Chapter 1
1.2.2 Bounded signals
$x(t)$ is bounded if
$\exists M \, [(0 < M < \infty) \wedge (\forall t \, |x(t)| \leq M)]$
(got upper n lower range limit)
1.2.3 Absolutely integrable signals
$x(t)$ is absolutely integrable if
$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$
1.2.6 Energy and Power Signals
Energy signals
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad (1.3a)$$
$x(t)$ is an energy signal $\iff 0 < E < \infty$ (1.3b)
Power signals
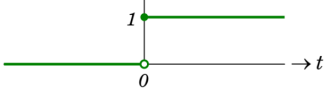$$P = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} |x(t)|^2 dt \qquad (1.4a)$$
$x(t)$ is a power signal $\iff 0 < P < \infty$ (1.4b)
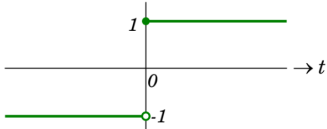If $x(t)$ is a periodic signal, average power may be computed by
$$\frac{1}{T} \int_0^T |x(t)|^2 dt$$

- Energy signals have 0 average power, bc E = finite implies P = 0
- Power signals have infinite total energy, bc P = finite implies $E = \infty$
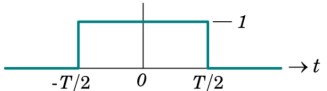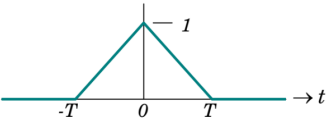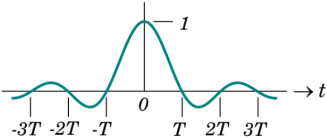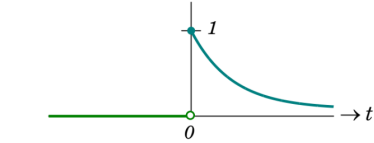- All bounded periodic signals are power signals

u(t):

sgn(t):

rect$\left(\frac{t}{T}\right)$:

tri$\left(\frac{t}{T}\right)$:
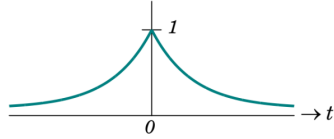
sinc$\left(\frac{t}{T}\right)$:

$e^{-\alpha t} u(t)$:

$e^{-\alpha|t|}$:

Chapter 2
2.1 Time-domain Operations
2.1.5 Convolution of 2 Signals
$x(t) * y(t) = \int_{-\infty}^{\infty} x(\alpha) y(t - \alpha) \, d\alpha$
Properties of Dirac-δ:
1. Symmetry: $\delta(t) = \delta(-t)$ (2.3)
2. Sampling:
   $x(t)\delta(t - \lambda) = x(\lambda)\delta(t - \lambda)$ (2.4)
3. Sifting
   $$\int_{-\infty}^{\infty} x(t)\delta(t - \lambda) dt = x(\lambda) \qquad (2.5)$$
4. Replication
   $x(t) * \delta(t - \lambda) = x(t - \lambda)$ (2.6)

Convolution with Dirac-δ Comb function
$$x_p(t) = x(t) * \sum_n \delta(t - nT)$$
$$= \sum_n x(t - nT)$$

Multiplication with the Dirac-δ Comb function
Used for sampling
$$x_s(t) = x(t) \times \sum_n \delta(t - nT)$$
$$= \sum_n x(t) \times \delta(t - nT)$$
$$= \sum_n x(nT)\delta(t - nT)$$

Chapter 3
3.2 Spectrum of a Sinusoid
Spectrum of a Complex Exponential Signal
$\tilde{x}(t) = \mu e^{j(2\pi f_0 t + \phi)} = \mu e^{j\phi} \times e^{j2\pi f_0 t}$,
Spectrum of a Cosine Signal
$\mu \cos(2\pi f_0 t + \phi)$
$$= \frac{\mu}{2} e^{j\phi} e^{j2\pi f_0 t} + \frac{\mu}{2} e^{j(-\phi)} e^{j2\pi(-f_0)t}$$
Spectrum of a Sine Signal
$$\mu \sin(2\pi f_0 t + \phi) = \frac{\mu}{2} e^{j(\phi - 0.5\pi)} e^{j2\pi f_0 t}$$
$$+ \frac{\mu}{2} e^{j(-\phi + 0.5\pi)} e^{j2\pi(-f_0)t}$$
Complex exponential Fourier Series
$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T_p}$$
$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_p t} \qquad (3.1a)$$
$$c_k = \frac{1}{T_p} \int_{t_0}^{t_0+T_p} x_p(t) e^{-j2\pi kt/T_p} dt, \, k \in \mathbb{Z} \qquad (3.1b)$$

