

规约算法

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Reduction Operations

Multiple values are reduced into a single value

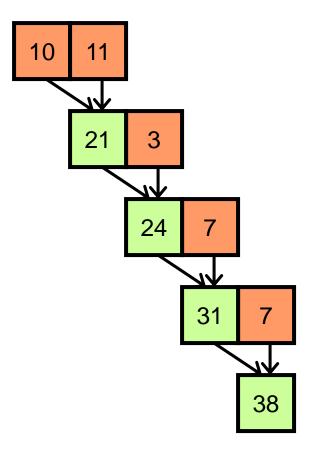
- ADD, MUL, AND, OR,

10 11 12 13
$$\rightarrow$$
 46

Useful primitive

Easy enough to allow us to focus on optimization techniques

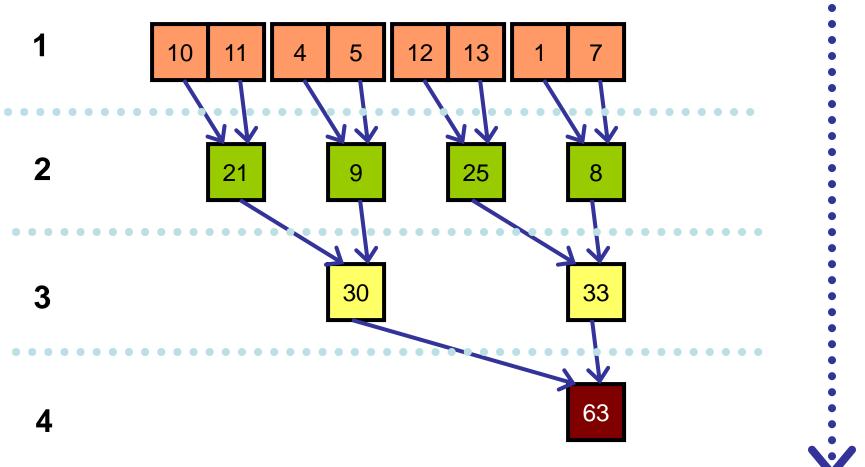
Sequential Reduction



- Start with the first two elements --> partial result
- Process the next element
- O(N)

Parallel Reduction

Pair-wise reduction in steps – Tree-like

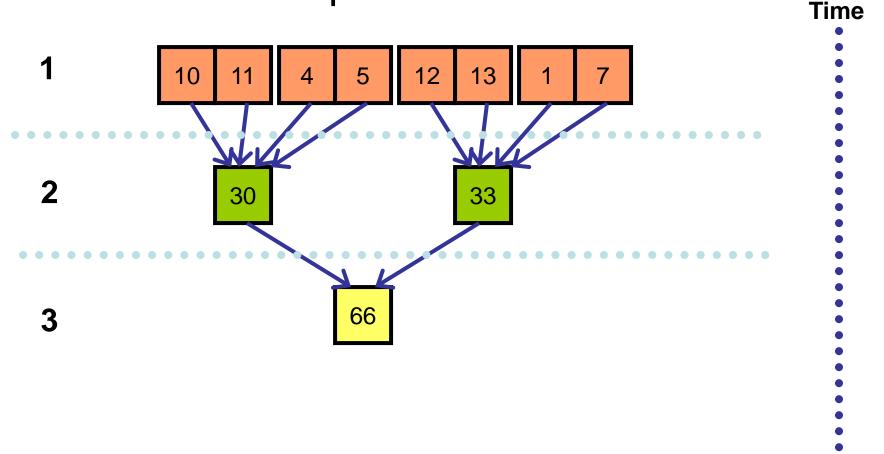


Time

• log₂ N steps / N amount of work

Parallel Reduction – Different Degree-Trees possible

Pair-wise reduction in steps – Tree-like



• log₄ N steps / N amount of work

CUDA Strategy

- Single Block:
 - Use Tree-Like approach
- Multiple Blocks?
 - Not a necessity
 - one thread can always process many elements
 - But, will suffer from low utilization
 - Utilize GPU resources
 - Useful for large arrays
- Each block processes a portion of the input



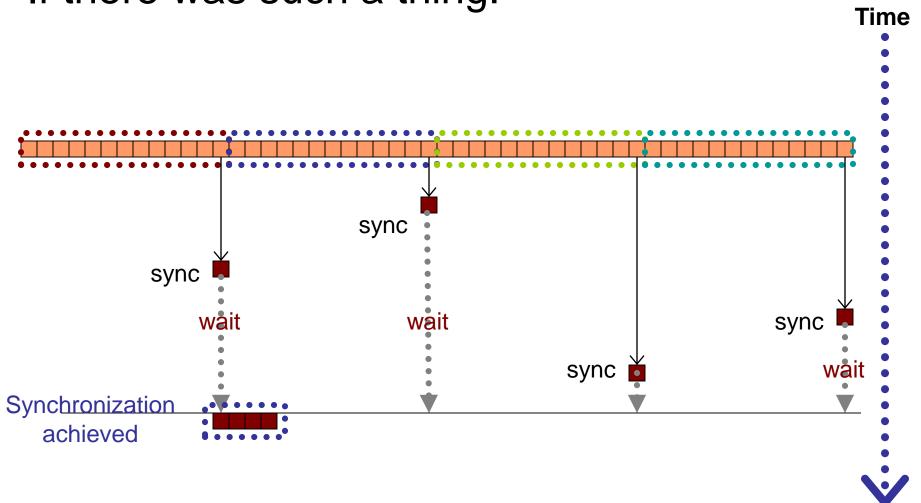
How about multiple blocks

 How do we communicate results across **Time** blocks?

- The key problem is synchronization:
 - How do we know that each block has finished?

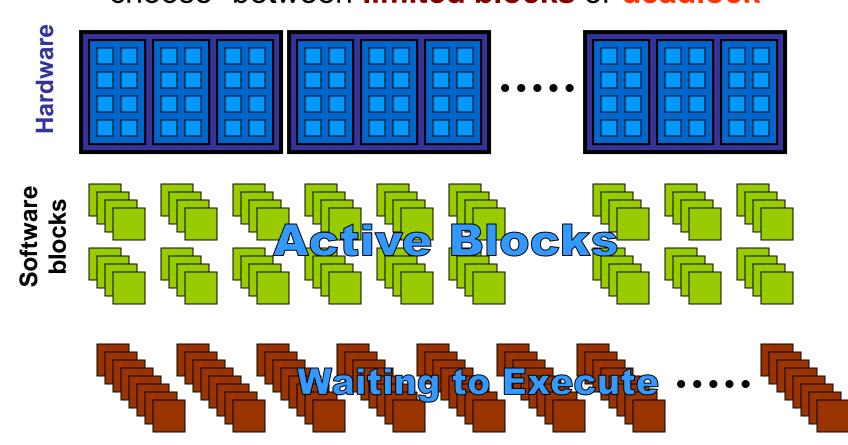
Global Synchronization

• If there was such a thing:

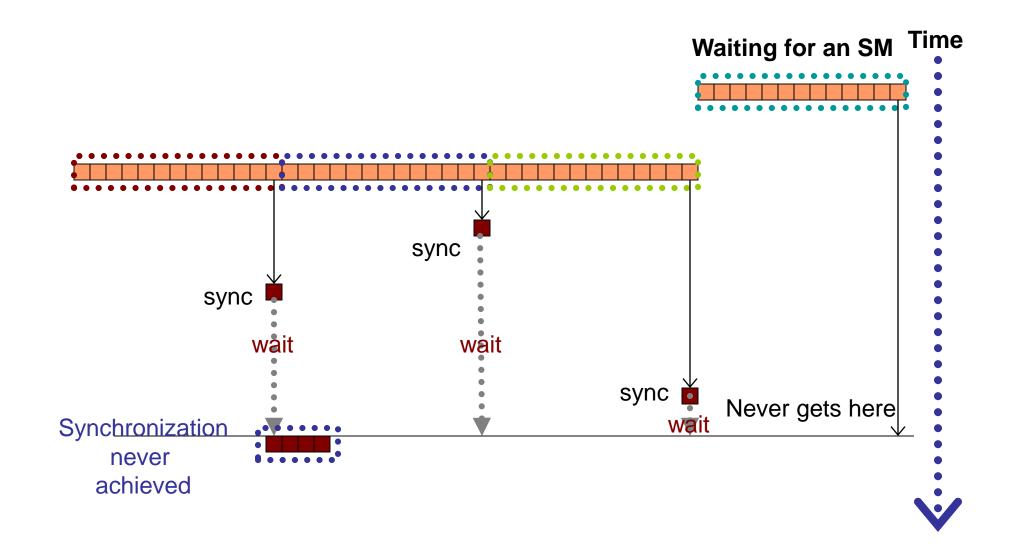


The Problem with Global Synchronization

- CUDA does not support it
 - One reason:
 - it's expensive to implement
 - Another reason:
 - "choose" between limited blocks or deadlock



Deadlock

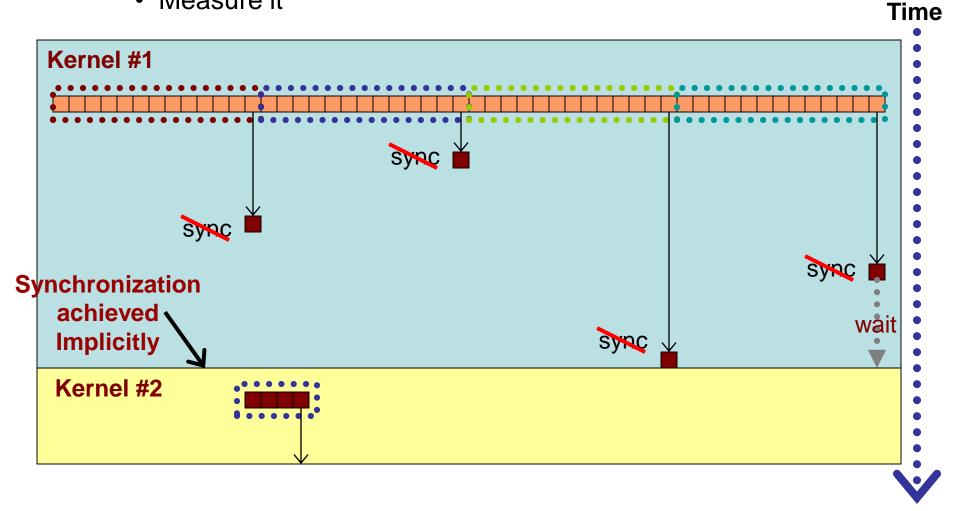


The Problem with Global Synchronization / Summary

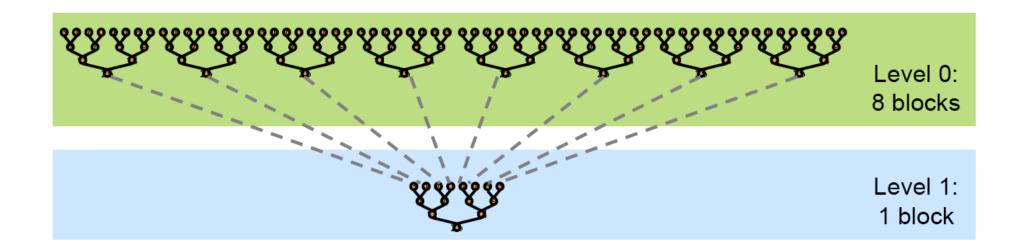
- If there was global sync
 - Global sync after each block
 - Once all blocks are done, continue recursively
- CUDA does not have global sync:
 - Expensive to support
 - Would limit the number of blocks
 - Otherwise deadlock will occur
 - Once a block gets assigned to an SM it stays there
 - Each SM can take only 8 blocks
 - So, at most #SMs x 8 blocks could be active at any given point of time
- Solution: Decompose into multiple kernels

Decomposing into Multiple Kernels

- Implicit Synchronization between kernel invocations
 - Overhead of launching a new kernel non-negligible
 - Don't know how much
 - Measure it



Reduction: Big Picture



- The code for all levels is the same
- The same kernel code can be called multiple times

Optimization Goal

Get maximum GPU performance



- Two components:
 - Compute Bandwidth: GFLOPs
 - Memory Bandwidth: GB/s

Optimization Goals Cont.

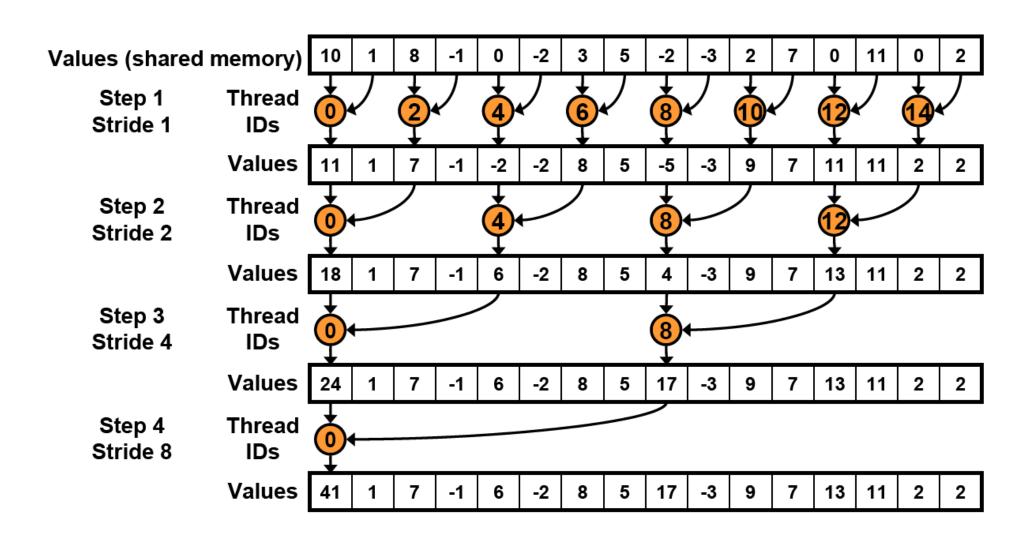
- Reductions typically have low arithmetic intensity
 - FLOPs/element loaded from memory
- So, bandwidth will be the limiter

