Euler's formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

Chapter 1

1.2.2 Bounded signals

$x(t)$ is bounded if

$\exists M\ [(0 < M < \infty) \wedge (\forall t\ |x(t)| \le M)]$

(got upper n lower range limit)

1.2.3 Absolutely integrable signals

$x(t)$ is absolutely integrable if

$$\int_{-\infty}^{\infty} |x(t)|dt < \infty$$

1.2.6 Energy and Power Signals

Energy signals

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \tag{1.3a}$$

$$x(t) \text{ is an energy signal} \iff 0 < E < \infty \tag{1.3b}$$

Power signals

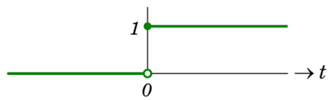
$$P = \lim_{\tau\to\infty} \frac{1}{2\tau}\int_{-\tau}^{\tau} |x(t)|^2 dt \tag{1.4a}$$

$$x(t) \text{ is a power signal} \iff 0 < P < \infty \tag{1.4b}$$

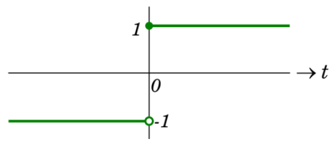
If $x(t)$ is a periodic signal, average power may be computed by $\frac{1}{T}\int_0^T |x(t)|^2 dt$

- Energy signals have 0 average power, bc E = finite implies P = 0
- Power signals have infinite total energy, bc P = finite implies $E = \infty$
- All bounded periodic signals are power signals

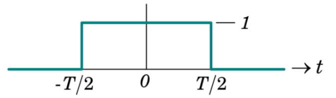
u(t):



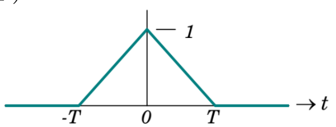
sgn(t):



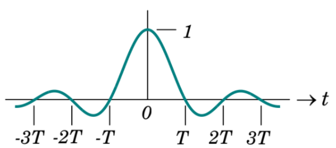
rect$\left(\frac{t}{T}\right)$:



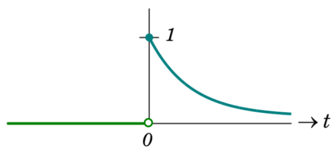
tri$\left(\frac{t}{T}\right)$:



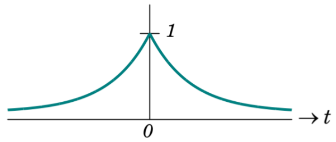
sinc$\left(\frac{t}{T}\right)$:



$e^{-\alpha t}u(t)$:



$e^{-\alpha|t|}$:



Chapter 2

2.1 Time-domain Operations

2.1.5 Convolution of 2 Signals

$x(t) * y(t) = \int_{-\infty}^{\infty} x(\alpha)y(t-\alpha)\,d\alpha$

Properties of Dirac-δ:

1. Symmetry: $\delta(t) = \delta(-t)$ (2.3)
2. Sampling:
$$x(t)\delta(t-\lambda) = x(\lambda)\delta(t-\lambda) \tag{2.4}$$
3. Sifting
$$\int_{-\infty}^{\infty} x(t)\delta(t-\lambda)dt = x(\lambda) \tag{2.5}$$
4. Replication
$$x(t) * \delta(t-\lambda) = x(t-\lambda) \tag{2.6}$$

Convolution with Dirac-δ Comb function

$$x_p(t) = x(t) * \sum_n \delta(t-nT)$$

$$= \sum_n x(t-nT)$$

Multiplication with the Dirac-δ Comb function

Used for sampling

$$x_s(t) = x(t) \times \sum_n \delta(t-nT)$$

$$= \sum_n x(t) \times \delta(t-nT)$$

$$= \sum_n x(nT)\delta(t-nT)$$

Chapter 3

3.2 Spectrum of a Sinusoid

Spectrum of a Complex Exponential Signal

$$\tilde{x}(t) = \mu e^{j(2\pi f_0 t+\phi)} = \mu e^{j\phi} \times e^{j2\pi f_0 t},$$

