Thesis:Financial Networks and Contagion

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Abstract

The modern financial system forms complex financial network due to the high connections, leading to the systemic risk because of financial contagion caused by external shocks. Especially in recent several years, the development of network technology prompts researches on financial contagion, which has been a new study field. This project focuses on the interconnected network based mainly on the loan relationship among large banks, referring to some existing researches, presenting a scale-free network model to analyze the contagion process considering variable factors like network capitalization, exposure levels, and the level of interconnectedness when there are default cascades. In this work, first download the data needed, for example the balance sheet, liabilities, instruments of systemically important financial institutions. Second simulate an external shock on one node and analyze the contagion process as well as its impact on the whole financial system, presenting it in a form of graph. Besides, it is assumed that the probability of contagion between two banks is proportional to their assets, liabilities, etc. Then apply 30 specific banks to this model, coming up with a method to indicate the systemic risk. What need to be specified is that all of the chosen banks are big banks, which means they could all be considered as hubs, amplifying the impact of default.

key words： financial contagion, scale-free network, systemic risk

JEL classification: C63 D85 G21

1. Introduction

In contemporary society, the impact of financial crisis on economy has been increasingly severe. Along with the development of such crisis, theories and researches on how to efficiently resolve the influence and loss caused by financial crisis also rapidly develop, and the complex network theory is one of valid methods.

Economic integration and financial globalization has gradually brought a fact that financial institutions have become increasingly dependent on each other, and interbank assets and liabilities are formed into a complex financial network. Thus, a small fluctuation on financial market is more likely to become a crisis.

A financial system contains various market aspects, like the security market, currency market, futures market with different duties and structure. What's more, the performance of the whole system is not simply indicated by the summation of subsystems, but is a nonlinear impact.

Since the 1990s computer science promoted the rapid development of studies on complex network, leading the wide application in biology, information, physics, etc. Network has a strong ability to explain phenomena of social and economical system. For instance, Albert had notices that the scale-free network will be pretty flexible while being randomly attacked, but fragile when hubs are attacked.

The new method of studying financial contagion in a network perspective serves as an important tool to understand the real financial system and the form and contagion mechanism of financial systemic risk and its dynamic property. It could be used to analyze the contagion paths, the potential asset losses, and decide which institution or country is vital and more susceptible to risk for the whole system.

The financial crisis in 2008 indicated the fact that nowadays financial institutions are interconnected with each other, showing the fact that banks that are too-connected-to-fail are as significant as banks that are too-big-to-fail. There is no doubt that the development in same sectors will increase the connections of interbank liabilities, making the interbank trading network more closely as well as raising the risk. Therefore it is necessary to analyze the financial contagion.

A financial network treated institutions as nodes and the transactions between them as edges, explaining the entire financial system clearly. As for the project, we first created a financial network model using download data, choosing 30 large banks and simulating the interbank assets and liabilities based on known information. Second simulating and analyzing the contagion process caused by one initial bank.

2. Literature Review

The periodic financial crisis indicates that the financial system is now increasingly integrated into the global capital circulation and financial service network. The theory of financial network is vital to assessing the stability of financial system, explaining the nature of systemic risk contagion in the system and the reason of lacking liquidity for banks.

Allen and Gale(2000) did a groundbreaking study on the risk of financial contagion among interbank networks of same sectors, and they started to use network or graph models to do their research.

They interpreted the financial crisis from the perspective of liquidity, studying the relationship between an incomplete market and the fluctuation of prices. Because the supply and demand of liquidity is inflexible in short term, even a slight uncertainty could lead to huge fluctuations in asset prices. They came up with a solution to study the interbank exposure risk under a structure of financial network and found that the incomplete market is more susceptible to contagion. Allen et al. Indicated that in order to reduce the systemic risk, one bank will transmit its proportion of loss on portfolio to other banks through a tightly-connected interbank network. They found that due to the different degree of interconnection and liquidity in the interbank market, banks respond differently to systemic risk.

Whereas the real financial network somtimes differs from their structures under specific conditions like “over-exposure”.

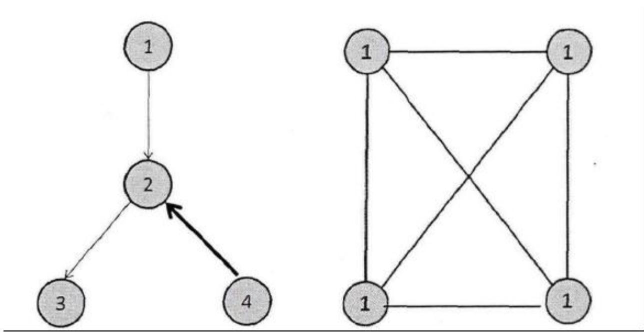
Gai and Kapadia(2010) studied shocks, the network framework and the influence of liquidity on the probability of contagion and financial stability utilizing the complex network and structure, finding that the financial system is stable as well as fragile. The probability of contagion would be low before the initial shock affects banks due to the high interbank connection, while once contagion happens, this special property could be one factor to increase the probability of default cascade.[2] Ladley(2011) also showed that in the environment of static network, for small shock, the high connection might contribute to the stability of network, however, for relatively large shock, this property could amplify the impact of the initial shock.

Eisenberg and Noe(2001) came up with a model indicating that nodes represent financial institutions(banks) and explaining their interbank liabilities. For shocks on nodes, they provide an approach to compute and clarify the interbank exposure as well as banks’ loss, which is important for analyzing the financial contagion. Their model showed how to decide the node considered as the one suffering the initial shock causing the default cascade, whose number and scale are relative with the structure of network.

Caccioli et al.(2012) considered the degree of nodes and the heterogeneity of assets, replacing the Erdos-Renyi random network used by Gai and Kapadia with scale-free network having bilateral risk exposure to analyse the impact of the heterogeneity of nodes. They indicated that when the initial default bank is specified, the scale-free network will reduce the probability of contagion, but not affect the degree of contagion and such network would be more fragile when high degree node is chosen. They also found that in Erdos-Renyi network, disassortativity would reduce the probability of contagion while assortativity will increase the instability of financial system. The reason of this phenomenon might be that disassortative nodes with more edges(hubs) are relatively more stable and those small nodes tend to connect with them, which reduce the probability of contagion, whereas in assortative network, small nodes tend to connect with counterparts similar to themselves.

Battiston et al.(2012) created a model on financial stability, analyzing the impact of different levels of interbank connection on the default cascade of financial network. They hold the view that incomplete information will lead the problem on liquidity. Actually, their model considered not only the impact of incomplete information, but the impact of change on asset price, indicating that the impact of the discrete level of one single bank on systemic risk is not quite clear, which means the structure of network and the level of financial stability should be highly paid attention on while considering how to improve the ability of resisting risk.

Graph could be used to explain the theory of complex network. It is like the figure shown below:



Graph is a collection of nodes and edges(Easley and Kleinberg, 2010), through which the practical or logical relationship of these nodes in the network can be clearly indicated. Different connecting way will form different network structure. The graph on the left is directed, and the right one is undirected, which only represent the relationship between each node without showing the direction of cash flows.

The typical scale-free network model is presented by Barabasi and Albert(1999). While studying the World-Wide-Web, they found that the distribution of nodes is heterogeneous and is connected by several high degree nodes, which is named as scale-free model.[7]

Systemic risk now has not been defined uniformly. De Bandt and Hartmann(2000) summarized quantities of different definitions. There is a common property in these definitions that systemic risk is a series of terrible economical results caused by an event like financial shocks or a default of financial institutions.

3. Data

In this project, the data covers 30 large banks. In order to make sure each chosen bank are is enough to be a hub, I choose the first 30 banks ranked by consolidated assets posting on FRSR(Federal Reserve Statistical Release). The specific data comes from FFIEC(The Federal Financial Institutions Examination Council), composed by five banking regulations, giving clear reports of most of banks. The data used in this project is collected quarterly and the latest one is on December 12th, 2017. Also, tons of classes are included in such report and I choose some of them to mainly represent the financial condition of those large uniform banks, including the balance sheet, which contains assets and liabilities, and derivative instruments. In this project it is assumed that all of the banks could be hubs and have connections with each other.

