论题 1-10 作业

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1 [UD] Problem 13.3

(a) No. Because both $(1,\sqrt{3})$ and $(1,-\sqrt{3})$ are elements of f, however, $\sqrt{3} \neq -\sqrt{3}$.

(b) No. Because for $x = 0$, there does not exist $y \in \mathbb{R}$, such that $y = 1/(x+1)$.

(c) Yes. Because for all $(x,y) \in \mathbb{R}^2$, there exists a unique real number z such that $z = x+y$.

(d) Yes. Because for every closed interval of real numbers $[a,b]$, there exists a unique real number a, such that $([a,b],a) \in f$.

(e) Yes. Because for every $(n,m) \in \mathbb{N}\times\mathbb{N}$, there exists a unique real number m, such that $((n,m),m) \in f$.

(f) Yes. Because for every real number x, there exists a real number y, such that $y = 0$ when $x \geq 0$ or $y = x$ when $x < 0$, i.e. $(x,y) \in f$.

(g) No. Because both $(6,7)$ and $(6,5)$ are elements of f, however, $7 \neq 5$.

(h) Yes. Because for every circle c in the plane $\mathbb{R}^2$, there exists a unique real number C, such that C is the circumference of c.

(i) Yes. Because for every polynomial with real coefficients p, p is differentiable, thus there exists a unique polynomial p', such that p' is the derivative of p.

(j) Yes. Because for every polynomial p, p is integrable on $[0,1]$, thus there exists a unique number I such that $I = \int_0^1 p(x)dx$.

2 [UD] Problem 13.4

We know that $A\cap\mathbb{N}$ is either an empty or a nonempty set. In the case that $A\cap\mathbb{N}$ is empty, there exists a unique integer -1, such that $(A,-1) \in f$. In the case that $A\cap\mathbb{N}$ is nonempty, $A\cap\mathbb{N}$ is a subset of $\mathbb{N}$. By well-ordering principle of $\mathbb{N}$, $\min(A\cap\mathbb{N})$ exists, so there exist a unique integer $\min(A\cap\mathbb{N})$, such that $(A,\min(A\cap\mathbb{N})) \in f$. Therefore f is a well-defined function.

3 [UD] Problem 13.5

(a) For all $x \in X$, either $x \in A$ or $x \in X \setminus A$ holds, so there exists a unique number y ($y = 1$ when $x \in A$ and $y = 0$ when $x \in X \setminus A$), such that $y = \chi_A$. Therefore χ_A is a function.

(b) The domain is X. The range is $\{0\}$ when $A = \varnothing$, $\{1\}$ when $A = X$, and $\{0,1\}$ when $A \neq \varnothing$ and $A \neq X$.

4 [UD] Problem 13.7

For every real number $y \neq 1/2$, let $(x-5)/(2x-3) = y$, and we get $x = (3y-5)/(2y-1) \neq 3/2$, which is an element of the domain. So $\operatorname{ran}(f) = \mathbb{R} \setminus \{1/2\}$. $\square$

5 [UD] Problem 13.11

No. For every $x \in A$, there may not exist y such that $(x,y) \in f$. Even though for every $x \in A$ there exists y such that $(x,y) \in f$, we cannot make sure that there only exists one y such that $(x,y) \in f$.

6 [UD] Problem 13.13

The only relation is $\{(x,y) \in X^2 : x = y\}$. By the reflexion of the equivalence, any relation on X is superset of $\{(x,y) \in X^2 : x = y\}$. Assume there exists relation X' such that $X' \setminus X \neq \varnothing$, let (a,b) be an element of X' such that $a \neq b$. However, (b,b) is an element of X' but $a \neq b$, so X' is not a function.

7 [UD] Problem 14.8

(a) Not one-to-one. $f(1) = f(-1) = 1/2$ but $1 \neq -1$.
Not onto. The range is $(0,1]$.

(b) Not one-to-one. $sin 0 = sin \pi = 0$ but $0 \neq \pi$.
Not onto. The range is $[-1,1]$.

(c) Not one-to-one. $f(1,2) = f(2,1) = 2$ but $(1,2) \neq (2,1)$.
Onto.

(d) Not one-to-one. $f((1,0),(0,0)) = f((0,0),(0,0)) = 0$ but $((1,0),(0,0)) \neq ((0,0),(0,0))$.
Onto.

(e) Not one-to-one. $f((0,0),(0,0)) = f((1,1),(1,1)) = 0$ but $((0,0),(0,0)) \neq ((1,1),(1,1))$.
Not onto. The range is $[0,+\infty)$.

