问题求解(I) 第一次作业

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不会的题目：

论题 1-3 [ES] Problem 17.18 (b)

论题 1-1 作业

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1 [UD] Problem 1.2

1. 题目要求找到符合以下条件的大写字母构成的单词：（1）正着读和反着读是一样的；（2）绕着中心旋转 180° 后，仍然正着读和反着读是一样的。

2. 要满足条件（1），这个单词必须是回文串。要满足条件（2），这个单词中的所有字母旋转 180° 后仍然是一个字母。这样的字母有：H, I, N, O, S, X, Z。

3. 根据上面的分析，找到一个满足所有条件的常见的英语单词：**NOON**。

2 [UD] Problem 1.3

1. “anagram”就是改变字母排列顺序得到的单词。题目要求将每一条中的字母重新排列，得到一个单词。前三条是地理概念上的地名，第四条是你可能住的地方。

2. 可以统计词组中各条目中字母出现的次数，然后到字典中查找具有相同各字母出现次数的单词即可。英语中地名首字母一般是大写，因此前三条只需查找那些首字母大写的单词即可。

3. 根据上面的分析可以找到答案：

   (a) **VANCOUVER**

   (b) **PENNSYLVANIA**

   (c) **PHILADELPHIA**

   (d) **DORMITORY**

4. 其他方法：当字母数量不多时，可列出这些字母所有可能的排列，再到字典中查找。这样就避免了在整本字典中逐条查找，较为节约时间。

3 [UD] Problem 1.4

1. 有 n 支球队进行单败淘汰制锦标赛，总共需要组织多少场比赛？单败淘汰赛制的规则是，所有参赛队两两配对进行比赛，败者淘汰，胜者继续两两配对比赛，如此反复直到决出冠军为止。

2. 注意到当且经当 n 是 2 的正整数次幂时，单败淘汰赛制才能组织。第一轮比赛共有 n 支球队，需要组织 $\frac{n}{2}$ 场比赛；第二轮比赛共有 $\frac{n}{2}$ 支球队，需要组织 $\frac{n}{2}$ 场比赛……直到决赛（第 $\log_2 n$ 轮）只剩 2 支球队，需要组织 1 场比赛。

3. 根据上面的分析，可计算出答案：当 n 是 2 的正整数次幂时，总比赛场次为：

$$\frac{n}{2}+\frac{n}{4}+\cdots+1=\sum_{i=1}^{\log_2 n} 2^{i-1}=2^{\log_2 n}-1=n-1$$

4 [UD] Problem 1.5

1. 假设你住在一幢奇怪的房子里，里面有 7 扇一模一样的、紧关着的门，其中有一扇通向浴室。在第一次尝试时找到浴室的概率是大于、小于还是等于第三次尝试时找到浴室的概率？

2. 第一次尝试时，有 7 扇门可能通向浴室，并且由于每扇门完全相同，因而是等可能的；第三次尝试时只有 5 扇门可能通向浴室，并且也是等可能的。

3. 因此，第一次尝试就找到浴室的概率为 $\frac{1}{7}$，第三次尝试时找到浴室的概率为 $\frac{1}{5}$，答案是小于。

5 [UD] Problem 1.6

1. 题目的意思是，给出一条通过字母移位编码得到的信息，要求解出原来的信息是什么。

```
RDSXCVIWTDGNXHUJCLTLXAAATPGCBDGTPQDJIXIAPITG
```

2. 可以将上述字符串中的所有字母移位 1 至 25，找出有合理意义的那一条即可。

3. 经过 1 至 25 次移位后，所有的字符串为：

```
SETYDWJXUEHOYIVKDMUMYBBBUQHDCEHUQREKJYJBQJUH
TFUZEXKYVFIPZJWLENVNZCCCVRIEDFIVRSFLKZKCRKVI
UGVAFYLZWGJQAKXMFOWOADDDWSJFEGJWSTGMLALDSLWJ
VHWBGZMAXHKRBLYNGPXPBEEEXTKGFHKXTUHNMBMETMXK
WIXCHANBYILSCMZOHQYQCFFFYULHGILYUVIONCNFUNYL
XJYDIBOCZJMTDNAPIRZRDGGGZVMIHJMZVWJPODOGVOZM
YKZEJCPDAKNUEOBQJSASEHHHAWNJIKNAWXKQPEPHWPAN
ZLAFKDQEBLOVFPCRKTBTFIIIBXOKJLOBXYLRQFQIXQBO
AMBGLERFCMPWGQDSLUCUGJJJCYPLKMPCYZMSRGRJYRCP
BNCHMFSGDNQXHRETMVDVHKKKDZQMLNQDZANTSHSKZSDQ
```

CODINGTHEORYISFUNWEWILLLEARNMOREABOUTITLATER

DPEJOHUIFPSZJTGVOXFXJMMMFBSONPSFBCPVUJUMBUFS

EQFKPIVJGQTAKUHWPYGYKNNNGCTPOQTGCDQWVKVNCVGT

FRGLQJWKHRUBLVIXQZHZLOOOHDUQPRUHDERXWLWODWHU

GSHMRKXLISVCMWJYRAIAMPPPIEVRQSVIEFSYXMXPEXIV

HTINSLYMJTWDNXKZSBJBNQQQJFWSRTWJFGTZYNYQFYJW

IUJOTMZNKUXEOYLATCKCORRRKGXTSUXKGHUAZOZRGZKX

JVKPUNAOLVYFPZMBUDLDPSSSLHYUTVYLHIVBAPASHALY

KWLQVOBPMWZGQANCVEMEQTTTMIZVUWZMIJWCBQBTIBMZ

LXMRWPCQNXAHRBODWFNFRUUUNJAWVXANJKXDCRCUJCNA

MYNSXQDROYBISCPEXGOGSVVVOKBXWYBOKLYEDSDVKDOB

NZOTYRESPZCJTDQFYHPHTWWWPLCYXZCPLMZFETEWLEPC

OAPUZSFTQADKUERGZIQIUXXXQMDZYADQMNAGFUFXMFQD

PBQVATGURBELVFSHAJRJVYYYRNEAZBERNOBHGVGYNGRE

QCRWBUHVSCFMWGTIBKSKWZZZSOFBACFSOPCIHWHZOHSF

其中粗体那一行有实际意义，将单词分割开，并加上标点符号，得到：

CODING THEORY IS FUN, WE WILL LEARN MORE ABOUT IT LATER.

