Final Presentation

Robo-risk and Copula CoVaR of SPY

STEVENS
INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY®

1870

Group 17:

Xinhang Wang

Qi Yuan

Shihao Zhang

Advisor:

Khaldoun Khashanah

Map

- Part I, Overview, Data Processing and Risk Measure
- Part II, ARMA-GARCH Model and Vine Copula
- Part III, Conclusion

Part I

Overview, Data Processing and Risk Measure

STEVENS
INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY®
1870

Overview

What is Our Project about

- Main Purpose: Investigate the relationship between interdependence among components and systemic risk

- Motivation: Find a new and effective method to measure systemic risk that can serve as a useful analytical tool for financial stability monitoring

Our data source

- Source: yahoo finance, from 2007-01-01 to 2016-01-01
- Selected SPY + 10 industry sector ETFs (11 ETFs index * 2266 trading days)
- Sectors ETFs symbols from SPDR and iShares US

SPY - S&P 500 Index

Sector 1: XLY - Consumer Discretionary

Sector 2: XLP - Consumer Staples

Sector 3: XLE - Energy

Sector 4: XLF - Financial

Sector 5: XLV - Health Care

Sector 6: XLI - Industrial

Sector 7: XLB - Materials

Sector 8: XLK - Technology

Sector 9: XLU - Utilities

Sector 10: IYZ - Telecommunications(iShares US ETF)

Data weightings

- S&P 500 Sector Weightings, our data is selected from 2007-01-01 to 2016-01-01
- Source: https://seekingalpha.com/article/4180955-s-and-p-500-sector-weightings-report-june-2018

S&P 500 Sector Weightings (%): 1995-2016

Sector	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Technology	9.39	12.31	12.24	17.78	29.18	21.23	17.72	14.63	17.73	16.05	15.10	15.14	16.82	15.40	19.76	18.65	19.02	18.95	18.58	19.83	20.69	20.77
Financials	13.14	15.07	16.91	15.67	13.02	17.34	17.75	20.46	20.63	20.64	21.29	22.27	17.39	12.97	14.38	16.06	13.43	15.63	16.17	16.64	16.47	14.81
Health Care	10.82	10.38	11.35	12.01	9.31	14.36	14.36	14.77	13.31	12.68	13.34	12.03	12.00	14.92	12.64	10.91	11.85	12.05	13.01	14.11	15.16	13.63
Cons. Discret.	12.97	11.70	11.95	12.43	12.70	10.28	13.14	13.26	11.29	11.90	10.81	10.62	8.41	8.39	9.60	10.63	10.67	11.40	12.54	11.94	12.89	12.03
Industrials	12.63	12.75	11.65	9.97	9.91	10.57	11.29	11.50	10.96	11.79	11.35	10.84	11.52	11.06	10.30	10.95	10.69	10.13	10.93	10.45	10.05	10.27
Cons. Staples	12.80	12.62	12.36	11.28	7.17	8.10	8.19	9.48	11.00	10.48	9.45	9.25	10.27	13.06	11.36	10.63	11.54	10.64	9.79	9.87	10.06	9.37
Energy	9.14	9.18	8.51	6.41	5.55	6.57	6.34	5.99	5.79	7.16	9.31	9.82	12.94	13.14	11.51	12.03	12.27	11.04	10.24	8.45	6.50	7.56
Materials	6.05	5.68	4.53	3.11	3.00	2.30	2.61	2.82	3.05	3.09	2.99	2.96	3.36	2.97	3.60	3.74	3.50	3.62	3.50	3.17	2.76	2.84
Utilities	4.53	3.69	3.39	3.04	2.21	3.79	3.09	2.84	2.85	2.94	3.36	3.55	3.62	4.20	3.72	3.30	3.87	3.47	2.93	3.25	2.99	3.17
Real Estate																						2.85
Telecom	8.53	6.62	7.12	8.29	7.94	5.46	5.50	4.25	3.38	3.27	3.01	3.51	3.66	3.88	3.15	3.11	3.17	3.07	2.31	2.29	2.43	2.66

Sector Statistics Summary

	mean	std	p-value(mean)	skewness	skewness test statistics	p-value(skewness)	kurtosis	kurtosis test statistics	p-value(kurtosis)	JBtest.X.squared
SPY.nort	0.000244	0.013560	0.391652	-0.085372	-1.659080	1.902900	12.434366	120.822544	0	14632.8683
XLY.nort	0.000372	0.014895	0.234617	-0.379548	-7.376003	2.000000	6.540475	63.552640	0	4103.8291
XLP.nort	0.000394	0.009079	0.038945	-0.336755	-6.544383	2.000000	5.426430	52.727666	0	2830.7236
XLE.nort	0.000101	0.019586	0.805977	-0.514467	-9.997979	2.000000	10.636750	103.355431	0	10806.6421
XLF.nort	-0.000106	0.023562	1.170171	-0.057517	-1.117757	1.736329	11.420915	110.975019	0	12344.1894
XLV.nort	0.000410	0.011223	0.082358	-0.280403	-5.449261	2.000000	10.987863	106.767132	0	11454.6019
XLI.nort	0.000264	0.014704	0.393016	-0.239575	-4.655808	1.999997	5.550550	53.933712	0	2938.4713
XLB.nort	0.000193	0.016975	0.589169	-0.290788	-5.651070	2.000000	6.571680	63.855859	0	4120.0450
XLK.nort	0.000329	0.013772	0.255626	0.051849	1.007607	0.313643	8.933803	86.808194	0	7554.4705
XLU.nort	0.000222	0.012049	0.381325	0.307380	5.973511	0.000000	11.116276	108.014894	0	11729.1328
IYZ.nort	0.000100	0.014843	0.749610	-0.137230	-2.666873	1.992344	9.011392	87.562111	0	7692.3074

- Data normality rejected, fat tail, high peak
- Showing highly autocorrelation, and ARCH effect

  by doing acf, pacf and McLeod-Li test

Before data processing

- The sum of weighted sector return: $R_t = \sum_{i=1}^{10} w_{i,t} \times X_{i,t}$ while $X_{i,t}$ = sector i return at time t
- The dynamic systemic risk indicator (SRI): $SRI_t = \sum_{i=1}^{10} w_{i,t} \times |X_{i,t} - \hat{X}_{i,t}|$

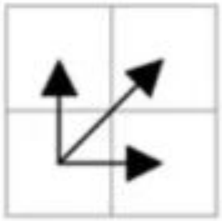
  Reference: Measuring Systemic Risk by Kuan-Heng Chen, 2017
- The idea of Dynamic Time Warping:

  DTW is a time series alignment algorithm developed originally for speech recognition.