Spectrum of a Cosine Signal

$$\mu\cos(2\pi f_0 t + \phi)$$

$$= \frac{\mu}{2}e^{j\phi}e^{j2\pi f_0 t} + \frac{\mu}{2}e^{j(-\phi)}e^{j2\pi(-f_0)t}$$

Spectrum of a Sine Signal

$$\mu\sin(2\pi f_0 t + \phi) = \frac{\mu}{2}e^{j(\phi-0.5\pi)}e^{j2\pi f_0 t}$$

$$+ \frac{\mu}{2}e^{j(-\phi+0.5\pi)}e^{j2\pi(-f_0)t}$$

Complex exponential Fourier Series

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T_p}$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kf_p t} \tag{3.1a}$$

$$c_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t)e^{-j2\pi kt/T_p}dt,\ k\in\mathbb{Z} \tag{3.1b}$$

Trigonometric Fourier Series

$$x_p(t) = a_0 + 2\sum_{k=1}^{\infty}[a_k\cos(2\pi kt/T_p) + b_k\sin(2\pi kt/T_p)]$$

$$a_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t)\cos(2\pi kt/T_p)dt;\ k\ge 0$$

$$b_k = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} x_p(t)\sin(2\pi kt/T_p)dt;\ k > 0 \tag{3.2}$$

Chapter 4

4.1 Fourier Transform

Forward Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \tag{4.1a}$$

Inverse Fourier Transform

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \tag{4.1b}$$

Spectrum of exponentially decaying pulse

$$x(t) = Ae^{-\alpha t}u(t)$$

Assume $\alpha > 0$

$$X(f) = \frac{A}{\alpha + j2\pi f}$$

4.3 Spectral properties of a REAL signal

- If $x(t)$ is **REAL** ($x^*(t) = x(t)$), then
  - $X(f)$ is conjugate symmetric ($X^*(f) = X(f)$)
  - $|X(f)|$ is even ($|X(f)| = |X(-f)|$)
  - $\angle X(f)$ is odd ($\angle X(f) = -\angle X(-f)$)
- If $x(t)$ is **REAL** and **EVEN** ($x^*(t) = x(t) \wedge x(-t) = x(t)$), then
  - $X(f)$ is real ($X^*f = X(f)$)
  - $X(f)$ is even ($X(-f) = X(f)$)
- If $x(t)$ is **REAL** and **ODD** ($x^*(t) = x(t) \wedge x(-t) = -x(t)$), then
  - $X(f)$ is imaginary ($X^*(f) = -X(f)$)
  - $X(f)$ is odd ($X(-f) = -X(f)$)

The above can apply to Fourier series coefficients of periodic signals too:

- $x_p(t)$ is **REAL**
  - c_k is conjugate symmetric ($c_k^* = c_{-k}$)
  - $|c_k|$ has even symmetry ($|c_k| = |c_{-k}|$)
  - $\angle c_k$ has odd symmetry ($\angle c_k = -\angle c_{-k}$)
- $x_p(t)$ is **REAL** and **EVEN**
  - c_k is real ($c_k^* = c_k$)
  - c_k is even ($c_k = c_{-k}$)
- $x_p(t)$ is **REAL** and **ODD**
  - c_k is imaginary ($c_k^* = -c_k$)
  - c_k is odd ($c_k = -c_{-k}$)

4.4 Spectrum of Signals that are not Absolutely Integrable

$$\Im\{K\delta(t)\} = \int_{-\infty}^{\infty} K\delta(t)e^{-j2\pi ft}dt = K \tag{4.13}$$