6 [UD] Problem 1.8

1. 有 12 枚看起来一样的硬币，其中有 1 枚是假的，其重量与真的不同。给一架天平，要求用最少的称量次数找出那枚假币。

2. 考虑一个较简单的情况：只有 4 枚硬币，其他条件相同。将这 4 枚硬币依次编号为 1, 2, 3, 4 。首先，将 1 号和 2 号硬币放在天平左右两端称量，若

   (a) 重量相等，说明 1, 2 是真的， 3, 4 是假的。这时将 1 号和 3 号硬币放在天平左右两端称量，若

      i. 重量相等，说明 4 是假的；

      ii. 重量不相等，说明 3 是假的。

   (b) 重量不相等，说明 3, 4 是真的， 1, 2 是假的。这时将 1 号和 3 号硬币放在天平左右两端称量，若

      i. 重量相等，说明 2 是假的；

      ii. 重量不相等，说明 1 是假的。

上面的讨论表明，要从 4 枚硬币中找出假币，称 2 次即可。

3. 下面讨论 12 枚硬币的情况。将其依次编号为 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 。将 1, 2, 3, 4 放在天平左端， 5, 6, 7, 8 放在天平右端称量，若
   (a) 重量相等，说明假币在 9, 10, 11, 12 中。用 2. 中的方法找出即可。
   (b) 天平左端较重。将 1, 2, 5 放在天平左端， 3, 6, 9 放在天平右端称量，若
      i. 天平左端较重，则 1, 2 可能是假币且较重，6 也可能是假币且较轻。将 1, 6 放在天平左端， 9, 10 放在天平右端，若
         A. 重量相等，则 2 是假的；
         B. 天平左端较重，则 1 是假的；
         C. 天平右端较重，则 6 是假的。
      ii. 天平右端较重，则 3 可能是假币且较重，5 也可能是假币且较轻。将 3, 5 放在天平左端， 9, 10 放在天平右端，若
         A. 天平左端较重，则 3 是假的；
         B. 天平右端较重，则 5 是假的。
      iii. 重量相等，则 4 可能是假币且较重，7, 8 也可能是假币且较轻。将 4, 7 放在天平左端， 9, 10 放在天平右端，若
         A. 重量相等，则 8 是假的；
         B. 天平左端较重，则 4 是假的；
         C. 天平右端较重，则 7 是假的。
   (c) 天平右端较重。将 1, 2, 5 放在天平左端， 3, 6, 9 放在天平右端称量，若
      i. 天平左端较重，则 5 可能是假币且较重，3 也可能是假币且较轻。将 3, 5 放在天平左端， 9, 10 放在天平右端，若
         A. 天平左端较重，则 5 是假的；
         B. 天平右端较重，则 3 是假的。
      ii. 天平右端较重，则 6 可能是假币且较重，1, 2 也可能是假币且较轻。将 1, 6 放在天平左端， 9, 10 放在天平右端，若
         A. 重量相等，则 2 是假的；
         B. 天平左端较重，则 6 是假的；
         C. 天平右端较重，则 1 是假的。
      iii. 重量相等，则 7, 8 可能是假币且较重，4 也可能是假币且较轻。将 4, 7 放在天平左端， 9, 10 放在天平右端，若
         A. 重量相等，则 8 是假的；
         B. 天平左端较重，则 7 是假的；
         C. 天平右端较重，则 4 是假的。

上面的讨论表明，称量 3 次即可从这 12 枚硬币中找出假币。下面证明不可能以少于 3 次的称量，从 12 枚硬币中找出假币。假设称量 2 次即可找出假币，每次称量的结果有左边重、右边重、重量相等这 3 种，称 2 次一共可能的结果组合有 $3\times3=9$ 种，而一共有 12 枚硬币是假币，从而至少有 1 种结果组合不能确定哪一枚硬币是假币。综上，从 12 枚硬币中找出假币的最少称量次数为 **3 次**。

论题 1-2 作业

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1 [UD] Problem 2.1

	Antecedent	Conclusion
(a)	It is raining.	I will stay at home.
(b)	The baby cries.	I wake up.
(c)	The fire alarm goes off.	I wake up.
(d)	x is odd.	x is prime.
(e)	x is odd.	x is prime.
(f)	You have an invitation.	You can come to the party.
(g)	The bell rings.	I leave the house.

2 [UD] Problem 2.5

Truth table:

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$(\mathbf{P} \to (\neg\mathbf{R} \vee \mathbf{Q})) \wedge \mathbf{R}$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

The statement form is neither a tautology nor a contradiction.

3 [UD] Problem 2.6

(a) I won't do my homework or I won't pass this class.

(b) Seven is not an integer or seven is not even.

(c) T is continuous and T is not bounded.

(d) I can't eat dinner and I can't go to the show.

(e) x is odd and x is not prime.

(f) x is odd and x is not prime.

(g) I am not home, and Sam won't answer the phone or he won't tell you how to reach me.

(h) The stars are green or the white horse is shining, and the world isn't eleven feet wide.

4 [UD] Problem 2.7

(a) $\neg(\neg P) \leftrightarrow P$

(b) $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$

(c) $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$

(d) $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$

5 [UD] Problem 2.8

$(P \wedge Q) \vee R$

6 [UD] Problem 2.10

(a) It is not the case that it doesn't snow and it is sunny.

(b) It doesn't snow and it is sunny.

7 [UD] Problem 2.11

(a) Let A represent "I am a truth teller", B represent "each person living on this island is either a truth teller or a liar" which is true. The statement $(A \rightarrow B) \leftrightarrow A$, which is equivalent to $\mathbf{T} \leftrightarrow A$ should be true, so A must be true, i.e. Arnie is a **truth-teller**.

(b) Let A represent "I am a truth teller", and B represent "Barnie is a truth teller". The statement $(A \rightarrow B) \leftrightarrow A$ should be true.

A	**B**	$(\mathbf{A} \rightarrow \mathbf{B}) \leftrightarrow \mathbf{A}$
T	T	T
T	F	F
F	T	F
T	F	F

So Arnie and Barnie are **both truth-tellers**.

8 [UD] Problem 3.2

(a) Contrapositive: If you don't live in a white house, then you aren't the President of the United States.
Converse: If you live in a white house, then you are the President of the United States.

(b) Contrapositive: If you don't need eggs, then you are not going to bake a soufflé.
Converse: If you need eggs, then you are going to bake a soufflé.

(c) Contrapositive: If x is not an integer, then x is not a real number.
Converse: If x is an integer, then x is a real number.

(d) Contrapositive: If $x^2 \geq 0$, then x is not a real number.
Converse: If $x^2 < 0$, then x is a real number.

9 [UD] Problem 3.6

Let P represents Matilda eats cereal, Q represent bread and R represent yogurt. Then the following four statements are all true: $(P \wedge Q) \rightarrow R$, $(Q \vee R) \rightarrow P$, $\neg(P \wedge R)$, $Q \vee P$.

