论题 1-6 作业

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1 [UD] Problem 6.7

- $B \setminus A$;
- $(A \cup B) \setminus (A \cap B)$;
- $A \cup B \cup C$;
- $(B \cap C) \setminus A$;
- $((A \cap B) \cup (A \cap C) \cup (B \cap C)) \setminus (A \cup B \cup C)$.

2 [UD] Problem 6.16

(a) For every n in A, $n = x^2$ where x is an integer, therefore n is an integer, i.e. $n \in B$, so $A \subseteq B$. $\square$

(b) For every t in A, t is a real number, there exists a real number $x = t/2$, such that $t = 2x$, so $t \in B$. Therefore $A \subseteq B$. $\square$

(c) For every point (x, y) in A, we have $y = (5-3x)/2$, therefore $2y + 3x = 5$, which means that (x, y) is also in B. So $A \subseteq B$. $\square$

3 [UD] Problem 6.17

(a) A is a proper subset of B. For every (x, y) in A, we have $xy > 0$, so both x and y are nonzero, thus $x^2 + y^2 > 0$, therefore A is a subset of B. However, $(1, -1)$ is an element of B, but not an element of A, so A is a proper subset of B. $\square$

(b) A is a proper subset of B. By *theorem*6.10, we have $A \subseteq$. However, $(0, 0)$ is an element of B, but not an element of A, so A is a proper subset of B. $\square$

4 [UD] Problem 7.1

(a) For every x in universe, by definition of complement, if $x \in A$, then $x \notin A^c$ and if $x \notin A^c$ then $x \in (A^c)^c$, therefore we have if $x \in A$, then $x \in (A^c)^c$, i.e. A is a subset of $(A^c)^c$. $(A^c)^c$ is a subset of A likewise. So $(A^c)^c = A$. $\square$

(b) For every x in $A \cap (B \cup C)$, we have $x \in A$, and $x \in B$ or C, so $x \in A$ and B or $x \in A$ and C, thus $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$. For every x in $(A \cap B) \cup (A \cap C)$, we have $x \in A$ and B or $x \in A$ and C, so $x \in A$, and $x \in B$ or C, thus $(A \cap B) \cup (A \cap C)$ is a subset of $A \cap (B \cup C)$. So $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. $\square$

(c) For every x in $X \setminus (A \cap B)$, we have $x \in X$ and, $x \notin A$ or $x \notin B$, thus $x \in X$ and $x \notin A$, or $x \in X$ and $x \notin B$, therefore $X \setminus (A \cap B)$ is a subset of $(X \setminus A) \cup (X \setminus B)$. For every x in $(X \setminus A) \cup (X \setminus B)$, we have $x \in X$ and $x \notin A$, or $x \in X$ and $x \notin B$, thus $x \in X$ and, $x \notin A$ or $x \notin B$, so $(X \setminus A) \cup (X \setminus B)$ is a subset of $X \setminus (A \cap B)$. Therefore $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$. $\square$

(d) Since A, B are subsets of X, for every $x \in X$, if $x \in A$ then $x \in B$ and if $x \notin B$ then $x \notin A$ are equivalent, so $A \subseteq B$ if and only if $(X \setminus B) \subseteq (X \setminus A)$. $\square$

(e) If $A \cap B = B$, then for every x, $x \in B$ and $x \in A$ and B are equivalent, so $x \in B$ implies $x \in A$, i.e. A is a subset of B. If $B \subseteq A$, for every x, $x \in B$ implies $x \in A$, thus $x \in B$ and $x \in A$ and B are equivalent, so $A \cap B = B$. Therefore, $A \cap B = B$ if and only if $B \subseteq A$. $\square$

5 [UD] Problem 7.8

(a) (ii);

(b) (i), (ii), (iii), (iv), (v);

(c) For every x in $(A \cap B) \setminus C$, we have $x \in A$ and B and $x \notin C$, so $x \in A$ and $x \notin C$, and $x \in B$ and $x \notin C$, thus $(A \cap B) \setminus C$ is a subset of $(A \setminus C) \cap (B \setminus C$. Likewise $(A \setminus C) \cap (B \setminus C$ is a subset of $(A \cap B) \setminus C$. Therefore $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$. $\square$

6 [UD] Problem 7.9

(a) For every x in $A \setminus B$, we have $x \in A$ and $x \notin B$, so $A \setminus B$ and B are disjoint. $\square$

(b) For every x in $A \cup B$, we have $x \in A$ or $x \in B$, so $x \in A$, or $x \in B$ and $x \notin A$, therefore $A \cup B$ is a subset of $(A \setminus B) \cup B$. For every x in $(A \setminus B) \cup B$, we have $x \in A$, or $x \in B$ and $x \notin A$, so $x \in A$ or $x \in B$, therefore $(A \setminus B) \cup B$ is a subset of $A \cup B$. So $A \cup B = (A \setminus B) \cup B$ $\square$

7 [UD] Problem 7.10

This statement is false. Here is a counterexample. Let $A = \{1,2\}$, $B = \{1\}$ and $C = \{2\}$, then $A \cup B = A \cup C$, but $B \neq C$. $\square$

8 [UD] Problem 7.11

This statement is true. We know that for every x, $x \in S$ if and only if $x \cap S = x$. For every $x \in A$, let $Y = x$, then $B \cap Y = A \cap Y = x$, so $x \in B$, thus A is a subset of B. B is a subset of A likewise. So the statement is true. $\square$