The part of data used in the project is presented below:

	Bank	Net Income/$	Total Bank Capital/$	Total Assets/$	Total Liabilities/$
1	JPMORGAN CHASE BANK, NATIONAL ASSOCIATION	18,930,000	211,846,000	2,140,778,000	1,928,619,000
2	WELLS FARGO BANK, NATIONAL ASSOCIATION	21,317,000	166,558,000	1,747,354,000	1,568,846,000
3	BANK OF AMERICA, NATIONAL ASSOCIATION	21,184,000	207,784,000	1,751,524,000	1,542,060,000
4	CITIBANK, N.A.	1,103,000	142,866,000	1,385,697,000	1,230,831,000
5	SUNTRUST BANK	2,382,813	24,552,833	201,637,519	175,834,099
6	HSBC BANK USA, NATIONAL ASSOCIATION	-232,763	23,263,105	180,371,724	152,870,180
7	GOLDMAN SACHS BANK USA	1,414,000	25,546,000	164,539,000	136,955,000
8	FIFTH THIRD BANK	2,293,128	16,887,506	140,077,697	122,443,369
9	KEYBANK NATIONAL ASSOCIATION	1,420,090	15,170,771	135,758,439	119,623,708
10	NORTHERN TRUST COMPANY, THE	1,157,600	9,223,936	138,163,151	127,621,426
11	ALLY BANK	2,131,000	16,963,000	137,474,000	120,511,000
12	MORGAN STANLEY BANK, N.A.	2,212,000	15,129,000	129,707,000	114,578,000
13	REGIONS BANK	1,372,976	16,103,151	123,325,220	106,227,269
14	MANUFACTURERS AND	1,461,270	14,312,146	118,072,176	102,361,064

	TRADERS TRUST COMPANY				
15	MUFG UNION BANK, NATIONAL ASSOCIATION	685,522	16,397,698	118,537,345	101,289,517
16	CAPITAL ONE, NATIONAL ASSOCIATION	1,196,322	37,756,198	290,651,177	252,894,748
17	BMO HARRIS BANK NATIONAL ASSOCIATION	446,943	15,441,813	109,372,608	93,530,795
18	HUNTINGTON NATIONAL BANK, THE	1,314,680	11,313,863	104,052,030	91,152,433
19	BANK OF THE WEST	437,616	12,125,302	89,765,667	77,620,599
20	COMPASS BANK	423,053	12,092,248	86,504,843	73,386,128
21	FIRST REPUBLIC BANK	757,660	7,818,301	87,780,507	79,185,122
22	SANTANDER BANK, NATIONAL ASSOCIATION	220,731	13,522,396	74,450,413	60,596,205
23	COMERICA BANK	784,566	7,408,789	71,609,090	63,645,456
24	ZB, NATIONAL ASSOCIATION	598,188	7,613,929	66,080,511	58,466,582
25	SILICON VALLEY BANK	446,769	3,762,542	50,387,875	46,625,333
26	DEUTSCHE BANK TRUST COMPANY AMERICAS	175,000	9,058,000	43,390,000	34,332,000
27	SIGNATURE BANK	387,209	4,031,691	43,119,702	38,830,630
28	EAST WEST BANK	502,273	3,830,696	37,120,068	33,289,372
29	BOKF, NATIONAL ASSOCIATION	331,552	3,255,912	32,217,466	28,951,554
30	SYNOVUS BANK	319,337	3,229,426	31,106,198	27,876,772

4. Methodology

Recent years, the systemic crisis caused by bankruptcy has reveals the importance of analyzing systemic risk, which influence the whole financial system rather than an individual one. Generally speaking, the systemic risk comes from several shocks like the inflation, the fluctuation of interest and foreign rates, and the financial contagion, which is vitally significant to the systemic risk.

4.1. Financial network

The constructed network bases on the interbank exposure, considering the network capitalization as well as the interbank interconnectedness. Two ways are used for present a financial network: Graph and Matrix.

Complex network theory uses graph to present network and is a set of a series of elements,which are called nodes, with linkages between them, called edges. It clearly indicates the practical or logical relationship among nodes and different connected way will form different structure of network.

Since the edge in the financial network might represent the bankroll trading volume, a matrix could be used in this method. For example, consider a financial network consisted with N banks, x_{ij} means the capital flow from bank i to bank j and x_{ji} means the capital flowing from bank j to bank i. Thus the interbank network consisted of N banks can form a N*N matrix:

$$X = \begin{bmatrix} 0 & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i1} & \cdots & 0 & \cdots & x_{iN} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & 0 \end{bmatrix}$$

Since a bank cannot trade with itself, the values on diagonal are all 0. The sum of each row of the matrix means the total asset of bank i, that is, $A_i = \sum_{j=1}^{n} x_{ij}$, and the sum of each column means the total liability of bank j, that is, $L_j = \sum_{i=1}^{n} x_{ij}$.

In this matrix, only A and L are given, while the specific x_{ij} is unknown. In order to estimate interbank exposure, it is necessary to make an assumption of the distribution of loans of banks.

In the project, the nodes of this financial network are all banks, indicating that the linkages mean loan relationships between each two banks and the data of bank-to-bank assets and liabilities of two banks is needed. Whereas the real bank-to-bank data is relatively hard to get, the maximum entropy method is used to estimate interbank deposit and loan and create network as well as the corresponding matrix. Network created in this way is completely connected, which is contrary to the fact that the real-world financial network is more likely to be a small-world or a scale-free one, meaning that the maximum entropy method will always overestimate the degree of risk contagion. For scale-free network, it is needed to preset several specific x_{ij} as 0 when there is no relationship between two banks and as 1 when there is a relationship between two banks. RAS algorithm is helpful while estimating x_{ij} according to A_i and L_j.

4.2. The Degree of Nodes

The degree of node could be presented by the amount of edges one node connects to others. There are two kinds of degree: the in-degree and the out-degree. The former one means liabilities flows into the bank and the latter one means assets flowing out from the bank, which can also be presented as follows:

$$\text{the in-degree: } d_j^{in} = \sum_i L_{ij} \text{ ;}$$

$$\text{the out-degree: } d_i^{out} = \sum_j A_{ij} \text{ .}$$

A high degree of node means this node has a tight connection with other one. In the perspective of contagion, a high degree indicates the importance of the node that has a strong impact on much more nodes and the whole network than others.

In scale-free network, a node with more linkages is called a hub. Typically, a network with many hubs seems to be more sparse, whereas one with less hubs is more dense because of more edges in the network.

4.3. Scale-free Network

When creating a network, it is necessary to decide the structure of network in advance if the real transaction data is unknown.

In the project, a scale-free network with N nodes and M directed edges is constructed.

4.4. Financial Contagion

When considering the systemic risk, two problems are always paid attention to: the loss caused by bank i and the number of banks affected by bank i.

Suppose that one bank in the network is affected by a shock and default. This bank is considered as an initial bank, whose default will cause loss on assets of its counterparts and lead further contagion of systemic risk. The whole process of contagion is :

1) For bank i, since its debtor cannot pay the liabilities, the interbank assets will decrease. Let S_i^0 be the set of its debtor, the loss of interbank assets can be presented as $LTA_i = LGD_k \sum_{k \in S_i^0} x_{ik}$. LGD means the loss given default.

$$LGD=1-R$$

R is the recovery rate.

2) Bank i need to compensate its asset loss. If it is unable to cover the loss, which means $LGD_k \sum_{k \in S_i^0} x_{ik} > E_i$, bank i will be bankrupt. E_i is the capital of bank i. The bankruptcy of other more banks caused by the initial bank is called the first round contagion and the set of these banks is set to be K^1, which satisfies the formula:

$$K_i^1 = \{i \mid LGD_k \sum_{k \in S_I^0} x_{ik} > E_i\}$$

The set of bankrupt banks becomes $S^1 = S^0 \bigcup K^1$. $K^1 = \bigcup K_i^1$.

3) The second round contagion is the same as step 1 and 2, causing more banks become bankrupt, until the model is stable and no more bank breaks.

In a scale-free network, a node with a high degree is more possible to be affected.

To analyze the effect of the structure of such network, models are needed to created. In this project, we use CI and CS to represent the contagion index and the size of default cascades:

$$CI = \beta_0 + \beta_1 d + \beta_2 L + \beta_3 A + \beta_4 Ass + \varepsilon$$
$$CS = \beta_0 + \beta_1 d + \beta_2 L + \beta_3 A + \beta_4 Ass + \varepsilon$$

CS also means K, the amount of default banks.

d is the degree of node; L is bank's liability and A is asset, which represents the level of risk exposure. Ass means the size or scale of bank, presented by the proportion of one bank asset to the total assets of the whole network.