  Use as a technique comparing the similarity of two time series

$$d_{ij} = d(x_i, y_j)$$

$$D(i,j) = d_{ij} + min\{\, D(i-1,j), D(i,j-1), D(i-1,j-1) \,\}$$



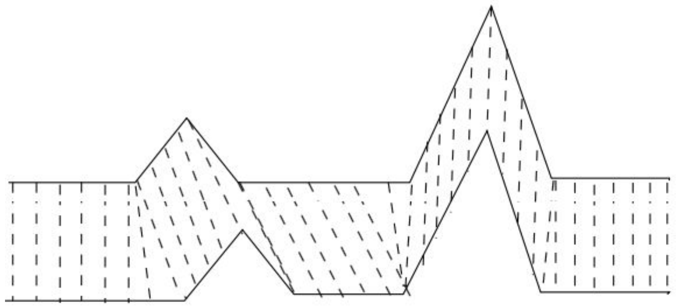


Figure 1. The market return and the sum of weighted sector return

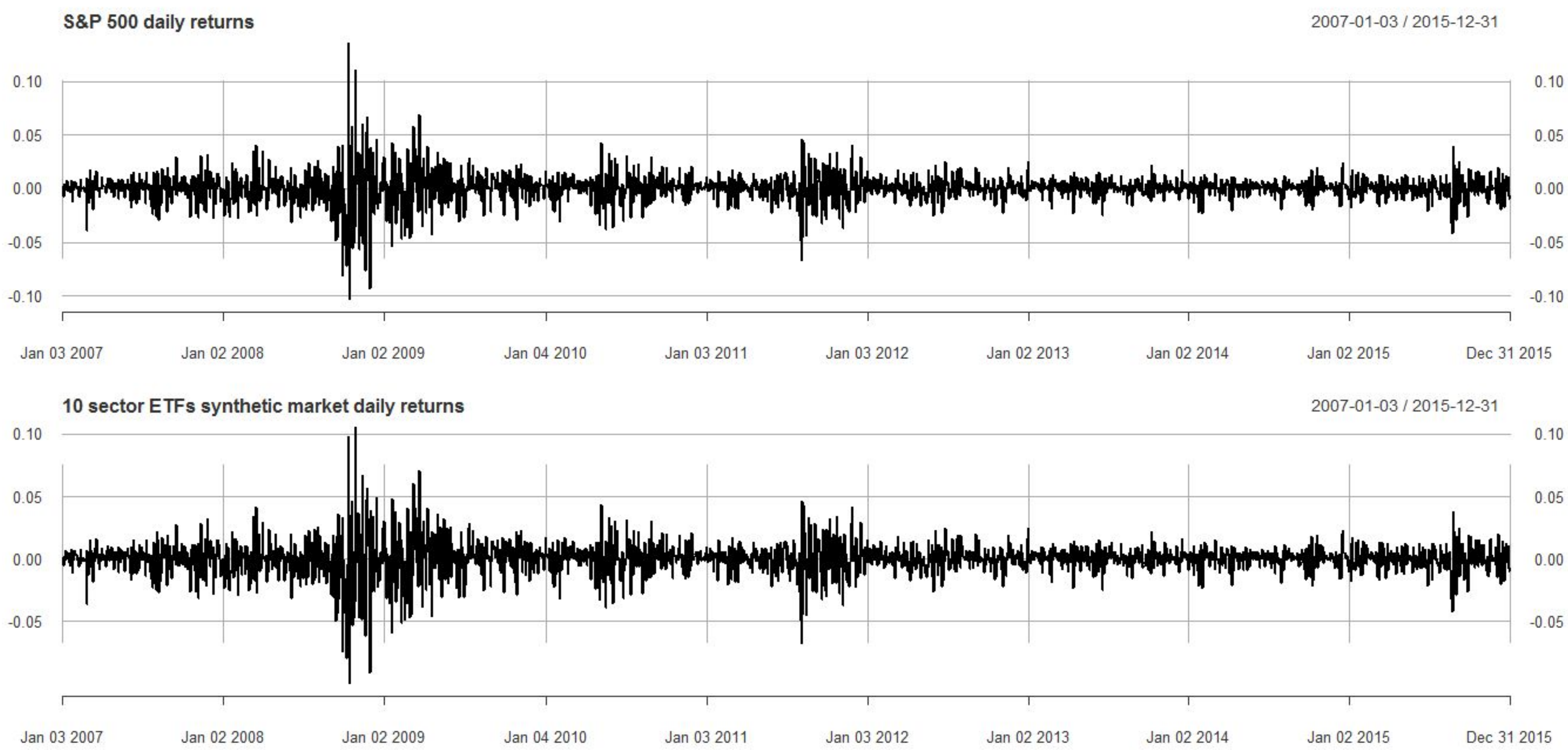


Figure 2. S&P 500 index, SRI and VIX