Trigonometric Fourier Series
$$x_p(t) = a_0 + 2 \sum_{k=1}^{\infty} [a_k \cos(2\pi kt/T_p)$$
$$+ b_k \sin(2\pi kt/T_p)]$$
$$a_k = \frac{1}{T_p} \int_{t_0}^{t_0+T_p} x_p(t) \cos(2\pi kt/T_p) dt; k \geq 0$$
$$b_k = \frac{1}{T_p} \int_{t_0}^{t_0+T_p} x_p(t) \sin(2\pi kt/T_p) dt; k > 0$$
(3.2)

Chapter 4
4.1 Fourier Transform
Forward Fourier Transform
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \qquad (4.1a)$$
Inverse Fourier Transform
$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \qquad (4.1b)$$
Spectrum of exponentially decaying pulse
$$x(t) = A e^{-\alpha t} u(t)$$
$$\text{Assume } \alpha > 0$$
$$X(f) = \frac{A}{\alpha + j2\pi f}$$

4.3 Spectral properties of a REAL signal
- If $x(t)$ is **REAL** ($x^*(t) = x(t)$), then
  - $X(f)$ is conjugate symmetric ($X^*(f) = X(f)$)
  - $|X(f)|$ is even ($|X(f)| = |X(-f)|$)
  - $\angle X(f)$ is odd ($\angle X(f) = -\angle X(-f)$)
- If $x(t)$ is **REAL** and **EVEN** ($x^*(t) = x(t) \wedge x(-t) = x(t)$), then
  - $X(f)$ is real ($X^* f = X(f)$)
  - $X(f)$ is even ($X(-f) = X(f)$)
- If $x(t)$ is **REAL** and **ODD** ($x^*(t) = x(t) \wedge x(-t) = -x(t)$), then
  - $X(f)$ is imaginary ($X^*(f) = -X(f)$)
  - $X(f)$ is odd ($X(-f) = -X(f)$)

The above can apply to Fourier series coefficients of periodic signals too:
- $x_p(t)$ is **REAL**
  - c_k is conjugate symmetric ($c_k^* = c_{-k}$)
  - $|c_k|$ has even symmetry ($|c_k| = |c_{-k}|$)
  - $\angle c_k$ has odd symmetry ($\angle c_k = -\angle c_{-k}$)
- $x_p(t)$ is **REAL** and **EVEN**
  - c_k is real ($c_k^* = c_k$)
  - c_k is even ($c_k = c_{-k}$)
- $x_p(t)$ is **REAL** and **ODD**
  - c_k is imaginary ($c_k^* = -c_k$)
  - c_k is odd ($c_k = -c_{-k}$)

4.4 Spectrum of Signals that are not Absolutely Integrable
$$\Im\{K\delta(t)\} = \int_{-\infty}^{\infty} K\delta(t) e^{-j2\pi ft} dt = K \quad (4.13)$$
By duality, $\Im\{K\} = K\delta(f)$
4.4.1 Spectrm of Unit Step and Signum function
$$\Im\{u(t)\} = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$
$$\Im\{\text{Sgn}(t)\} = \frac{1}{j\pi f}$$
4.4.2 Continuous-Frequency Spectrum of Periodic Signals
The following make use of the fact that
$$\Im\{k\} = K\delta(f) \qquad (4.14)$$
DC
$$x_{dc}(t) = K$$
$$X_{dc}(f) = \Im\{k\} = K\Im\{1\} = K\delta(f)$$

Complex Exponential
$$\tilde{x}(t) = K e^{j2\pi f_0 t}$$
$$\tilde{X}(f) = \Im\{K e^{j2\pi f_0 t}\} = K\delta(f - f_0)$$
Cosine
$$\Im\{K\cos(2\pi f_0 t)\}$$
$$= \frac{K}{2}\delta(f - f_0) + \frac{K}{2}\delta(f + f_0)$$
Sine
$$\Im\{K\sin(2\pi f_0 t)\}$$
$$= \frac{K}{j2}\delta(f - f_0) - \frac{K}{j2}\delta(f + f_0)$$
where
$$\begin{cases} |X_s(f)| &= \frac{K}{2}\delta(f - f_0) \\ &+ \frac{K}{2}\delta(f + f_0) \\ \angle X_s(f) &= \begin{cases} -\pi/2, & f = f_0 \\ \pi/2, & f = -f_0 \end{cases} \end{cases}$$