By duality, $\Im\{K\} = K\delta(f)$

4.4.1 Spectrm of Unit Step and Signum function

$$\Im\{u(t)\} = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

$$\Im\{\text{Sgn}(t)\} = \frac{1}{j\pi f}$$

4.4.2 Continuous-Frequency Spectrum of Periodic Signals

The following make use of the fact that

$$\Im\{k\} = K\delta(f) \tag{4.14}$$

DC

$$x_{dc}(t) = K$$

$$X_{dc}(f) = \Im\{k\} = K\Im\{1\} = K\delta(f)$$

Complex Exponential

$$\tilde{x}(t) = Ke^{j2\pi f_0 t}$$

$$\tilde{X}(f) = \Im\{Ke^{j2\pi f_0 t}\} = K\delta(f-f_0)$$

Cosine

$$\Im\{K\cos(2\pi f_0 t)\}$$

$$= \frac{K}{2}\delta(f-f_0) + \frac{K}{2}\delta(f+f_0)$$

Sine

$$\Im\{K\sin(2\pi f_0 t)\}$$

$$= \frac{K}{j2}\delta(f-f_0) - \frac{K}{j2}\delta(f+f_0)$$

$$\text{where}\begin{cases} |X_s(f)| &= \frac{K}{2}\delta(f-f_0) + \frac{K}{2}\delta(f+f_0) \\ \angle X_s(f) &= \begin{cases}-\pi/2, & f = f_0 \\ \pi/2, & f = -f_0\end{cases}\end{cases}$$

Arbitrary periodic signals

Let $x_p(t)$ be a periodic signal with period T_p and fundamental frequency f_p

$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k\delta(f-kf_p) \tag{4.16}$$

4.4.2.1 Spectrum of Dirac-δ Comb function

$$\text{comb}_\lambda(t) \triangleq \sum_n \delta(t-n\lambda)$$

$$c_k = \frac{1}{\lambda}$$

$$\Im\{\text{comb}_\lambda(t)\} = \text{COMB}_\lambda(f)$$

$$= \frac{1}{\lambda}\sum_k \delta(f-k/\lambda)$$

Chapter 5

5.1 Energy Spectral Density (ESD)

Total energy of a signal $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \text{ (Joules)} \tag{5.1}$$

Rayleigh Energy Theorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df, \tag{5.2}$$

Energy Spectral Density

$$E_x(f) = |X(f)|^2 \text{ Joules Hz}^{-1} \tag{5.3}$$

Properties of $E_x(f)$

1. $E_x(f)$ is a real function of f
2. $E_x(f) \ge 0\quad \forall f$
3. $E_x(f)$ is an even function of f if $x(t)$ is real.

5.2 Power Spectral Density (PSD)

In the time-domain, the average power of a signal $x(t)$ is defined as

$$P = \lim_{\tau\to\infty}\frac{1}{2\tau}\int_{-\tau}^{\tau} |x(t)|^2 dt \tag{5.4}$$

Windowed version of $x(t)$:

$$x_W(t) = x(t)\text{rect}\left(\frac{t}{2W}\right) \tag{5.5}$$

Parseval Power Theorem

$$P = \lim_{W\to\infty}\frac{1}{2W}\int_{-W}^{W} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} \lim_{W\to\infty}\frac{1}{2W}|X_W(f)|^2 df \tag{5.9}$$

Power Spectral Density

$$P_x(f) = \lim_{W\to\infty}\frac{1}{2W}|X_W(f)|^2 \text{ Watts Hz}^{-1} \tag{5.10}$$

Properties of $P_x(f)$

1. $P_x(f)$ is a real function of f
2. $P_x(f) \ge 0\quad \forall f$
3. $P_x(f)$ is an even function of f if $x(t)$ is real.

5.2.1 PSD of Periodic Signals

From chapter 4 equation 4.16:

$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k\delta(f-kf_p)$$

PSD of $x_p(t)$

$$P_x(f) = \sum_{k=-\infty}^{\infty} |c_k|^2\delta(f-kf_p) \tag{5.12}$$

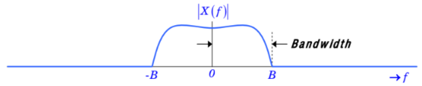
Average power of $x_p(t)$

$$P = \int_{-\infty}^{\infty} P_x(f)df = \sum_{k=-\infty}^{\infty} |c_k|^2 \tag{5.13}$$

