论题 1-2 作业

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1 [UD] Problem 2.1

	Antecedent	Conclusion
(a)	It is raining.	I will stay at home.
(b)	The baby cries.	I wake up.
(c)	The fire alarm goes off.	I wake up.
(d)	x is odd.	x is prime.
(e)	x is odd.	x is prime.
(f)	You have an invitation.	You can come to the party.
(g)	The bell rings.	I leave the house.

2 [UD] Problem 2.5

Truth table:

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$(\mathbf{P} \to (\neg\mathbf{R} \vee \mathbf{Q})) \wedge \mathbf{R}$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

The statement form is neither a tautology nor a contradiction.

3 [UD] Problem 2.6

(a) I won't do my homework or I won't pass this class.

(b) Seven is not an integer or seven is not even.

(c) T is continuous and T is not bounded.

(d) I can't eat dinner and I can't go to the show.

(e) x is odd and x is not prime.

(f) x is odd and x is not prime.

(g) I am not home, and Sam won't answer the phone or he won't tell you how to reach me.

(h) The stars are green or the white horse is shining, and the world isn't eleven feet wide.

4 [UD] Problem 2.7

(a) $\neg(\neg P) \leftrightarrow P$

(b) $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$

(c) $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$

(d) $(P \to Q) \leftrightarrow (\neg P \vee Q)$

5 [UD] Problem 2.8

$(P \wedge Q) \vee R$

6 [UD] Problem 2.10

(a) It is not the case that it doesn't snow and it is sunny.

(b) It doesn't snow and it is sunny.

7 [UD] Problem 2.11

(a) Let A represent "I am a truth teller", B represent "each person living on this island is either a truth teller or a liar" which is true. The statement $(A \to B) \leftrightarrow A$, which is equivalent to $\mathbf{T} \leftrightarrow A$ should be true, so A must be true, i.e. Arnie is a **truth-teller**.

(b) Let A represent "I am a truth teller", and B represent "Barnie is a truth teller". The statement $(A \to B) \leftrightarrow A$ should be true.

A	**B**	$(\mathbf{A} \to \mathbf{B}) \leftrightarrow \mathbf{A}$
T	T	T
T	F	F
F	T	F
T	F	F

So Arnie and Barnie are **both truth-tellers**.

8 [UD] Problem 3.2

(a) Contrapositive: If you don't live in a white house, then you aren't the President of the United States.
Converse: If you live in a white house, then you are the President of the United States.

(b) Contrapositive: If you don't need eggs, then you are not going to bake a soufflé.
Converse: If you need eggs, then you are going to bake a soufflé.

(c) Contrapositive: If x is not an integer, then x is not a real number.
Converse: If x is an integer, then x is a real number.

(d) Contrapositive: If $x^2 \geq 0$, then x is not a real number.
Converse: If $x^2 < 0$, then x is a real number.

9 [UD] Problem 3.6

Let P represents Matilda eats cereal, Q represent bread and R represent yogurt. Then the following four statements are all true: $(P \wedge Q) \rightarrow R$, $(Q \vee R) \rightarrow P$, $\neg(P \wedge R)$, $Q \vee P$.

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{R}$	$(\mathbf{P} \wedge \mathbf{Q}) \rightarrow \mathbf{R}$	$(\mathbf{Q} \vee \mathbf{R}) \rightarrow \mathbf{P}$	$\neg(\mathbf{P} \wedge \mathbf{R})$	$\mathbf{Q} \vee \mathbf{P}$
T	T	T	T	T	F	T
T	T	F	F	T	T	T
T	F	T	T	T	F	T
T	F	F	T	T	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	F

We can conclude from the truth table that the four statements are all true if and only if P is true and Q, R are false. So Matilda eats **cereal** on Monday.

10 [UD] Problem 3.7

(a)

Letter	Substatement
P	The coat is green.
Q	The moon is full.
R	The cow jumps over the moon.

Original statement: $P \rightarrow (Q \vee R)$

(b) Contrapositive: $(\neg Q \wedge \neg R) \to \neg P$

If the moon isn't full and the cow doesn't jump over it, then the coat isn't green.

(c) Converse: $(Q \vee R) \to P$

If the moon is full or the cow jumps over it, then the coat is green.