- For the ASUS ENGTX280 OC
 - 512-bit interface, 1.14GHz DDR3
 - $-512/8 \times 1.14 \times 2 = 145.92$ GB/s

Reduction #1: Strategy

- Load data:
 - Each thread loads one element from global memory to shared memory
- Actual Reduction: Proceed in logN steps
 - A thread reduces two elements
 - The first two elements by the first thread
 - The next two by the next thread
 - And so, on
 - At the end of each step:
 - Deactivate half of the threads
 - Terminate: when one thread left
- Write back to global memory

Reduction Steps



Reduction #1 Code: Interleaved Accesses

```
global__ void reduce0(int *g_idata, int *g_odata) {
extern __shared__ int sdata[];
// each thread loads one element from global to shared mem
unsigned int tid = threadldx.x;
unsigned int i = blockldx.x*blockDim.x + threadldx.x;
sdata[tid] = g_idata[i];
 _syncthreads();
// do reduction in shared mem
for (unsigned int s=1; s < blockDim.x; s *= 2) { // step = s x 2
     if (tid \% (2*s) == 0) { // only threadIDs divisible by the step participate
             sdata[tid] += sdata[tid + s];
      syncthreads();
// write result for this block to global mem
if (tid == 0) g_odata[blockldx.x] = sdata[0];
```

Performance for kernel #1

Time	(2^{22})	ints)
------	------------	-------

Bandwidth

K	\bigcirc	rn	el	1	
$-\Gamma N$	し				

interleaved addressing with divergent branching

8.054 ms

2.083 GB/s

Note: Block size = 128 for all experiments

Caveat:

Bandwidth calculation:

results are for a G80 processor

Each block processes 128 elements and does:

128 reads & 1 write
We only care about global memory

At each kernel/step:

N (element reads) + N / 128 (element writes)

Every kernel/step reduces input size by 128x

next set N = N / 128

So for:

N = 4194304

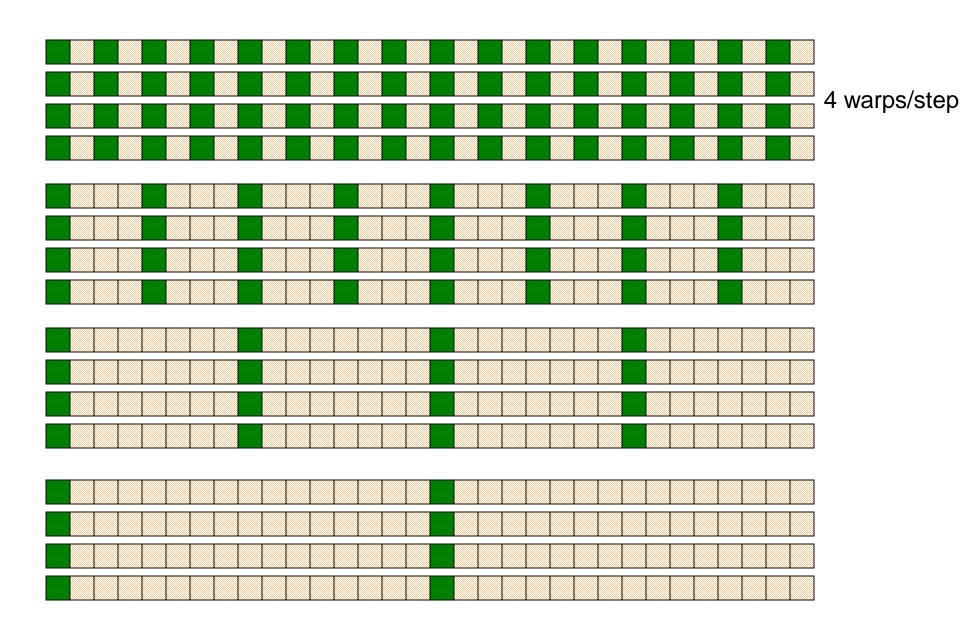
Accesses = 4227394

Each access is four bytes

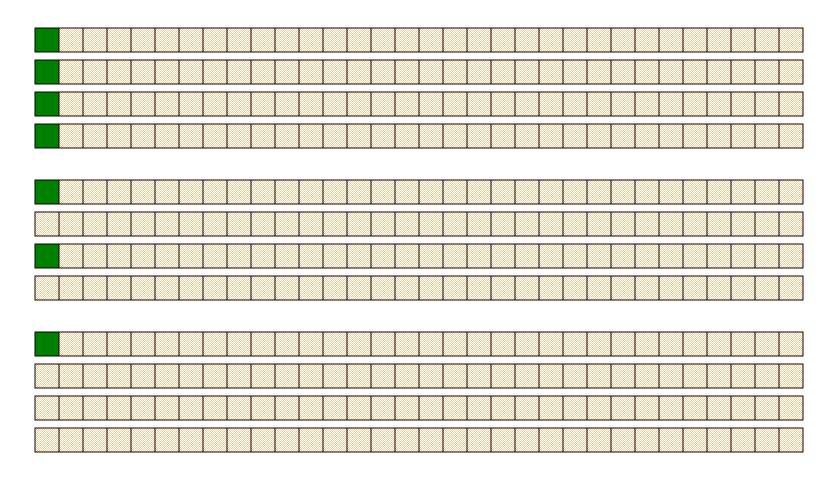
Reduction #1 code: Interleaved Accesses

```
_global___ void reduce0 (int *g_idata, int *g_odata) {
extern __shared__ int sdata[];
// each thread loads one element from global to shared mem
unsigned int tid = threadldx.x;
unsigned int i = blockldx.x*blockDim.x + threadldx.x;
sdata[tid] = g_idata[i];
  syncthreads();
// do reduction in shared mem
for (unsigned int s=1; s < blockDim.x; s *= 2) {
     if (tid % (2*s) == 0) {
                                          Highly divergent code
            sdata[tid] += sdata[tid + s];
                                            leads to very poor
                                                performance
     __syncthreads();
// write result for this block to global mem
if (tid == 0) g_odata[blockldx.x] = sdata[0];
```

Divergent Branching: Warp Control Flow



Divergent Branching: Warp Control Flow



Reduction #2: Interleaved Addr./non-divergent branching

Replace the divergent branching code:

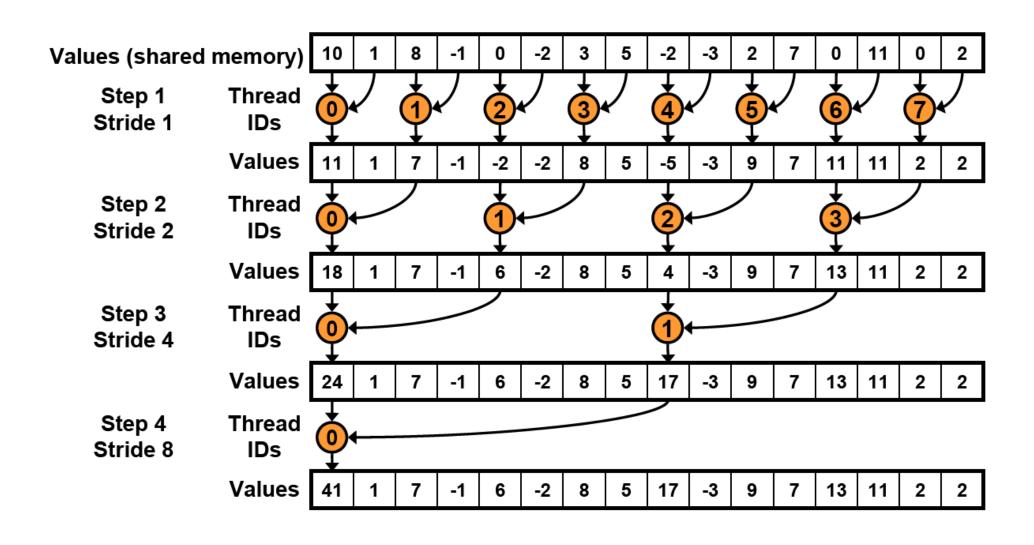
```
// do reduction in shared mem
for (unsigned int s=1; s < blockDim.x; s *= 2) {
      if (tid % (2*s) == 0) {
            sdata[tid] += sdata[tid + s];
      }
      __syncthreads();
}</pre>
```

With strided index and non-divergent branch

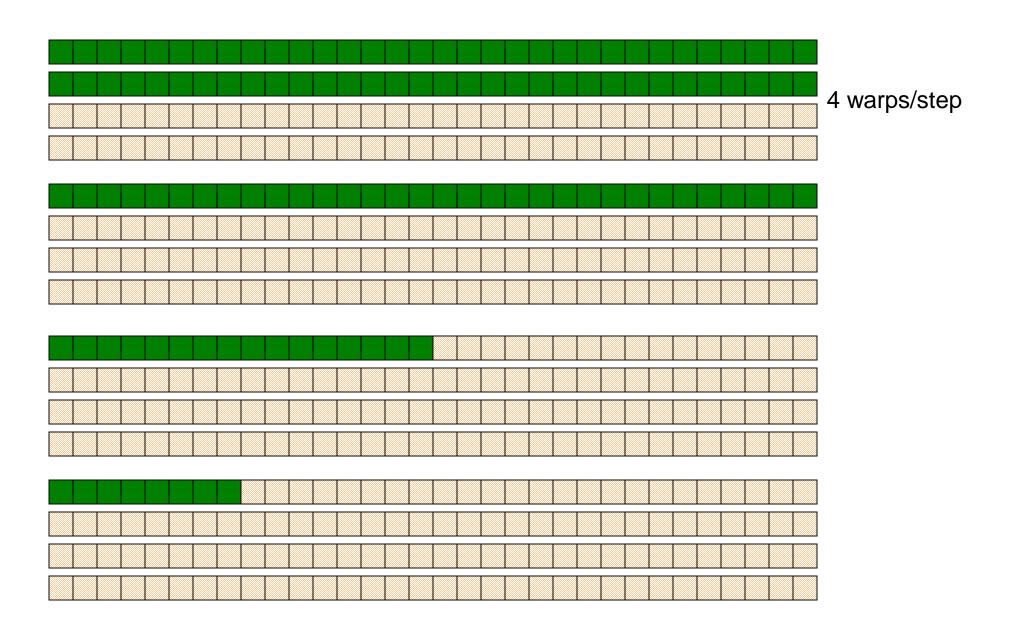
```
// do reduction in shared mem
for (unsigned int s=1; s < blockDim.x; s *= 2) {
    int index = 2 * s * tid;

    if (index < blockDim.x / s) {
        sdata[index] += sdata[index + s];
    }
    __syncthreads();
}</pre>
```

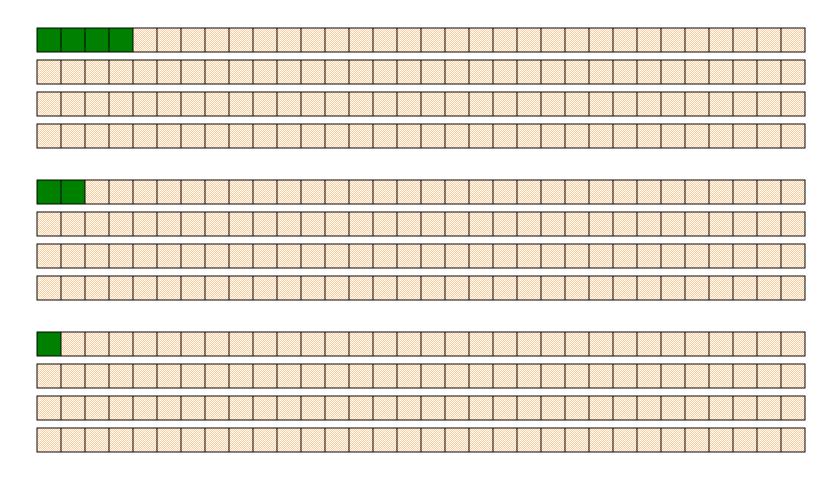
Reduction #2: Access Pattern



Reduction #2: Warp control flow



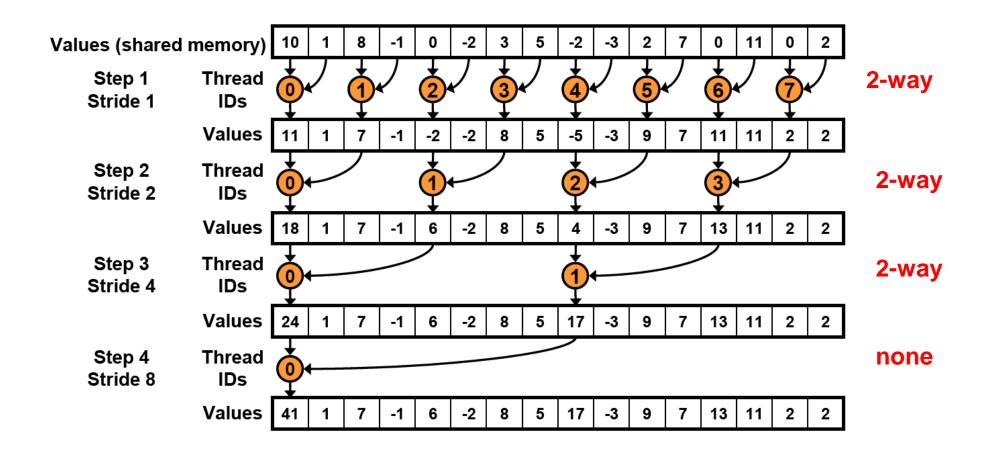
Reduction #2: Warp control flow



Performance for 4M element reduction

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing non-divergent branching	3.456 ms	4.854 GB/s	2.33x	2.33x

