcal{L}\left\{y'''\right\} &= s^3\mathcal{L}\{y\} - s^2y(0) - sy'(0)\\ &\quad - y''(0)\end{aligned}$$
$$\begin{aligned}\mathcal{L}\left\{y''''\right\} &= s^4\mathcal{L}\{y\} - s^3y(0) - s^2y'(0)\\ &\quad - sy''(0) - y'''(0)\end{aligned}$$

## Chapter 7

**7.1 Impulse Response**

Impulse response, $h(t)$: The response/output when the input is a unit impulse, $\delta(t)$.
$$\delta(t) \to \text{LTI system} \to h(t)$$
where
$$h(t) = \mathbf{T}[\delta(t)] \tag{7.1}$$
$$\mathbf{T}[x(t)] = y(t) = x(t) * h(t) \tag{7.5}$$

**7.1.1 Step Response**

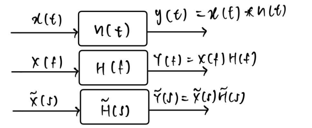
Step response: the output of the system when input is unit step function
$$\begin{aligned}u(t) \to h(t) \to o(t) &= \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau\\ &= \int_{-\infty}^{t} h(\tau)d\tau\end{aligned}$$
Step response equals integration of impulse response:
$$o(t) = \int_{-\infty}^{t} h(\tau)d\tau$$
Impulse response equals differentiation of step response:
$$h(t) = \frac{d}{dt}o(t)$$

**7.2 Frequency Response**

Frequency response ($H(f)$): The Fourier transform of the system impulse response $h(t)$
$$H(f) = \Im\{h(t)\} = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt \tag{7.6}$$
$$Y(f) = X(f)\cdot H(f) \tag{7.7}$$
$$H(f) = |H(f)|e^{j\angle H(f)} \tag{7.8}$$
where $|H(f)|$ is called the magnitude response and $\angle H(f)$ is called the phase response of the system.

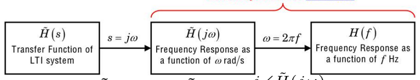
**7.3 Transfer Function**

Transfer function $\tilde H(s)$: Laplace transform of $h(t)$
$$\tilde H(s) = \mathcal{L}\{h(t)\} = \int_0^\infty h(t)e^{-st}dt \tag{7.9}$$
where $s = \sigma + j\omega$ is a complex variable.
$$y(t) = x(t) * h(t)$$
$$\tilde Y(s) = \tilde X(s)\cdot \tilde H(s) \tag{7.10}$$




**7.4 Relationship between Transfer Function and Frequency Response**

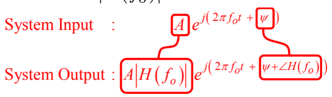
Substituting $s = j\omega$ into (7.9), we get
$$\tilde H(s)\Big|_{s=j\omega} = \tilde H(j\omega) = \int_0^\infty h(t)e^{-j\omega t}dt \tag{7.11}$$
Sub $\omega = 2\pi f$ into (7.11):
$$\tilde H(j\omega)\Big|_{\omega=2\pi f} = \int_0^\infty h(t)e^{-j2\pi ft}dt \tag{7.12}$$
For causal LTI systems, $\forall t < 0\ h(t) = 0$. Hence (7.6) and (7.12) are equivalent.
$$H(f) = \tilde H(j\omega)\Big|_{\omega=2\pi f} \tag{7.13}$$

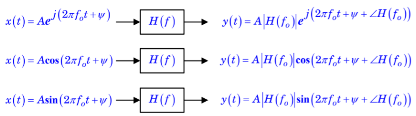



$$\tilde H(j\omega) = |\tilde H(j\omega)|e^{j\angle\tilde H(j\omega)} \tag{7.14}$$
where $|\tilde H(j\omega)|$ is called the magnitude response and $\angle\tilde H(j\omega)$ is called the phase response of the system.

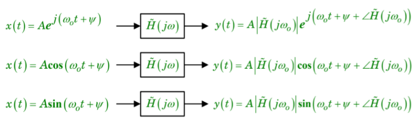
**7.4 Sinusoidal Response at Steady-State**

Let system input at steady-state be
$$x(t) = Ae^{j(2\pi f_0 t + \psi)} \tag{7.15}$$
Then
$$X(f) = Ae^{j\psi}\delta(f - f_0) \tag{7.16}$$
$$Y(f) = A\,|H(f_0)|\,e^{j(\psi + \angle H(f_0))}\delta(f - f_0) \tag{7.17}$$
$$\begin{aligned}y(t) &= \Im^{-1}\{Y(f)\}\\ &= A\,|H(f_0)|\,e^{j(2\pi f_0 t + \psi + \angle H(f_0))}\end{aligned} \tag{7.18}$$




Steady-state Sinusoidal Response of a LTI System in *f*-domain




Steady-state Sinusoidal Response of a LTI System in $\omega$-domain

**7.6 LTI Systems Described by Differential Equations**

LTI systems represented by linear constant-coefficient differential equations have the general form
$$\sum_{n=0}^{N} a_n\frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} b_m\frac{d^m x(t)}{dt^m} \tag{7.21}$$
where $x(t)$ is input, $y(t)$ is output, and $a_n$, $b_m$ are real constants.