(d) Negation: $P \wedge \neg Q \wedge \neg R$

The coat is green, and the moon isn't full, and the cow doesn't jump over it.

(e) The original statement and its contrapositive are equivalent.

11 [UD] Problem 3.8

(a) Truth table:

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{P \to Q}$	$\mathbf{P \to (Q \vee \neg P)}$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

(b) The two statements are equivalent.

12 [UD] Problem 3.9

First, consider the chocolate. Let P_1, Q_1, R_1, S_1 represent the French, the Swiss, the German and the American recipes use semisweet chocolate respectively, and exactly three of them are true. These statements should be true: $\neg(Q_1 \leftrightarrow R_1)$, $\neg(R_1 \leftrightarrow S_1)$. Construct the truth table:

$\mathbf{P_1}$	$\mathbf{Q_1}$	$\mathbf{R_1}$	$\mathbf{S_1}$	$\neg(\mathbf{Q_1 \leftrightarrow R_1})$	$\neg(\mathbf{R_1 \leftrightarrow S_1})$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	T	T	F
F	T	T	T	F	F

So the French, the Swiss and the American recipes use semisweet chocolate.

Second, consider the flour. Let P_2, Q_2, R_2, S_2 represent the French, the Swiss, the German and the American recipes use very little flour respectively, and exactly three of them are true as well. These statements should be true: $R_2 \leftrightarrow S_2$, $\neg(P_2 \leftrightarrow S_2)$. Construct the truth table:

$\mathbf{P_2}$	$\mathbf{Q_2}$	$\mathbf{R_2}$	$\mathbf{S_2}$	$\mathbf{R_2 \leftrightarrow S_2}$	$\neg(\mathbf{P_2 \leftrightarrow S_2})$
T	T	T	F	F	F
T	T	F	T	F	T
T	F	T	T	T	T
F	T	T	T	T	F

So the French, the German and the American recipes use very little flour.

Third, consider the sugar. Let P_3, Q_3, R_3, S_3 represent the French, the Swiss, the German and the American recipes use less than 1/4 cup sugar, and still exactly three of them are true. The statement should be true: $\neg(R_3 \wedge S_3)$. Construct the truth table:

$\mathbf{P_3}$	$\mathbf{Q_3}$	$\mathbf{R_3}$	$\mathbf{S_3}$	$\neg(\mathbf{R_3} \wedge \mathbf{S_3})$
T	T	T	F	T
T	T	F	T	T
T	F	T	T	F
F	T	T	T	F

So the French and the Swiss recipes use less than 1/4 cup sugar. Since each of the four has at least two of the qualities, we can determine that the American recipe also uses less than 1/4 cup sugar.

	French	Swiss	German	American
use semisweet chocolate	$\surd$	$\surd$	$\times$	$\surd$
use very little flour	$\surd$	$\times$	$\surd$	$\surd$
use less than 1/4 cup sugar	$\surd$	$\surd$	$\surd$	$\times$

So, Karl's favorite recipe is the **French** one.

13 [UD] Problem 3.10

The contrapositive of the statement is: if n is even, then $3n$ is even.

If n is even, then there exists an integer k s.t. $n = 2k$. Therefore, $3n = 3(2k) = 2(3k)$ where $3k$ is an integer, so $3n$ is even. $\square$

14 [UD] Problem 3.11

Suppose $\sqrt{2x} = k$ is an integer, then $2x = k^2$ is even.

The contrapositive of "if k^2 is even, then k is even" is "if k is odd, then k^2 is odd". Let $k = 2m+1$ where m is an integer, then $k^2 = 4m^2 + 4m + 1 = 2(2m^2 + 2m) + 1$ is odd. So the statement is true.

We have proved that k is even, i.e. there exists an integer n s.t. $k = 2n$. So $k^2 = 4n^2 = 2x$, i.e. $x = 2n^2$ is even, but by assumption x is odd, which leads to a contradiction. So $\sqrt{2x}$ is not an integer. $\square$

15 [UD] Problem 4.1

(a) $\forall x, \exists y, x = 2y$.

(b) $\forall y, \exists x, x = 2y$.

(c) $\forall x, \forall y, x = 2y$.

(d) $\exists x, \exists y, x = 2y$.

(e) $\exists x, \exists y, x = 2y$.