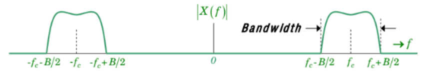
5.3 Bandwidth

5.3.1 Bandlimited Signals

Lowpass signal



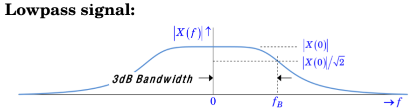
Bandpass signal



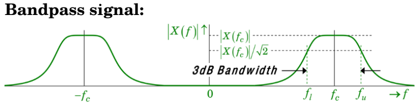
5.3.2 Signals with Unrestricted Band

5.3.2.1 3dB Bandwidth

Lowpass signal:

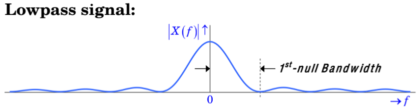


Bandpass signal:

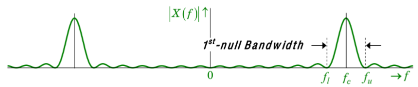


5.3.2.2 1st-null Bandwidth

Lowpass signal:



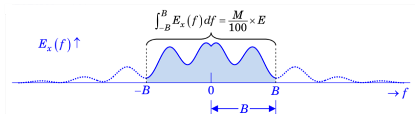
Bandpass signal:



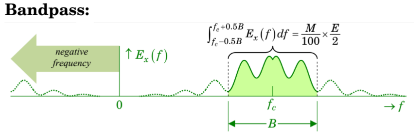
5.3.2.3 M% Energy Containment Bandwidth

Smallest bandwidth that contains at least M% of the total signal energy $E = \int_{-\infty}^{\infty} E_x(f)\,df$

Lowpass:



Bandpass:

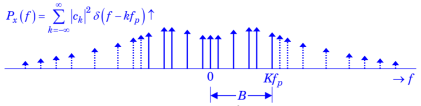


5.3.2.4 M% Power Containment Bandwidth

The smallest bandwidth that contains at least M% of the average signal power. For a periodic signal, the aerage power is given by

$$P=\int_{-\infty}^{\infty}P_x(f)df=\sum_{k=-\infty}^{\infty}|c_k|^2$$

where f_p(Hz) is the fundamental frequency and c_k's are the Fourier series coefficients.



$$B=Kf_p$$

where K is the smallest positive integer that satisfies

$$\sum_{k=-k}^{K}|c_k|^2\geq\frac{M}{100}\times P$$

Chapter 6

6.1 Systems

6.2 Classification of Systems

6.2.1 Systems with Memory and Without Memory

Memoryless: output at a given time is dependent on only the input at that time.

Otherwise, the system has memory / is dynamic.

6.2.2 Causal and Noncausal Systems

Causal (or non-anticipative): Its output, $y(t)$, at the present time depends on only the present and/or past values of its input, $x(t)$.

$\therefore$ not possible for a causal system to produce an output before an input is applied. $\therefore\ \forall t<0\ y(t)=0$.

6.2.3 Stable and Unstable Systems

BIBO stable (bounded-input/bounded-output): For every bounded input $x(t)$ where

$$\forall t\ |x(t)|\leq k \tag{6.2}$$

the system produces a bounded output $y(t)$ where

$$\forall t\ |y(t)|\leq L \tag{6.3}$$

in which K and L are positive constants.

6.2.4 Linear and Nonlinear Systems

Linear system satisfies the following:

$$\begin{aligned}\mathbf{T}[\alpha_1x_1(t)+\alpha_2x_2(t)]\\=\alpha_1\mathbf{T}[x_1(t)]+\alpha_2\mathbf{T}[x_2(t)]\\=\alpha_1y_1(t)+\alpha_2y_2(t)\end{aligned} \tag{6.6}$$

(6.6) is known as the superposition property.

Important property of linear systems:

$x(t)=0\implies y(t)=0$

6.2.5 Time-Invariant and Time-Varying Systems

Time-invariant: a time shift (delay or advance) in the input signal, $x(t)$, causes the same time shift in the output signal, $y(t)$.

$$\mathbf{T}[x(t-\tau)]=y(t-\tau) \tag{6.7}$$

A time-varying system is one which does not satisfy (6.7).

Laplace Transform

$$\tilde{F}(s)=\mathcal{L}\{f(t)\}=\int_0^{\infty}f(t)e^{-st}dt \tag{6.8}$$

where s is a complex variable.