Arbitrary periodic signals
Let $x_p(t)$ be a periodic signal with period T_p and fundamental frequency f_p
$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k \delta(f - kf_p) \qquad (4.16)$$
4.4.2.1 Spectrum of Dirac-δ Comb function
$$\text{comb}_\lambda(t) \triangleq \sum_n \delta(t - n\lambda)$$
$$c_k = \frac{1}{\lambda}$$
$$\Im\{\text{comb}_\lambda(t)\} = \text{COMB}_\lambda(f)$$
$$= \frac{1}{\lambda}\sum_k \delta(f - k/\lambda)$$

Chapter 5
5.1 Energy Spectral Density (ESD)
Total energy of a signal $x(t)$ is defined as
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \text{ (Joules)} \qquad (5.1)$$
Rayleigh Energy Theorem
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df, \quad (5.2)$$
Energy Spectral Density
$$E_x(f) = |X(f)|^2 \text{ Joules Hz}^{-1} \qquad (5.3)$$
Properties of $E_x(f)$
1. $E_x(f)$ is a real function of f
2. $E_x(f) \geq 0 \quad \forall f$
3. $E_x(f)$ is an even function of f if $x(t)$ is real.
5.2 Power Spectral Density (PSD)
In the time-domain, the average power of a signal $x(t)$ is defined as
$$P = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} |x(t)|^2 dt \qquad (5.4)$$
Windowed version of $x(t)$:
$$x_W(t) = x(t)\text{rect}\left(\frac{t}{2W}\right) \qquad (5.5)$$
Parseval Power Theorem
$$P = \lim_{W \to \infty} \frac{1}{2W} \int_{-W}^{W} |x(t)|^2 dt$$
$$= \int_{-\infty}^{\infty} \lim_{W \to \infty} \frac{1}{2W} |X_W(f)|^2 df \qquad (5.9)$$
Power Spectral Density
$$P_x(f) = \lim_{W \to \infty} \frac{1}{2W} |X_W(f)|^2 \text{ Watts Hz}^{-1}$$
$$\qquad (5.10)$$
Properties of $P_x(f)$
1. $P_x(f)$ is a real function of f
2. $P_x(f) \geq 0 \quad \forall f$
3. $P_x(f)$ is an even function of f if $x(t)$ is real.
5.2.1 PSD of Periodic Signals

From chapter 4 equation 4.16:
$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k \delta(f - kf_p)$$
PSD of $x_p(t)$
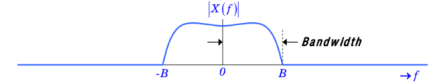$$P_x(f) = \sum_{k=-\infty}^{\infty} |c_k|^2 \delta(f - kf_p) \qquad (5.12)$$
Average power of $x_p(t)$
$$P = \int_{-\infty}^{\infty} P_x(f) df = \sum_{k=-\infty}^{\infty} |c_k|^2 \qquad (5.13)$$
5.3 Bandwidth
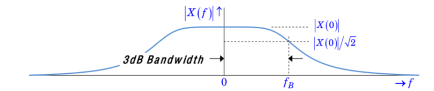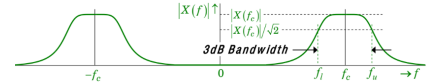### 5.3.1 Bandlimited Signals
Lowpass signal

Bandpass signal

5.3.2 Signals with Unrestricted Band
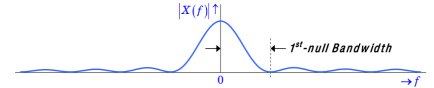#### 5.3.2.1 3dB Bandwidth
Lowpass signal:

Bandpass signal:

5.3.2.2 1st-null Bandwidth
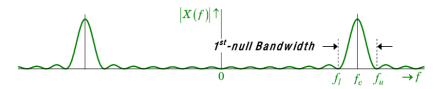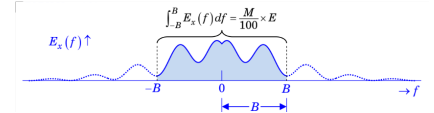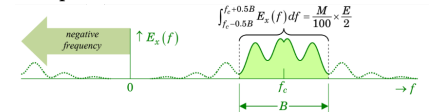**Lowpass signal:**

Bandpass signal:

5.3.2.3 M% Energy Containment Bandwidth
Smallest bandwidth that contains at least M% of the total signal energy $E = \int_{-\infty}^{\infty} E_x(f) \, df$
Lowpass:
$\int_{-B}^{B} E_x(f) df = \frac{M}{100} \times E$

Bandpass:
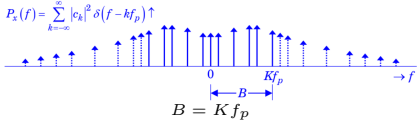$\int_{f_c-0.5B}^{f_c+0.5B} E_x(f) df = \frac{M}{100} \times \frac{E}{2}$

5.3.2.4 M% Power Containment Bandwidth

The smallest bandwidth that contains at least M% of the average signal power. For a periodic signal, the aerage power is given by

$$P = \int_{-\infty}^{\infty} P_x(f)df = \sum_{k=-\infty}^{\infty} |c_k|^2$$

where f_p(Hz) is the fundamental frequency and c_k's are the Fourier series coefficients.

$$B = Kf_p$$

where K is the smallest positive integer that satisfies

$$\sum_{k=-k}^{K} |c_k|^2 \geq \frac{M}{100} \times P$$

Chapter 6
6.1 Systems
6.2 Classification of Systems
6.2.1 Systems with Memory and Without Memory
Memoryless: output at a given time is dependent on only the input at that time.
Otherwise, the system has memory / is dynamic.
6.2.2 Causal and Noncausal Systems
Causal (or non-anticipative): Its output, $y(t)$, at the present time depends on only the present and/or past values of its input, $x(t)$.
∴ not possible for a causal system to produce an output before an input is applied. ∴ $\forall t < 0\ y(t) = 0$.
6.2.3 Stable and Unstable Systems
BIBO stable (bounded-input/bounded-output): For every bounded input $x(t)$ where

$$\forall t\ |x(t)| \leq k \tag{6.2}$$

the system produces a bounded output $y(t)$ where

$$\forall t\ |y(t)| \leq L \tag{6.3}$$

in which K and L are positive constants.
6.2.4 Linear and Nonlinear Systems
Linear system satisfies the following:

$$\mathbf{T}[\alpha_1 x_1(t) + \alpha_2 x_2(t)]$$
$$= \alpha_1 \mathbf{T}[x_1(t)] + \alpha_2 \mathbf{T}[x_2(t)] \tag{6.6}$$
$$= \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

(6.6) is known as the superposition property.
Important property of linear systems:
$$x(t) = 0 \implies y(t) = 0$$
6.2.5 Time-Invariant and Time-Varying Systems
Time-invariant: a time shift (delay or advance) in the input signal, $x(t)$, causes the same time shift in the output signal, $y(t)$.

$$\mathbf{T}[x(t - \tau)] = y(t - \tau) \tag{6.7}$$

A time-varying system is one which does not satisfy (6.7).
Laplace Transform

$$\bar{F}(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt \tag{6.8}$$

where s is a complex variable.
Inverse Laplace Transform

$$f(t) = \mathcal{L}^{-1}\{\bar{F}(s)\} = \frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty} \bar{F}(s)ds \tag{6.9}$$

$$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$$
$$\mathcal{L}\{y'''\} = s^3\mathcal{L}\{y\} - s^2y(0) - sy'(0)$$
$$\qquad - y''(0)$$
$$\mathcal{L}\{y''''\} = s^4\mathcal{L}\{y\} - s^3y(0) - s^2y'(0)$$
$$\qquad - sy''(0) - y'''(0)$$

Chapter 7
7.1 Impulse Response
Impulse response, $h(t)$: The response/output when the input is a unit impulse, $\delta(t)$.

$$\delta(t) \to \text{LTI system} \to h(t)$$

where

$$h(t) = \mathbf{T}[\delta(t)] \tag{7.1}$$
$$\mathbf{T}[x(t)] = y(t) = x(t) * h(t) \tag{7.5}$$

7.1.1 Step Response
Step response: the output of the system when input is unit step function

$$u(t) \to h(t) \to o(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau$$
$$= \int_{-\infty}^{t} h(\tau)d\tau$$

Step response equals integration of impulse response:

$$o(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

Impulse response equals differentiation of step response:

$$h(t) = \frac{d}{dt}o(t)$$

7.2 Frequency Response
Frequency response ($H(f)$): The Fourier transform of the system impulse response $h(t)$

$$H(f) = \Im\{h(t)\} = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt \tag{7.6}$$
$$Y(f) = X(f) \cdot H(f) \tag{7.7}$$
$$H(f) = |H(f)|e^{j\angle H(f)} \tag{7.8}$$

where $|H(f)|$ is called the magnitude response and $\angle H(f)$ is called the phase response of the system.
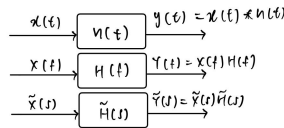### 7.3 Transfer Function
Transfer function $\tilde{H}(s)$: Laplace transform of $h(t)$

$$\tilde{H}(s) = \mathcal{L}\{h(t)\} = \int_0^\infty h(t)e^{-st}dt \tag{7.9}$$

where $s = \sigma + j\omega$ is a complex variable.

$$y(t) = x(t) * h(t)$$
$$\tilde{Y}(s) = \tilde{X}(s) \cdot \tilde{H}(s) \tag{7.10}$$

7.4 Relationship between Transfer Function and Frequency Response
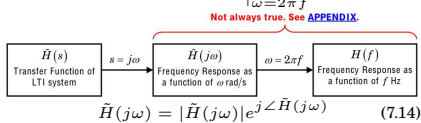Substituting $s = j\omega$ into (7.9), we get

$$\tilde{H}(s)\Big|_{s=j\omega} = \tilde{H}(j\omega) = \int_0^\infty h(t)e^{-j\omega t}dt \tag{7.11}$$

Sub $\omega = 2\pi f$ into (7.11):

$$\tilde{H}(j\omega)\Big|_{\omega=2\pi f} = \int_0^\infty h(t)e^{-j2\pi ft}dt \tag{7.12}$$

For causal LTI systems, $\forall t < 0\ h(t) = 0$. Hence (7.6) and (7.12) are equivalent.

$$H(f) = \tilde{H}(j\omega)\Big|_{\omega=2\pi f} \tag{7.13}$$

$$\tilde{H}(j\omega) = |\tilde{H}(j\omega)|e^{j\angle\tilde{H}(j\omega)} \tag{7.14}$$

where $|\tilde{H}(j\omega)|$ is called the magnitude response and $\angle\tilde{H}(j\omega)$ is called the phase response of the system.
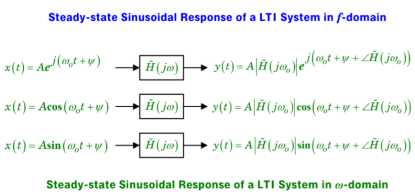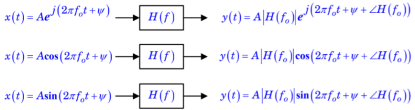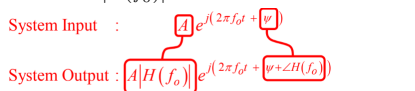### 7.4 Sinusoidal Response at Steady-State
Let system input at steady-state be

$$x(t) = Ae^{j(2\pi f_0 t + \psi)} \tag{7.15}$$

Then

$$X(f) = Ae^{j\psi}\delta(f - f_0) \tag{7.16}$$

$$Y(f) = A|H(f_0)|e^{j(\psi+\angle H(f_0))}\delta(f - f_0) \tag{7.17}$$

$$y(t) = \Im^{-1}\{Y(f)\}$$
$$= A|H(f_0)|e^{j(2\pi f_0 t + \psi + \angle H(f_0))} \tag{7.18}$$

System Input : $Ae^{j(2\pi f_0 t + \boxed{\psi})}$

System Output : $\boxed{A|H(f_0)|}e^{j(2\pi f_0 t + \boxed{\psi + \angle H(f_0)})}$

Steady-state Sinusoidal Response of a LTI System in f-domain

Steady-state Sinusoidal Response of a LTI System in ω-domain

7.6 LTI Systems Described by Differential Equations
LTI systems represented by linear constant-coefficient differential equations have the general form