The contagion index is divided into two level: higher than 5% or lower than 5%. When index greater than 5%, it is considered to be exposed to a high level of systemic risk. If the probability that CI is greater than 5% is high, the systemic risk will correspondingly have a high probability to seriously affect the network system. We then create a logistic model to analyze the impact of the degree of node, interbank exposure and the size of banks on systemic risk, that is,

$$\log it[p(CI > 5\%)] = \beta_0 + \beta_2 Deg + \beta_2 L + \beta_3 A + \beta_4 Ass + \beta_5 CS + \varepsilon .$$

In this project, we predetermine one bank to get affected by an external shock, calculating the probability of transforming the risk to other node, and then simulate the contagion process.

5. Disccusion

5.1. Construct a network

In reality, the interbank exposure is not public, thus the actual structure of the interbank market is unknown. In the project, the maximum entropy method is used to estimate the exposure level. The RAS algorithm requires that the total assets and liabilities of banks should be equal, whereas we only choose 30 banks and cannot satisfy this requirement. In this circumstance, the problem that the RAS algorithm is not convergent and the result is not entirely perfect is usual.

According to this method, the interbank exposure matrix is given by $x_{ij} = a_i l_j$ and elements in diagonal equal to zero. In complex network, we should first decide the connected relationship between banks in order to better set the interbank matrix.

In this project we first construct a basic undirected scale-free network based on BA network model with an algorithm showing below:

1) Given n_o nodes;

2) Increment: add a new node and m (m< n_o) edges in each time step;

3) The connected node chosen by new nodes follows the probability $\prod k_i / \sum_j k_j$. k_i is the degree of the chosen node. Through the conception of the scale-free network, we assume that the node with large degree has low average path length and large clustering coefficient. We randomly allocate 30 banks to the nodes based on their sizes. High degree nodes represent banks with large sizes.

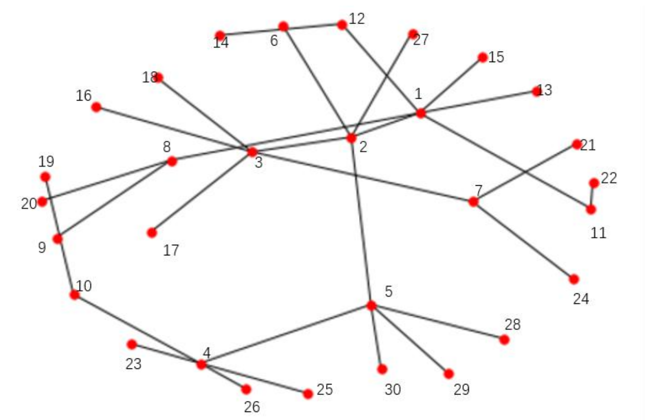


Figure 1 BA scale-free network

Number	Bank name	degree

1	JPMORGAN CHASE BANK, NATIONAL ASSOCIATION	6
2	WELLS FARGO BANK, NATIONAL ASSOCIATION	5
3	BANK OF AMERICA, NATIONAL ASSOCIATION	5
4	CITIBANK, N.A.	5
5	SUNTRUST BANK	5
6	HSBC BANK USA, NATIONAL ASSOCIATION	3
7	GOLDMAN SACHS BANK USA	3
8	FIFTH THIRD BANK	3
9	KEYBANK NATIONAL ASSOCIATION	3
10	NORTHERN TRUST COMPANY, THE	2
11	ALLY BANK	2
12	MORGAN STANLEY BANK, N.A.	2
13	REGIONS BANK	1
14	MANUFACTURERS AND TRADERS TRUST COMPANY	1
15	MUFG UNION BANK, NATIONAL ASSOCIATION	1
16	CAPITAL ONE, NATIONAL ASSOCIATION	1
17	BMO HARRIS BANK NATIONAL ASSOCIATION	1
18	HUNTINGTON NATIONAL BANK, THE	1
19	BANK OF THE WEST	1
20	COMPASS BANK	1
21	FIRST REPUBLIC BANK	1
22	SANTANDER BANK, NATIONAL ASSOCIATION	1
23	COMERICA BANK	1
24	ZB, NATIONAL ASSOCIATION	1
25	SILICON VALLEY BANK	1
26	DEUTSCHE BANK TRUST COMPANY AMERICAS	1
27	SIGNATURE BANK	1

28	EAST WEST BANK	1
29	BOKF, NATIONAL ASSOCIATION	1
30	SYNOVUS BANK	1

The degree of nodes is vitally significant. In this project, a node with a larger degree indicates that the position of this node in the whole network is more important. By analyzing the distribution of degrees of one network, we could better understand the interconnection of the network.

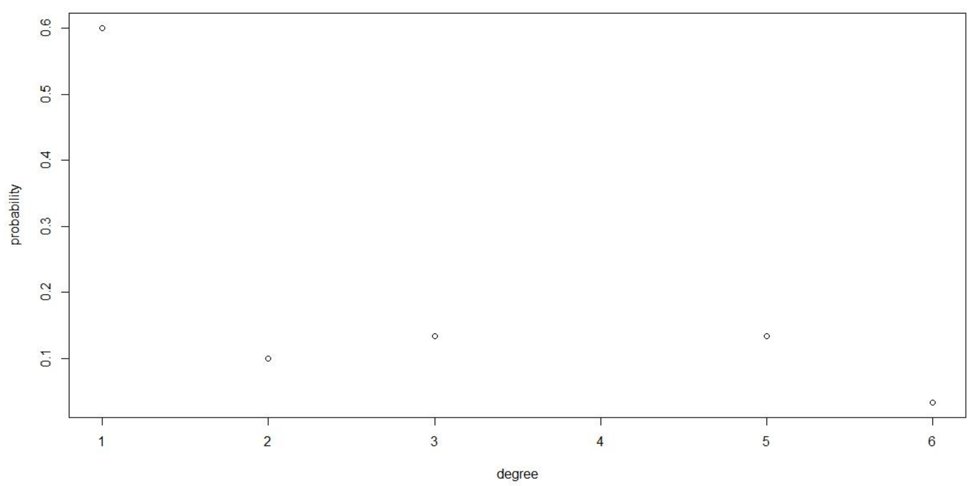


Figure 2 The degree distribution of the scale-free network of 30 banks

From the figure above, we could roughly assume that the connections of this network follows the power law distribution. The conclusion would be more accurate with more sample data. It can be concluded that the minority of nodes occupies most connected relationship of the network, which is the main property of scale-free network.

The interbank matrix is constructed based on the given scale-free network. And the result of the simulation of the interbank exposure of the 30 chosen banks is given by the table below (here use numbers to substitute banks and probabilities to represent exposures).

from	to	exposure	from	to	exposure	total
1	8	0. 007056538	8	1	1. 05E-02	1. 76E-02
1	12	1. 30E-02	12	1	1. 84E-02	3. 14E-02
1	2	1. 04E-01	2	1	0. 14554269	2. 49E-01
1	11	0. 008504922	11	1	1. 26E-02	2. 11E-02
1	13	0. 01219454	13	1	1. 75E-02	2. 97E-02
1	15	0. 0116277	15	1	1. 69E-02	2. 85E-02
2	5	3. 47E-173	5	2	1. 51E-175	3. 49E-173
2	3	0. 097761815	3	2	7. 22E-02	1. 70E-01
2	6	6. 76E-04	6	2	1. 03E-03	1. 71E-03
2	27	0. 004457627	27	2	3. 09E-03	7. 54E-03
3	16	0. 02903147	16	3	0. 041324712	7. 04E-02
3	17	0. 01073702	17	3	0. 01555057	2. 63E-02
3	7	0. 004549596	7	3	0. 007592157	1. 21E-02
3	18	0. 010464	18	3	0. 014794092	2. 53E-02
4	26	0. 003941199	26	4	0. 01302423	1. 70E-02
4	25	0. 005352433	25	4	0. 01512476	2. 05E-02
4	5	1. 67E-02	5	4	0. 05017966	6. 69E-02
4	23	0. 007306286	23	4	0. 02149466	2. 88E-02
4	10	1. 47E-02	10	4	0. 04147198	5. 61E-02
5	28	3. 82E-03	28	5	1. 28E-03	5. 11E-03
5	29	3. 32E-03	29	5	1. 12E-03	4. 44E-03
5	30	3. 20E-03	30	5	1. 08E-03	4. 28E-03
6	14	1. 18E-02	14	6	1. 68E-02	2. 85E-02
6	12	1. 33E-04	12	6	8. 53E-05	2. 18E-04
7	24	6. 71E-03	24	7	0. 006752304	1. 35E-02

7	21	9.09E-03	21	7	0.008969674	1.81E-02
8	20	8.42E-03	20	8	0.006192224	1.46E-02
8	9	9.70E-04	9	8	0.000807329	1.78E-03
9	10	2.65E-91	10	9	1.18E-93	2.66E-91
9	19	8.91E-03	19	9	1.28E-02	2.17E-02
11	22	6.96E-03	22	11	0.005329339	1.23E-02

Utilizing the results and simulations above, we can construct a more specific scale-free network.

