论题 2-17 作业

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1 [CZ] Problem 4.2

Suppose, to the contrary, that there exists a connected graph G, all of whose vertices have even degrees, that contains a bridge, named e. Then, $G-e$ contains exactly two components. For one of its components, it has exactly one odd vertex, which is incident to e in G. Hence, the sum of the degrees over all vertices in the component is odd, leading to contradiction. Therefore, every connected graph all of whose vertices have even degrees contains no bridge.

2 [CZ] Problem 4.3

Obviously, uv is a $u-v$ path. Suppose there exists another $u-v$ path, denoted by P, then $uv \notin P$. Note that $uv \cup P$ is a cycle, where uv lies. Hence, uv is not a bridge, which leads to contradiction. Therefore, if uv is a bridge in a graph G, then there is a unique $u-v$ path in G.

3 [CZ] Problem 4.5

(a) $n-1$.

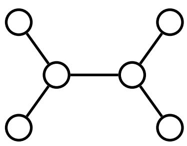
Because every edge of G is bridge, i.e. every edge lies on no cycle, there is no cycle in the connected graph G. Therefore, G is a tree of order n, and its size is $n-1$.

(b) $n-k$.

For every component of G, it is a connected graph, where every edge is a bridge. Apply the conclusion we obtained in (a), the size of each component is the order of it minus 1. Summing all the sizes up, we get that the size of G is $n-k$, where k is the number of the components.

4 [CZ] Problem 4.16

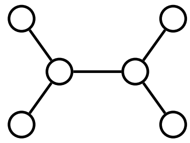
(a)



(b) Let n be the order of the tree, then the size of the tree is $n-1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

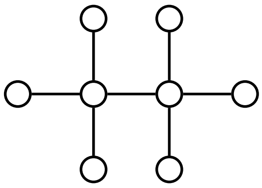
$$\frac{2}{3}n \times 1 + \frac{1}{3}n \times 3 = 2(n-1).$$

Solve this equation we get $n = 6$. So the only possible tree is:



5 [CZ] Problem 4.17

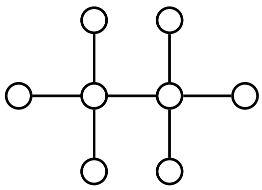
(a)



(b) Let n be the order of the tree, then the size of the tree is $n-1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$n \times 75\% \times 1 + n \times 25\% \times 4 = 2(n-1).$$

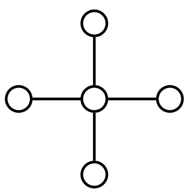
Solve this equation we get that $n = 8$. The only possible tree is:

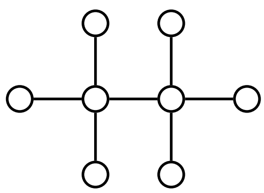


(c) Let n be the order of the tree, $m > 1$ be the degree of the remaining vertices, then the size of the tree is $n-1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$n \times 75\% \times 1 + n \times 25\% \times m = 2(n-1).$$

The integer solutions of this equation are: $n = 4, m = 3$ and $n = 8, m = 4$. The corresponding trees are:

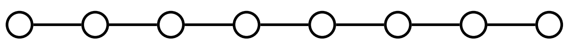




(d) Let n be the order of the tree, $m > 1$ be the degree of the remaining vertices, then the size of the tree is $n-1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$n \times 75\% \times 1 + n \times 25\% \times m = 2(n-1).$$

The only integer solution of this equation is $n = 8, m = 2$. The only possible tree is:



6 [CZ] Problem 4.18

Let n be the order of the tree, m be number of vertices of degree 3, then the size of the tree is $n-1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$(n-m)\times 1+m\times 3=2(n-1)$$

after some algebra we get $m=(n-2)/2$, i.e. T contains $(n-2)/2$ vertices of degree 3.

7 [CZ] Problem 4.19

(a) Substitute $\sum_i n_i$ for n in $2(n-1)=\sum_i in_i$, we get $\sum_i 2n_i - 2 = \sum_i in_i$, i.e. $0 = 2+\sum_i (i-2)n_i$. Therefore

$$n_1 = 2+n_3+2n_4+3n_5+4n_6+\cdots$$

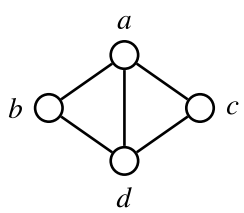
(b) Replace $n_i (i>1)$ with the corresponding numbers, we get

$$n_1 = 2+5+2\times 2 = 11$$

i.e. T has 11 end vertices.

8 [CZ] Problem 4.19

(a) This statement is false. Here is a counterexample:



In this graph, there are three cycles: $(a,b,d),(a,c,d),(a,b,d,c)$, 4 vertices and 5 edges. Therefore, $m=5<n+2=6$.

(b) This statement is true.

Let $n>0$ be the order of the tree, $r\geq 0$ be the degree of each vertex, then the size of the tree is $n-1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$nr = 2(n-1).$$

Integer solutions of this equation are: $n=1, r=0$ and $n=2, r=1$. Therefore, the only possible trees are:

9 [CZ] Problem 4.21

For every vertex v in $\overline{C_{n+2}}$, we have $\deg v = n-1$, therefore $\delta(\overline{C_{n+2}}) = n-1$. By Theorem 4.9, T is isomorphic to a subgraph of $\overline{C_{n+2}}$.

10 [CZ] Problem 4.22

The size of a tree T of order n is $n-1$. The size of K_n has $n(n-1)/2$. Therefore, the size of $\overline{T}$ is $n(n-1)/2-(n-1)=(n-2)(n-1)/2$, which is equal to the size of K_{n-1}.

11 [CZ] Problem 4.23

Let n denote the order of T, then the size of T is $n-1$, and the size of $\overline{T}$ is $n(n-1)/2-(n-1)=(n-2)(n-1)/2$. Since $\overline{T}$ is also a tree, the size of $\overline{T}$ is the order of $\overline{T}$ minus 1, i.e.

$$(n-2)(n-1)/2=n-1.$$

Solve this equation for n, we get $n=1$ or $n=4$. For $n=1$, the tree is trivial. For $n=4$, the only possible tree whose complement is still a tree is: