

论题 2-17 作业

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1 [CZ] Problem 4.2

Suppose, to the contrary, that there exists a connected graph G , all of whose vertices have even degrees, that contains a bridge, named e . Then, $G - e$ contains exactly two components. For one of its components, it has exactly one odd vertex, which is incident to e in G . Hence, the sum of the degrees over all vertices in the component is odd, leading to contradiction. Therefore, every connected graph all of whose vertices have even degrees contains no bridge.

2 [CZ] Problem 4.3

Obviously, uv is a $u - v$ path. Suppose there exists another $u - v$ path, denoted by P , then $uv \notin P$. Note that $uv \cup P$ is a cycle, where uv lies. Hence, uv is not a bridge, which leads to contradiction. Therefore, if uv is a bridge in a graph G , then there is a unique $u - v$ path in G .

3 [CZ] Problem 4.5

(a) $n - 1$.

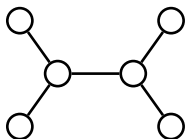
Because every edge of G is bridge, i.e. every edge lies on no cycle, there is no cycle in the connected graph G . Therefore, G is a tree of order n , and its size is $n - 1$.

(b) $n - k$.

For every component of G , it is a connected graph, where every edge is a bridge. Apply the conclusion we obtained in (a), the size of each component is the order of it minus 1. Summing all the sizes up, we get that the size of G is $n - k$, where k is the number of the components.

4 [CZ] Problem 4.16

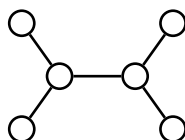
(a)



(b) Let n be the order of the tree, then the size of the tree is $n - 1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

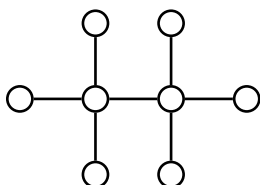
$$\frac{2}{3}n \times 1 + \frac{1}{3}n \times 3 = 2(n - 1).$$

Solve this equation we get $n = 6$. So the only possible tree is:



5 [CZ] Problem 4.17

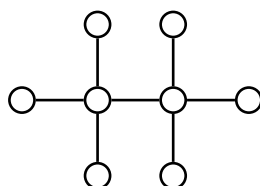
(a)



(b) Let n be the order of the tree, then the size of the tree is $n - 1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$n \times 75\% \times 1 + n \times 25\% \times 4 = 2(n - 1).$$

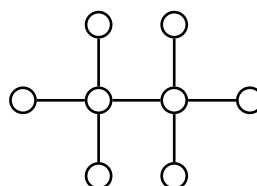
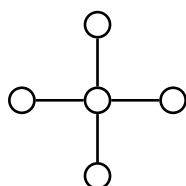
Solve this equation we get that $n = 8$. The only possible tree is:



(c) Let n be the order of the tree, $m > 1$ be the degree of the remaining vertices, then the size of the tree is $n - 1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$n \times 75\% \times 1 + n \times 25\% \times m = 2(n - 1).$$

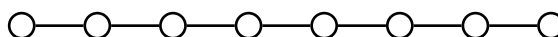
The integer solutions of this equation are: $n = 4, m = 3$ and $n = 8, m = 4$. The corresponding trees are:



(d) Let n be the order of the tree, $m > 1$ be the degree of the remaining vertices, then the size of the tree is $n - 1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$n \times 75\% \times 1 + n \times 25\% \times m = 2(n - 1).$$

The only integer solution of this equation is $n = 8, m = 2$. The only possible tree is:



6 [CZ] Problem 4.18

Let n be the order of the tree, m be number of vertices of degree 3, then the size of the tree is $n - 1$. The sum of degrees over all vertices is twice the size of the tree, i.e.

$$(n - m) \times 1 + m \times 3 = 2(n - 1)$$

after some algebra we get $m = (n - 2)/2$, i.e. T contains $(n - 2)/2$ vertices of degree 3.

7 [CZ] Problem 4.19

(a) Substitute $\sum_i n_i$ for n in $2(n - 1) = \sum_i in_i$, we get $\sum_i 2n_i - 2 = \sum_i in_i$, i.e. $0 = 2 + \sum_i (i - 2)n_i$. Therefore

$$n_1 = 2 + n_3 + 2n_4 + 3n_5 + 4n_6 + \cdots$$

(b) Replace $n_i (i > 1)$ with the corresponding numbers, we get

$$n_1 = 2 + 5 + 2 \times 2 = 11$$

i.e. T has 11 end vertices.