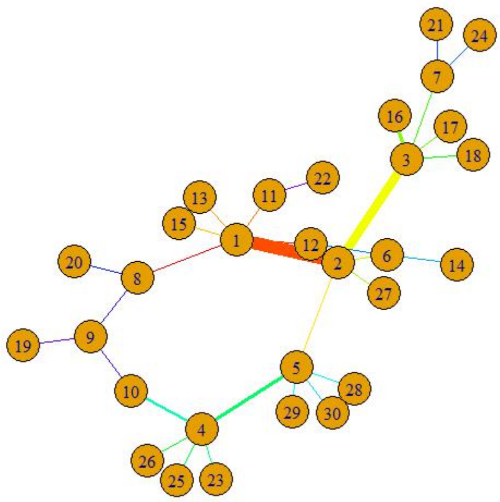


Figure 3 The scale-free network of 30 nodes

From the network structure, bank 1, 5, 2, 4 have relatively larger degree, showing tight connections with other nodes.

5.2. Contagion

Assume that the interbank contagion is unpredictable, and happens in a short time, indicating that the lending model stays same. In this case, we do not consider other methods of contagion, and what we have analyzed is mainly based on connections generated by lending relationships.

As is mentioned before, in a network with N nodes, once a bank j has a shock, the loss of defaulting is absorbed the capital of bank i k_i (x_{ji} is liability from j to i) . If $k_i < \lambda x_{ji}$, bank i become bankrupt. λ is the loss given default. This is the first round contagion. If x_{ih}, x_{jh} are the liabilities from i and k to h, in the first round contagion, the bankruptcy of bank j cause the loss of bank h, λx_{jh} . Assume that in the first round contagion, bank h has not become bankrupt, then bank h will still lose λx_{ih}. If $k_h < \lambda(x_{jh} + x_{ih})$, bank h would become bankrupt. The contagion will continue in the network.

Susceptible Infected Recovered Model

The mode of risk contagion in financial market depends on not only the properties of pathogenic carried by the risk, but the cycle of the infection, the structure of the financial network, etc. In SIR model, the risk spreads through a contact network. Each node represents a financial institution, and edges between nodes indicate the connected relationship. Thus, the initial risk from one institution is possible to spread to others.

In SIR model, every node will go through three conditions: susceptible, infective, removal. We set “susceptible” institutions as S, “infective” institutions as I, and institutions that has recovery or are bankrupt as R.

1) At the beginning, all of the financial institutions (nodes) are susceptible.

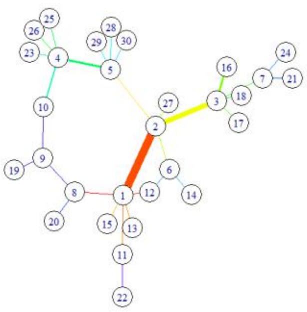
2) Several nodes confront shocks and change into infective condition, I.

3) Nodes in I condition have infectivity to their connected institutions in some specific probabilities.

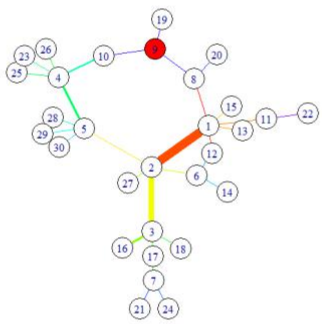
4) After t time, infective institutions will be no longer with infectivity and infected by risk, going into R condition.

At first, the institutions being infected is just a few, whereas the ratio of infected ones to the total number of nodes will increase as time t increases, following a rule of “S” curve till a peak, which is the most severe period during a financial crisis.

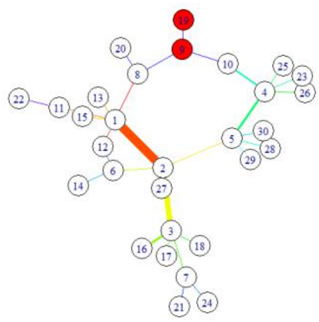
In the project, we simulate one node being infected, and the probability of contagion of each edge is proportional to the ratio of banks’ total capitals to assets, checking the contagion process of each time point. In the simulation, we use white color to show the susceptible nodes and red color to show the infective nodes.



Day 0



Day 1



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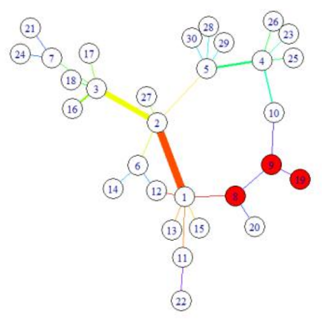

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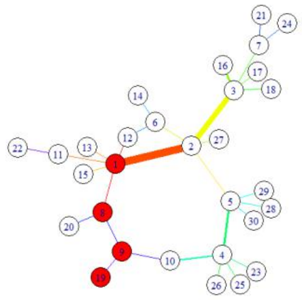

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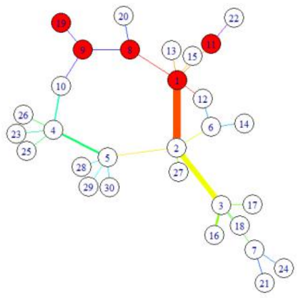
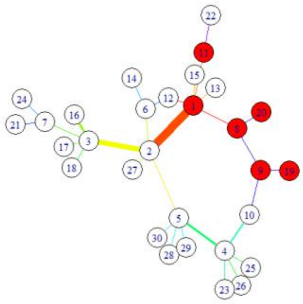


Day 17



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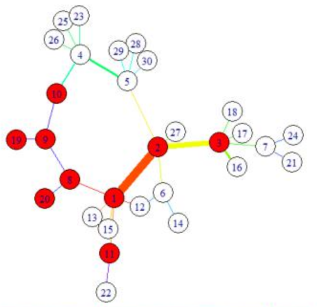

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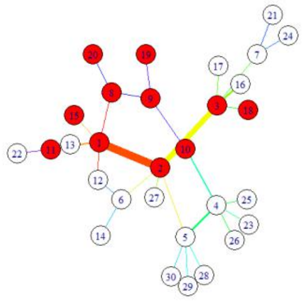

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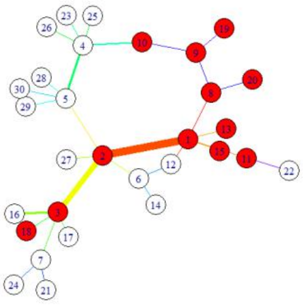

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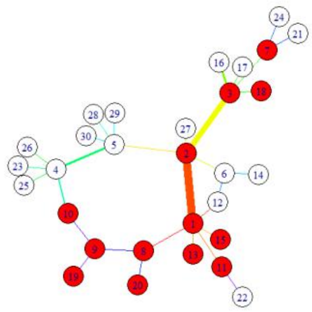

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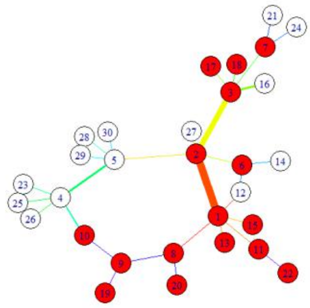

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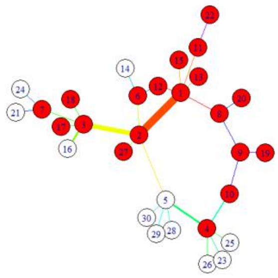

