论题 2-2 作业

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1 [CS] P9 Problem 9

Since

$$\binom{n}{2} = \frac{n(n-1)}{2},$$

we have

$$n\binom{n-1}{2} = \frac{n(n-1)(n-2)}{2}$$

and

$$\binom{n}{2}(n-1) = \frac{n(n-1)(n-2)}{2},$$

therefore,

$$n\binom{n-1}{2} = \binom{n}{2}(n-1).$$

Consider choosing one member as the president and two other members as a committee. If we choose the president first, then choose the committee, there are $n\binom{n-1}{2}$ different ways. If we choose the committee first, then choose the president, there are $\binom{n}{2}(n-1)$ different ways. Because the number of ways has nothing to do with the order we choose, we get $n\binom{n-1}{2} = \binom{n}{2}(n-1)$.

2 [CS] P9 Problem 13

Let P_i be the set of the pennies I receive on Day i. By supposition, $|P_1| = 1$ and $|P_{i+1}| = 2|P_i|$. Therefore $|P_i| = 2^{i-1}$. For every positive integer n, $P_1, P_2, \cdots, P_n$ are disjoint sets. By the sum principle, the number of pennies I have on Day 20 is

$$\left|\bigcup_{i=1}^{20} P_i\right| = 1 + 2 + \cdots + 2^{20} = 2^{21} - 1 = 2\,097\,151,$$

and the number of pennies I have on day n is

$$\left|\bigcup_{i=1}^{n} P_i\right| = 1 + 2 + \cdots + 2^{n} = 2^{n+1} - 1.$$