Reduction #2: Problem

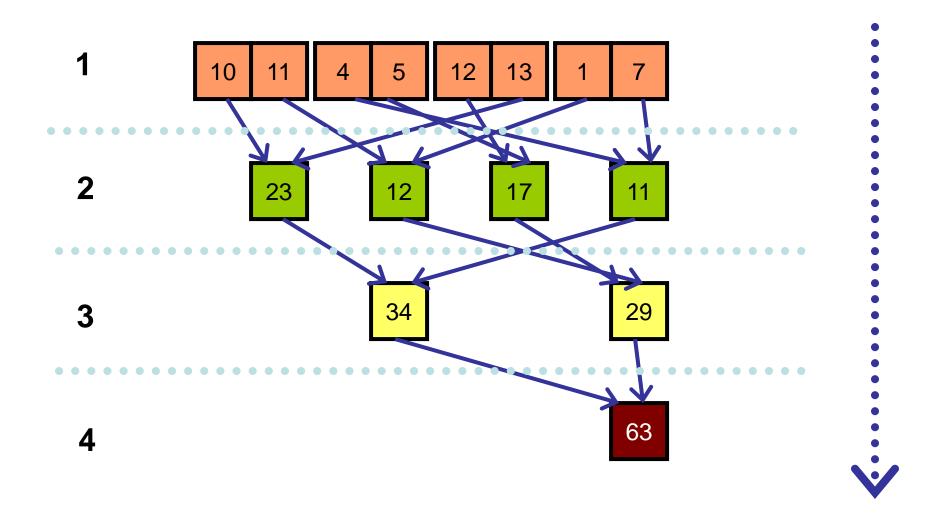


2-way bank conflicts at every step

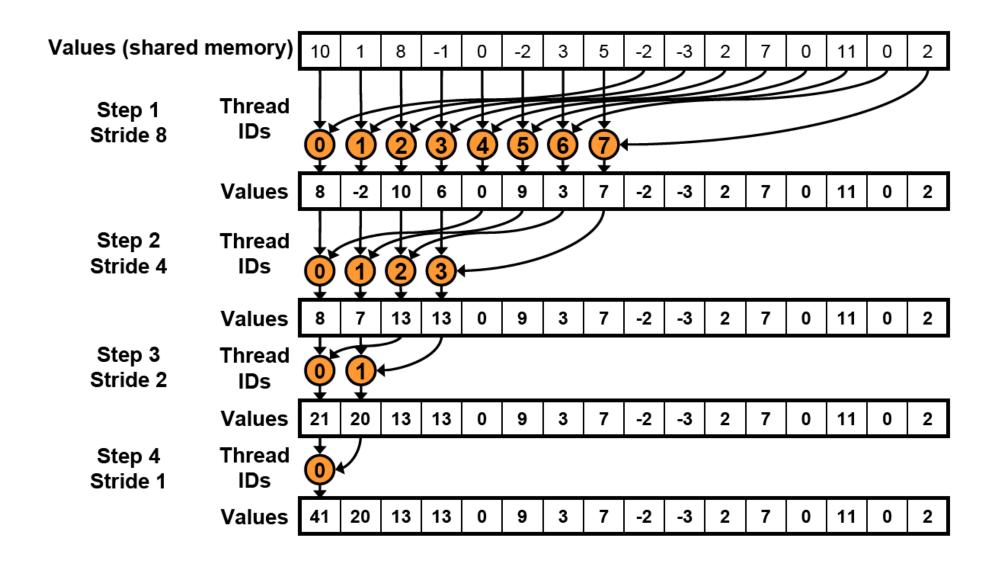
Recall there are more than 16 threads

To see the conflicts see what happens with 128 threads

Observe: Arbitrary Unique Pairs OK



Reduction #3: Sequential Accesses



Eliminates bank conflicts

Reduction #3: Code Changes

Replace stride indexing in the inner loop:

```
// do reduction in shared mem
for (unsigned int s=1; s < blockDim.x; s *= 2) {
    int index = 2 * s * tid;

    if (index < blockDim.x == 0) {
        sdata[index] += sdata[index + s];
    }
    __syncthreads();
}</pre>
```

With reversed loop and threadID-based indexing:

```
// do reduction in shared mem
for (unsigned int s = blockDim.x/2; s > 0; s /= 2) {
    if (tid < s) {
        sdata[tid] += sdata[tid + s];
    }
    __syncthreads();
}</pre>
```

Performance for 4M element reduction

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing non-divergent branching	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x

Reduction #3: Bad resource utilization

- All threads read one element
- First step: half of the threads are idle
- Next step: another half becomes idle

```
// do reduction in shared mem
for (unsigned int s = blockDim.x/2; s > 0; s /= 2) {
      if (tid < s) {
            sdata[tid] += sdata[tid + s];
      }
      __syncthreads();
}</pre>
```

Reduction #4: Read two elements and do the first step

Original: Each thread reads one element

```
// each thread loads one element from global to shared mem unsigned int tid = threadldx.x; unsigned int i = blockldx.x*blockDim.x + threadldx.x; sdata[tid] = g_idata[i]; __syncthreads();
```

Read two and do the first reduction step:

```
// each thread loads two elements from global to shared mem
// end performs the first step of the reduction
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x* blockDim.x * 2 + threadIdx.x;
sdata[tid] = g_idata[i] + g_idata[i + blockDim.x];
__syncthreads();
```

Performance for 4M element reduction

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing non-divergent branching	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x
Kernel 4: first step during global load	0.965 ms	17.377 GB/s	1.78x	8.34x

Reduction #4: Still way off

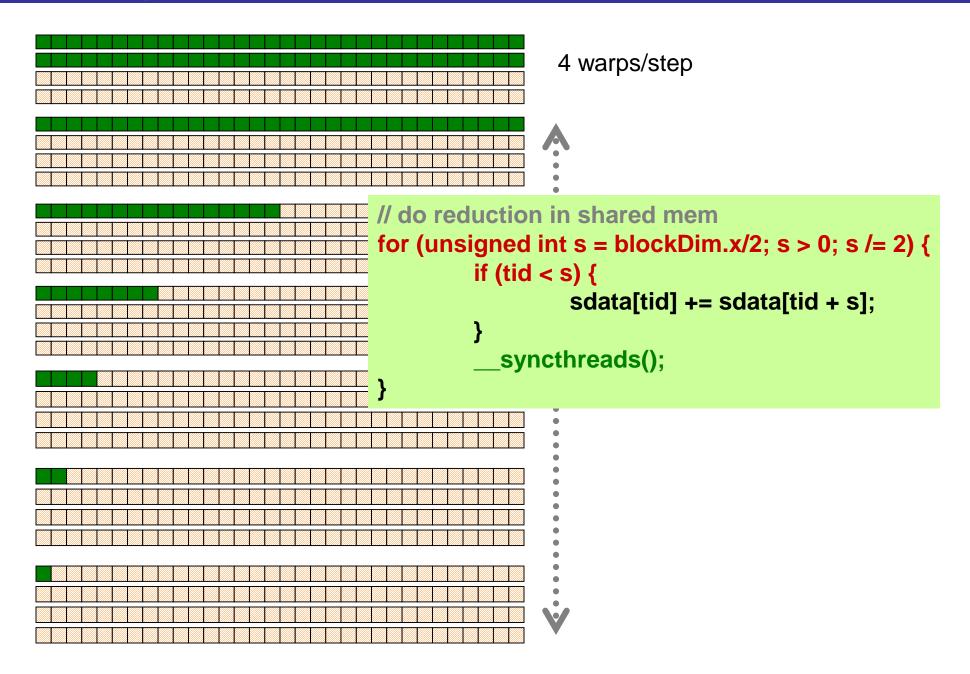
- Memory bandwidth is still underutilized
 - We know that reductions have low arithmetic density
- What is the potential bottleneck?
 - Ancillary instructions that are not loads, stores, or arithmetic for the core computation
 - Address arithmetic and loop overhead

