论题 1-11 作业

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1 [DH] Problem 4.1

(a) $S \leftarrow 0$;
for i going from 1 to N do the following:
    if $A[i,1] > A[A[i,2],1]$ then do the following:
        $S \leftarrow S + A[i,1]$;
output S.

(b) Suppose the root of the binary tree is R.

$S \leftarrow 0$;
$P \leftarrow R$;
$N \leftarrow$ the content of the first offspring of R;
if the content of $R >$ **get the salary of Nth employee** do the following:
    $S \leftarrow S$ + the content of R;
while P has a second offspring do the following:
    $P \leftarrow$ the second offspring of P;
    $N \leftarrow$ the content of the first offspring of S;
    if the content of $P >$ **get the salary of Nth employee** do the following:
        $S \leftarrow S$ + the content of P;
output S.

subroutine **get the salary of Nth employee**
$T \leftarrow R$;
do the following $N-1$ times:
    $T \leftarrow$ the second offspring of T;
$T \leftarrow$ the second offspring of T;
return the content of T;

2 [DH] Problem 4.2

(a) $S \leftarrow 0$;

call **add**(T, 0);

output S.

subroutine **add**(P, x)

$S \leftarrow S + x$;

$N \leftarrow 1$;

while P has an Nth offspring do the following:

call **add**(the Nth offspring of P, $x+1$);

$N \leftarrow N + 1$;

return.

(b) $S \leftarrow 0$;

call **count**(T, 0);

output S;

subroutine **count**(P, x)

if $x = K$ then do the following:

$S \leftarrow S + 1$;

return;

$N \leftarrow 1$;

while P has an Nth offspring do the following:

call **count**(the Nth offspring of P, $x+1$);

$N \leftarrow N + 1$;

return.

(c) $R \leftarrow$ false;

call **check**(T, 0);

output R.

subroutine **check**(P, x)

if x is even then do the following:

if P doesn't have a first offspring then do the following:

$R \leftarrow$ true;

return;

$N \leftarrow 1$;

while P has an Nth offspring do the following:

call **check**(the Nth offspring of P, $x+1$);

$N \leftarrow N+1$;

return.

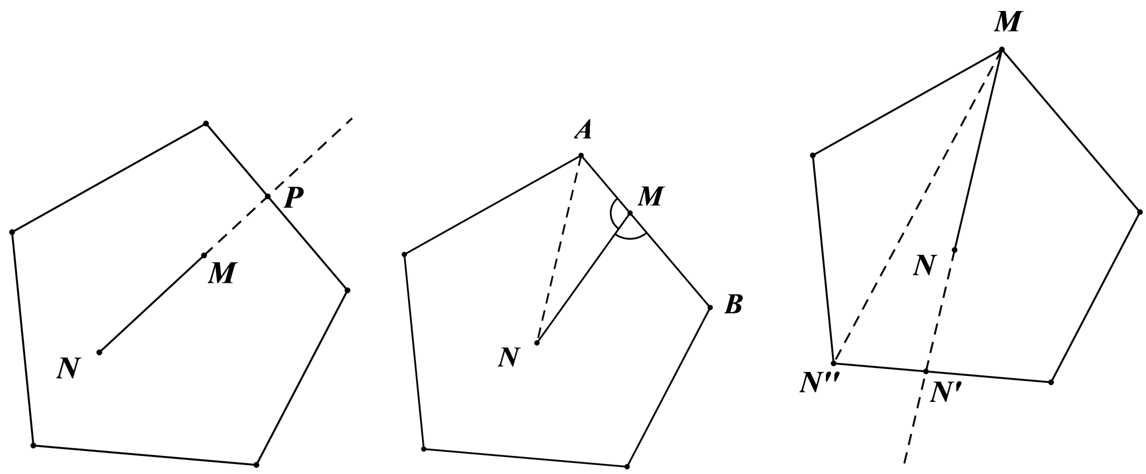
3 [DH] Problem 4.8

Suppose that the maximal distance between any two points on a polygon occurs between M and N. First, regard N as an arbitrary fixed point, and consider point M.

Case 1: M is in the polygon. Extend NM cutting the polygon at E (Figure 2(a)). NP is longer than than NM.

Case 2: M is on one edge of the polygon, but M is not a vertex (Figure 2(b)). Let the edge where M is on be AB. At least one of $\angle NMA$ and $\angle NMB$ is not less than 90 degrees. Assume, WLOG, that $\angle NMA \geq 90^\circ$. By the law of sines, we get $NA > NM$.

Now, we have proved that for arbitrary N, the length of NM is maximal when M is a vertex of the polygon. Consider point N, we can prove that the length of MN is maximal when N is a vertex of the polygon likewise (Figure 2(c)). Hence, the maximal distance between any two points on a polygon occurs between two of the vertices. □



(a) (b) (c)

Figure 2: the distance of two points on a polygon

4 [DH] Problem 4.9

Language: C++

The first line of the input contains a positive integer n, giving the number of the vertices of the polygon. The following $n+1$ lines of the input contains the coordinates of the vertices. The x-coordinate and y-coordinate are separated by a space.

```
#include <iostream>
#include <cmath>
#include <algorithm>
using namespace std;

int n;
double x[1000], y[1000];

double dist(int i1, int i2)
{
    return hypot(x[i1 % n] - x[i2 % n], y[i1 % n] - y[i2 % n]);
}

int main()
{
    double ans = 0;
    int j, k;
    cin >> n;
    for (int i = 0; i < n; i++)
        cin >> x[i] >> y[i];
    j = n;
    k = n + 1;
    while (k < 2*n)
    {
        while (!(dist(j, k) > dist(j, k - 1) && dist(j, k) > dist(j,
k + 1)))
            k++;
        ans = max(ans, dist(j, k));
        j++;
    }
    cout << ans << endl;
    return 0;
}
```

5 [DH] Problem 4.11

Suppose the vector is named V.