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$(\mathbf{P} \wedge \mathbf{Q}) \rightarrow \mathbf{R}$	$(\mathbf{Q} \vee \mathbf{R}) \rightarrow \mathbf{P}$	$\neg(\mathbf{P} \wedge \mathbf{R})$	$\mathbf{Q} \vee \mathbf{P}$
T	T	T	T	T	F	T
T	T	F	F	T	T	T
T	F	T	T	T	F	T
T	F	F	T	T	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	F

We can conclude from the truth table that the four statements are all true if and only if P is true and Q, R are false. So Matilda eats **cereal** on Monday.

10 [UD] Problem 3.7

(a)

Letter	Substatement
P	The coat is green.
Q	The moon is full.
R	The cow jumps over the moon.

Original statement: $P \rightarrow (Q \vee R)$

(b) Contrapositive: $(\neg Q \wedge \neg R) \to \neg P$

If the moon isn't full and the cow doesn't jump over it, then the coat isn't green.

(c) Converse: $(Q \vee R) \to P$

If the moon is full or the cow jumps over it, then the coat is green.

(d) Negation: $P \wedge \neg Q \wedge \neg R$

The coat is green, and the moon isn't full, and the cow doesn't jump over it.

(e) The original statement and its contrapositive are equivalent.

11 [UD] Problem 3.8

(a) Truth table:

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{P \to Q}$	$\mathbf{P \to (Q \vee \neg P)}$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

(b) The two statements are equivalent.

12 [UD] Problem 3.9

First, consider the chocolate. Let P_1, Q_1, R_1, S_1 represent the French, the Swiss, the German and the American recipes use semisweet chocolate respectively, and exactly three of them are true. These statements should be true: $\neg(Q_1 \leftrightarrow R_1)$, $\neg(R_1 \leftrightarrow S_1)$. Construct the truth table:

$\mathbf{P_1}$	$\mathbf{Q_1}$	$\mathbf{R_1}$	$\mathbf{S_1}$	$\neg(\mathbf{Q_1} \leftrightarrow \mathbf{R_1})$	$\neg(\mathbf{R_1} \leftrightarrow \mathbf{S_1})$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	T	T	F
F	T	T	T	F	F

So the French, the Swiss and the American recipes use semisweet chocolate.

Second, consider the flour. Let P_2, Q_2, R_2, S_2 represent the French, the Swiss, the German and the American recipes use very little flour respectively, and exactly three of them are true as well. These statements should be true: $R_2 \leftrightarrow S_2$, $\neg(P_2 \leftrightarrow S_2)$. Construct the truth table:

$\mathbf{P_2}$	$\mathbf{Q_2}$	$\mathbf{R_2}$	$\mathbf{S_2}$	$\mathbf{R_2 \leftrightarrow S_2}$	$\neg(\mathbf{P_2 \leftrightarrow S_2})$
T	T	T	F	F	F
T	T	F	T	F	T
T	F	T	T	T	T
F	T	T	T	T	F

So the French, the German and the American recipes use very little flour.

Third, consider the sugar. Let P_3, Q_3, R_3, S_3 represent the French, the Swiss, the German and the American recipes use less than 1/4 cup sugar, and still exactly three of them are true. The statement should be true: $\neg(R_3 \wedge S_3)$. Construct the truth table:

$\mathbf{P_3}$	$\mathbf{Q_3}$	$\mathbf{R_3}$	$\mathbf{S_3}$	$\neg(\mathbf{R_3 \wedge S_3})$
T	T	T	F	T
T	T	F	T	T
T	F	T	T	F
F	T	T	T	F

So the French and the Swiss recipes use less than 1/4 cup sugar. Since each of the four has at least two of the qualities, we can determine that the American recipe also uses less than 1/4 cup sugar.

	French	Swiss	German	American
use semisweet chocolate	$\surd$	$\surd$	$\times$	$\surd$
use very little flour	$\surd$	$\times$	$\surd$	$\surd$
use less than 1/4 cup sugar	$\surd$	$\surd$	$\surd$	$\times$

So, Karl's favorite recipe is the **French** one.

13 [UD] Problem 3.10

The contrapositive of the statement is: if n is even, then $3n$ is even.

If n is even, then there exists an integer k s.t. $n = 2k$. Therefore, $3n = 3(2k) = 2(3k)$ where $3k$ is an integer, so $3n$ is even. $\square$

14 [UD] Problem 3.11

Suppose $\sqrt{2x} = k$ is an integer, then $2x = k^2$ is even.

The contrapositive of "if k^2 is even, then k is even" is "if k is odd, then k^2 is odd". Let $k = 2m+1$ where m is an integer, then $k^2 = 4m^2 + 4m + 1 = 2(2m^2 + 2m) + 1$ is an odd. So the statement is true.

We have proved that k is even, i.e. there exists an integer n s.t. $k = 2n$. So $k^2 = 4n^2 = 2x$, i.e. $x = 2n^2$ is even, but by assuption x is odd, which leads to a contradiction. So $\sqrt{2x}$ is not an integer. $\square$

15 [UD] Problem 4.1

(a) $\forall x, \exists y, x = 2y$.

(b) $\forall y, \exists x, x = 2y$.

(c) $\forall x, \forall y, x = 2y$.

(d) $\exists x, \exists y, x = 2y$.

(e) $\exists x, \exists y, x = 2y$.

16 [UD] Problem 4.5

The universe is $\mathbb{R}$ throughout.

(a) There exists $x \in \mathbb{R}$ such that $x^2 \leq 0$.

(b) There exists an odd integer which is zero.

(c) I am hungry and I don't eat chocolate.

(d) There is a girl who likes every boy.

(e) For all x the inequality $g(x) \leq 0$ holds.

(f) There exits x, for all y we have $xy \neq 1$.

(g) For all y, there exists x such that $xy \neq 0$.

(h) There exists x such that $x \neq 0$ and for all y the inequality $xy \neq 1$ holds.

(i) There exists x such that $x > 0$ and there exists y such that $xy^2 < 0$.

(j) There exists $\varepsilon > 0$, for all $\delta > 0$, $|x-1| < \delta$ and $|x^2 - 1| \geq \varepsilon$ hold.

(k) There exists a real number M, for every real number N, there exists $n > N$ such that $|f(n)| \leq M$.

17 [UD] Problem 4.7

(a) The negation of the statement is

$$\begin{aligned}
&\neg(\forall x, ((x \in \mathbb{Z} \wedge \neg(\exists y, (y \in \mathbb{Z} \wedge x = 7y))) \rightarrow (\exists z, (z \in \mathbb{Z} \wedge x = 2z)))) \\
=&\exists x, ((x \in \mathbb{Z} \wedge \neg(\exists y, (y \in \mathbb{Z} \wedge x = 7y))) \wedge \neg(\exists z, (z \in \mathbb{Z} \wedge x = 2z))) \\
=&\exists x, ((x \in \mathbb{Z} \wedge \forall y, \neg(y \in \mathbb{Z} \wedge x = 7y)) \wedge \forall z, \neg(z \in \mathbb{Z} \wedge x = 2z)) \\
=&\exists x, ((x \in \mathbb{Z} \wedge \forall y, (y \notin \mathbb{Z} \vee x \neq 7y)) \wedge \forall z, (z \notin \mathbb{Z} \vee x \neq 2z))
\end{aligned}$$

(b) For all x, if x is an integer and there does not exist y such that y is an integer and $x = 7y$, then there exists z such that z is an integer and $x = 2z$.