Unroll loops to eliminate these "extra" instructions

Unrolling the last warp

- At every step the number of active threads halves
 - When s <=32 there is only one warp left</p>
- Instructions are SIMD-synchronous within a warp
 - They all happen in lock step
 - No need to use __syncthreads()
 - We don't need "if (tid < s)" since it does not save any work
 - All threads in a warp will "see" all instructions whether they execute them or not
- Unroll the last 6 iterations of the inner loop
 - $-s \le 32$

Reduction #2: Warp control flow



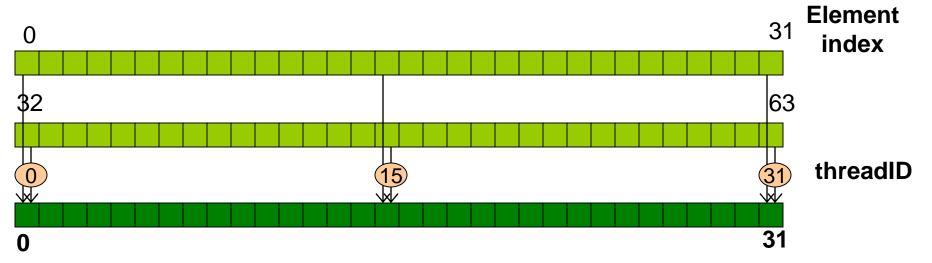
Reduction #5: Unrolling the last 6 iterations

```
// do reduction in shared mem
for (unsigned int s = blockDim.x/2; s > 32; s /= 2) {
      if (tid < s) {
               sdata[tid] += sdata[tid + s];
        _syncthreads();
if (tid <32)
      sdata[tid] += sdata[tid + 32];
      sdata[tid] += sdata[tid + 16];
      sdata[tid] += sdata[tid + 8];
      sdata[tid] += sdata[tid + 4];
      sdata[tid] += sdata[tid + 2];
      sdata[tid] += sdata[tid + 1];
```

- This saves work in all warps not just the last one
 - Without unrolling all warps execute the for loop and if statement

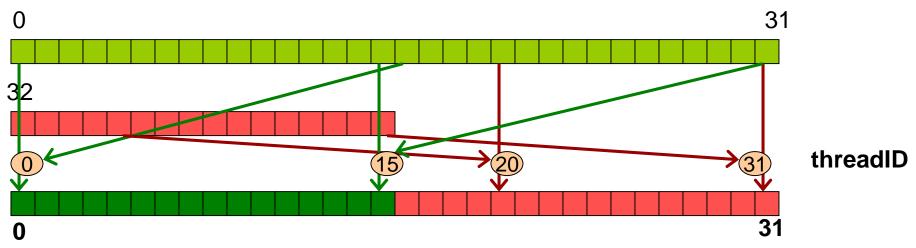
- Keep in mind:
- #1: Warp execution proceeds in lock step for all threads
 - All threads execute the same instruction
 - So:
 - sdata[tid] += sdata[tid + 32];
 - Becomes:
 - Read into a register: sdata[tid]
 - Read into a register: sdate[tid+32]
 - Add the two
 - Write: sdata[tid]
- #2: Shared memory can provide up to 16 words per cycle
 - If we don't use this capability it just gets wasted

• sdata[tid] += sdata[tid + 32];

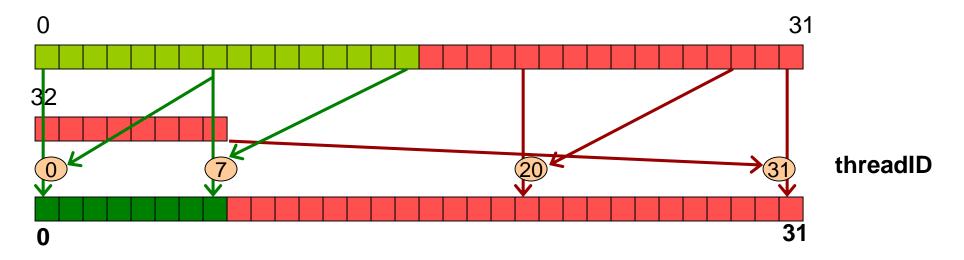


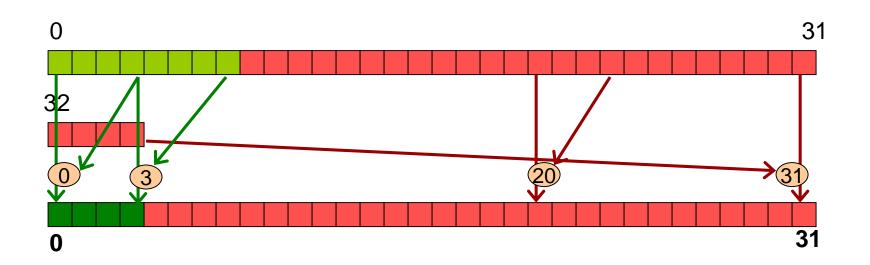
All threads doing useful work

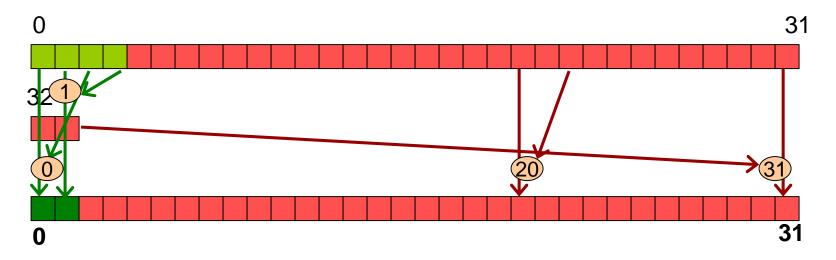
sdata[tid] += sdata[tid + 16];

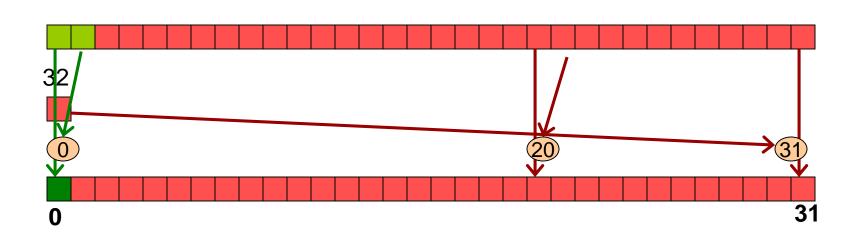


- Half of the threads, 16-31, are doing useless work
- At the end they "destroy" elements 16-31
- Elements 16-31 are inputs to threads 0-14
- But threads 0-15 read them before they get written by threads 16-31
 - All reads proceed in "parallel" first
 - All writes proceed in "parallel" last
- So, no correctness issues
- But, threads 16-31 are doing useless work
 - The units and bandwidth are there → no harm (only power)