$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m} \tag{7.21}$$

where $x(t)$ is input, $y(t)$ is output, and a_n, b_m are real constants.
7.6.1 Transfer Function

$$\tilde{H}(s) = K\frac{\left(\frac{s}{z_1}+1\right)\left(\frac{s}{z_2}+1\right)\cdots\left(\frac{s}{z_M}+1\right)}{\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\cdots\left(\frac{s}{p_N}+1\right)}$$

$$K = \frac{a_0}{b_0}$$
$$\tag{7.23b}$$

$$\tilde{H}(s) = K'\frac{(s+z_1)(s+z_2)\cdots(s+z_M)}{(s+p_1)(s+p_2)\cdots(s+p_N)}$$
$$\tag{7.23c}$$

$$K = \frac{b_M}{a_N}$$

$\forall n \in \{1, 2, \ldots, N\}$
- $\tilde{H}(-p_n) = \infty$
- $-p_n$ are called **poles** of $\tilde{H}(s)$

$\forall m \in \{1, 2, \ldots, M\}$
- $\tilde{H}(-z_m) = 0$
- $-z_m$ are called **zeros** of $\tilde{H}(s)$

The system is said to have N poles and M zeros, and the difference $N - M$ is called pole-zero excess.
7.6.2 System Stability
BIBO Stable
- All system poles lying on the left-half s-plane
- $h(t)$ will converge to 0 as t tends to infinity
$\lim_{t\to\infty} h(t) = 0$
Marginally Stable
- One or more **non-repeated** system poles lying on the imaginary axis of the s-plane and no system pole lying on the right half s-plane.
- $h(t)$ will not "blow up" and become unbounded, but neither will it converge to zero as t tends to infinity.
$\lim_{t\to\infty} |h(t)| \neq \infty$ and $\lim_{t\to\infty} h(t) \neq 0$
Unstable (Case 1)
- One or more system poles lying on the right-half s-plane

- $h(t)$ will "blow up" and become unbounded as t tends to infinity
$\lim_{t\to\infty} |h(t)| = \infty$
Unstable (Case 2)
- One or more repeated system poles lying on the imaginary axis
- $h(t)$ will "blow up" and become unbounded as t tends to infinity
$\lim_{t\to\infty} |h(t)| = \infty$
7.7 First Order System (Standard Form)
7.7.1 Differential Eqn, Transfer Func, Impulse Response and Step Response
- Differential equation:

$$T\frac{dy(t)}{dt} + y(t) = Kx(t) \tag{7.26}$$

where
  - $x(t)$: system input
  - $y(t)$: system output
  - K: DC gain
  - T: time-constant
- Transfer Function $\tilde{H}(s)$:

$$Ts\tilde{Y}(s) + \tilde{Y}(s) = K\tilde{X}(s)$$
$$\to \tilde{H}(s) = \frac{\tilde{Y}(s)}{\tilde{X}(s)} = \frac{K}{Ts+1} \tag{7.27}$$

Pole: $s_1 = -\frac{1}{T}$
- Impulse Response $h(t)$

$$h(t) = \mathcal{L}^{-1}\{\tilde{H}(s)\} = \frac{K}{T}e^{-t/T}u(t)$$

- Step Response $o(t)$

$$o(t) = \int_{-\infty}^{t} h(\tau)d\tau = \mathcal{L}^{-1}\left\{\frac{1}{s}\tilde{H}(s)\right\}$$
$$= K\left[1 - e^{-t/T}\right]u(t)$$

7.8 Second Order System (Standard Form) 7.8.1 Differential Eqn and Transfer Func
- Differential equation:

$$\frac{d^2 y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = K\omega_n^2 x(t) \tag{7.28}$$

where
  - $x(t)$: system input
  - $y(t)$: system output
  - ζ: damping ratio
  - ω_n: undamped natural frequency (when $\zeta < 1$)
  - K: DC gain
- Transfer function $\tilde{H}(s)$