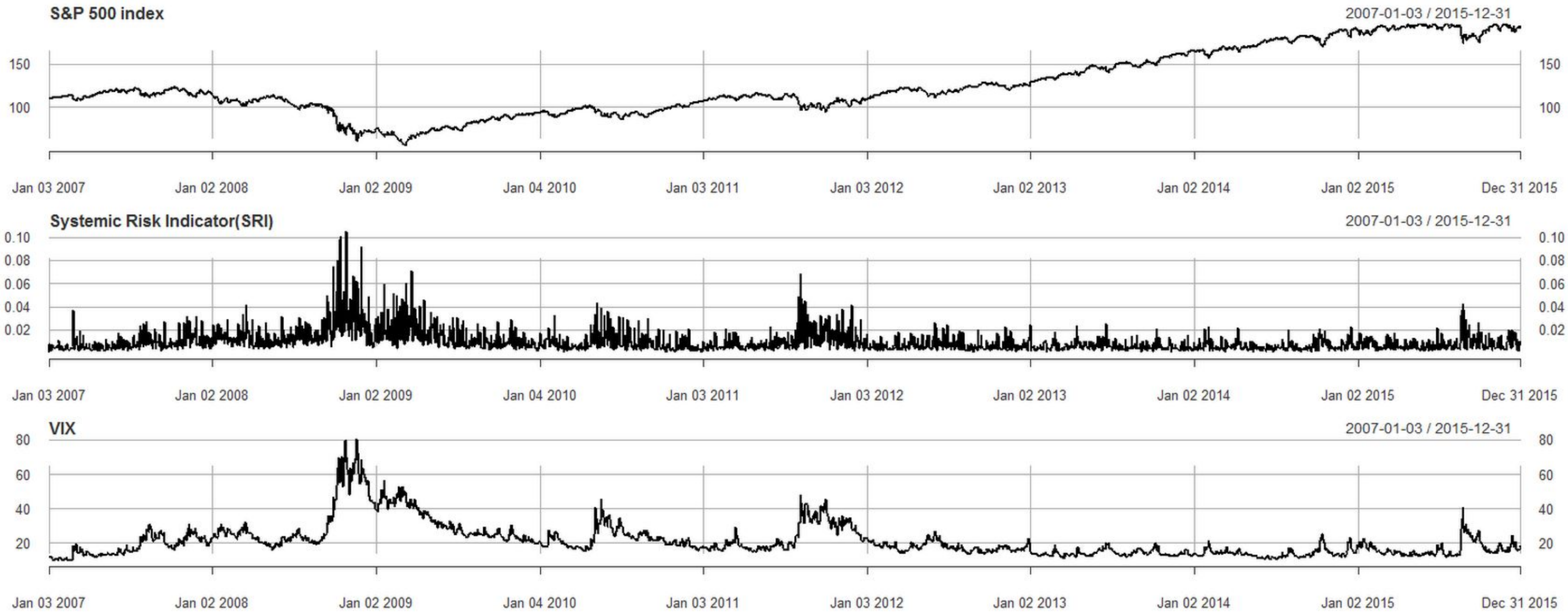
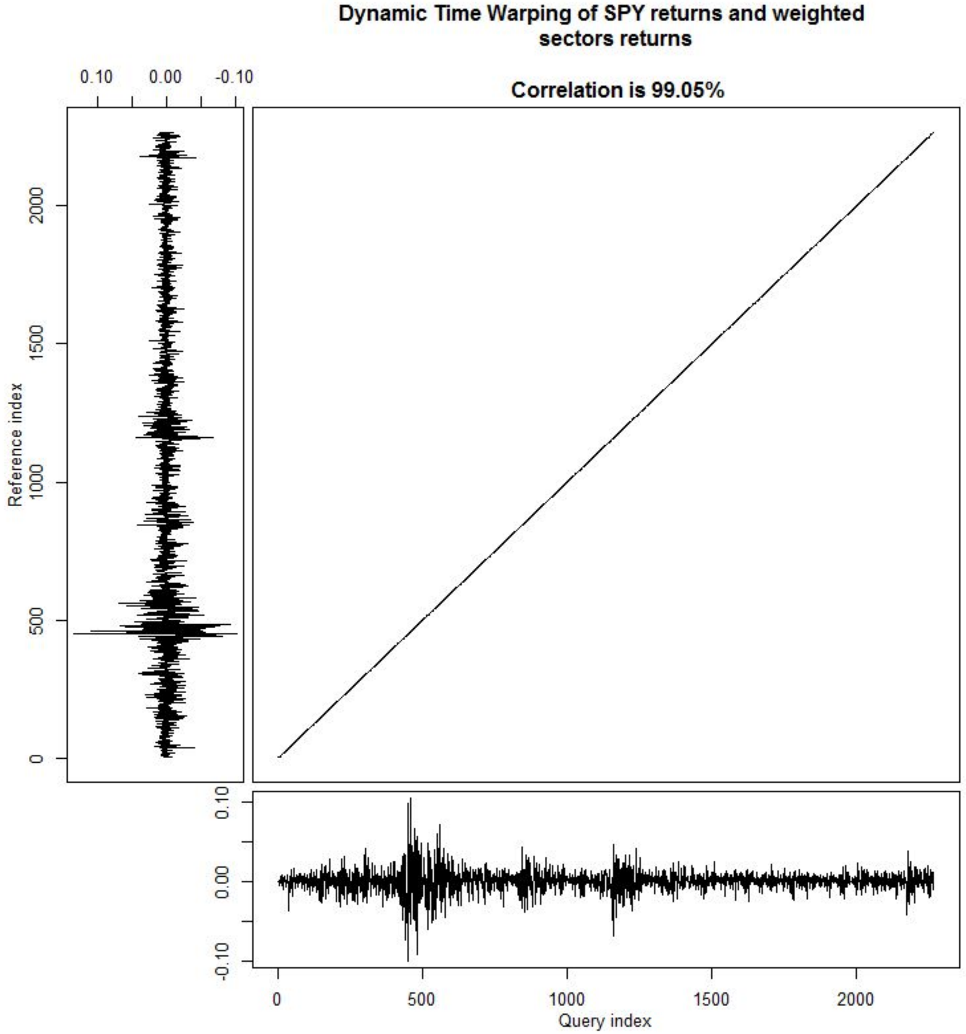
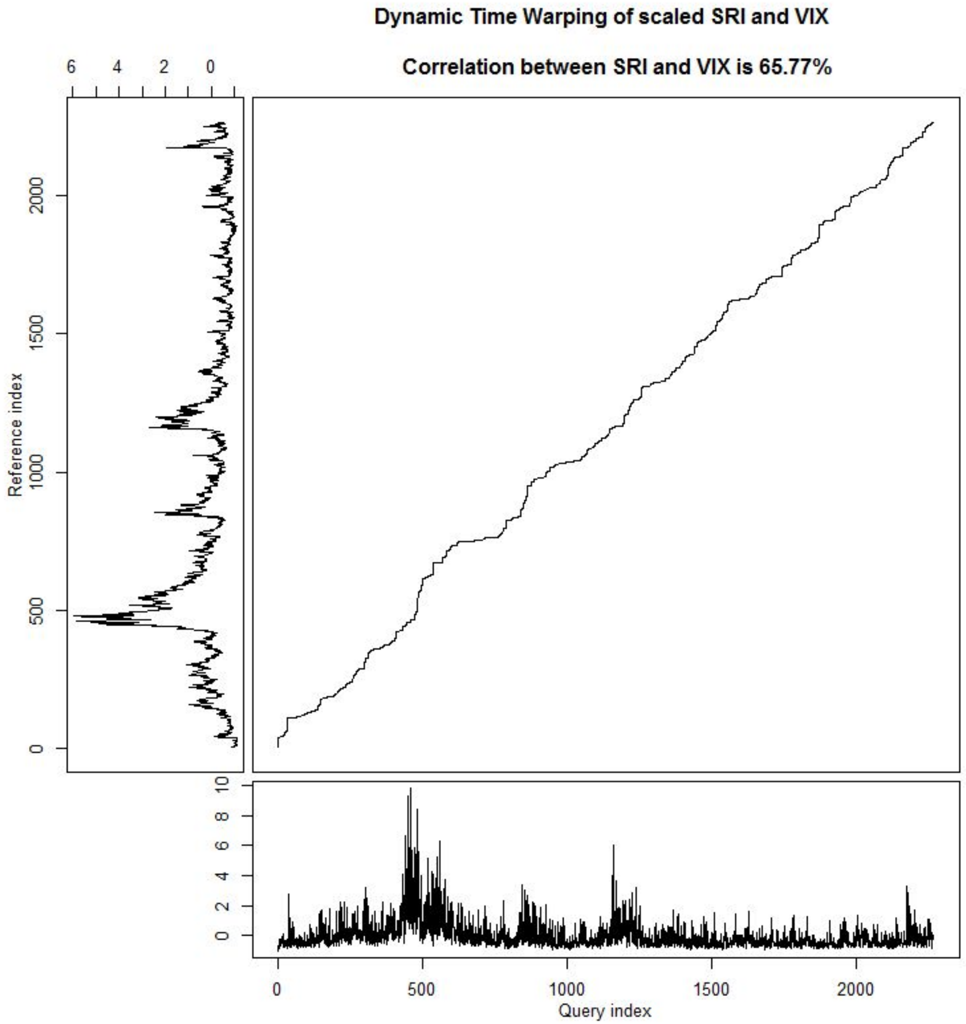


Figure 3. Dynamic Time Warping comparing





Part II

ARMA-GARCH Model and Vine Copula

STEVENS
INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY®

ARMA-EGARCH Model

- **ARMA(p,q)-EGARCH(1,1) Model with the skewed Student's t distributed innovation**