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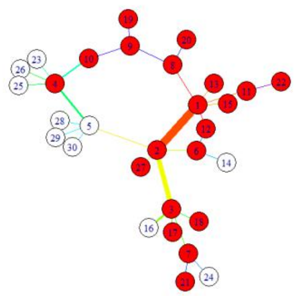

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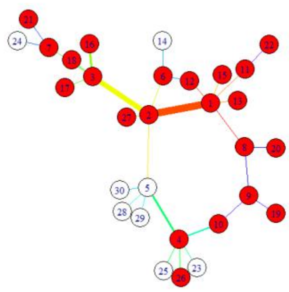

Day 34

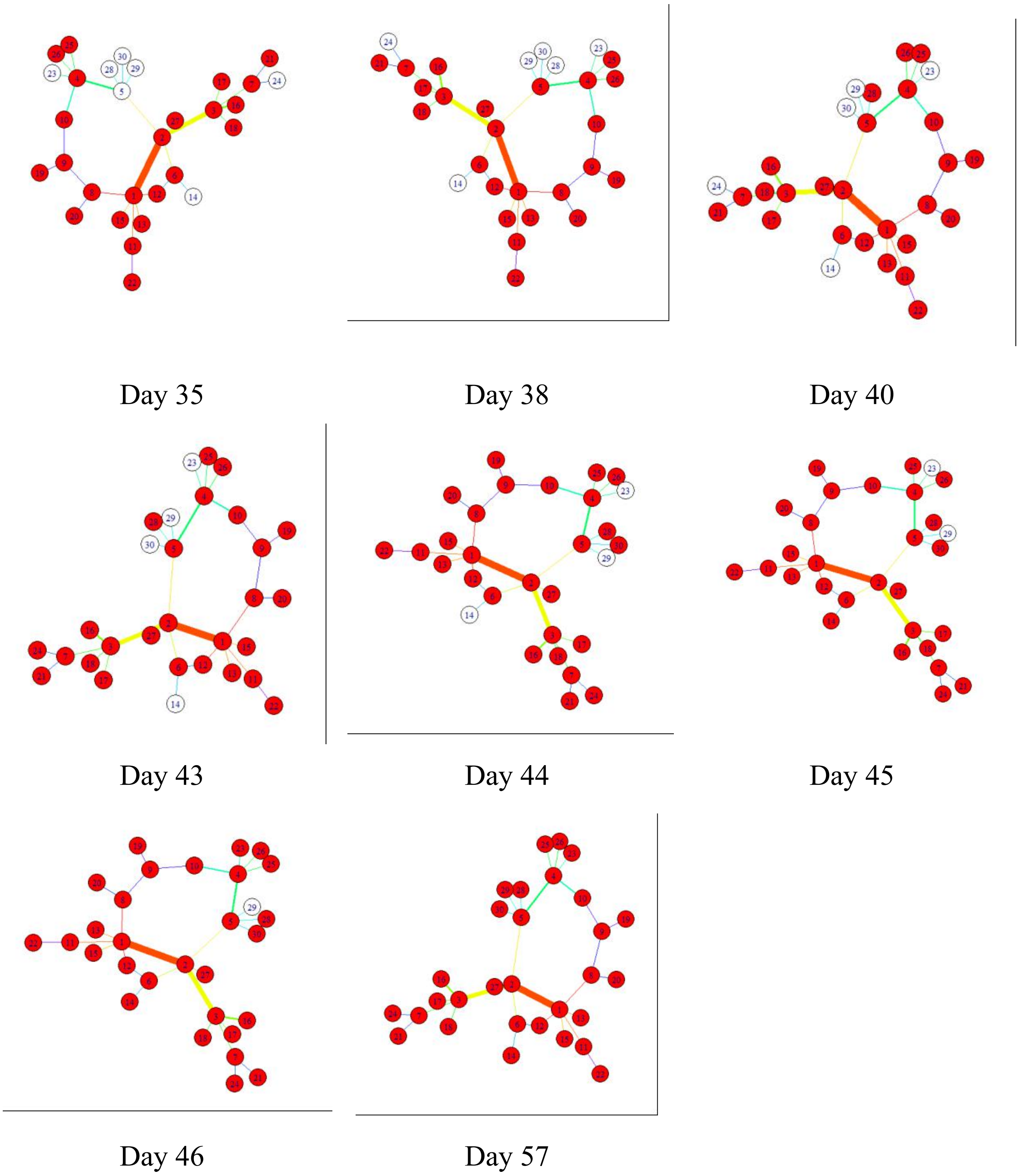

Figure 4 Simulation of contagion process

From figures showing above, it is indicated that in the scale-free network, once an initial bank is infected, even though the probability of contagion is slight, the risk will spread to the whole network system with a high speed of contagion. Besides, in the scale-free network, we can see that nodes with high degree are almost infected by 30 days, while other nodes with small degrees as well as relatively long distance to the initial infected bank are less possible to be infective.

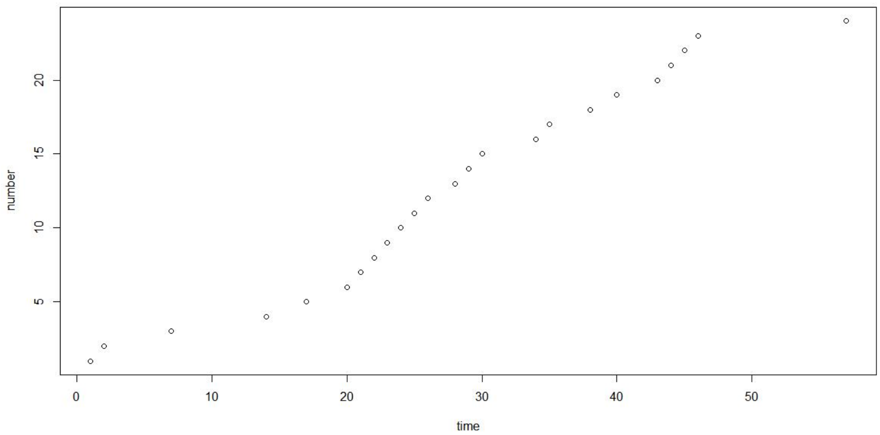

Figure 5 Spread speed of infection in the scale-free network

In reality, the condition is similar to the simulation we have came up with. The lending relationships between banks leading to the tight connections among them. When one bank becomes bankruptcy, relevant institutions can easily spread risks and crises they have encountered to others connected to them, which making the entire financial market fragile.

6. Conclusion

The project analyzes the mechanism of financial contagion from the perspective of network based on the balance sheet, combining the complex network theory and the contagion of risk, simulating the distribution of degree of each node based on the power law distribution.

Through the analysis of Susceptible Infected Recovered Model, we came up with the result that high degree nodes are more likely to be infected, that is, in scale-free network, the more important the node is, the easier for it to get affected. Thus, for the important nodes in a scale-free network, it is appropriate to protect these nodes preferentially to avoid the infection as well as the probability of spreading risks to other nodes. At the beginning of the contagion, the spread speed keeps going up till the maximum value. Thus, it is better to avoid the infection at the initial period.

According to the heterogeneity of scale-free network, in order to avoid the infection, we could select nodes protected purposely, that is, to protect a minority of important nodes. In the process of controlling risk, the financial market should be kept stable.

What's more, to ensure enough capitals also makes sense. Nodes in networks should find a way to increase the ability of withstanding shocks, that is, a bank should guarantee a minimum required level of capitals. Banks always utilize the leverage to earn high profits along with generating high systemic risks. Because of the constrain of credit limit, banks should optimize their asset structures in a limited range.

However, strategies mentioned above is only for a short term. Among different structures of networks, the scale-free network is easier to transmit risks. More hubs will increase the speed of risk contagion. From my perspective, if the financial market network develops towards the small work structure, the contagion process might be restricted to some extent.

Further research

In the project, we construct an "undirected" scale-free network. More questions are needed to be discussed.

1) With the lending relationship between banks, a directed scale-free network could be considered to better explain the contagion process. More factors could be considered during the research like different risk sources.

2) The real financial network is extremely complex. More researches are required to make

sure whether a network model could be fitted. In this project, we construct the network only based on some basic models and part of the results are based on roughly simulations. Improved model could be used further. Also, if more complete data can be gotten, better research could be done then.

While simulating the contagion process, we use SIR model. But in the project we have not indicated removal nodes. Besides, most of the discussion is based on existing researches. Further studies like the impact of the structure of financial system on risks and how to deal with the difficulties of risk control cause by unpredictable and sudden changes.

