论题 2-14 作业

姓名：陈劭源　　学号：161240004

1 [TC] Problem 21.1-2

"If": we use mathematical induction to prove this. For base step, a and a are in the same set, and a and a are in the same connected component. For induction step, if S is a set, then S is either a set with only one element, or a set which is a union of two disjoint sets, say S_1 and S_2, connected by edge (u, v) where $u \in S_1$ and $v \in S_2$. In the latter case, for every two elements x and y in S, if both x and y are in S_1 or S_2, then x and y are in the same connected component by induction hypothesis. Otherwise, assume $x \in S_1$, $y \in S_2$, then x has a path to u, v has a path to v, and there exists edge (u, v), thus x and y are connected. Therefore, if two elements are in the same set, they are in the same connected component.

"Only if": if a and b are in the same connected component, then there exists a path from a to b. After the procedure executed, all edges in the path have been processed and all these vertices have been united, thus a and b are in the same set.

2 [TC] Problem 21.1-3

There are $|E|$ edges, and for every edge, FIND-SET is called twice, thus FIND-SET is called $2|E|$ times in all.

The edges, where UNION is performed, constitute the spanning trees of the connected components. For the ith connected component, assume there are $|V_i|$ vertices, then its spanning tree has $|V_i| - 1$ edges. Therefore, there are $|V| - k$ edges in the spanning trees, so UNION is called $|V| - k$ times in all.

3 [TC] Problem 21.2-1

MAKE-SET()

4 [TC] Problem 21.2-3

5 [TC] Problem 21.2-6

6 [TC] Problem 21.3-1

7 [TC] Problem 21.3-2

FIND-SET(x)

```
1  y = x
2  while y.p ≠ y
3      y = y.p
4  z = x
5  while z ≠ y
6      x = z.p
7      z.p = y
8      z = x
9  return y
```

8 [TC] Problem 21.3-3

9 [TC] Problem 21-1