(a) $M_1 \leftarrow$ **find maximum of first** N **elements**;

$I \leftarrow 1$;

do the following while $V[I] \neq M_1$:

  $I \leftarrow I+1$;

for I going from I to $N-1$ do the following:

  $V[I] \leftarrow V[I+1]$;

$M_2 \leftarrow$ **find maximum of first** $N-1$ **elements**;

output M_1, M_2.

subroutine **find maximum of first** n **elements**

  $A \leftarrow V[1]$;

  for i going from 2 to n do the following:

    if $V[i] > A$ do the following:

      $A \leftarrow V[i]$;

  return A.

(b) $M_1 \leftarrow$ **find maximum from** 1**th to** N**th element**;

$I \leftarrow 1$;

do the following while $V[I] \neq M_1$:

  $I \leftarrow I+1$;

for I going from I to $N-1$ do the following:

  $V[I] \leftarrow V[I+1]$;

$M_2 \leftarrow$ **find maximum from** 1**th to** $(N-1)$**th element**;

output M_1, M_2.

subroutine **find maximum from** m**th to** n**th element**;

  if $m = n$ then do the following:

    return $V[m]$;

  $p \leftarrow \lfloor (m+n)/2 \rfloor$;

  $T_1 \leftarrow$ **find maximum from** m**th to** p**th element**;

  $T_2 \leftarrow$ **find maximum from** $(p+1)$**th to** n**th element**;

  if $T_1 > T_2$ do the following:

    return T_1;

  otherwise do the following:

    return T_2.

6 [DH] Problem 4.12

Suppose there are M nodes and N edges in the graph, the nodes are numbered from 1 to M and the edges are stored in vector V. Every edge T support three operations: get the number of the first node it connects(T.first), get the number of the second node it connects(T.second) and get the length of the node(T.length). Let U be an empty vector of integers. The output is the edges constituting the minimal spanning tree.

call **initialize**;
$m \leftarrow 0$;
$i \leftarrow 1$;
while $m < M - 1$ do the following:
    if **find** $V[i]$.first $\neq$ **find** $V[i]$.second then do the following:
        call **union** $V[i]$.first **and** y.first;
        output $V[i]$;
        $m \leftarrow m + 1$;
    $i \leftarrow i + 1$.

subroutine **initialize**
    for i going from 1 to N do the following:
        $U[i] = i$;

subroutine **find** x
    if $U[x] = x$ then do the following:
        return x;
    $t \leftarrow$ **find** $U[x]$;
    $U[x] \leftarrow t$;
    return t.

subroutine **union** x **and** y
    $p \leftarrow$ **find** x;
    $q \leftarrow$ **find** y;
    $U[p] \leftarrow q$.

subroutine **quicksort from** a **to** b
    if $a \geq b$ then do the following:
        return;
    $p \leftarrow$ **partition from** a **to** b;
    call **quicksort from** a **to** $p - 1$;
    call **quicksort from** $p + 1$ **to** b.

subroutine **partition from** a **to** b

call **swap** $\lfloor (a+b)/2 \rfloor$ **and** L;

$L \leftarrow a$;

for i going from a to $b-1$ do the following:

if $V[i]$.length $< V[b]$.length do the following:

call **swap** i **and** L;

$L \leftarrow L+1$;

call **swap** b **and** L;

return L.

subroutine **swap** a **and** b

$t \leftarrow V[a]$;

$V[a] \leftarrow V[b]$;

$V[b] \leftarrow t$;

return.

7 [DH] Problem 4.13

(a) Let R be an empty vector of integers, S be an empty two-dimensional array of integers.

for i going from 0 to C do the following:

$R[i] \leftarrow 0$;

for j going from 1 to N do the following:

$S[i][j] = 0$;

for i going from 1 to N do the following:

for j going from 1 to $Q[i]$ do the following:

for k going down from C to $W[i]$ do the following:

if $R[j-W[i]] + P[i] > R[j]$ do the following:

$R[j] \leftarrow R[j-W[i]] + P[i]$;

for l going from 1 to i do the following:

$S[j][l] \leftarrow S[j-W[i]][l]$;

$S[j][i] \leftarrow S[j][i]+1$;

output $S[C]$.

(b) The output is [0, 1, 3, 2, 1]. The total profit of the knapsack is 194.

8 [DH] Problem 4.14

(a) Let S be an empty vector of real numbers.

while $C \neq 0$ do the following:

$t \leftarrow$ **find best material**;

if $W[t] \times Q[t] < C$ then do the following:

$C \leftarrow C - W[t] \times Q[t]$;

$Q[t] \leftarrow 0$;

$S[t] \leftarrow Q[t]$;

otherwise do the following:

$Q[t] \leftarrow Q[t] - C/W[t]$;

$S[t] \leftarrow C/W[t]$;

$C \leftarrow 0$;

output S.

subroutine **find best material**

$i \leftarrow 1$;

while $Q[i] = 0$ do the following:

$i \leftarrow i + 1$;

$t \leftarrow i$;

for i going from $i+1$ to N do the following:

if $Q[i] > 0$ and $P[i]/W[i] > P[t]/W[t]$ then do the following:

$t \leftarrow i$;

return t.

(b) The output is [0, 1, 1.8, 5, 1]. The total profit of the knapsack is 200.