$$s^2\tilde{Y}(s) + 2\zeta\omega_n s\tilde{Y}(s) + \omega_n^2\tilde{Y}(s) = K\omega_n^2\tilde{X}(s)$$
$$\implies \tilde{H}(s) = \frac{\tilde{Y}(s)}{\tilde{X}(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Poles: $s_{1,2} = -\omega_n\zeta \pm \omega_n\left(\zeta^2 - 1\right)^{1/2}$
- Overdamped system: distinct real poles, $\zeta > 1$
- Critically damped system: repeated real poles, $\zeta = 1$
- Underdamped system: conjugate complex poles, $0 < \zeta < 1$
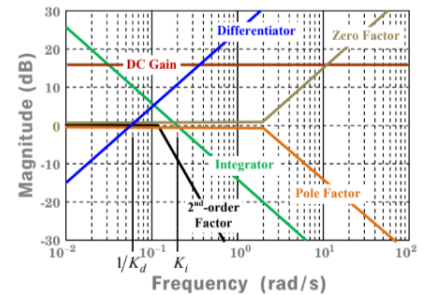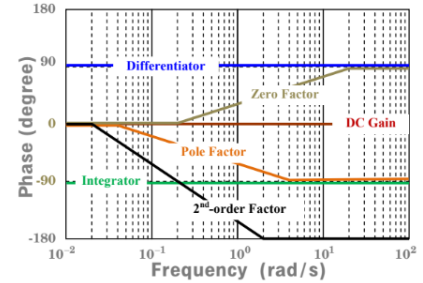- Undamped system: conjugate imaginary poles, $\zeta = 0$
Blue stuff:
$$\sin(\omega_0 t)u(t): +\frac{j}{4}[\delta(f + f_0) - \delta(f - f_0)]$$
$$\cos(\omega_0 t)u(t): +\frac{1}{4}[\delta(f + f_0) + \delta(f - f_0)]$$
Chapter 8
1. $\tilde{H}(s) = K_{dc}$: DC gain (constant)
2. $\tilde{H}(s) = K_d s$: differentiator with gain K_d
3. $\tilde{H}(s) = K_i/s$: integrator with gain K_i
4. $\tilde{H}(s) = s/z_m + 1$: zero factor with unity DC gain ($\tilde{H}(0) = 1$)
5. $\tilde{H}(s) = \frac{1}{s/p_n+1}$: pole factor with unity DC gain
6. $\tilde{H}(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$: 2nd-order factor with unity DC gain

8.2 Asymptotic Behavior of Bode Plots

Asymptotic phase of phase plot
High frequency:
$$\text{Pole-zero excess} \times (-90°) \tag{8.4a}$$
Low frequency:
$$\left[\text{No. of }\int dt - \text{No. of }\frac{d}{dt}\right] \times (-90°) \tag{8.4b}$$
Asymptotic slope of magnitude plot
High frequency:
$$[\text{Pole-zero excess}] \times (-20\text{ dB/decade}) \tag{8.5a}$$
Low frequency:
$$\left[\text{No. of }\int dt - \text{No. of }\frac{d}{dt}\right] \times (-20\text{ dB/decade}) \tag{8.5b}$$

Chapter 9
9.1 Idealized LTI filters
Ideal Low-Pass Filter (LPF)
- Frequency response: $H(f) = A\operatorname{rect}\left(\frac{f}{2B}\right)$
- Impulse response: $h(t) = 2AB\operatorname{sinc}(2Bt)$
Ideal Band-Pass Filter (BPF)
- Frequency response:
$$H(f) = A\left[\operatorname{rect}\left(\frac{f+f_0}{B}\right) + \operatorname{rect}\left(\frac{f-f_0}{B}\right)\right]$$
- Impulse response:
$$h(t) = 2AB\operatorname{sinc}(Bt)\cos(2\pi f_0 t)$$
9.2 Continuous-time Sampling and Reconstruction of Signals
Nyquist Sampling Theorem:
Nyquist sampling frequency / Nyquist rate $f_s = 2f_m$
9.3 Sampling Bandpass Signal below Nyquist
(a) Overlapping spectral images ($f_c > 0.5B$, symmetric about f_c and f_{-c})
$$f_s = 2f_c/k; \quad k = 1, 2, \ldots, \lfloor 2f_c/B \rfloor \tag{9.2a}$$
(b) Un-aliased spectral images ($f_c > 1.5B$)

$$\frac{2f_c + B}{k + 1} \leq f_s \leq \frac{2f_c - B}{k};$$
$$k = 1, 2, \ldots, \left\lfloor\frac{2f_c - B}{2B}\right\rfloor \tag{9.2b}$$