Inverse Laplace Transform

$$f(t)=\mathcal{L}^{-1}\left\{\tilde{F}(s)\right\}=\frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty}\tilde{F}(s)ds \tag{6.9}$$

$$\mathcal{L}\left\{y''\right\}=s^2\mathcal{L}\{y\}-sy(0)-y'(0)$$

$$\mathcal{L}\left\{y'''\right\}=s^3\mathcal{L}\{y\}-s^2y(0)-sy'(0)-y''(0)$$

$$\mathcal{L}\left\{y''''\right\}=s^4\mathcal{L}\{y\}-s^3y(0)-s^2y'(0)-sy''(0)-y'''(0)$$

Chapter 7

7.1 Impulse Response

Impulse response, $h(t)$: The response/output when the input is a unit impulse, $\delta(t)$.

$$\delta(t)\to\text{LTI system}\to h(t)$$

where

$$h(t)=\mathbf{T}[\delta(t)] \tag{7.1}$$

$$\mathbf{T}[x(t)]=y(t)=x(t)*h(t) \tag{7.5}$$

7.1.1 Step Response

Step response: the output of the system when input is unit step function

$$u(t)\to h(t)\to o(t)=\int_{-\infty}^{\infty}h(\tau)u(t-\tau)d\tau$$

$$=\int_{-\infty}^{t}h(\tau)d\tau$$

Step response equals integration of impulse response:

$$o(t)=\int_{-\infty}^{t}h(\tau)d\tau$$

Impulse response equals differentiation of step response:

$$h(t)=\frac{d}{dt}o(t)$$

7.2 Frequency Response

Frequency response ($H(f)$): The Fourier transform of the system impulse response $h(t)$

$$H(f)=\Im\{h(t)\}=\int_{-\infty}^{\infty}h(t)e^{-j2\pi ft}dt \tag{7.6}$$

$$Y(f)=X(f)\cdot H(f) \tag{7.7}$$

$$H(f)=|H(f)|e^{j\angle H(f)} \tag{7.8}$$

where $|H(f)|$ is called the magnitude response and $\angle H(f)$ is called the phase response of the system.

7.3 Transfer Function

Transfer function $\tilde{H}(s)$: Laplace transform of $h(t)$

$$\tilde{H}(s)=\mathcal{L}\{h(t)\}=\int_0^{\infty}h(t)e^{-st}dt \tag{7.9}$$

where $s=\sigma+j\omega$ is a complex variable.

$$y(t)=x(t)*h(t)$$

$$\tilde{Y}(s)=\tilde{X}(s)\cdot\tilde{H}(s) \tag{7.10}$$

$$x(t)\longrightarrow\boxed{h(t)}\longrightarrow y(t)=x(t)*h(t)$$

$$X(f)\longrightarrow\boxed{H(f)}\longrightarrow Y(f)=X(f)H(f)$$

$$\tilde{X}(s)\longrightarrow\boxed{\tilde{H}(s)}\longrightarrow \tilde{Y}(s)=\tilde{X}(s)\tilde{H}(s)$$

7.4 Relationship between Transfer Function and Frequency Response

Substituting $s=j\omega$ into (7.9), we get

$$\tilde{H}(s)\Big|_{s=j\omega}=\tilde{H}(j\omega)=\int_0^{\infty}h(t)e^{-j\omega t}dt \tag{7.11}$$

Sub $\omega=2\pi f$ into (7.11):

$$\tilde{H}(j\omega)\Big|_{\omega=2\pi f}=\int_0^{\infty}h(t)e^{-j2\pi ft}dt \tag{7.12}$$

For causal LTI systems, $\forall t<0\ h(t)=0$. Hence (7.6) and (7.12) are equivalent.

$$H(f)=\tilde{H}(j\omega)\Big|_{\omega=2\pi f} \tag{7.13}$$

Not always true. See APPENDIX.

