Version Space Algebra in Program Synthesis

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Version Space

In machine learning, the **version space method** proposed by Mitchell [4] learns a Boolean function from given positive/negative examples. Lau et. al. extended Mitchell's version space concept to functions with arbitrary range and defined **version space algebra** [3]. Using these techniques, they developed SMARTedit, a *programming by demonstration* application for repetitive text editing tasks.

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Version space algebra is especially useful in

1. programming by demonstration (PbD);
2. synthesizing action sequences (e.g., text-editing scripts [4], robot control programs [5]).

It can even be used to synthesize shell scripts [6] and simple python programs [1].

Terminology

Hypothesis a function (from attribute space to label space) in machine learning; a program (from input space to output space) in program synthesis.

Hypothesis space the set of all functions can be learned by a learning algorithm; the set of all programs can be produced by a synthesizer (= search space).

Version Space the set of all hypotheses in a hypothesis space H consistent with all training examples in a training set D, referred as $VS_{H,D}$.

Example: Binary Classification with Rectangle

Problem: given a set of points with binary labels in a plane, find a rectangle that separates all positive points and all negative sample points.

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- Hypothesis: all functions $h : \mathbb{R}^2 \to \{0, 1\}$
- Hypothesis space: $\{h | \{(x, y) : h(x, y) = 1\} = [A, B] \times [C, D]\}$

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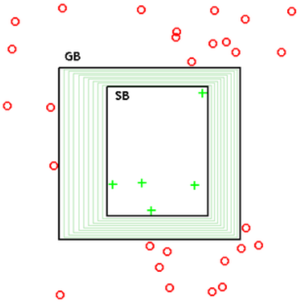

Candidate Elimination Algorithm

The *candidate elimination algorithm* is used to compute the version space $VS_{H,D}$ given hypothesis space H and training set D.

Algorithm 1 Candidate elimination algorithm

Input: hypothesis space H; training set D
Output: version space $VS_{H,D}$

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VS ← H
for each training sample (x, y) in D do
    VS ← {h ∈ VS | h(x) = y}        ▷ Refinement step
end for
return VS
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4: end for
5: return VS
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Maintaining the version space as a list of hypotheses is often infeasible, because the size of the hypothesis space may be very large. We need to find a *compressed* representation of a version space to speed up the refinement step.

Hypotheses Space as a Poset

We may define a partial order between the hypotheses in a hypothesis space.

For Boolean-valued functions, a canonical partial order has been defined.

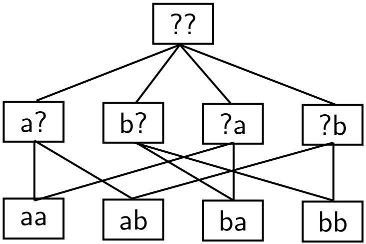
Definition (generality order)

For hypotheses with Boolean values, we say h_1 is **more general** than h_2 ($h_1 \succeq h_2$) if $h_2(x) \to h_1(x)$ for all x in input space. The induced partial order in a hypothesis space is called the **generality order**.

For arbitrary-valued functions, the partial order may be defined by the application designer.

Example: the "ab?" Language

- Hypothesis: $\{a, b\}^2 \to \{0, 1\}$
- Hypothesis space: $\{a, b, ?\}^2$, where ? matches any character
- Hasse diagram of the generality order:



Boundary-Set Representability

With a partial order defined on the hypothesis space, we may use two antichains G, S (called *boundaries*) to represent a version space.

Definition (boundary-set representability)

A version space VS of a partially-ordered hypothesis space H is **boundary-set representable (BSR)**, if it can be written as the form

$$VS = \{h \in H | \exists h_g \in G, \exists h_s \in S, h_g \succeq h \succeq h_s\}$$

where G, S are two antichains of H.

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Examples for the “ab?” language:

- $D = \{(ab, 1)\}$: $G = \{??\}$, $S = \{ab\}$;
- $D = \{(aa, 0)\}$: $G = \{b?, ?a\}$, $S = \{ab, ba, bb\}$;
- $D = \{(aa, 1), (ab, 1)\}$: $G = S = \{a?\}$.

Boundary-Set Representability

Not all version spaces are BSR. Hirsh showed that **convexity** and **definiteness** are a necessary and sufficient condition for a version space being BSR [2].

A subset C of a partially ordered set $(S, \preceq)$ is

- **convex**, if: $x \preceq y \preceq z$ $(x, z \in C, y \in S)$ implies $y \in C$;
- **definite**, if: for every $y \in C$, there exists $x \in \min\{C\}$ (minimal elements of C) and $z \in \max\{C\}$ (maximal elements of C), such that $x \preceq y \preceq z$.

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Theorem (version space representation theorem)

Every version space of a Boolean hypothesis space with generality order is BSR.

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Example:

- Hypothesis: $\Sigma^* \rightarrow \{0, 1\}$ where $\Sigma = \{a, b, c\})$
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2. $D = \{(\text{aba},1),(\text{ba},0)\}$, $G = \{a*\}$, $S = \{aba*\}$;
3. $D = \{(\text{aba},1),(\text{ba},0),(\text{abc},1)\}$, $G = \{a*\}$, $S = \{ab*\}$;

Version Space Algebra

Lau et. al. defined **version space algebra**, which allows us to build complex version space from simple, atomic version spaces.

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Definition (version space union)

Given two hypothesis spaces H_1, H_2 with the same input and output spaces, the **union** of two version spaces $VS_{H_1,D} \cup VS_{H_2,D}$ is defined as $VS_{H_1 \cup H_2,D}$.

Definition (version space join)

Given two version spaces VS_{H_1,D_1} and VS_{H_2,D_2}, the **independent join** of the two spaces $VS_{H_1,D_1} \bowtie VS_{H_2,D_2}$ is defined as

$$\{\langle h_1, h_2\rangle | h_1 \in VS_{H_1,D_1}, h_2 \in VS_{H_2,D_2}\},$$

where $\langle h_1, h_2\rangle(x, y) \equiv (h_1(x), h_2(y))$.

Version Space Algebra

The version space algebra is analogous to the context-free grammar, in the following sense:

VSA	CFG
atomic version space	terminal symbol
compound version space	non-terminal symbol
version space union	“or” of two production rules
version space join	concatenation of two symbols

Note: compound version spaces can be represented by representing all its components simultaneously; thus compound version space can be updated by updating all components individually.

Example: SMARTedit

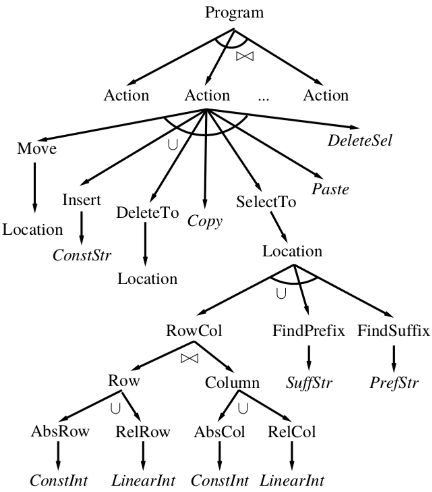

Figure: Version space structure for SMARTedit

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