论题 1-9 作业

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1 [UD] Problem 10.2

(a) $\{(1,1),(2,2),(3,3),(4,4),(5,5)\}$;

(b) $\{(1,1),(2,2),(2,3),(3,3),(3,4),(4,4),(5,5)\}$;

(c) $\{(1,2),(2,1)\}$

(d) $\{(1,2),(2,3),(3,4),(4,5),(5,1)\}$;

2 [UD] Problem 10.4

Yes. First, it is reflexive, because $x_1 - y_1 = x_2 - y_2 = 0$ are even when $(x_1, x_2) = (y_1, y_2)$. Second, it is symmetry because both $x_1 - y_1$ and $x_2 - y_2$ are even if and only if $y_1 - x_1$ and $y_2 - x_2$ are even. Third, it is transitive, because that both $x_1 - y_1$ and $y_1 - z_1$ are even implies $x_1 - z_1$ is even, and $x_2 - z_2$ is even likewise.

3 [UD] Problem 10.5

"If": for all $a \in E_x$, we have $a \sim x$, since $x \sim y$, we get $a \sim y$, therefore $a \in E_y$. Hence E_x is a subset of E_y. Likewise E_y is a subset of E_x. So $E_x = E_y$.

"Only if": by the definition of equivalence class, $E_x = \{a \in X : x \sim a\}$, since $y \in E_y$ and $E_x = E_y$, we have $y \in E_x$, that is, $x \sim y$. $\square$

4 [UD] Problem 10.8

(a) Yes. The equivalence class is $\{\sum_{i=0}^{n} a_i x^i : a_0 = 0\}$.

(b) Yes. E_r is the set of all the polynomials of degree 1.

(c) No. Because it is not reflexive.

5 [UD] Problem 11.3

(a) Yes. A_r represents a plane, on which the sum of the coordinates of a point is r.

(b) Yes. A_r represents a sphere whose center is the origin and its radius is $|r|$.

6 [UD] Problem 11.7

(a) Yes. Obviously, for all $m \in \mathbb{N}$, A_m is nonempty. Since every polynomial has a degree, so $\bigcup\limits_{m\in\mathbb{N}} A_m = P$. Every polynomial has only one degree, so that for all $\alpha, \beta \in \mathbb{N}$, $A_\alpha = A_\beta$(when $\alpha = \beta$) or $A_\alpha \cap A_\beta = \varnothing$ (when $\alpha \neq \beta$) holds. Therefore, A_m determine a partition.

(b) Yes. For all $c \in \mathbb{R}$, there exists a polynomial such that $p(0) = c$, so A_c is always nonempty. We have that $\bigcup\limits_{c\in\mathbb{R}} A_c = P$. For every polynomial, $p(0)$ is a constant, so that for all $\alpha, \beta \in \mathbb{R}$, $A_\alpha = A_\beta$(when $\alpha = \beta$) or $A_\alpha \cap A_\beta = \varnothing$ (when $\alpha \neq \beta$) holds.

(c) No. Consider A_x and A_{x^2}, x^2 is an element of both, however, $A_x \neq A_{x^2}$ because x is an element of the former one but not an element of the latter one.

(d) No. Consider A_0 and A_1, $x^2 - x$ is an element of both, however, $A_0 \neq A_1$ because x is an element of the former one but not an element of the latter one.

7 [UD] Problem 11.8

First, for all $\alpha \in I \cup J$, A_a is nonempty because $\{A_\alpha : \alpha \in I\}$ and $\{A_\alpha : \alpha \in J\}$ are both nonempty. Second, for every real number x, there exists $\alpha \in I \cup J$ ($\alpha \in I$ when $x > 0$ and $\alpha \in J$ when $x \leq 0$), therefore $\bigcup\limits_{\alpha\in I\cup J} A_\alpha = \mathbb{R}$. Third, for all $\alpha, \beta \in I$ (or J), $A_\alpha = A_\beta$ or $A_\alpha \cap A_\beta$ holds, and for all $\alpha \in I$ and $\beta \in J$, $A_\alpha \cap A_\beta = \varnothing$, therefore for all $\alpha, \beta \in I \cup J$, $A_\alpha = A_\beta$ or $A_\alpha \cap A_\beta$ holds. Hence $\{A_\alpha : \alpha \in I \cup J\}$ is a partition of $\mathbb{R}$.

8 [UD] Problem 11.9

(a) No. Let $X = \{1,2,3\}$, and $\{A_\alpha : \alpha \in I\} = \{\{1\},\{2,3\}\}$ be a partition of X. Let $B = \{1,2\} \subseteq X$ such that $B \cap \{1\} \neq \varnothing$ and $B \cap \{2,3\} \neq \varnothing$. However, $\{A_\alpha \cap B : \alpha \in I\} = \{\{1\},\{2\}\}$ is not a partition of B, because $\bigcup\limits_{\alpha\in I} A_\alpha \cap B = \{1,2\} \neq X$.

(b) No. Let $X = \{1,2,3\}$, and $\{A_\alpha : \alpha \in I\} = \{\{1\},\{2\},\{3\}\}$ be a partition of X. However, $\{X \setminus A_\alpha : \alpha \in I\} = \{\{1,2\},\{1,3\},\{2,3\}\}$ is not a partition of X because $\{1,2\} \neq \{1,3\}$ and $\{1,2\} \cap \{1,3\} \neq \varnothing$.