Performance for 4M element reduction

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
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Kernel 4: first step during global load	0.965 ms	17.377 GB/s	1.78x	8.34x
Kernel 5: Unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x

Reduction #6: Complete Unrolling

- If we knew the number of iterations at compile time, we could completely unroll the reduction
 - Block size is limited to 512
 - We can restrict our attention to powers-of-two block sizes
- We can easily unroll for a fixed block size
 - But we need to be generic
 - How can we unroll for block sizes we don't know at complile time?
- C++ Templates
 - CUDA supports C++ templates on device and host functions

Unrolling Using Templates

Specify block size as a function template parameter:

```
template <unsigned int blockSize>
__global___ void reduce5(int *g_data, int *g_odata)
```

Reduction #6: Completely Unrolled

```
if (blockSize >= 512) {
         if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads();
if (blockSize >= 256) {
         if (tid < 128) { sdata[tid] += sdata[tid + 128]; } __syncthreads();
if (blockSize >= 128) {
         if (tid < 64) { sdata[tid] += sdata[tid + 64]; } __syncthreads();
if (tid < 32) {
         if (blockSize >= 64) sdata[tid] += sdata[tid + 32];
         if (blockSize >= 32) sdata[tid] += sdata[tid + 16];
         if (blockSize >= 16) sdata[tid] += sdata[tid + 8];
         if (blockSize >= 8) sdata[tid] += sdata[tid + 4];
         if (blockSize >= 4) sdata[tid] += sdata[tid + 2];
         if (blockSize >= 2) sdata[tid] += sdata[tid + 1];
```

- Note: all code in RED will be evaluated at compile time.
- Results in a very efficient inner loop

What if block size is not known at compile time?

There are "only" 10 possibilities: switch (threads) case 512: reduce5<512><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 256: reduce5<256><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 128: reduce5<128><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 64: reduce5< 64><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 32: reduce5< 32><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break; case 16: reduce5< 16><<< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 8: reduce5< 8><< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 4: reduce5< 4><< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 2: reduce5< 2><< dimGrid, dimBlock, smemSize >>>(d_idata, d_odata); break; case 1: reduce5< 1><<< dimGrid, dimBlock, smemSize >>>(d idata, d odata); break;

Performance for 4M element reduction

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
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Kernel 5: Unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x
Kernel 6: Complete Unroll	0.381 ms	43.996 GB/s	1.41x	21.16x

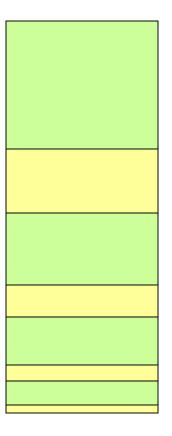
Can we improve?

copy data

O(N/P)

O(log N)

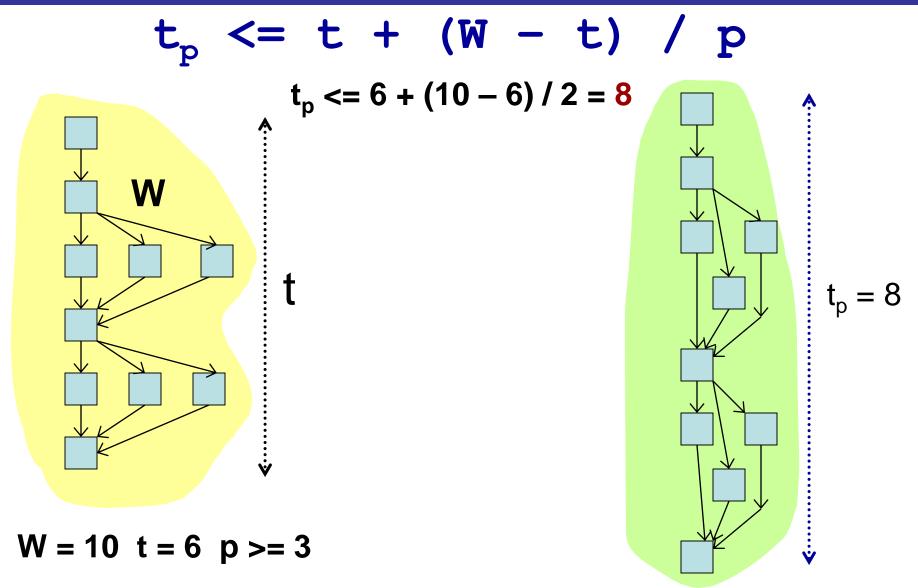
If we can choose P --> can we make this O(log N)



Can we improve?

- Two parts:
 - Loading elements + pair-wise reduction
 - O(*N/P*)
 - Tree-like reduction
 - O(log N)
- Overall: O(N/P + log N)
- Can we make the O(N/P) part O(log N)?
- What should be P?
 - N/log N
- So, first step should be:
 - load N/log N elements
 - do the first N/log reductions

Brent's theorem



If given infinite processors an algorithm takes t steps then given p processors it should take at most t_p steps

Can we improve the algorithm?

- Work or "element complexity": W
 - total number of operations performed collectively by all processors
 - Time to execute on a single processor
- Time Step: All operations that have no unresolved dependencies are performed in parallel
- Depth or "step complexity": t
 - time to complete all time steps
 - time to execute if we had infinite processors
- Computation time on P Processors: t_p
 - time to execute when there are P processors

Brent's Theorem

$$t_p \ll t + (W - t) / p$$

- p = 1, $t_p = W$ (seq. algorithm)
- $p \rightarrow inf, t_p \rightarrow t (limit)$
- if sequential --> t = W

- Work or "element complexity": W
- Depth or "step complexity": t
- Computation time on P Processors: tp

What is Brent's theorem useful for?

- Tells us whether an algorithm can be improved for sure
- Example:
 - summing the elements of an array
 - for (i=0; i < N); i++) sum = sum + a[i];
 - W = N, t = N
 - $t_p = N + (N N) / p = N$
 - No matter what, this will take N time

Brent's theorem example application

- Summing the elements recursively
 - -((a[0] + a[1]) + ((a[2] + a[3])) ...
 - t = log N
 - -W=N
- T = log(N) + (N log(N))/p

- Conclusions:
 - No implementation can run faster than O(log N).
 - Given N processors, there is an algorithm of log N time
 - Given $N/\log N$ processors, there is an algorithm of ~2 x log N = O(log N) time
 - Given 1 processor, there is an algorithm that takes N time

Brent's theorem contd.

- Cost: P x Time Complexity
 - $-P = 1 \times O(N)$ time
 - $-P = log N \times O(log N) time$
 - $-P = N \times O(log N)$ time
 - The implementations with 1 or log N processors, therefore are cost optimal,
 - The implementation with N processors is not.
- It is important to remember that Brent's theorem does not tell us how to implement any of these algorithms in parallel
 - it merely tells us what is possible
 - The Brent's theorem implementation may be hideously ugly compared to the naive implementation

Parallel Reduction Complexity

- Log(N) parallel steps, each step S does N/2^S independent operations
 - Step Complexity: O(log N)
- For $N=2^D$, performs $\sum_{S \in [1...D]} 2^{D-S} = N-1$ operations
 - Work Complexity is O(N) it is work-efficient
 - Does not perform more operations than a sequential algorithm
- With P threads physically in parallel (P processors), time complexity is O(N/P + log N)
 - N/P for global to shared
 - log N for the calculations
 - Compare to O(N) for sequential reduction

What about Cost?