**7.6.1 Transfer Function**
$$\tilde H(s) = K\frac{\left(\frac{s}{z_1}+1\right)\left(\frac{s}{z_2}+1\right)\dots\left(\frac{s}{z_M}+1\right)}{\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\dots\left(\frac{s}{p_N}+1\right)}$$
$$K = \frac{a_0}{b_0} \tag{7.23b}$$
$$\tilde H(s) = K'\frac{(s+z_1)(s+z_2)\dots(s+z_M)}{(s+p_1)(s+p_2)\dots(s+p_N)} \tag{7.23c}$$
$$K = \frac{b_M}{a_N}$$
$\forall n \in \{1, 2, \dots, N\}$
- $\tilde H(-p_n) = \infty$
- $-p_n$ are called **poles** of $\tilde H(s)$

$\forall m \in \{1, 2, \dots, M\}$
- $\tilde H(-z_m) = 0$
- $-z_m$ are called **zeros** of $\tilde H(s)$

The system is said to have $N$ poles and $M$ zeros, and the difference $N - M$ is called pole-zero excess.

**7.6.2 System Stability**

**BIBO Stable**
- All system poles lying on the left-half s-plane
- $h(t)$ will converge to 0 as $t$ tends to infinity $\lim_{t\to\infty} h(t) = 0$

**Marginally Stable**
- One or more **non-repeated** system poles lying on the imaginary axis of the s-plane and no system pole lying on the right half s-plane.
- $h(t)$ will "blow up" and become unbounded, but neither will it converge to zero as $t$ tends to infinity. $\lim_{t\to\infty} |h(t)| \ne \infty$ and $\lim_{t\to\infty} h(t) \ne 0$

**Unstable (Case 1)**
- One or more system poles lying on the right-half s-plane
- $h(t)$ will "blow up" and become unbounded as $t$ tends to infinity $\lim_{t\to\infty} |h(t)| = \infty$

**Unstable (Case 2)**
- One or more repeated system poles lying on the imaginary axis
- $h(t)$ will "blow up" and become unbounded as $t$ tends to infinity $\lim_{t\to\infty} |h(t)| = \infty$

**7.7 First Order System (Standard Form)**

**7.7.1 Differential Eqn, Transfer Func, Impulse Response and Step Response**
- Differential equation:
$$T\frac{dy(t)}{dt} + y(t) = Kx(t) \tag{7.26}$$
where
  - $x(t)$: system input
  - $y(t)$: system output
  - $K$: DC gain
  - $T$: time-constant
- Transfer Function $\tilde H(s)$:
$$T s\tilde Y(s) + \tilde Y(s) = K\tilde X(s)$$
$$\to \tilde H(s) = \frac{\tilde Y(s)}{\tilde X(s)} = \frac{K}{Ts+1} \tag{7.27}$$
Pole: $s_1 = -\frac{1}{T}$
- Impulse Response $h(t)$
$$h(t) = \mathcal{L}^{-1}\left\{\tilde H(s)\right\} = \frac{K}{T}e^{-t/T}u(t)$$
- Step Response $o(t)$
$$\begin{aligned}o(t) &= \int_{-\infty}^{t} h(\tau)d\tau = \mathcal{L}^{-1}\left\{\frac{1}{s}\tilde H(s)\right\}\\ &= K\left[1 - e^{-t/T}\right]u(t)\end{aligned}$$