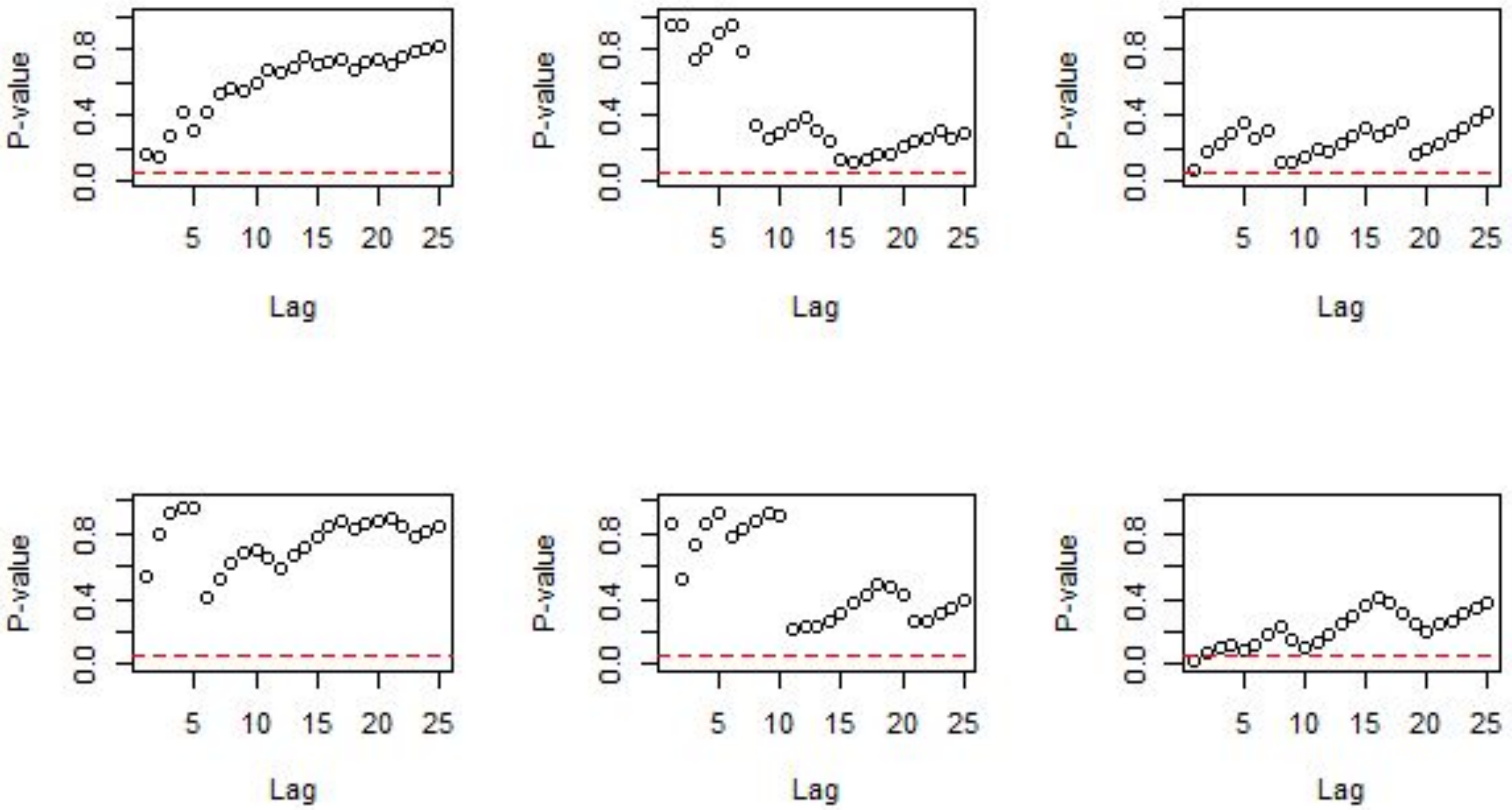
$$r_t = \mu_t + \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$\ln(\sigma_t{}^2) = \kappa_t + \alpha_t z_{t-1} + \zeta_t(|z_{t-1}| - E\{[z_{t-1}]\}) + \beta_t \ln(\sigma_{t-1}{}^2)$$

- **skewed student's t density function**

$$p(z|\zeta, f) = \frac{2}{\xi + \frac{1}{x_i}}\{f(\frac{z}{\xi})I_{[0,\infty)}(z) + f(\xi z)I_{[0,\infty)}(z)\}$$

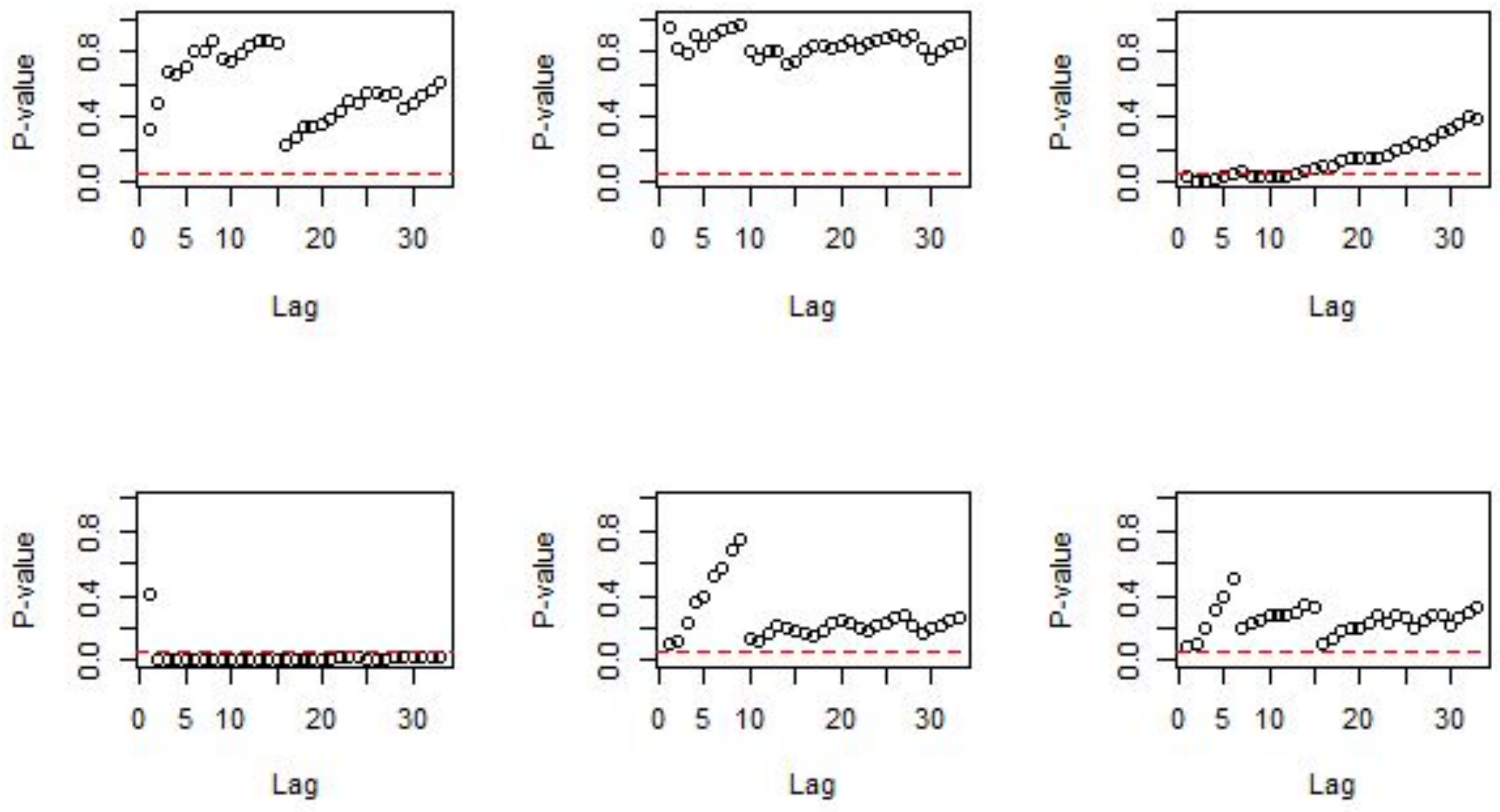
ARCH Test for ARMA(1,1)-EGARCH(1,1) Model



1. XLY: Consumer Discretionary
2. XLP: Consumer Staples
3. XLE: Energy
4. XLF: Financials
5. XLV: Healthcare
6. XLI: Industrials

Data from 2008-01-01 to 2009-05-01

ARCH Test for ARMA(1,1)-EGARCH(1,1) Model



1. XLY: Consumer Discretionary
2. XLP: Consumer Staples
3. XLE: Energy
4. XLF: Financials
5. XLV: Healthcare
6. XLI: Industrials