(f) One-to-one.

Not onto. The range is $A \times \{b\}$.

(g) One-to-one.

Onto.

(h) Not one-to-one. $f(X) = f(B) = B$ but $X \neq B$.

Not onto. The range is $\mathcal{P}(X \setminus B)$.

(i) One-to-one.

Not onto. The range is $(0, +\infty)$.

8 [UD] Problem 14.12

$f(x) = \dfrac{(d-c)x + cb - da}{b-a} (x \in [a,b])$.

One-to-one: Let $f(x_1) = f(x_2)$, we have $\dfrac{(d-c)x_1 + cb - da}{b-a} = \dfrac{(d-c)x_2 + cb - da}{b-a}$. Multiplying $b-a$ and cancelling on both sides, we have $x_1 = x_2$.

Onto: Let $c \leq f(x) \leq d$, that is $c \leq \dfrac{(d-c)x + cb - da}{b-a} \leq d$. Multiplying $b-a$ and cancelling on both sides, we have $a \leq x \leq b$. It means, for every $x \in [a,b]$, there exists y, such that $y = f(x)$, thus $f(x)$ is onto.

Since $f(x)$ is both one-to-one and onto, $f(x)$ is a bijection. $\square$

9 [UD] Problem 14.13

ϕ is a function from $F([0,1])$ to $\mathbb{R}$. Because for all $f \in F([0,1])$, there exists a unique real number y, such that $y = f(0)$.

ϕ is not one-to-one. Let $f_1(x) = 0 \in F([0,1])$, $f_2(x) = x \in F([0,1])$, we have that $\phi(f_1) = \phi(f_2)$, however, $f_1 \neq f_2$ because $f_1(1) \neq f_2(1)$.

ϕ is onto. For every real number a, there exists $f_0(x) = a \in F([0,1])$, such that $\phi(f_0) = a$.

10 [UD] Problem 14.15

For all $x \in \mathbb{R}$, since $f(x)$ is defined on $\mathbb{R}$, there exists a unique real number $y = f(x) \cdot f(x)$, such that $y = (f \cdot f)(x)$, therefore $f \cdot f$ is a function. $\square$

(a) Yes. $f(x) = e^x$.

(b) No. $\mathrm{ran}(f \cdot f) = \{a^2 : a \in \mathrm{ran}(f)\}$.

11 [UD] Problem 15.1

	$(f\circ g)(x)$	$\mathrm{dom}(f\circ g)$	$\mathrm{ran}(f\circ g)$	$(g\circ f)(x)$	$\mathrm{dom}(g\circ f)$	$\mathrm{ran}(g\circ f)$
(a)	$1/(1+x^2)$	$\mathbb{R}$	$(0,1]$	$1/(1+x)^2$	$\mathbb{R}\setminus\{-1\}$	$\mathbb{R}^+$
(b)	x	$\mathbb{R}^+$	$\mathbb{R}^+$	$\lvert x\rvert$	$\mathbb{R}$	$[0,+\infty)$
(c)	$1/(x^2+1)$	$\mathbb{R}$	$(0,1]$	$(1/x^2)+1$	$\mathbb{R}\setminus\{0\}$	$(1,+\infty)$
(d)	$\lvert x\rvert$	$\mathbb{R}$	$[0,+\infty)$	$\lvert x\rvert$	$\mathbb{R}$	$[0,+\infty)$

12 [UD] Problem 15.6

(a) $$(f\circ g)(x)=f(g(x))=\frac{\dfrac{3+2x}{1-x}-3}{\dfrac{3+2x}{1-x}+2}=\frac{\dfrac{5x}{1-x}}{\dfrac{5}{1-x}}=x,$$

$$(g\circ f)(x)=g(f(x))=\frac{3+2\dfrac{x-3}{x+2}}{1-\dfrac{x-3}{x+2}}=\frac{\dfrac{5x}{x+2}}{\dfrac{5}{x+2}}=x.$$

(b) (Theorem 15.4) Let $f:A\to B$ be a bijective function, and f^{-1} be the inverse of f, then $f\circ g=i_B$, and $g\circ f=i_A$.

13 [UD] Problem 15.7

14 [UD] Problem 15.11

15 [UD] Problem 15.12

16 [UD] Problem 15.13

17 [UD] Problem 15.14

18 [UD] Problem 15.15

19 [UD] Problem 15.20

20 [UD] Problem 16.19

21 [UD] Problem 16.20

22 [UD] Problem 16.21

23 [UD] Problem 16.22