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Appendix

Figure 1 simple scale-free network

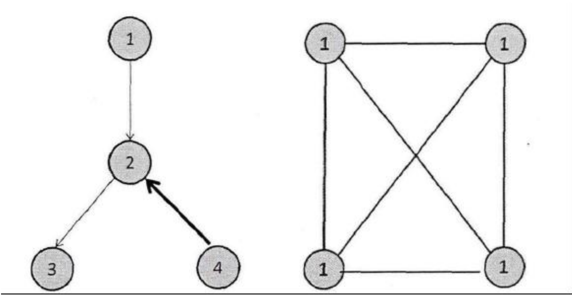


Figure 2 BA scale-free network

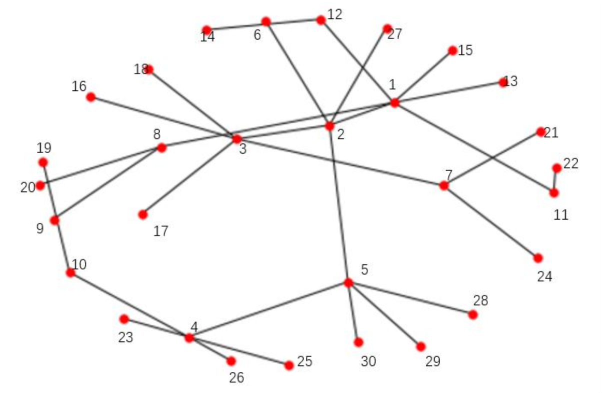


Figure 3 The degree distribution of the scale-free network of 30 banks

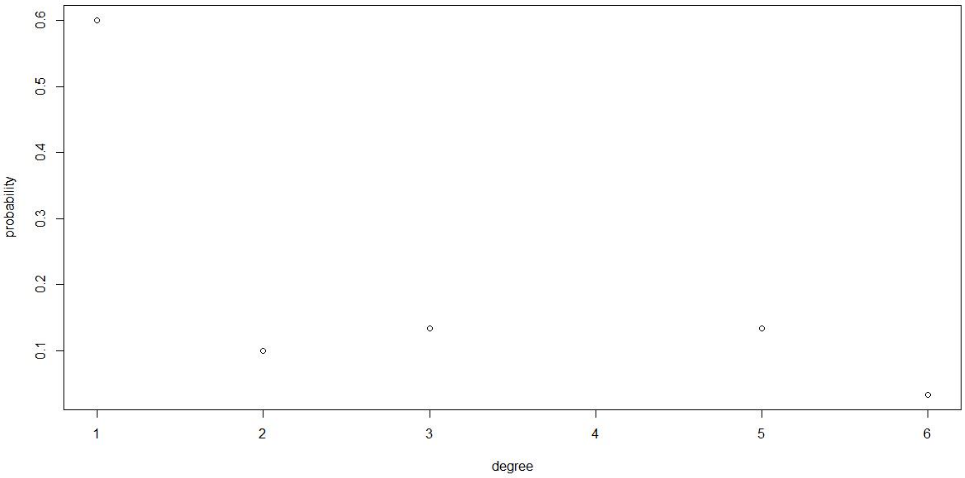


Figure 4 The scale-free network of 30 nodes

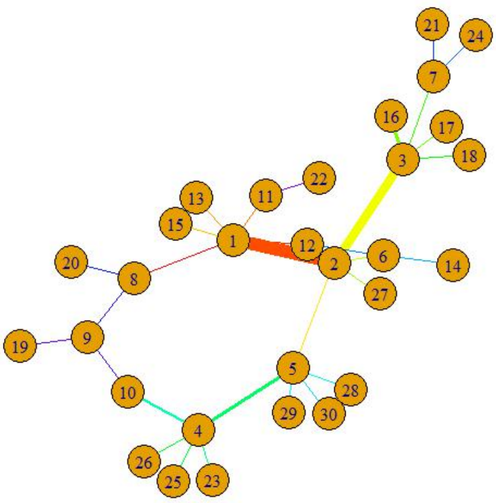
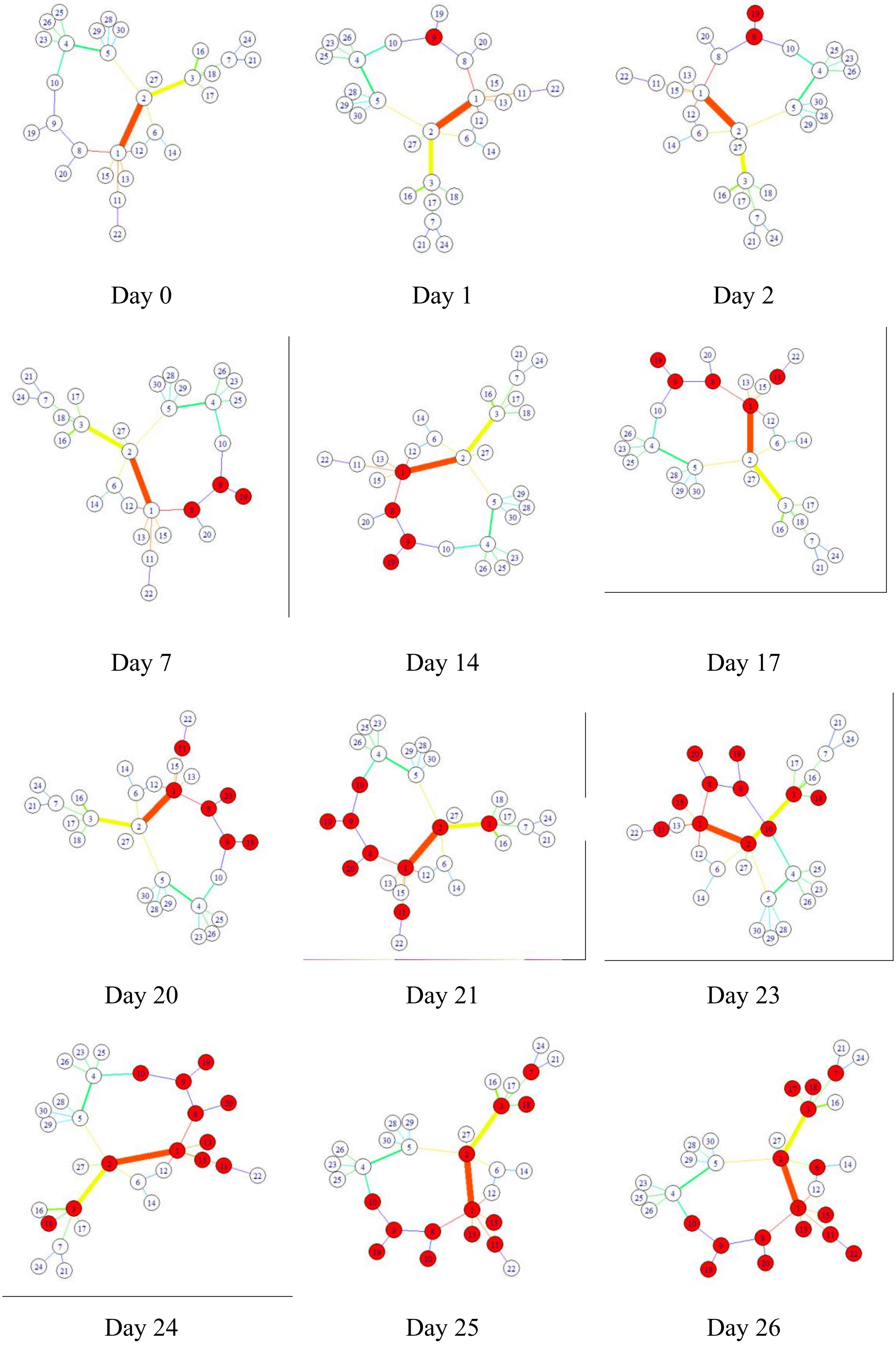