Data from 2007-01-01 to 2016-01-01

ARMA-GJRGARCH Model

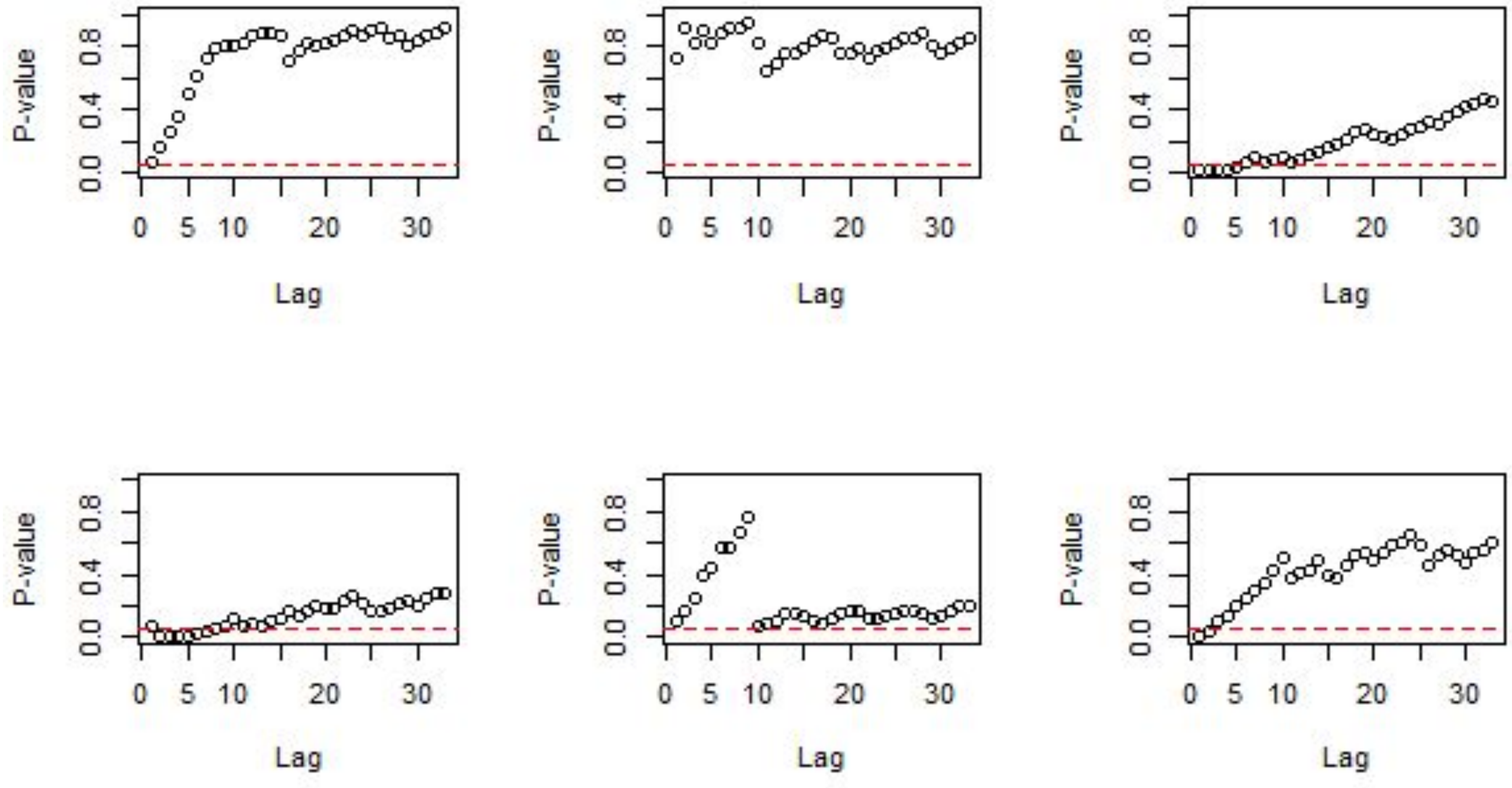
- **ARMA-GJRGARCH Model**

$$r_t = \mu_t + \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$\sigma_t^{\,2} = \omega + \sum_{i=1}^{p} \alpha_i \sigma^2{}_{t-i} + \sum_{j=1}^{q} \beta_j \varepsilon^2{}_{t-j} + \sum_{j=1}^{q} \gamma_j 1_{t-j} \varepsilon^2{}_{t-j}, \varepsilon_t = z_t \sigma_t$$

where $1_{t-j} = 1 \; if \varepsilon_{t-j} < 0, otherwise 1_{t-j} = 0$

ARCH Test for ARMA(1,1)-GJRGARCH(1,1) Model



1. XLY: Consumer Discretionary
2. XLP: Consumer Staples
3. XLE: Energy
4. XLF: Financials
5. XLV: Healthcare
6. XLI: Industrials

Data from 2007-01-01 to 2016-01-01

Results for ARMA(1,1)-GJRGARCH(1,1) Model

	XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	IYZ	SPY
mu	0.00048	0.00042	0.00028	0.00031	0.00051	0.00032	0.00018	0.00048	0.00037	0.00018	0.00040
ar1	0.67421	0.61564	-0.50339	0.57847	0.56443	0.60213	0.71414	0.59233	0.73866	0.73349	0.59798
ma1	-0.72344	-0.68839	0.45211	-0.65215	-0.62272	-0.64036	-0.75645	-0.64190	-0.77753	-0.74976	-0.66827
omega	2.26E-06	1.88E-06	2.98E-06	2.81E-06	3.95E-06	2.11E-06	1.59E-06	3.18E-06	2.09E-06	2.76E-06	2.51E-06
alpha1	0.01623	0.00688	0.01660	0.04292	0.00000	0.00000	0.00384	0.00000	0.03744	0.00145	0.00000
beta1	0.89851	0.88362	0.91510	0.88377	0.86930	0.90886	0.92843	0.88614	0.90535	0.92335	0.87201
gamma1	0.14397	0.16422	0.11087	0.13622	0.17551	0.15690	0.12096	0.18234	0.07328	0.10980	0.22121
skew	0.83854	0.86231	0.85673	0.90147	0.85758	0.83781	0.81058	0.84012	0.87569	0.86452	0.81998
DF	9.41401	10.35482	13.88961	6.72865	8.93365	11.42184	10.10042	7.44099	9.86119	9.25129	6.52130
ARCH	0	0	0	0	0	0	0	0	0	0	0
J-B test	1	1	1	1	1	1	1	1	1	1	1