(c) The negation is true. Consider the original one, we have a counterexample: let $x = 5$, x is an integer and x is not a multiple of 7. However, x is not a multiple of 2 as well, which is contradictory to the original statement. So the negation is true.

18 [UD] Problem 4.9

This joke reflects the fact that physicists and chemists, especially chemists, tend to use a lot of inductions, rather than mathematicians, who use deductive reasoning in their work. Logically speaking, a statement concluded by (incomplete) inductive reasoning is not always true, for you can't prove that there doesn't exist a counterexample, no matter how obvious the statement is. In this joke, we can't say in logic that all the cows in Switzerland are brown until we see every single cow in Switzerland, and we can't say in logic that the cow we've seen is brown until we see the other side of the cow.

19 [UD] Problem 4.13

(a) Yes. "If don't love Sam, then I don't love Bill" is the contrapositive of "If I love Bill, then I love Sam". They are logically equivalent, and the former one is true, so the latter one is also true.

(b) No. A logical implication is false if and only if its antecedent is true and its conclusion is false. In this case, antecedent is "Susie goes to the ball in the red dress" which is false, and the conclusion is "I will stay home", which could be either true or false according to the implication.

(c) Yes. The contrapositive of (1) is if for all real number m, $m \leq l$, then l is not a positive real number. We can conclude from (2) that the antecedent is true when $l = t$, so the conclusion is also true, i.e. t is not positive.

(d) Yes. Let P represent "every little breeze seems to whisper Louise" and Q represent "my name is Igor". The statement $P \vee Q$ is true and Q is false, so P must be true.

(e) No. Statement (2) is equivalent to "every house on my street is not blue", which means the antecedent of statement (1) is false, so statement (3) can be either true or false.

(f) Yes. The contrapositive of (1) is "if $y \geq 1/5$, then $x \leq 5$", and the antecedent is true, so the conclusion, $x \leq 5$ is also true.

(g) No. When the antecedent is false, the conclusion can be either true or false.

(h) Yes. The contrapositive of (1) is "if $y \leq z$, then $y \leq x$ or $y \leq 0$, and the antecedent is true, so the conclusion is also true.

论题 1-3 作业

姓名：陈劭源　　　　学号：161240004

1 [UD] Problem 6.12

Not always. Let $x = 1$ and $y = -2$, since x and y are both nonzero, they are in S. However, $x \# y = x + y + 1 = 1 - 2 + 1 = 0$ is zero, which means $x \# y$ is not in S.

2 [UD] Problem 6.14

(a) Let $a = 1$, $b = 0$, $c = -4$, where a, b, c are all integers with a is nonzero. When $x = 2$, the equality $ax^2 + bx + c = x^2 - 4 = 0$ holds, therefore $2 \in A$. □

(b) Let $a = 1$, $b = 0$, $c = -2$, where a, b, c are all integers with a is nonzero. When $x = \sqrt{2}$, we have $x^2 - 2 = 0$, so $\sqrt{2} \in A$. □

(c) $\sqrt[3]{2}$.

(d) Let $p = a$, $q = b$, $r = c$. When a, b, c are integers with at least one of them is nonzero, p, q, r are rational numbers with at least one of them is nonzero. For every real number x, if $ax^2 + bx + c = 0$, then $px^2 + qx + c = 0$, so we have $A \subseteq B$.
By the definition of rational numbers, there exists integers $m_p, n_p, m_q, n_q, m_r, n_r$ with $n_p, n_q, n_r \neq 0$ such that $p = \dfrac{m_p}{n_p}$, $q = \dfrac{m_q}{n_q}$ and $r = \dfrac{m_r}{n_r}$. Since at least one of p, q, r is nonzero, at least one of m_p, m_q, m_r is nonzero. Let $a = pn_pn_qn_r = m_pn_qn_r$, $b = qn_pn_qn_r = m_qn_pn_r$, $c = rn_pn_qn_r = m_rn_pn_q$, so a, b, c are all integers and with at least one of them is nonzero. For every real number x, if $px^2 + qx + r = 0$, then $n_pn_qn_r(px^2 + qx + r) = ax^2 + bx + c = 0$, so we have $B \subseteq A$.
By the definition of equality of sets, we get $A = B$. □

(e) By the definition of rational numbers, for every number $x \in \mathbb{Q}$, there exists two integers m, n with $n \neq 0$ such that $x = \dfrac{m}{n}$. Let $a = 0$, $b = n$, $c = -m$ with c is nonzero, and we have $ax^2 + bx + c = 0$, so we get $\mathbb{Q} \subseteq A$. □

3 [UD] Problem 6.15

(a) A is the graph of the inequality $y \neq 0$ with respect to x and y. In other words, A is a collection of points in a plane whose y-coordinate is nonzero.

(b) Since (x,y), (z,w) are elements of A, we have $y \neq 0$ and $w \neq 0$. Thus, $wy \neq 0$, which means $(xw+zy, wy)$ is again an object in A. $\square$

(c) $(a,b) \diamond (x,y) = (ay+bx, by) = (x,y)$ for every (x,y) in A. Compare the coefficients of x, y, we get $a = 0$ and $b = 1$, so the element is $(0,1)$.

(d) For every element (a,b) in A, if we regard a as the numerator, and b as the denominator, we can find that this "new" addition shows the addition of two fractions:

$$\frac{x}{y} + \frac{z}{w} = \frac{xw+zy}{wy}.$$

4 [UD] Problem 6.18

No. Consider $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, we have $\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1 \leq 1$ and $\left|\frac{\sqrt{2}}{2}\right| + \left|\frac{\sqrt{2}}{2}\right| = \sqrt{2} > 1$, so $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is an element of the first set and it is not an element of the second set. Therefore the second set is not a subset of the first set, and thus the two sets are not equal. $\square$

5 [UD] Problem 17.11

(a) Take $m = 0$ and $n = 1$, we have $g(1) = g(0)g(1)$. Cancel the positive real number $g(1) = a$ on both sides, we get $g(0) = 0$. $\square$

(b) Let $P(n)$ denote that $g(n) = a^n$. For the base step, we have to check $g(1) = 1$, this is obviously true because this is one of the properties of g.