Figure 5 Simulation of contagion process

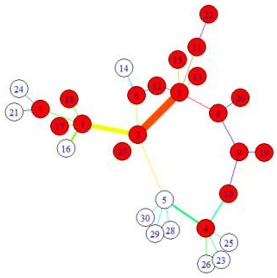


Day 0 Day 1 Day 2

Day 7 Day 14 Day 17

Day 20 Day 21 Day 23

Day 24 Day 25 Day 26



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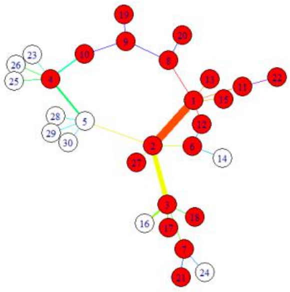
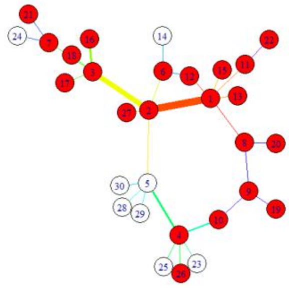
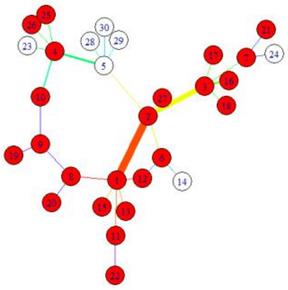


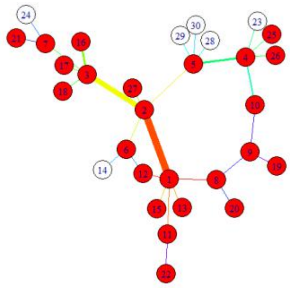
Day 30



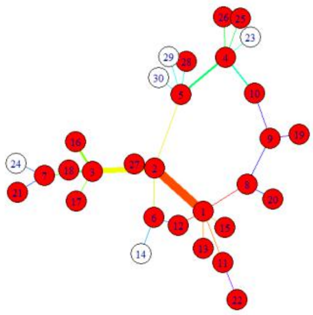
Day 34



Day 35



Day 38



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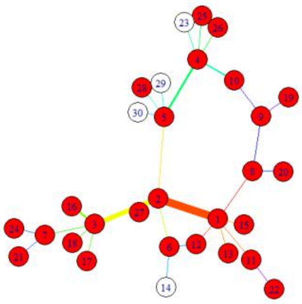
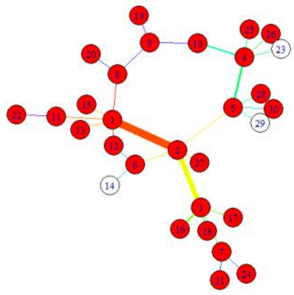
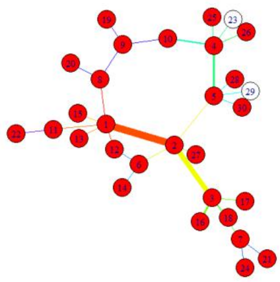


Day 43



Day 44



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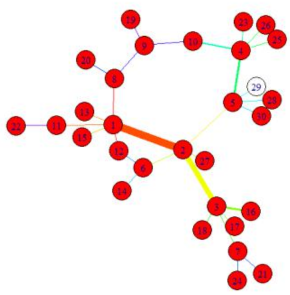
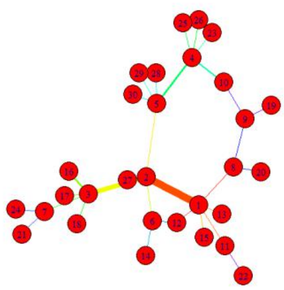


Day 46



Day 57

Table 1 Balance sheets of 30 large banks

	Bank	Net Income/$	Total Bank Capital/$	Total Assets/$	Total Liabilities/$
1	JPMORGAN CHASE BANK, NATIONAL ASSOCIATION	18,930,000	211,846,000	2,140,778,000	1,928,619,000
2	WELLS FARGO BANK, NATIONAL ASSOCIATION	21,317,000	166,558,000	1,747,354,000	1,568,846,000
3	BANK OF AMERICA, NATIONAL ASSOCIATION	21,184,000	207,784,000	1,751,524,000	1,542,060,000
4	CITIBANK, N.A.	1,103,000	142,866,000	1,385,697,000	1,230,831,000
5	SUNTRUST BANK	2,382,813	24,552,833	201,637,519	175,834,099
6	HSBC BANK USA, NATIONAL ASSOCIATION	-232,763	23,263,105	180,371,724	152,870,180
7	GOLDMAN SACHS BANK USA	1,414,000	25,546,000	164,539,000	136,955,000
8	FIFTH THIRD BANK	2,293,128	16,887,506	140,077,697	122,443,369
9	KEYBANK NATIONAL ASSOCIATION	1,420,090	15,170,771	135,758,439	119,623,708
10	NORTHERN TRUST COMPANY, THE	1,157,600	9,223,936	138,163,151	127,621,426
11	ALLY BANK	2,131,000	16,963,000	137,474,000	120,511,000
12	MORGAN STANLEY BANK, N.A.	2,212,000	15,129,000	129,707,000	114,578,000
13	REGIONS BANK	1,372,976	16,103,151	123,325,220	106,227,269
14	MANUFACTURERS AND TRADERS TRUST	1,461,270	14,312,146	118,072,176	102,361,064

	COMPANY				
15	MUFG UNION BANK, NATIONAL ASSOCIATION	685,522	16,397,698	118,537,345	101,289,517
16	CAPITAL ONE, NATIONAL ASSOCIATION	1,196,322	37,756,198	290,651,177	252,894,748
17	BMO HARRIS BANK NATIONAL ASSOCIATION	446,943	15,441,813	109,372,608	93,530,795
18	HUNTINGTON NATIONAL BANK, THE	1,314,680	11,313,863	104,052,030	91,152,433
19	BANK OF THE WEST	437,616	12,125,302	89,765,667	77,620,599
20	COMPASS BANK	423,053	12,092,248	86,504,843	73,386,128
21	FIRST REPUBLIC BANK	757,660	7,818,301	87,780,507	79,185,122
22	SANTANDER BANK, NATIONAL ASSOCIATION	220,731	13,522,396	74,450,413	60,596,205
23	COMERICA BANK	784,566	7,408,789	71,609,090	63,645,456
24	ZB, NATIONAL ASSOCIATION	598,188	7,613,929	66,080,511	58,466,582
25	SILICON VALLEY BANK	446,769	3,762,542	50,387,875	46,625,333
26	DEUTSCHE BANK TRUST COMPANY AMERICAS	175,000	9,058,000	43,390,000	34,332,000
27	SIGNATURE BANK	387,209	4,031,691	43,119,702	38,830,630
28	EAST WEST BANK	502,273	3,830,696	37,120,068	33,289,372
29	BOKF, NATIONAL ASSOCIATION	331,552	3,255,912	32,217,466	28,951,554
30	SYNOVUS BANK	319,337	3,229,426	31,106,198	27,876,772

Table 2 Degrees of banks in the constructed network

Number	Bank name	degree
1	JPMORGAN CHASE BANK, NATIONAL ASSOCIATION	6
2	WELLS FARGO BANK, NATIONAL ASSOCIATION	5
3	BANK OF AMERICA, NATIONAL ASSOCIATION	5
4	CITIBANK, N.A.	5
5	SUNTRUST BANK	5
6	HSBC BANK USA, NATIONAL ASSOCIATION	3
7	GOLDMAN SACHS BANK USA	3
8	FIFTH THIRD BANK	3
9	KEYBANK NATIONAL ASSOCIATION	3
10	NORTHERN TRUST COMPANY, THE	2
11	ALLY BANK	2
12	MORGAN STANLEY BANK, N.A.	2
13	REGIONS BANK	1
14	MANUFACTURERS AND TRADERS TRUST COMPANY	1
15	MUFG UNION BANK, NATIONAL ASSOCIATION	1
16	CAPITAL ONE, NATIONAL ASSOCIATION	1
17	BMO HARRIS BANK NATIONAL ASSOCIATION	1
18	HUNTINGTON NATIONAL BANK, THE	1
19	BANK OF THE WEST	1
20	COMPASS BANK	1
21	FIRST REPUBLIC BANK	1
22	SANTANDER BANK, NATIONAL ASSOCIATION	1
23	COMERICA BANK	1
24	ZB, NATIONAL ASSOCIATION	1
25	SILICON VALLEY BANK	1
26	DEUTSCHE BANK TRUST COMPANY AMERICAS	1