Vine Copula

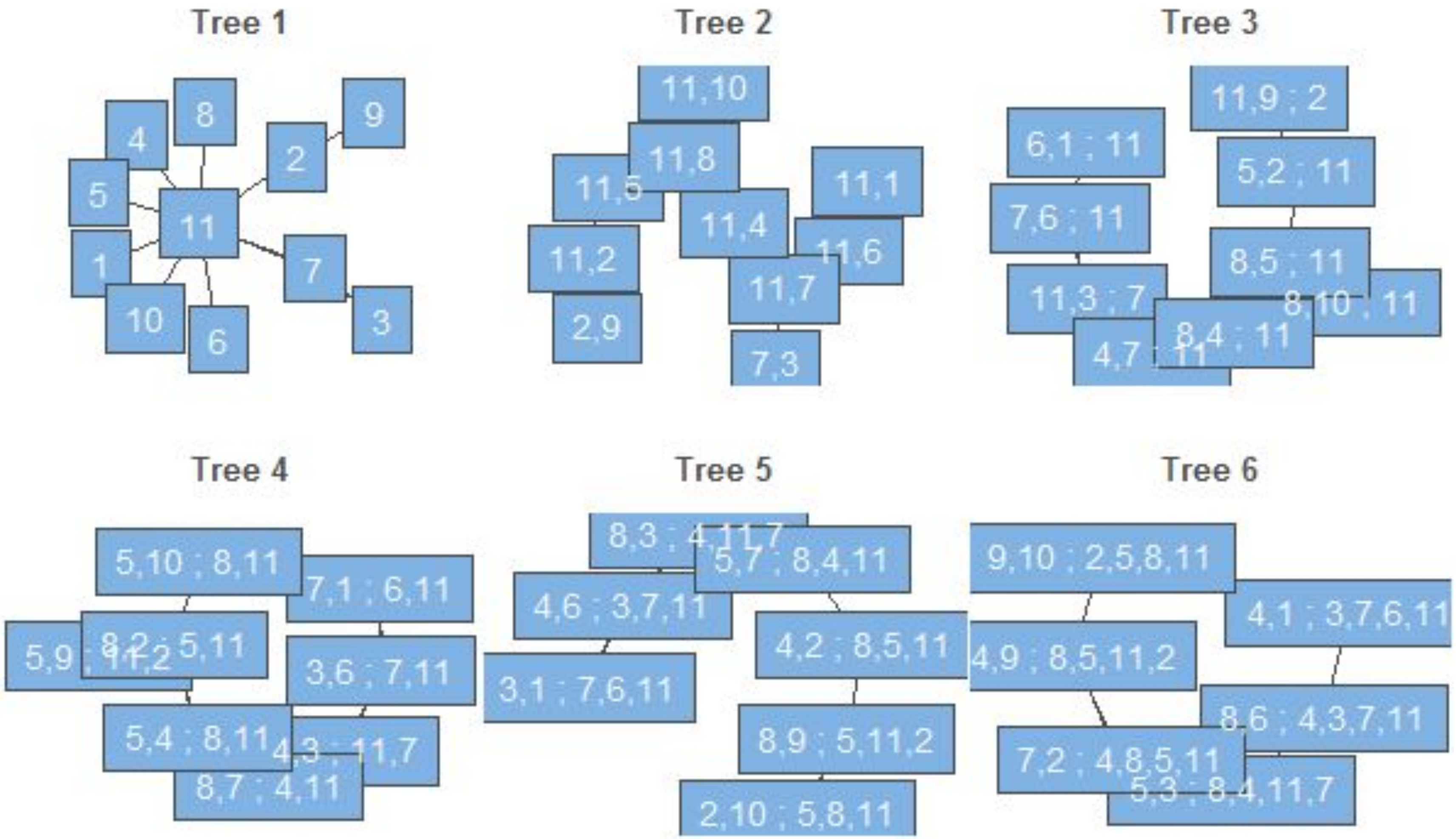
Copula

- Sklar's Theorem $H(x) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$

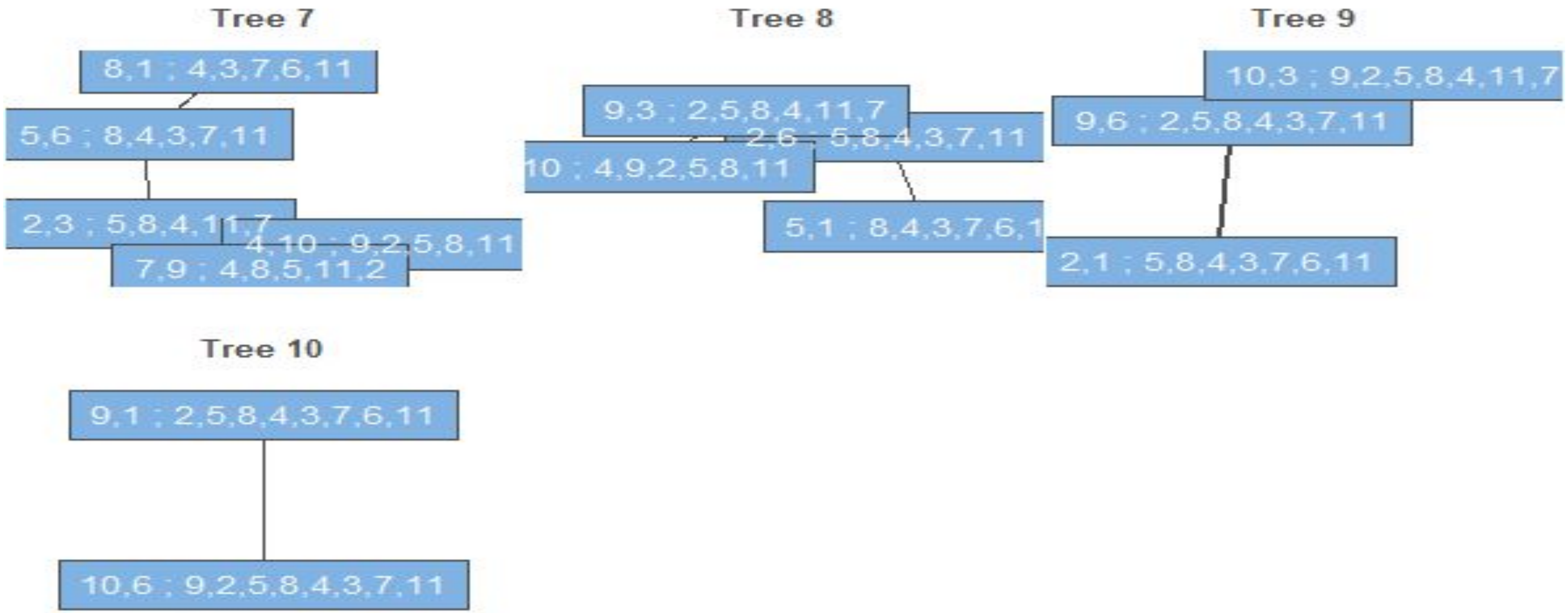
Vine Copula

- The vine copula method allows a joint distribution to be built from bivariate and conditional bivariate copulas arranged together according to the graphical structure of a regular vine
- VineCopula RVineStructureSelect()