16 [UD] Problem 4.5

The universe is $\mathbb{R}$ throughout.

(a) There exists $x \in \mathbb{R}$ such that $x^2 \leq 0$.

(b) There exists an odd integer which is zero.

(c) I am hungry and I don't eat chocolate.

(d) There is a girl who likes every boy.

(e) For all x the inequality $g(x) \leq 0$ holds.

(f) There exits x, for all y we have $xy \neq 1$.

(g) For all y, there exists x such that $xy \neq 0$.

(h) There exists x such that $x \neq 0$ and for all y the inequality $xy \neq 1$ holds.

(i) There exists x such that $x > 0$ and there exists y such that $xy^2 < 0$.

(j) There exists $\varepsilon > 0$, for all $\delta > 0$, $|x-1| < \delta$ and $|x^2-1| \geq \varepsilon$ hold.

(k) There exists a real number M, for every real number N, there exists $n > N$ such that $|f(n)| \leq M$.

17 [UD] Problem 4.7

(a) The negation of the statement is

$$
\begin{aligned}
&\neg(\forall x, ((x \in \mathbb{Z} \wedge \neg(\exists y, (y \in \mathbb{Z} \wedge x = 7y))) \to (\exists z, (z \in \mathbb{Z} \wedge x = 2z))))\\
=&\exists x, ((x \in \mathbb{Z} \wedge \neg(\exists y, (y \in \mathbb{Z} \wedge x = 7y))) \wedge \neg(\exists z, (z \in \mathbb{Z} \wedge x = 2z)))\\
=&\exists x, ((x \in \mathbb{Z} \wedge \forall y, \neg(y \in \mathbb{Z} \wedge x = 7y)) \wedge \forall z, \neg(z \in \mathbb{Z} \wedge x = 2z))\\
=&\exists x, ((x \in \mathbb{Z} \wedge \forall y, (y \notin \mathbb{Z} \vee x \neq 7y)) \wedge \forall z, (z \notin \mathbb{Z} \vee x \neq 2z))
\end{aligned}
$$

(b) For all x, if x is an integer and there does not exist y such that y is an integer and $x = 7y$, then there exists z such that z is an integer and $x = 2z$.

(c) The negation is true. Consider the original one, we have a counterexample: let $x = 5$, x is an integer and x is not a multiple of 7. However, x is not a multiple of 2 as well, which is contradictory to the original statement. So the negation is true.

18 [UD] Problem 4.9

This joke reflects the fact that physicists and chemists, especially chemists, tend to use a lot of inductions, rather than mathematicians, who use deductive reasoning in their work. Logically speaking, a statement concluded by (incomplete) inductive reasoning is not always true, for you can't prove that there doesn't exist a counterexample, no matter how obvious the statement is. In this joke, we can't say in logic that all the cows in Switzerland are brown until we see every single cow in Switzerland, and we can't say in logic that the cow we've seen is brown until we see the other side of the cow.

19 [UD] Problem 4.13

(a) Yes. "If don't love Sam, then I don't love Bill" is the contrapositive of "If I love Bill, then I love Sam". They are logically equivalent, and the former one is true, so the latter one is also true.

(b) No. A logical implication is false if and only if its antecedent is true and its conclusion is false. In this case, antecedent is "Susie goes to the ball in the red dress" which is false, and the conclusion is "I will stay home", which could be either true or false according to the implication.

(c) Yes. The contrapositive of (1) is if for all real number m, $m \leq l$, then l is not a positive real number. We can conclude from (2) that the antecedent is true when $l = t$, so the conclusion is also true, i.e. t is not positive.

(d) Yes. Let P represent "every little breeze seems to whisper Louise" and Q represent "my name is Igor". The statement $P \vee Q$ is true and Q is false, so P must be true.

(e) No. Statement (2) is equivalent to "every house on my street is not blue", which means the antecedent of statement (1) is false, so statement (3) can be either true or false.

(f) Yes. The contrapositive of (1) is "if $y \geq 1/5$, then $x \leq 5$", and the antecedent is true, so the conclusion, $x \leq 5$ is also true.

(g) No. When the antecedent is false, the conclusion can be either true or false.

(h) Yes. The contrapositive of (1) is "if $y \leq z$, then $y \leq x$ or $y \leq 0$, and the antecedent is true, so the conclusion is also true.