$$\boxed{\begin{matrix}\tilde{H}(s)\\ \text{Transfer Function of}\\ \text{LTI system}\end{matrix}}\xrightarrow{s=j\omega}\boxed{\begin{matrix}\tilde{H}(j\omega)\\ \text{Frequency Response as}\\ \text{a function of }\omega\text{ rad/s}\end{matrix}}\xrightarrow{\omega=2\pi f}\boxed{\begin{matrix}H(f)\\ \text{Frequency Response as}\\ \text{a function of }f\text{ Hz}\end{matrix}}$$

$$\tilde{H}(j\omega)=|\tilde{H}(j\omega)|e^{j\angle\tilde{H}(j\omega)} \tag{7.14}$$

where $|\tilde{H}(j\omega)|$ is called the magnitude response and $\angle\tilde{H}(j\omega)$ is called the phase response of the system.

7.4 Sinusoidal Response at Steady-State

Let system input at steady-state be

$$x(t)=Ae^{j(2\pi f_0t+\psi)} \tag{7.15}$$

Then

$$X(f)=Ae^{j\psi}\delta(f-f_0) \tag{7.16}$$

$$Y(f)=A\,|H(f_0)|\,e^{j(\psi+\angle H(f_0))}\delta(f-f_0) \tag{7.17}$$

$$\begin{aligned}y(t)&=\Im^{-1}\{Y(f)\}\\&=A\,|H(f_0)|\,e^{j(2\pi f_0t+\psi+\angle H(f_0))}\end{aligned} \tag{7.18}$$

System Input : $\boxed{A}\,e^{j(2\pi f_ot+\boxed{\psi})}$

System Output : $\boxed{A\left|H(f_o)\right|}\,e^{j(2\pi f_ot+\boxed{\psi+\angle H(f_o)})}$

$$x(t)=Ae^{j(2\pi f_ot+\psi)}\longrightarrow\boxed{H(f)}\longrightarrow y(t)=A\left|H(f_o)\right|e^{j(2\pi f_ot+\psi+\angle H(f_o))}$$

$$x(t)=A\cos(2\pi f_ot+\psi)\longrightarrow\boxed{H(f)}\longrightarrow y(t)=A\left|H(f_o)\right|\cos(2\pi f_ot+\psi+\angle H(f_o))$$

$$x(t)=A\sin(2\pi f_ot+\psi)\longrightarrow\boxed{H(f)}\longrightarrow y(t)=A\left|H(f_o)\right|\sin(2\pi f_ot+\psi+\angle H(f_o))$$

Steady-state Sinusoidal Response of a LTI System in f-domain

$$x(t)=Ae^{j(\omega_ot+\psi)}\longrightarrow\boxed{\tilde{H}(j\omega)}\longrightarrow y(t)=A\left|\tilde{H}(j\omega_o)\right|e^{j(\omega_ot+\psi+\angle\tilde{H}(j\omega_o))}$$

$$x(t)=A\cos(\omega_ot+\psi)\longrightarrow\boxed{\tilde{H}(j\omega)}\longrightarrow y(t)=A\left|\tilde{H}(j\omega_o)\right|\cos(\omega_ot+\psi+\angle\tilde{H}(j\omega_o))$$

$$x(t)=A\sin(\omega_ot+\psi)\longrightarrow\boxed{\tilde{H}(j\omega)}\longrightarrow y(t)=A\left|\tilde{H}(j\omega_o)\right|\sin(\omega_ot+\psi+\angle\tilde{H}(j\omega_o))$$

Steady-state Sinusoidal Response of a LTI System in ω-domain

7.6 LTI Systems Described by Differential Equations

LTI systems represented by linear constant-coefficient differential equations have the general form

$$\sum_{n=0}^{N}a_n\frac{d^ny(t)}{dt^n}=\sum_{m=0}^{M}b_m\frac{d^mx(t)}{dt^m} \tag{7.21}$$

where $x(t)$ is input, $y(t)$ is output, and a_n,b_m are real constants.