R Vine Structure



R Vine Structure

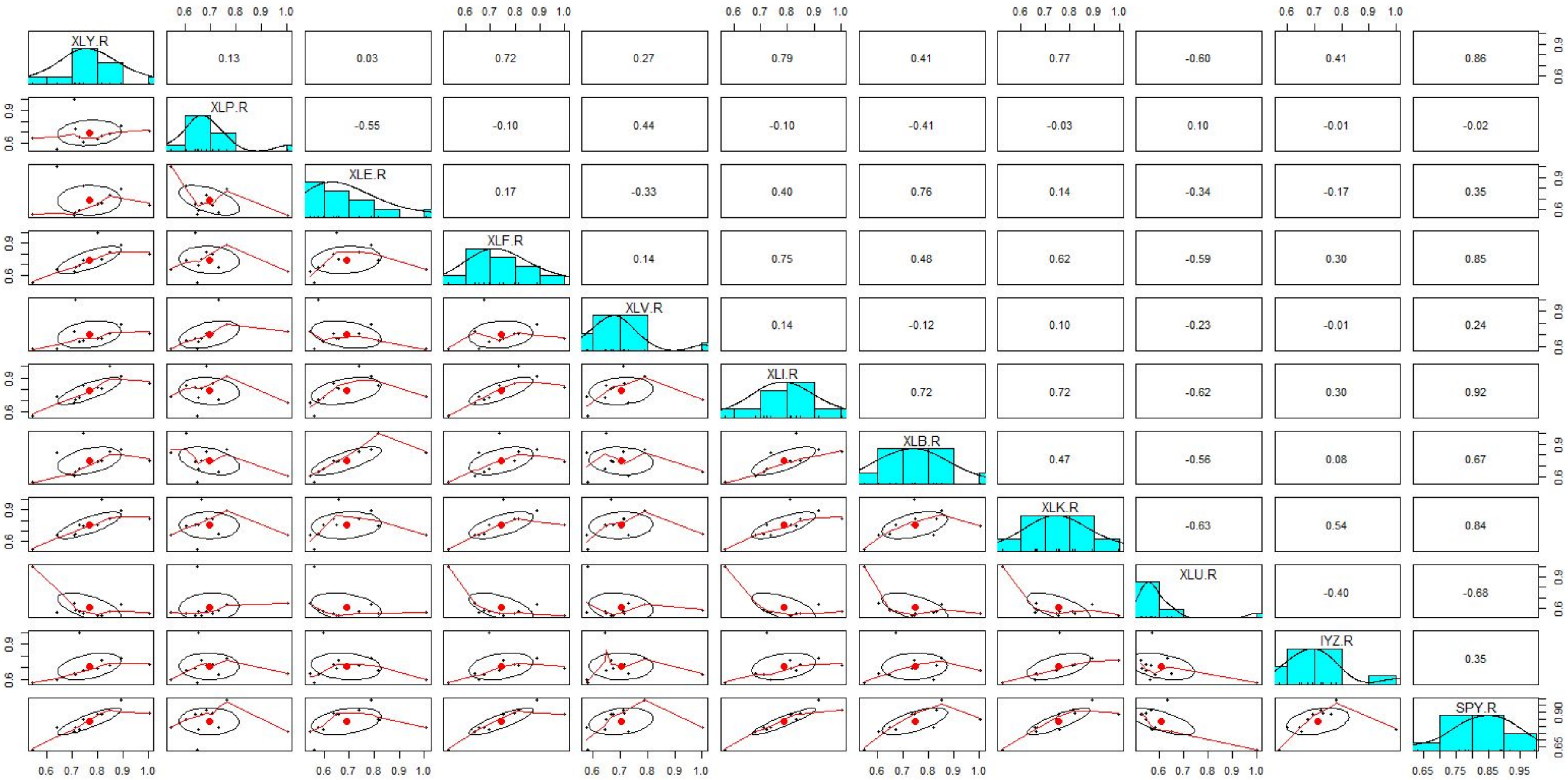



```
1 <-> XLY.R,   2 <-> XLP.R,   3 <-> XLE.R,   4 <-> XLF.R,   5 <-> XLV.R,   6 <-> XLI.R,
7 <-> XLB.R,   8 <-> XLK.R,   9 <-> IYZ.R,   10 <-> XLU.R
```

Part III

Conclusion

Correlation of residuals



Simulation of residuals of 11 tickers by R-Vine

1	0.130564274	0.687775163	0.597126683	0.31812020	0.552802284	0.28973877	0.218945025	0.631796409	0.022019860	0.70058457	0.38423649
2	0.510888671	0.733650869	0.569121870	0.46419615	0.590245038	0.79822332	0.583922504	0.150877022	0.749989626	0.11526066	0.35955709
3	0.487037645	0.413011945	0.119626208	0.59533393	0.928310992	0.44449553	0.474627398	0.386458103	0.866700893	0.74724024	0.18971866
4	0.488347908	0.103399969	0.605960335	0.99749522	0.062291681	0.07610942	0.717565841	0.333875331	0.007818201	0.56075828	0.61839627
5	0.437846551	0.088698188	0.616037869	0.53052466	0.011102918	0.34879241	0.055937292	0.048717984	0.607761268	0.34942104	0.85904153
6	0.144823770	0.824478913	0.851370670	0.08248476	0.691092792	0.22902685	0.085087711	0.600682903	0.516724291	0.01124300	0.69383186
7	0.484036992	0.820766280	0.745752793	0.17367818	0.517230415	0.47207439	0.546746357	0.471020165	0.707670200	0.83232546	0.58993445
8	0.805294614	0.425792569	0.612363086	0.32195750	0.754133231	0.73956740	0.454535343	0.848885013	0.047613125	0.98670013	0.74264736
9	0.044482685	0.314451239	0.693463314	0.92262799	0.190961077	0.83994915	0.437984880	0.708052035	0.652376648	0.95799853	0.47246898
10	0.268868573	0.057968405	0.805532870	0.64980571	0.636487239	0.21265146	0.352373565	0.233961352	0.280051753	0.09836433	0.95854754
11	0.234479670	0.922258761	0.605085758	0.38384818	0.516269261	0.89637156	0.837783013	0.199931589	0.712727759	0.60653571	0.18873599
12	0.945569989	0.896659580	0.672567035	0.34930142	0.263260784	0.36107033	0.256199943	0.903624897	0.598722508	0.40262266	0.74544625
13	0.411897472	0.788257332	0.107849692	0.88346788	0.874105783	0.62473868	0.424374177	0.208681542	0.332318622	0.17878411	0.32923430
14	0.170825711	0.818918588	0.773957497	0.63933489	0.757560155	0.30377908	0.212314855	0.335819305	0.897523063	0.69808685	0.46038803

…

VINE-COPULA ARMA-EGARCH MODEL

- ARMA-EGARCH MODEL:

$$r_t = \mu_t + \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$$

$$\ln\left(\sigma_t^2\right) = \kappa_t + \alpha_t z_{t-1} + \zeta_t \left(|z_{t-1}| - E\left\{[z_{t-1}]\right\}\right) + \beta_t \ln\left(\sigma_{t-1}^2\right)$$

	coef1	coef2	coef3	coef4	coef5	coef6	coef7	coef8	coef9	coef10	coef11
mu	-0.00100539	-0.0004552357	-0.001234189	-0.003031845	-0.001036569	-0.001629716	-0.001106006	-0.0008863284	-0.0007361939	-0.00100539	-0.001192917
ar1	0.49890633	0.3575723378	0.215467093	0.657195593	0.268613948	0.284799422	0.582172674	0.2673470178	0.5555673828	0.49890633	0.446062765
ma1	-0.59098704	-0.5381421179	-0.368434539	-0.784633493	-0.407272578	-0.421761829	-0.662126870	-0.4195397533	-0.6217874672	-0.59098704	-0.581662248
omega	-0.13555063	-0.2522312976	-0.194622023	-0.092475732	-0.432623158	-0.090939841	-0.158387928	-0.1532085567	-0.2505037563	-0.13555063	-0.176429099
alpha1	-0.09924672	-0.1105433862	-0.136112666	-0.124077304	-0.166515281	-0.106262422	-0.098145256	-0.0951545691	-0.1336305143	-0.09924672	-0.138696813
beta1	0.98185100	0.9710753100	0.972697399	0.985463791	0.948977350	0.988104635	0.978529224	0.9805877093	0.9705227063	0.98185100	0.977794119
gamma1	0.10664207	0.1317651148	0.095371319	0.206146111	0.174414189	0.084512897	0.135465768	0.1381620382	0.1898465317	0.10664207	0.141993461

