

Reading summary:

Hidden Markov Models for Zero-Inflated Poisson Counts with an Application to Substance Use

Weijia Xiong

6/1/2020

Data and Objective

- ▶ Data

45 participants. The TLFB daily usage for the 12 weeks prior to and 4 weeks after completion of the 2-day stress and cue-reactivity study.

- ▶ Objective

Whether study participation alters drug seeking behavior.

ZIP Hidden Markov Model for Poisson Counts

ZIP-HMM:

$$(Z_{it}|S_{it}) = \begin{cases} 0 & \text{w.p. } \pi_{it} \\ \text{Poisson } (\theta_{it}) & \text{w.p. } 1 - \pi_{it} \end{cases}$$
$$\log(\theta_{it}) = \lambda_{0i} + \lambda_{1i}S_{it-1}$$

Poisson distribution:

$$f(Z_{it}|\theta_{it}, S_{it} = 1, X_{it}, X_{TRT_{it}}) = \frac{1}{Z_{it}!} e^{-\theta_{it}} \theta_{it}^{Z_{it}}$$

Transition probabilities:

$$\begin{aligned} p_{it}^{00} &= P(S_{it} = 0 | S_{i,t-1} = 0, X_{it}, X_{TRT_{it}}) \\ p_{it}^{10} &= P(S_{it} = 1 | S_{i,t-1} = 0, X_{it}, X_{TRT_{it}}) \\ p_{it}^{01} &= P(S_{it} = 0 | S_{i,t-1} = 1, X_{it}, X_{TRT_{it}}) \\ p_{it}^{11} &= P(S_{it} = 1 | S_{i,t-1} = 1, X_{it}, X_{TRT_{it}}) \end{aligned}$$

Transitioning or remaining high:

$$p_{it}^{1S_{(i,t-1)}} = \text{logit}^{-1}(\beta_0 + \beta_1 S_{i,t-1} + \beta_2 X_{it} + \beta_3 X_{TRT_{it}})$$

Low-risk group:

$$\text{logit}(\pi_{it} | Y_{it}) = \alpha_0 + \alpha_1 Y_{it1} + \dots + \alpha_j Y_{itj}$$

Bayesian Inference

Estimation of exact posterior distribution:

- ▶ Gibbs sampler
- ▶ MCMC algorithms

Implement tool: WinBUGS

Comparison with other models

- ▶ conditional predictive ordinate statistic: $-\log(CPO)_i$ (Smaller)
- ▶ log pseudo-marginal likelihood: LPML (Larger)

1. Nested 1-state models

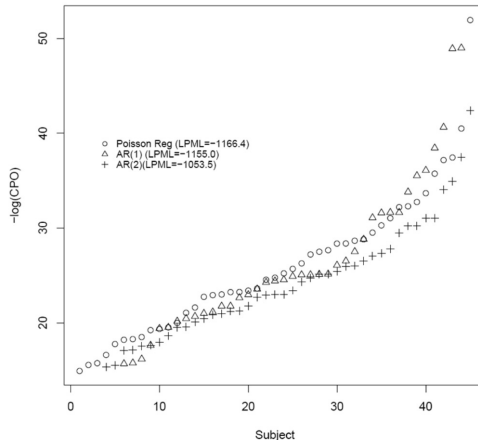


Figure 4. Relative model fits of nested models based on $-\log(CPO)_i$. The value in parenthesis is the LPML (larger is better).

Comparison with other models

2. Second order HMM

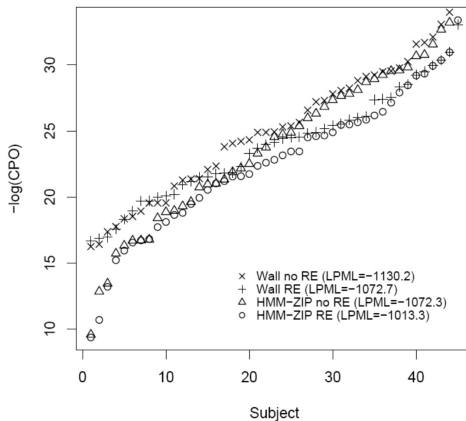


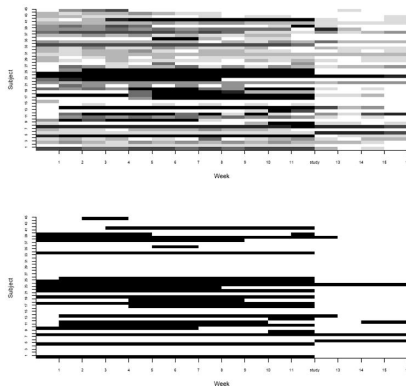
Figure 5.

Relative model fits of the 2-state hidden Markov models based on $-\log(CPO_i)$. The value in parenthesis is the LPML (larger is better).

Results

Top: the matrix of weekly drug use counts for each participant in the study.

Bottom: membership in the hidden state derived from posterior probabilities.



- ▶ $OR_{TRT0} = 0.06[0.01, 0.19]$
- ▶ Highly preventative effect: study-benefit (seen after week 12) in reducing drug-seeking behavior