Graph Structure in the Web — Revisited

or A Trick of the Heavy Tail

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Overall

- Crawled in 2012
- Containing 3.5 billion web pages and 128.7 billion links
- Analyzed features of the Web graph, including
  - degrees (indegree, outdegree)
  - components (weekly connected, strongly connected)
  - diameter and distances

Overall

Intuition

It is natural to treat the web as graph.

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By studying Web graph, we can

- design crawl strategies on the web
- improve PageRank algorithms
- understand the sociology of content creation on the web
- predict the evolution of the web

Bow-Tie Structure

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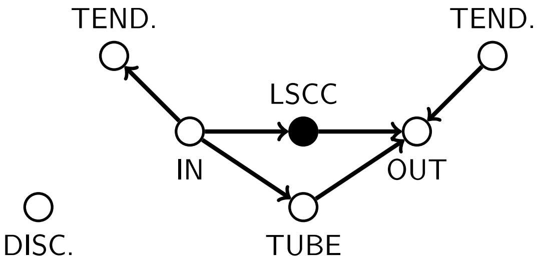


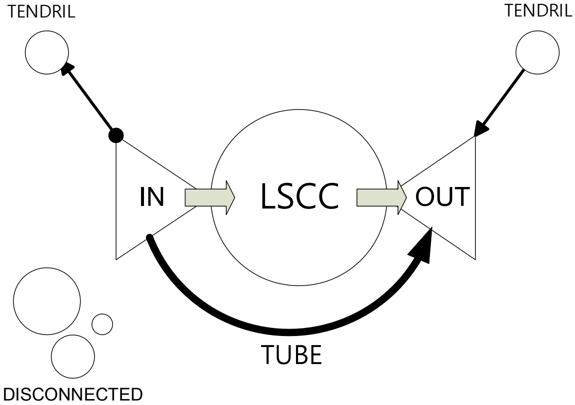
Bow-Tie Structure

Components of Bow-Tie Structure

- LSCC: large strongly connected components
- IN: nodes that can reach LSCC
- OUT: nodes that can be reached from LSCC
- TENDRILS: nodes that can either be reached from IN, or can reach OUT
- TUBES: nodes that lie on paths from IN to OUT, without passing LSCC
- DISCONNECTED: nodes that are not weakly connected to LSCC

Bow-Tie Structure

A Typical Bow-Tie Structure



Bow-Tie Structure

Comparison of Sizes of Bow-Tie Components

	Common Crawl 2012		Broder *et al.* (2000)	
Component	# nodes (k)	% nodes	# nodes (k)	% nodes
LSCC	1 827 543	51.28	56 464	27.74
IN	1 138 869	31.96	43 343	21.29
OUT	215 409	6.05	43 166	21.21
TENDRILS	164 465	4.61	43 798	21.52
TUBES	9 099	0.26	-	-
DISC.	208 217	5.84	16 778	8.24

Table: Comparison of sizes of bow-tie components

Bow-Tie Structure

Phenomena & Analysis

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Phenomena & Analysis

1. The size of LSCC has almost doubled.
   - The web has become more dense and connected.
2. The IN component has become much larger than OUT component in size.
   - Crawl methodology (esp. crawl seeds)
   - Small websites?

Bow-Tie Structure

Comparison between Page Graph and PLD Graph

	page graph		PLD graph	
Component	# nodes (M)	% nodes	# nodes (M)	% nodes
LSCC	1 828	51.28	22.3	51.94
IN	1 139	31.96	3.3	7.65
OUT	215	6.05	13.3	30.98
TENDRILS	164	4.61	0.5	1.20
TUBES	9	0.26	0.2	0.04
DISC.	208	5.84	3.5	8.20

Table: Comparison between Page Graph and PLD Graph

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- Linear in log-log plot

Degree Distribution

Why not Power Law?

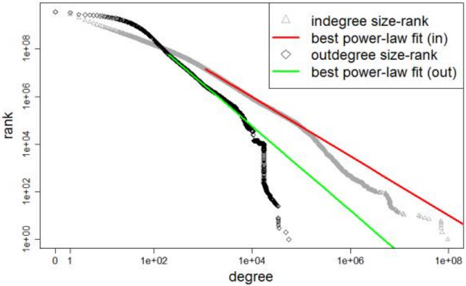

Figure: Log-log plot of degree distributions

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- The conclusion was drawn just by the approximate linear shape in log-log plot.
- The concavity in the left part cannot be explained.
  - There are not so much pages with few hyperlinks as expected.
- The data points in the right part deviate the line.
  - The number of pages with huge number of hyperlinks decreases rapidly as the number of links increases. (hyperpolynomial decrease)

Degree Distribution

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- technical limitations

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Why?

- technical limitations
- although the average degree has significantly increased by 5

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- The harmonic diameter is 24.43.
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- "Small-world network"

References

[1] R. Meusel, S. Vigna, O. Lehmberg, and C. Bizer. Graph structure in the Web — Revisited: A trick of the heavy tail. *Proceedings of WWW Companion '14*, 427–432, 2014.

[2] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, R. Stata, A. Tomkins and J. Wiener. Graph structure in the Web: experiments and models. *Computer Networks*, 33(1–6):309-320, 2000.

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[4] D. Donato, S. Leonardi, S. Millozzi, and P. Tsaparas. Mining the inner structure of the web graph. *WebDB*, 145-150, 2005.