For the induction step, assume that $P(n)$ holds for a positive integer n. That is $g(n) = a^n$. Apply the second property of g, we get $g(n+1) = g(n)g(1) = a^n \times a = a^{n+1}$. Thus, $P(n+1)$ holds.

By mathematical induction, we conclude that $g(n) = a^n$ for all $n \in (N)$. $\square$

6 [UD] Problem 17.13

Let $P(n)$ denote that $p(c) = 0$ implies $c = 0$ for every polynomial $p(x)$ of order n satisfies the conditions in the nontheorem. The implication $P(1) \to P(2)$ doesn't hold. When $n = 2$, $p(c)$ has only two factors, so if we remove one factor to form $q(x)$, $q(x)$ must be a 1-order polynomial and $q(c)$ doesn't have the factor $ac(a_1c+b_1)$.

7 [UD] Problem 17.14

Let $P(n)$ denote that "$Q(1), \ldots, Q(n)$" are all true. For the base step, let $n = 1$. $P(1)$ is certainly true because $Q(1)$ is true.

For the induction step, assume that $P(n)$ is true where n is a positive integer, i.e. $Q(1),\ldots,Q(n)$ are all true. By supposition (ii), we have that $Q(n+1)$ is true. $Q(n+1)$, along with $Q(1),\ldots,Q(n)$ are all true, thus $P(n+1)$ is true.

By mathematical induction, we conclude that $P(n)$ holds for all positive integers n. Therefore, $Q(n)$ holds for all positive integers n. $\square$

8 [UD] Problem 17.16

For the base step, we have to prove that the sum of all the interior angles of a triangle is 180°. Let $\triangle ABC$ be a triangle. Draw line l through point A and parallel to BC. By the property of parallel lines, we have that $\angle 1 = \angle C$, $\angle 3 = \angle B$, so the sum of all the interior angles of a triangle is $\angle A + \angle B + \angle C = \angle 2 + \angle 3 + \angle 1 = 180^\circ$.

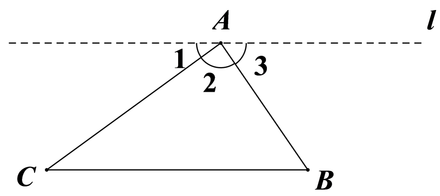

Figure 1: the sum of all the interior angles of a triangle is 180°

For the induction step, assume that the sum of all the interior angles of a convex polygon with n $(n \geq 3, n \in \mathbb{N})$ vertices is $(n-2)180^\circ$. Let $A_1A_2\cdots A_{n-1}A_nA_{n+1}$ be a convex polygon with $n+1$ vertices. Draw a line through A_1A_n, then $A_1A_nA_{n+1}$ is a triangle, of which the sum of all the interior angles is 180°, and a convex polygon with n vertices, of which the sum of all the interior angles is $(n-2)180^\circ$ by induction hypothesis. So the sum of all the interior angles of $A_1A_2\cdots A_{n-1}A_nA_{n+1}$ is $\angle A_1 + \angle A_2 + \cdots + \angle A_{n+1} = (\angle A_{n+1} + \angle A_{n+1}A_nA_1 + \angle A_{n+1}A_1A_n) + (\angle A_{n-1}A_nA_1 + \angle A_nA_1A_2 + \angle A_1 + \cdots + \angle A_{n-1}) = 180^\circ + (n-2)180^\circ = [(n+1)-2]180^\circ$.

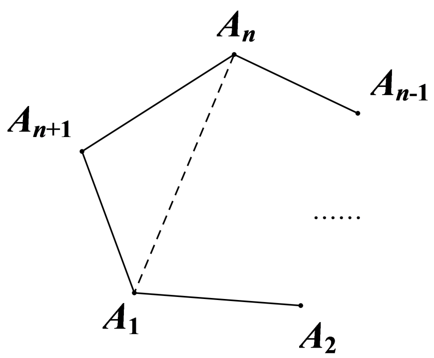

Figure 2: a convex polygon with $n+1$ vertices

By mathematical induction, we conclude that for all integers n where $n \geq 3$, the sum of all the interior angles of a convex polygon with n vertices is $(n-2)180^\circ$. $\square$

9 [UD] Problem 17.18

(a) $T_n = \dfrac{n(n+1)}{2}$.

Proof: The validity of the base step is obvious. Now assume that $T_n = \dfrac{n(n+1)}{2}$. We can find that $T_{n+1} = T_n + (n+1)$, so $T_{n+1} = \dfrac{n(n+1)}{2} + (n+1) = \dfrac{n(n+1)+2n+2}{2} = \dfrac{(n+1)(n+2)}{2}$.

By mathematical induction, we conclude that $T_n = \dfrac{n(n+1)}{2}$ holds for all positive integers n. $\square$

(b)

10 [UD] Problem 17.19

(a) $5! = 120$, $\dbinom{8}{3} = 56$, $\dbinom{8}{5} = 56$, $\dbinom{5}{2} = 10$, $\dbinom{5}{3} = 10$, $\dbinom{7}{0} = 1$, $\dbinom{7}{7} = 1$.

(b) $(m+1)^2$ equally spaced dots form a square with sides built of $m+1$ equally spaced dots. Divide these dots into four parts as the picture shows. There are m^2 dots in the upper-left part, m dots in the upper-right part, m dots in the lower-left part and one dot in the lower-right part. Thus $(m+1)^2 = m^2 + 2m + 1$.

(c) By the definition of binomial coefficient, we have

$$\begin{aligned}
\binom{n}{k-1} + \binom{n}{k} &= \frac{n!}{(k-1)!(n-k+1)!} + \frac{n!}{k!(n-k)!} \\
&= \frac{n!k}{k!(n-k+1)!} + \frac{n!(n-k+1)}{k!(n-k+1)!} \\
&= \frac{(n+1)!}{k!(n-k+1)!} \\
&= \frac{(n+1)!}{k!(n-k+1)!} \\
&= \binom{n+1}{k}
\end{aligned}$$

$\square$

(d) We use mathematical induction. For the base step, we have to check that $a+b = \dbinom{1}{0} b + \dbinom{1}{1} a$, which is certainly true.