7.6.1 Transfer Function

$$\tilde{H}(s)=K\frac{\left(\frac{s}{z_1}+1\right)\left(\frac{s}{z_2}+1\right)\dots\left(\frac{s}{z_M}+1\right)}{\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\dots\left(\frac{s}{p_N}+1\right)}$$

$$K=\frac{a_0}{b_0} \tag{7.23b}$$

$$\tilde{H}(s)=K'\frac{(s+z_1)(s+z_2)\dots(s+z_M)}{(s+p_1)(s+p_2)\dots(s+p_N)} \tag{7.23c}$$

$$K=\frac{b_M}{a_N}$$

$\forall n\in\{1,2,\dots,N\}$

- $\tilde{H}(-p_n)=\infty$
- $-p_n$ are called **poles** of $\tilde{H}(s)$

$\forall m\in\{1,2,\dots,M\}$

- $\tilde{H}(-z_m)=0$
- $-z_m$ are called **zeros** of $\tilde{H}(s)$

The system is said to have N poles and M zeros, and the difference $N-M$ is called pole-zero excess.

7.6.2 System Stability

BIBO Stable

- All system poles lying on the left-half s-plane
- $h(t)$ will converge to 0 as t tends to infinity $\lim_{t\to\infty}h(t)=0$

Marginally Stable

- One or more **non-repeated** system poles lying on the imaginary axis of the s-plane and no system pole lying on the right half s-plane.
- $h(t)$ will not "blow up" and become unbounded, but neither will it converge to zero as t tends to infinity. $\lim_{t\to\infty}|h(t)|\neq\infty$ and $\lim_{t\to\infty}h(t)\neq0$

Unstable (Case 1)

- One or more system poles lying on the right-half s-plane
- $h(t)$ will "blow up" and become unbounded as t tends to infinity $\lim_{t\to\infty}|h(t)|=\infty$

Unstable (Case 2)

- One or more repeated system poles lying on the imaginary axis
- $h(t)$ will "blow up" and become unbounded as t tends to infinity $\lim_{t\to\infty}|h(t)|=\infty$

7.7 First Order System (Standard Form)

7.7.1 Differential Eqn, Transfer Func, Impulse Response and Step Response

- Differential equation:
$$T\frac{dy(t)}{dt}+y(t)=Kx(t) \tag{7.26}$$
where
  - $x(t)$: system input
  - $y(t)$: system output
  - K: DC gain
  - T: time-constant
- Transfer Function $\tilde{H}(s)$:
$$Ts\tilde{Y}(s)+\tilde{Y}(s)=K\tilde{X}(s)$$
$$\to\tilde{H}(s)=\frac{\tilde{Y}(s)}{\tilde{X}(s)}=\frac{K}{Ts+1} \tag{7.27}$$
Pole: $s_1=-\frac{1}{T}$
- Impulse Response $h(t)$
$h(t)=\mathcal{L}^{-1}\left\{\tilde{H}(s)\right\}=\frac{K}{T}e^{-t/T}u(t)$
- Step Response $o(t)$
$$o(t)=\int_{-\infty}^{t}h(\tau)d\tau=\mathcal{L}^{-1}\left\{\frac{1}{s}\tilde{H}(s)\right\}$$
$$=K\left[1-e^{-t/T}\right]u(t)$$

7.8 Second Order System (Standard Form) 7.8.1 Differential Eqn and Transfer Func

- Differential equation:
$$\frac{d^2y(t)}{dt^2}+2\zeta\omega_n\frac{dy(t)}{dt}+\omega_n^2y(t)=K\omega_n^2x(t) \tag{7.28}$$
where
  - $x(t)$: system input
  - $y(t)$: system output
  - ζ: damping ratio
  - ω_n: undamped natural frequency (when $\zeta<1$)
  - K: DC gain
- Transfer function $\tilde{H}(s)$
$$s^2\tilde{Y}(s)+2\zeta\omega_ns\tilde{Y}(s)+\omega_n^2\tilde{Y}(s)=K\omega_n^2\tilde{X}(s)$$
$$\implies\tilde{H}(s)=\frac{\tilde{Y}(s)}{\tilde{X}(s)}=\frac{K\omega_n^2}{s^2+2\zeta\omega_ns+\omega_n^2}$$
Poles: $s_{1,2}=-\omega_n\zeta\pm\omega_n\left(\zeta^2-1\right)^{1/2}$
- Overdamped system: distinct real poles, $\zeta>1$
- Critically damped system: repeated real poles, $\zeta=1$
- Underdamped system: conjugate complex poles, $0<\zeta<1$
- Undamped system: conjugate imaginary poles, $\zeta=0$