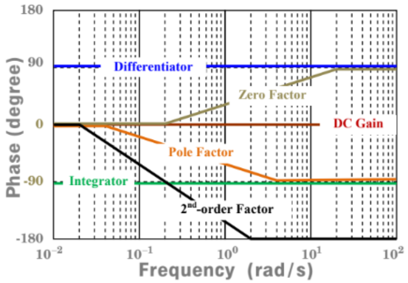
**7.8 Second Order System (Standard Form) 7.8.1 Differential Eqn and Transfer Func**
- Differential equation:
$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = K\omega_n^2 x(t) \tag{7.28}$$
where
  - $x(t)$: system input
  - $y(t)$: system output
  - $\zeta$: damping ratio
  - $\omega_n$: undamped natural frequency (when $\zeta < 1$)
  - $K$: DC gain
- Transfer function $\tilde H(s)$
$$s^2\tilde Y(s) + 2\zeta\omega_n s\tilde Y(s) + \omega_n^2\tilde Y(s) = K\omega_n^2\tilde X(s)$$
$$\implies \tilde H(s) = \frac{\tilde Y(s)}{\tilde X(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
Poles: $s_{1,2} = -\omega_n\zeta \pm \omega_n\left(\zeta^2 - 1\right)^{1/2}$
- Overdamped system: distinct real poles, $\zeta > 1$
- Critically damped system: repeated real poles, $\zeta = 1$
- Underdamped system: conjugate complex poles, $0 < \zeta < 1$
- Undamped system: conjugate imaginary poles, $\zeta = 0$

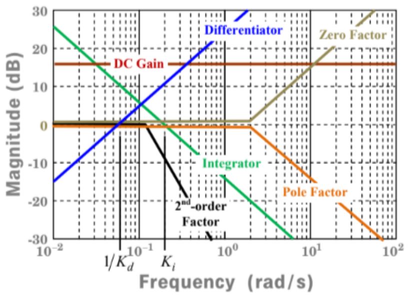
Blue stuff:
$\sin(\omega_0 t)u(t)$: $+\frac{j}{4}\left[\delta(f+f_0) - \delta(f-f_0)\right]$
$\cos(\omega_0 t)u(t)$: $+\frac{1}{4}\left[\delta(f+f_0) + \delta(f-f_0)\right]$

## Chapter 8

1. $\tilde H(s) = K_{dc}$: DC gain (constant)
2. $\tilde H(s) = K_d s$: differentiator with gain $K_d$
3. $\tilde H(s) = K_i/s$: integrator with gain $K_i$
4. $\tilde H(s) = s/z_m + 1$: zero factor with unity DC gain ($\tilde H(0) = 1$)
5. $\tilde H(s) = \frac{1}{s/p_n+1}$: pole factor with unity DC gain
6. $\tilde H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$: 2nd-order factor with unity DC gain

**8.2 Asymptotic Behavior of Bode Plots**




**Asymptotic phase of phase plot**

High frequency:
$$\text{Pole-zero excess} \times (-90^\circ) \tag{8.4a}$$
Low frequency:
$$\left[\text{No. of} \int dt - \text{No. of } \frac{d}{dt}\right] \times (-90^\circ) \tag{8.4b}$$

**Asymptotic slope of magnitude plot**

High frequency:
$$[\text{Pole-zero excess}] \times (-20\ \text{dB/decade}) \tag{8.5a}$$
Low frequency:
$$\left[\text{No. of} \int dt - \text{No. of } \frac{d}{dt}\right] \times (-20\ \text{dB/decade}) \tag{8.5a}$$

**Resonant frequency:** $\omega_r = \omega_n(1 - 2\zeta^2)^{0.5}$

**Resonant peak:** $\left|\tilde H(j\omega_r)\right| = \frac{K}{2\zeta(1-\zeta^2)^{0.5}}$

## Chapter 9

**9.1 Idealized LTI filters**

**Ideal Low-Pass Filter (LPF)**
- Frequency response: $H(f) = A\,\mathrm{rect}\left(\frac{f}{2B}\right)$
- Impulse response: $h(t) = 2AB\,\mathrm{sinc}\,(2Bt)$

**Ideal Band-Pass Filter (BPF)**
- Frequency response:
$$H(f) = A\left[\mathrm{rect}\left(\frac{f+f_0}{B}\right) + \mathrm{rect}\left(\frac{f-f_0}{B}\right)\right]$$
- Impulse response:
$$h(t) = 2AB\,\mathrm{sinc}\,(Bt)\cos(2\pi f_0 t)$$

**9.2 Continuous-time Sampling and Reconstruction of Signals**

**Nyquist Sampling Theorem:**
Nyquist sampling frequency / Nyquist rate $f_s = 2f_m$

**9.3 Sampling Bandpass Signal below Nyquist**

(a) Overlapping spectral images ($f_c > 0.5B$, symmetric about $f_c$ and $f_{-c}$)
$$f_s = 2f_c/k; \quad k = 1, 2, \dots, \lfloor 2f_c/B\rfloor \tag{9.2a}$$
(b) Un-aliased spectral images ($f_c > 1.5B$)
$$\frac{2f_c + B}{k+1} \le f_s \le \frac{2f_c - B}{k}; \quad k = 1, 2, \dots, \left\lfloor\frac{2f_c - B}{2B}\right\rfloor \tag{9.2b}$$