For the induction step, assume that

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

holds for every a and b. Then,

$$\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n \\
&= (a+b)\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} \\
&= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}b^{n-k} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k+1} \\
&= \sum_{k=1}^{n+1}\binom{n}{k-1}a^k b^{n-k+1} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k+1} \\
&= a^{n+1} + b^{n+1} + \sum_{k=1}^{n}\left[\binom{n}{k-1} + \binom{n}{k}\right]a^k b^{n-k+1} \\
&= a^{n+1} + b^{n+1} + \sum_{k=1}^{n}\binom{n+1}{k}a^k b^{n-k+1} \\
&= \sum_{k=0}^{n+1}\binom{n+1}{k}a^k b^{n-k+1}
\end{aligned}$$

By mathematical induction, we conclude that $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$. □

(e) Take $a=-1, b=1$ and apply binomial theorem, the left side is identically zero, the right side is $\sum_{k=0}^{n}\binom{n}{k}(-1)^k$, so $\sum_{k=0}^{n}\binom{n}{k}(-1)^k = 0$. □

11 [ES] Problem 24.4

Divide the square into four identical small squares as Figure 3 shows. Each small square includes its border. By the Pigeonhole Principle, there exists one small square who has at least two points. In one small square, the maximum of the distance of two points is $\sqrt{2}/2$, the length of the diagonal. So there always exist two points whose distance is at most $\sqrt{2}/2$. □

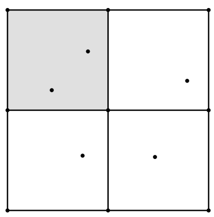

Figure 3: the square is divided into four small squares

12 [ES] Problem 24.6

Proposition Give nine distinct lattice points in three-dimensional space, at least one of the segments determined by these points has a lattice point as its midpoint.

Proof Classify the nine points into eight types by the parity of their coordinates:

$$\begin{array}{cccc} (even, even, even) & (even, even, odd) & (even, odd, even) & (even, odd, odd) \\ (odd, even, even) & (odd, even, odd) & (odd, odd, even) & (odd, odd, odd) \end{array}$$

By the Pigeonhole Principle, there exists two points who have the same parity. Let (a,b,c) and (d,e,f) be the two points, recall the midpoint formula, the coordinates of their midpoint is

$$\left(\frac{a+d}{2}, \frac{b+e}{2}, \frac{c+f}{2}\right).$$

$\frac{a+d}{2}$ is an integer because a and d shares the same parity. $\frac{b+e}{2}$ and $\frac{c+f}{2}$ are integers likewise. This proves that the midpoint is a lattice point. $\square$

13 [ES] Problem 24.8

```
 Language:  C++
#include <iostream>
#include <algorithm>
using namespace std;

int arr[1050];
int n = 0;
int s[1050];
int top = 0;

int main()
{
    int ans;
    int temp;
    while (cin >> arr[n]) n++;
    for (int i = 0; i < n; i++)
    {
        temp = upper_bound(s, s + top, arr[i]) - s;
        s[temp] = arr[i];
        top = max(temp + 1, top);
```

```
    }
    ans = top;
    top = 0;
    for (int i = 0; i < n; i++)
    {
        arr[i] = -arr[i];
        temp = upper_bound(s, s + top, arr[i]) - s;
        s[temp] = arr[i];
        top = max(temp + 1, top);
    }
    cout << max(ans, top) << endl;
    return 0;
}
```

论题 1-4 作业

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1 [DH] Problem 2.1

循环体包含两项操作：将当前指针指向的职员工资加入到总工资中；将当前指针指向下一个职员。在处理最后一个职员时，因为该职员已是最后一个，所以“将当前指针指向下一个职员”的操作是未定义的，所以应当单独处理。

2 [DH] Problem 2.2

(a) 冒泡排序完成第 k 次循环时，可以保证第 k 大元素处在正确的位置上。进行 $N-1$ 次循环后，前 $N-1$ 大数已经处在了正确的位置上，第 N 大数则只可能处在最后一个位置上，这恰好也是正确的位置。因此冒泡排序只要进行 $N-1$ 次外层循环即可。

(b) (1) $I \leftarrow 0$;

(2) do the following N times:

(2.1) point to the first element;

(2.2) do the following $N-I$ times:

(2.2.1) compare the element pointed to with the next element;

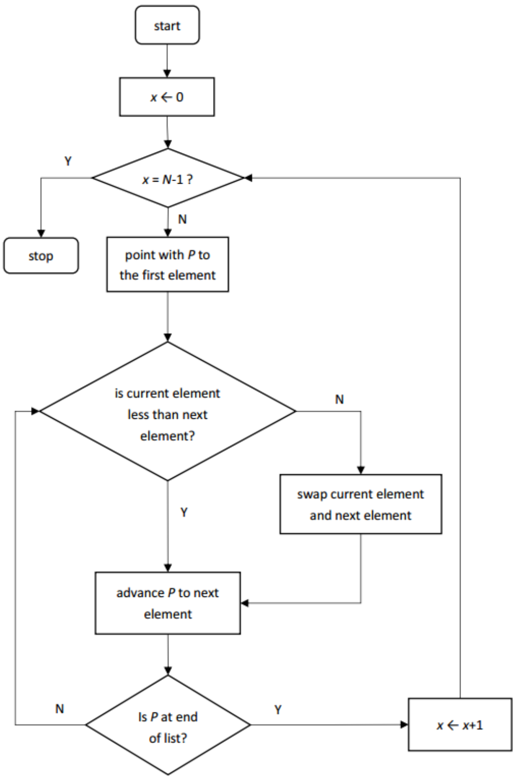
(2.2.2) if the compared elements are in the wrong order, exchange them;

(2.2.3) point to the next element;

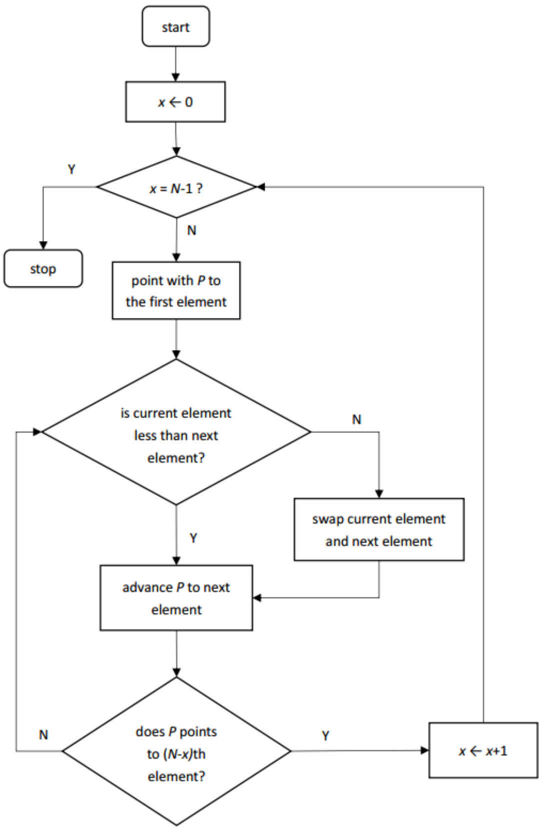
(2.3) $I \leftarrow I+1$.