27	SIGNATURE BANK	1
28	EAST WEST BANK	1
29	BOKF, NATIONAL ASSOCIATION	1
30	SYNOVUS BANK	1

Table 3 simulated interbank exposures

from	to	exposure	from	to	exposure	total
1	8	0. 007056538	8	1	1. 05E-02	1. 76E-02
1	12	1. 30E-02	12	1	1. 84E-02	3. 14E-02
1	2	1. 04E-01	2	1	0. 14554269	2. 49E-01
1	11	0. 008504922	11	1	1. 26E-02	2. 11E-02
1	13	0. 01219454	13	1	1. 75E-02	2. 97E-02
1	15	0. 0116277	15	1	1. 69E-02	2. 85E-02
2	5	3. 47E-173	5	2	1. 51E-175	3. 49E-173
2	3	0. 097761815	3	2	7. 22E-02	1. 70E-01
2	6	6. 76E-04	6	2	1. 03E-03	1. 71E-03
2	27	0. 004457627	27	2	3. 09E-03	7. 54E-03
3	16	0. 02903147	16	3	0. 041324712	7. 04E-02
3	17	0. 01073702	17	3	0. 01555057	2. 63E-02
3	7	0. 004549596	7	3	0. 007592157	1. 21E-02
3	18	0. 010464	18	3	0. 014794092	2. 53E-02
4	26	0. 003941199	26	4	0. 01302423	1. 70E-02
4	25	0. 005352433	25	4	0. 01512476	2. 05E-02
4	5	1. 67E-02	5	4	0. 05017966	6. 69E-02

4	23	0.007306286	23	4	0.02149466	2.88E-02
4	10	1.47E-02	10	4	0.04147198	5.61E-02
5	28	3.82E-03	28	5	1.28E-03	5.11E-03
5	29	3.32E-03	29	5	1.12E-03	4.44E-03
5	30	3.20E-03	30	5	1.08E-03	4.28E-03
6	14	1.18E-02	14	6	1.68E-02	2.85E-02
6	12	1.33E-04	12	6	8.53E-05	2.18E-04
7	24	6.71E-03	24	7	0.006752304	1.35E-02
7	21	9.09E-03	21	7	0.008969674	1.81E-02
8	20	8.42E-03	20	8	0.006192224	1.46E-02
8	9	9.70E-04	9	8	0.000807329	1.78E-03
9	10	2.65E-91	10	9	1.18E-93	2.66E-91
9	19	8.91E-03	19	9	1.28E-02	2.17E-02
11	22	6.96E-03	22	11	0.005329339	1.23E-02

R code

```
library(mipfp)

library(utils)

InitialM <- array(1,dim=c(30,30))

InitialM[3:7,1]<-0
InitialM[9:10,1]<-0
InitialM[14,1]<-0
InitialM[16:30,1]<-0
InitialM[4,2]<-0
InitialM[7:26,2]<-0
InitialM[28:30,2]<-0
InitialM[1,3]<-0
InitialM[3:6,3]<-0
InitialM[8:15,3]<-0
InitialM[19:30,3]<-0
InitialM[1:3,4]<-0
InitialM[6:9,4]<-0
InitialM[11:22,4]<-0
InitialM[24,4]<-0

InitialM[27:30,4]<-0
InitialM[1,5]<-0
InitialM[3:11,6]<-0
InitialM[13,6]<-0
InitialM[22:23,7]<-0
InitialM[25:30,7]<-0
```

```
InitialM[3,5]<-0
InitialM[6:27,5]<-0
InitialM[1,6]<-0
InitialM[15:30,6]<-0
InitialM[1:2,7]<-0
InitialM[4:20,7]<-0
InitialM[2:8,8]<-0
InitialM[10:19,8]<-0
InitialM[21:30,8]<-0

InitialM[1:7,9]<-0
InitialM[11:18,9]<-0
InitialM[20:30,9]<-0
InitialM[1:3,10]<-0
InitialM[5:8,10]<-0
InitialM[10:30,10]<-0
InitialM[2:21,11]<-0
InitialM[23:30,11]<-0
InitialM[2:5,12]<-0
InitialM[7:30,12]<-0
InitialM[2:30,13]<-0
InitialM[1:5,14]<-0
InitialM[7:30,14]<-0
InitialM[2:30,15]<-0
InitialM[1:2,16]<-0

InitialM[4:30,16]<-0
InitialM[1:2,17]<-0
InitialM[4:30,17]<-0
InitialM[1:2,18]<-0
InitialM[4:30,18]<-0
InitialM[1:8,19]<-0
InitialM[10:30,19]<-0
InitialM[1:7,20]<-0
InitialM[9:30,20]<-0
InitialM[1:6,21]<-0
InitialM[1:8,19]<-0
InitialM[10:30,19]<-0
InitialM[1:7,20]<-0
InitialM[9:30,20]<-0
InitialM[1:6,21]<-0

InitialM[1:8,19]<-0
InitialM[10:30,19]<-0
InitialM[1:7,20]<-0
InitialM[9:30,20]<-0
InitialM[1:6,21]<-0
InitialM[1:8,19]<-0
InitialM[10:30,19]<-0
InitialM[1:7,20]<-0
InitialM[9:30,20]<-0
InitialM[1:6,21]<-0
InitialM[8:30,21]<-0
InitialM[1:10,22]<-0
InitialM[12:30,22]<-0
InitialM[1:3,23]<-0
InitialM[5:30,23]<-0

InitialM[8:30,21]<-0
InitialM[1:10,22]<-0
InitialM[12:30,22]<-0
InitialM[1:3,23]<-0
InitialM[5:30,23]<-0
InitialM[1:6,24]<-0
InitialM[8:30,24]<-0
InitialM[1:3,25]<-0
InitialM[5:30,25]<-0
InitialM[1:3,26]<-0
InitialM[5:30,26]<-0
InitialM[1,27]<-0
InitialM[3:30,27]<-0
InitialM[1:4,28]<-0
InitialM[6:30,28]<-0
```

```
InitialM[5:30,26]<-0            InitialM[1:4,29]<-0
InitialM[1,27]<-0               InitialM[6:30,29]<-0
InitialM[3:30,27]<-0            InitialM[1:4,30]<-0
InitialM[1:4,28]<-0             InitialM[6:30,30]<-0
InitialM[6:30,28]<-0


for(i in 1:30) {
  InitialM[i,i] <- 0
}
SumOfRows <- c(2140778000, 1747354000, 1751524000, 1385697000,
201637519,180371724,164539000,140077697,135758439,138163151,137474000,129707000,123325220,
118072176,118537345,290651177,109372608,104052030,89765667,86504843,87780507,74450413,7160
9090,66080511,50387875,43390000,43119702,37120068,32217466,31106198)
SumOfCols <-
c(1928619000,1568846000,1542060000,1230831000,175834099,152870180,136955000,122443369,1196
23708,127621426,120511000,114578000,106227269,102361064,101289517,252894748,93530795,91152
433,77620599,73386128,79185122,60596205,63645456,58466582,46625333,34332000,38830630,33289
372,28951554,27876772)
TargetData<- list(SumOfRows, SumOfCols)
Target.list <- list(1,2)
RandomMatrix <- Ipfp(InitialM, Target.list, TargetData)     #shift to probabilities!
library(stats)
library(igraph)
library(sna)
library(PAFit)
setwd(dir='E:/')
data=read.csv("undirected.csv",header =TRUE)
```

```
head(data)
data_stru<-graph.data.frame(data,directed = FALSE)
g<-graph.data.frame(data,directed = FALSE)
plot(data_stru,edge.width=data$freq*50,edge.color=rainbow(40),edge.arrow.size=2)
V(data_stru)$color = "white"
V(data_stru)$color[23] = "red"
plot(data_stru,edge.width=data$freq*50,edge.color=rainbow(40),edge.arrow.size=2)
probability=c(0.6,0.1,0.133333333,0.133333333,0.033333333)
degree=c(1,2,3,5,6)
plot(degree,probability)
time<-c(1,2,7,14,17,20,21,22,23,24,25,26,28, 29,30, 34,35,38,40,43,44, 45,46, 57)
number<-c(1:24)
plot(time,number)
library(sna)
i=25
while(i<70){
   p=0.181629563
   coins = c(rep(1, p*1000), rep(0,(1-p)*1000))
   n = length(coins)
   rslt<-sample(coins, 1, replace=TRUE, prob=rep(1/n, n))
   i=i+1
   cat(rslt, i,"\n")
}
```