Blue stuff:

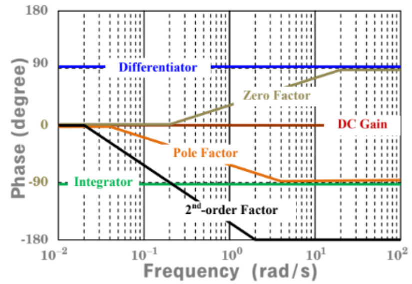
$\sin(\omega_0t)u(t)$: $+\frac{j}{4}\left[\delta(f+f_0)-\delta(f-f_0)\right]$

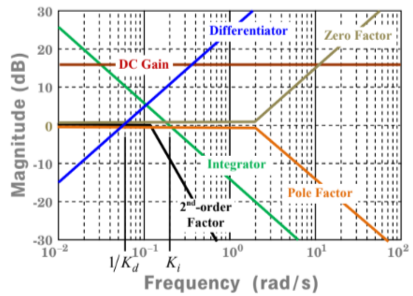
$\cos(\omega_0t)u(t)$: $+\frac{1}{4}\left[\delta(f+f_0)+\delta(f-f_0)\right]$

Chapter 8

1. $\tilde{H}(s)=K_{dc}$: DC gain (constant)
2. $\tilde{H}(s)=K_ds$: differentiator with gain K_d
3. $\tilde{H}(s)=K_i/s$: integrator with gain K_i
4. $\tilde{H}(s)=s/z_m+1$: zero factor with unity DC gain ($\tilde{H}(0)=1$)
5. $\tilde{H}(s)=\frac{1}{s/p_n+1}$: pole factor with unity DC gain
6. $\tilde{H}(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_ns+\omega_n^2}$: 2nd-order factor with unity DC gain

8.2 Asymptotic Behavior of Bode Plots





Asymptotic phase of phase plot

High frequency:

$$\text{Pole-zero excess}\times(-90^{\circ}) \tag{8.4a}$$

Low frequency:

$$\left[\text{No. of}\int dt-\text{No. of}\frac{d}{dt}\right]\times(-90^{\circ}) \tag{8.4b}$$

Asymptotic slope of magnitude plot

High frequency:

$$[\text{Pole-zero excess}]\times(-20\text{ dB/decade}) \tag{8.5a}$$

Low frequency:

$$\left[\text{No. of}\int dt-\text{No. of}\frac{d}{dt}\right]\times(-20\text{ dB/decade}) \tag{8.5a}$$

Chapter 9

9.1 Idealized LTI filters

Ideal Low-Pass Filter (LPF)

- Frequency response: $H(f)=A\,\mathrm{rect}\left(\frac{f}{2B}\right)$
- Impulse response: $h(t)=2AB\,\mathrm{sinc}\,(2Bt)$

Ideal Band-Pass Filter (BPF)

- Frequency response:
$H(f)=A\left[\mathrm{rect}\left(\frac{f+f_0}{B}\right)+\mathrm{rect}\left(\frac{f-f_0}{B}\right)\right]$
- Impulse response:
$h(t)=2AB\,\mathrm{sinc}\,(Bt)\cos(2\pi f_0t)$

9.2 Continuous-time Sampling and Reconstruction of Signals

Nyquist Sampling Theorem:

Nyquist sampling frequency / Nyquist rate $f_s=2f_m$

9.3 Sampling Bandpass Signal below Nyquist

(a) Overlapping spectral images ($f_c>0.5B$, symmetric about f_c and f_{-c})
$$f_s=2f_c/k;\quad k=1,2,\dots,\lfloor 2f_c/B\rfloor \tag{9.2a}$$
(b) Un-aliased spectral images ($f_c>1.5B$)
$$\frac{2f_c+B}{k+1}\leq f_s\leq\frac{2f_c-B}{k};\qquad k=1,2,\dots,\left\lfloor\frac{2f_c-B}{2B}\right\rfloor \tag{9.2b}$$