3 [DH] Problem 2.3

（流程图见下页）



(a) unimproved



(b) improved

Figure 4: flowchart for bubblesort

4 [DH] Problem 2.4

(a) (1) $S \leftarrow 0$;

(2) $P \leftarrow 1$;

(3) point to the first element of L

(4) do the following $N-1$ times:

(4.1) add the integer pointed to to S;

(4.2) if the number pointed to is odd, then

(4.2.1) multiply P by the number pointed to;

(4.3) point to the next element of L;

(5) add the integer pointed to to S;

(6) if the number pointed to is odd, then

(6.1) multiply P by the number pointed to.

(b) (1) $S \leftarrow 0$;

(2) $P \leftarrow 1$;

(3) point to the first element of L

(4) add the integer pointed to to S;

(5) if the number pointed to is odd, then

(5.1) multiply P by the number pointed to;

(6) if last element is pointed to, goto (9);

(7) point to the next element of L;

(8) goto (4);

(9) end.

5 [DH] Problem 2.5

(a) (1) $I \leftarrow 0$;

(2) while $I < N$ do the following;

(2.1) $\cdots\cdots$;

(2.2) $I \leftarrow I+1$.

(b) (1) $t \leftarrow$ false;

(2) while A is true and t is false do the following;

(2.1) $\cdots\cdots$;

(2.2) $t \leftarrow$ true;

(3) while A is false and t is false do the following;

(3.1) $\cdots\cdots$;

(3.2) $t \leftarrow$ true.

(c) (1) if A is true then do the following:

(1.1) $\cdots\cdots$;

(1.2) goto (1).

(d) (1) if A is true then do the following:

(1.1) repeat the following:

(1.1.1) $\cdots\cdots$;

(1.1) until A is false.

6 [DH] Problem 2.6

move A to C;
move A to B;
move C to B;
move A to C;
move B to A;
move B to C;
move A to C;
move A to B;
move C to B;
move C to A;
move B to A;
move C to B;
move A to C;
move B to A;
move C to B;
move A to C;
move B to A;
move B to C;
move A to C;
move B to A;
move C to B;
move C to A;
move B to A;
move B to C;
move A to C;
move A to B;
move C to B;

move A to C;

move B to A;

move B to C;

move A to C.

7 [DH] Problem 2.7

(a) (1) $I \leftarrow 1$;

(2) $P \leftarrow 1$;

(3) while $I \leq N$ do the following:

(3.1) $P \leftarrow P \times I$;

(3.2) $I \leftarrow I + 1$;

(4) output P.

(b) function **factorial of** i;

(1) if $i = 0$ then return 1;

(2) return $i\times$ **factorial of** $i - 1$.

(1) output **factorial of** N.

8 [DH] Problem 2.8

subroutine **while-do**;

(1) if A is true then return;

(2) $\cdots\cdots$;

(3) call **while-do**.

论题 1-5 作业

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1 [DH] Problem 2.10

Let T be a vector of Booleans.

(1) for I going from 1 to N do the following:
  (1.1) $T[I] \leftarrow$ false;
(2) for I going from 1 to N do the following:
  (2.1) if $P[I] < 1$ or $P[I] > N$ do the following:
    (2.2.1) output 'NO';
    (2.2.2) end;
  (2.2) $T[P[I]] =$ true;
(3) for I going from 1 to N do the following:
  (3.1) if $T[I] =$ false do the following:
    (3.1.1) output 'NO';
    (3.1.2) end;
(4) output 'YES'.

2 [DH] Problem 2.11

Let K be a vector of Booleans, L be a vector of integers which stores a permutation.

subroutine **produce permutation** I
(1) if $I = N$ do the following:
  (1.1) output R;
  (1.2) return;
(2) for i going from 1 to N do the following:
  (2.1) if $K[i]$ is false do the following:
    (2.1.1) $R[I] \leftarrow i$
    (2.1.2) $K[i] \leftarrow$ true;
    (2.1.3) call **produce permutation** $I+1$
    (2.1.4) $K[i] \leftarrow$ false.

(1) i going from 1 to N do the following:

(1.1) $K[i] \leftarrow$ false;

(2) call **produce permutation** 0.

3 [DH] Problem 2.12

(a) i. **read**(X), **push**(X,S), **read**(X), **push**(X,S), **read**(X), **print**(X), **pop**(X,S), **print**(X), **pop**(X,S), **print**(X)

ii. **read**(X), **push**(X,S), **read**(X), **push**(X,S), **read**(X), **print**(X), **read**(X), **print**(X), **pop**(X,S), **print**(X), **pop**(X,S), **print**(X)

iii. **read**(X), **push**(X,S), **read**(X), **push**(X,S), **read**(X), **print**(X), **read**(X), **push**(X,S), **read**(X), **print**(X), **read**(X), **push**(X,S), **read**(X), **print**(X), **pop**(X,S), **print**(X), **read**(X), **print**(X), **pop**(X,S), **print**(X), **read**(X), **print**(X), **pop**(X,S), **print**(X), **read**(X), **print**(X), **pop**(X,S), **print**(X)

(b) i. 要生成 (3, 1, 2) 这个排列，由于 3 是最先输出的，1, 2 依次在栈中，此时若要继续输出，必然是以 2,1 的形式输出，所以不可能用栈生成 (3, 1, 2) 这个排列。 □

ii. 要生成 (4, 5, 3, 7, 2, 1, 6) 这个排列，当输出 7 时，栈中剩余的元素依次为 1, 2, 6 ，下一个需要输出 2 ，但输出 2 之前 6 必须输出，从而不可能用栈生成 (4, 5, 3, 7, 2, 1, 6) 这个排列。 □

(c) 容易验证，以下排列可以用栈生成：

(1, 2, 3, 4) (1, 2, 4, 3) (1, 3, 2, 4) (1, 3, 4, 2) (1, 4, 3, 2) (2, 1, 3, 4) (2, 1, 4, 3)
(2, 3, 1, 4) (2, 3, 4, 1) (2, 4, 3, 1) (3, 2, 1, 4) (3, 2, 4, 1) (3, 4, 2, 1) (4, 3, 2, 1)

以下排列不能用栈生成：

(1, 4, 2, 3) (2, 4, 1, 3) (3, 1, 2, 4) (3, 1, 4, 2) (3, 4, 1, 2) (4, 1, 2, 3) (4, 1, 3, 2)
(4, 2, 3, 1) (4, 2, 1, 3) (4, 3, 1, 2)

所以共有 10 个排列不能用栈生成。

4 [DH] Problem 2.13

Let S be an empty stack. Assume that the length of the permutation is N.

function **test**

(1) $I \leftarrow 1$;

(2) for i going from 1 to N do the following:

(2.1) **read**(X);

(2.2) if $I \leq X$ do the following:

(2.2.1) for j going from I to X do the following:

(2.2.1.1) **push**(j,S);

(2.2.2) **pop**(X,S);

(2.2.3) $I \leftarrow X+1$;

(2.3) otherwise do the following:

(2.3.1) **pop**(Y,S);

(2.3.2) if $X \neq Y$ then return false;

(3) return true.

subroutine **print operations**

(1) $I \leftarrow 1$;

(2) for i going from 1 to N do the following:

(2.1) **read**(X);

(2.2) if $I \leq X$ do the following:

(2.2.1) for j going from I to X do the following:

(2.2.1.1) **print**("**read**(X)");

(2.2.1.2) **print**("**push**(X,S)");

(2.2.1.3) **push**(j,S);

(2.2.2) **print**("**pop**(X,S)");

(2.2.3) **print**("**print**(X)");

(2.2.4) **pop**(X,S);

(2.2.5) $I \leftarrow X+1$;

(2.3) otherwise do the following:

(2.3.1) **pop**(Y,S);

(2.3.2) **print**("**pop**(X,S)");

(2.3.3) **print**("**print**(X)").