- Cost of parallel algorithm
 - Processors x Time Complexity
 - Allocate threads instead of processors: O(N) threads
 - Time complexity is O(log N), so cost is O(N log N)
 - Not cost efficient
- Brent's theorem suggests O(N/log N) threads
 - Each thread does O(log N) sequential work
 - Then all O(Wlog N) threads cooperate for O(log N) steps
 - Cost = O((Nlog N) * log N) = O(N) → cost efficient
- Sometimes called algorithm cascading
 - Can lead to significant speedups in practice

Algorithm Cascading

- Combine sequential and parallel reduction
 - Each thread loads and sums multiple elements into shared memory
 - Tree-based reduction in shared memory
- Brent's theorem says each thread should sum O(log N) elements
 - i.e., 1024 to 2048 elements per block vs. 256
- In Mike Harris' experience, beneficial to push it even further
 - Possibly better latency hiding with more work per thread
 - More threads per block reduces levels in tree of recursive kernel invocations
 - High kernel launch overhead in last levels with few blocks
- On G80, best perf with 64-256 blocks of 128 threads
 - 1024-4096 elements per thread

Reduction #7: Multiple Adds / Thread

Replace load and add two elements

```
// each thread loads two elements from global to shared mem
// end performs the first step of the reduction
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x* blockDim.x * 2 + threadIdx.x;
sdata[tid] = g_idata[i] + g_idata[i + blockDim.x];
__syncthreads();
```

With a loop to add as many as necessary

```
unsigned int tid = threadldx.x;
unsigned int i = blockldx.x * blockSize * 2 + threadldx.x;
unsigned int gridSize = blockSize * 2 * gridDim.x;

sdata[tid] = 0;
while (i < n) {
    sdata[tid] += g_idata[i] + g_idata[i + blockSize];
    i += gridSize;
}
__syncthreads();</pre>
gridSize steps to achieve coalescing
```

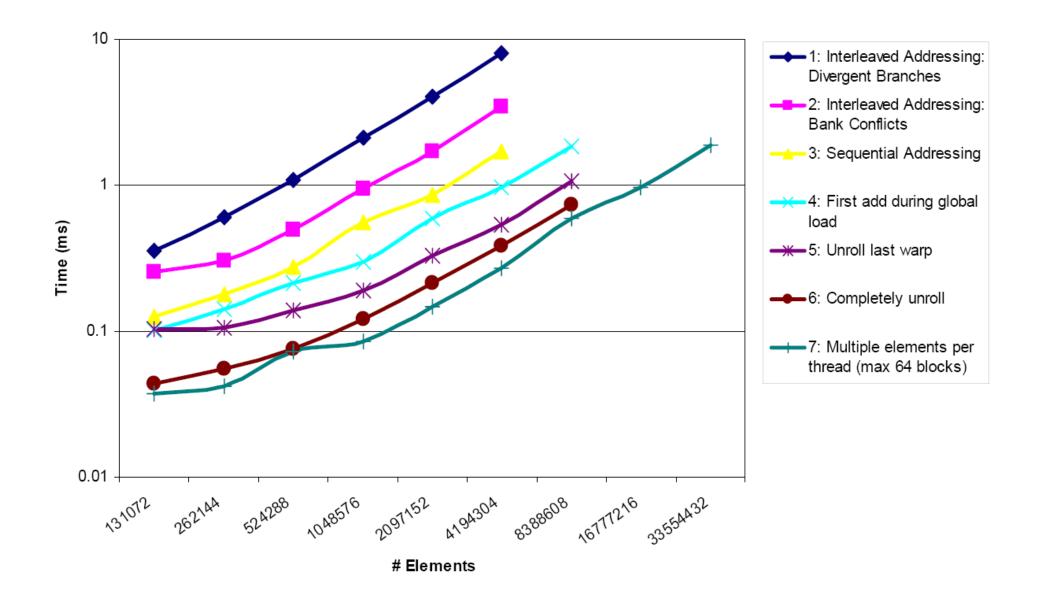
Performance for 4M element reduction

	Time (2 ²² ints)	Bandwidth	Step Speedup	Cumulative Speedup
Kernel 1: interleaved addressing with divergent branching	8.054 ms	2.083 GB/s		
Kernel 2: interleaved addressing non-divergent branching	3.456 ms	4.854 GB/s	2.33x	2.33x
Kernel 3: sequential addressing	1.722 ms	9.741 GB/s	2.01x	4.68x
Kernel 4: first step during global load	0.965 ms	17.377 GB/s	1.78x	8.34x
Kernel 5: Unroll last warp	0.536 ms	31.289 GB/s	1.8x	15.01x
Kernel 6: Complete Unroll	0.381 ms	43.996 GB/s	1.41x	21.16x
Kernel 7: multiple elements per thread	0.268 ms	62.671 GB/s	1.42x	30.04x

Final Optimized Kernel

```
template <unsigned int blockSize>
  _global___ void reduce6(int *g_idata, int *g_odata, unsigned int n)
     __shared__ int sdata[];
    unsigned int tid = threadldx.x;
    unsigned int i = blockldx.x*(blockSize*2) + tid;
    unsigned int gridSize = blockSize*2*gridDim.x;
    sdata[tid] = 0;
    do { sdata[tid] += g_idata[i] + g_idata[i+blockSize]; i += gridSize; } while (i < n);
     __syncthreads();
    if (blockSize \geq 512) { if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads(); }
    if (blockSize \geq 256) { if (tid < 128) { sdata[tid] += sdata[tid + 128]; } syncthreads(); }
    if (blockSize >= 128) { if (tid < 64) { sdata[tid] += sdata[tid + 64]; } __syncthreads(); }
    if (tid < 32) {
            if (blockSize >= 64) sdata[tid] += sdata[tid + 32];
            if (blockSize >= 32) sdata[tid] += sdata[tid + 16];
            if (blockSize >= 16) sdata[tid] += sdata[tid + 8];
            if (blockSize >= 8) sdata[tid] += sdata[tid + 4];
            if (blockSize >= 4) sdata[tid] += sdata[tid + 2];
            if (blockSize >= 2) sdata[tid] += sdata[tid + 1]:
    if (tid == 0) g_odata[blockldx.x] = sdata[0];
```

Performance comparison on G80



Optimization types

- Algorithmic optimizations
 - Changes to addressing, algorithm cascading
 - 11.84x speedup

- Code optimizations:
 - Loop unrolling
 - 2.54x speedup

Summary

- Understand CUDA performance characteristics
 - Memory coalescing
 - Divergent branching
 - Bank conflicts
 - Latency hiding
- Use peak performance metrics to guide optimization
- Understand parallel algorithm complexity theory
- Know how to identify type of bottleneck
 - e.g., memory, core computation, or instruction overhead
- Optimize your algorithm, then unroll loops
- Use template parameters to generate optimal code



THANK YOU