(1) if **test** is true then do the following:

(1.1) **print**("Yes");

(1.2) call **print operations**;

(2) otherwise do the following:

(2.1) **print**("No").

5 [DH] Problem 2.14

(a) i. Obtain by a queue: **read**(X), **add**(X,Q), **read**(X), **add**(X,Q), **read**(X), **print**(X), **remove**(X,Q), **print**(X), **remove**(X,Q), **print**(X)

Obtain by two stacks: **read**(X), **push**(X,S), **read**(X), **push**(X,S'), **read**(X),

$\mathbf{print}(X)$, $\mathbf{pop}(X,S)$, $\mathbf{print}(X)$, $\mathbf{pop}(X,S')$, $\mathbf{print}(X)$

ii. Obtain by a queue: $\mathbf{read}(X)$, $\mathbf{add}(X,Q)$, $\mathbf{read}(X)$, $\mathbf{add}(X,Q)$, $\mathbf{read}(X)$, $\mathbf{add}(X,Q)$, $\mathbf{read}(X)$, $\mathbf{print}(X)$, $\mathbf{read}(X)$, $\mathbf{print}(X)$, $\mathbf{remove}(X,Q)$, $\mathbf{add}(X,Q)$, $\mathbf{remove}(X,Q)$, $\mathbf{add}(X,Q)$, $\mathbf{remove}(X,Q)$, $\mathbf{print}(X)$, $\mathbf{read}(X)$, $\mathbf{add}(X,Q)$, $\mathbf{read}(X)$, $\mathbf{print}(X)$, $\mathbf{remove}(X,Q)$, $\mathbf{add}(X,Q)$, $\mathbf{remove}(X,Q)$, $\mathbf{print}(X)$, $\mathbf{remove}(X,Q)$, $\mathbf{add}(X,Q)$, $\mathbf{remove}(X,Q)$, $\mathbf{print}(X)$, $\mathbf{remove}(X,Q)$, $\mathbf{print}(X)$

Obtain by two stacks: $\mathbf{read}(X)$, $\mathbf{push}(X,S)$, $\mathbf{read}(X)$, $\mathbf{push}(X,S)$, $\mathbf{read}(X)$, $\mathbf{push}(X,S)$, $\mathbf{read}(X)$, $\mathbf{print}(X)$, $\mathbf{read}(X)$, $\mathbf{print}(X)$, $\mathbf{pop}(X,S)$, $\mathbf{print}(X)$, $\mathbf{read}(X)$, $\mathbf{push}(X,S')$, $\mathbf{read}(X)$, $\mathbf{print}(X)$, $\mathbf{pop}(X,S)$, $\mathbf{print}(X)$, $\mathbf{pop}(X,S)$, $\mathbf{print}(X)$, $\mathbf{pop}(X,S')$, $\mathbf{print}(X)$

(b) 用以下方法可只用一个队列生成任一排列：对于某一个要输出的数字，如果不在队列中，则将该数字之前的数字全部入队，然后直接输出该数字；如果在队列中，反复将队首数字出队后再入队“翻找”整个队列，直到找到为止，输出这个数字。对于排列中的每一个数字，依次重复上述操作，即可用一个队列输出任意排列。 □

(c) 用以下方法可只用两个队列生成任一排列：对于某一个要输出的数字，如果不在任一栈中，则将该数字之前的数字全部压入任何一个栈，然后直接输出该数字；如果在某个栈中，将该数字之上的数字依次弹出并压入到另一个之中，然后输出这个数字。对于排列中的每一个数字，依次重复上述操作，即可用两个栈输出任意排列。 □

6 [DH] Problem 2.15

Let S, S' be two empty stacks. Assume that the length of the permutation is N.

function $\mathbf{top}(S)$
(1) $\mathbf{pop}(t,S)$;
(2) $\mathbf{push}(t,S)$;
(3) return t.

(1) $I \leftarrow 1$;
(2) for i going from 1 to N do the following:
  (2.1) $\mathbf{read}(X)$;
  (2.2) if $I \leq X$ do the following:
    (2.2.1) for j going from I to X do the following:
      (2.2.1.1) **print**(”$\mathbf{read}(X)$”);
      (2.2.1.2) **print**(”$\mathbf{push}(X,S)$”);
      (2.2.1.3) $\mathbf{push}(j,S)$;
    (2.2.2) **print**(”$\mathbf{pop}(X,S)$”);
    (2.2.3) **print**(”$\mathbf{print}(X)$”);

(2.2.4) **pop**(X,S);

(2.2.5) $I \leftarrow X+1$;

(2.3) otherwise do the following:

(2.3.1) while **is-empty**(S') is false do the following:

(2.3.1.1) **print**("**pop**(X,S')");

(2.3.1.2) **print**("**push**(X,S)");

(2.3.1.3) **pop**(X,S');

(2.3.1.4) **push**(X,S);

(2.3.2) while **top**(S) $\neq I$ do the following:

(2.3.2.1) **print**("**pop**(X,S)");

(2.3.2.2) **print**("**push**(X,S')");

(2.3.2.3) **pop**(X,S);

(2.3.2.4) **push**(X,S');

(2.3.3) **pop**(X,S);

(2.3.4) **print**("**pop**(X,S)");

(2.3.5) **print**("**print**(X)").

7 [DH] Problem 2.16

(a) Let T be an empty binary search tree, L be a list of integers, N be the number of integers in L.

subroutine **add** X **to** S

(1) if S is empty then do the following:

(1.1) $S \leftarrow X$;

(1.2) return;

(2) if $X < S$ then do the following:

(2.1) **add** X **to left**(S);

(3) otherwise do the following:

(3.1) **add** X **to right**(S).

(1) for i going from 1 to N do the following:

(1.1) **add** $L[i]$ **to** T.

(b) Let T be a binary search tree.

subroutine **visit** S

(1) if S is empty then return;

(2) **visit right**(S);

(3) output S;

(4) **visit left**(S).

(1) **visit left**(T).