COMPUTATION OF SYSTEMIC RISK

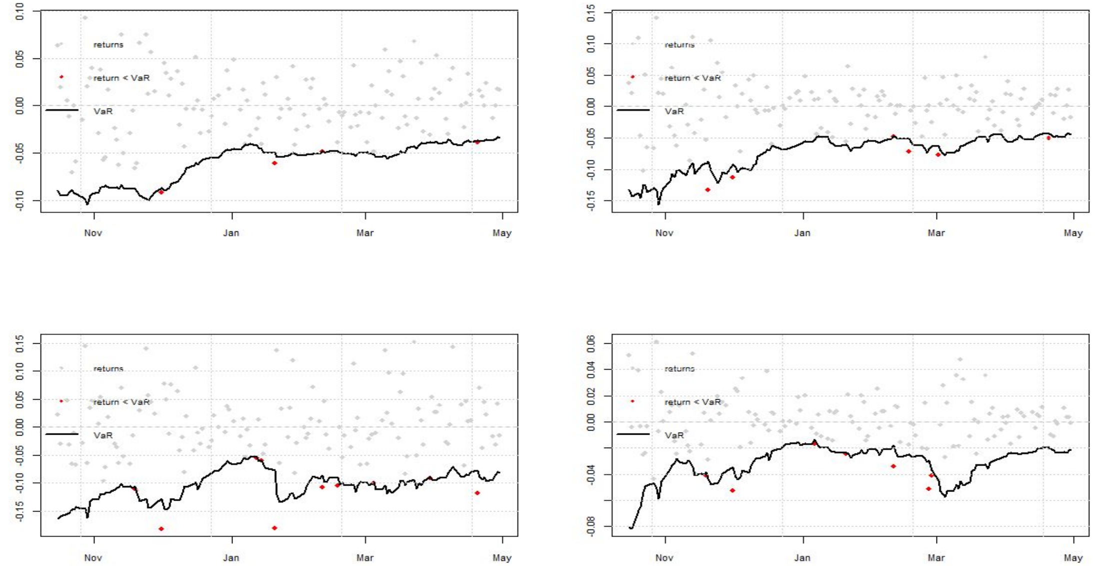
- DYNAMIC COPULA COVAR: measuring systemic risk & determining the major risk contributors

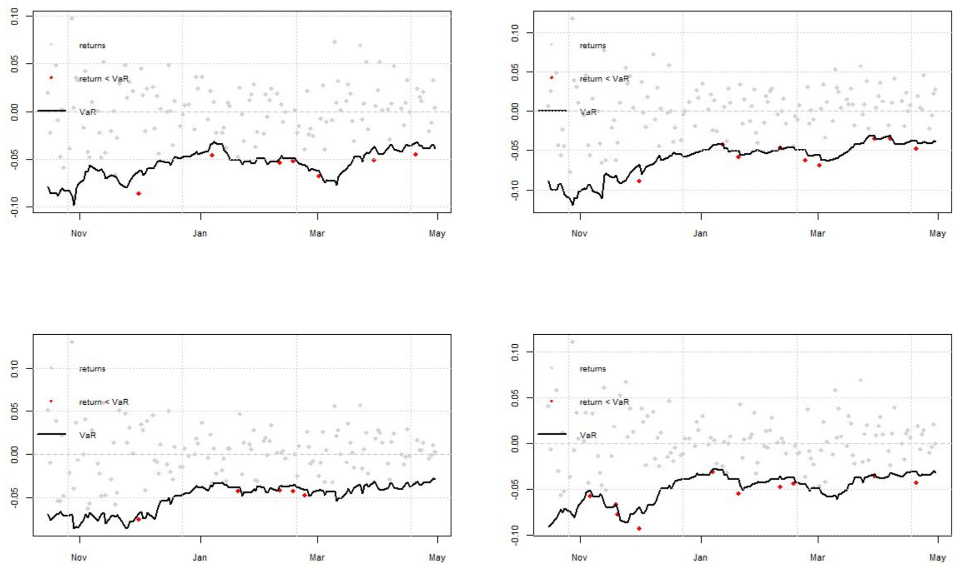
$$P\left[X^s \leq CoVaR^{s|m} | X^m \leq VaR^m\right] = 1 - \alpha$$

- Vine Copula-Based Model (high-dimensional distribution)

10DAY ROLLING FORECAST OF COVAR (2008-2009)

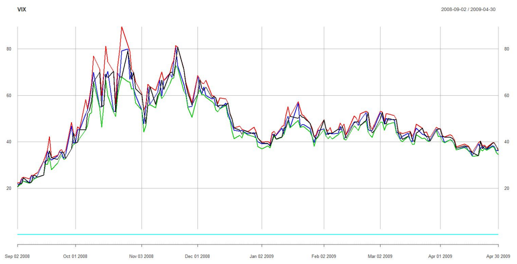
BASED ON ARMA(1,1)-EGARCH(1,1) MODEL





1. XLY 2. XLE
3. XLF 4. XLV

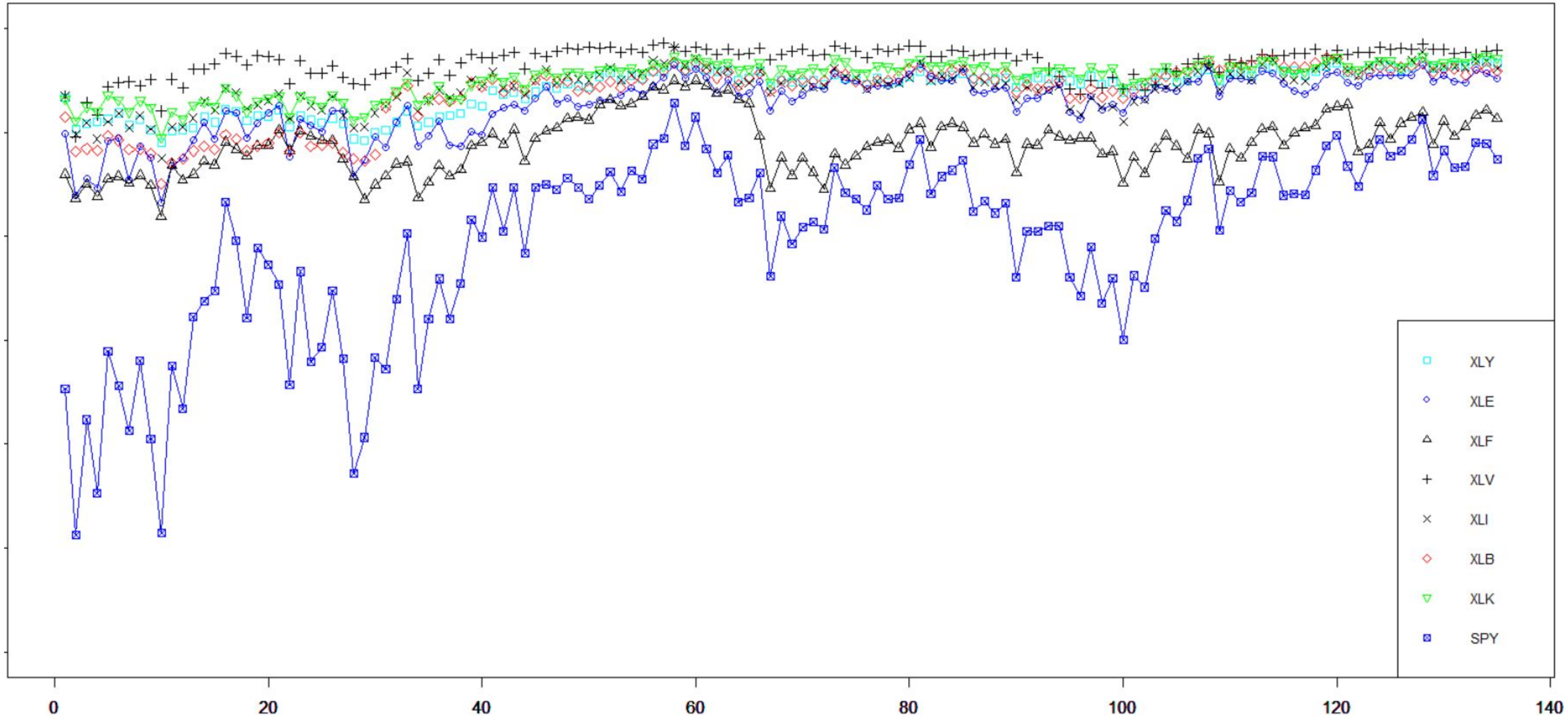
5. XLI 6. XLB
7. XLK 8. SPY



CONCLUSION: ONE-DAY AHEAD ROLLING COVAR (95%)

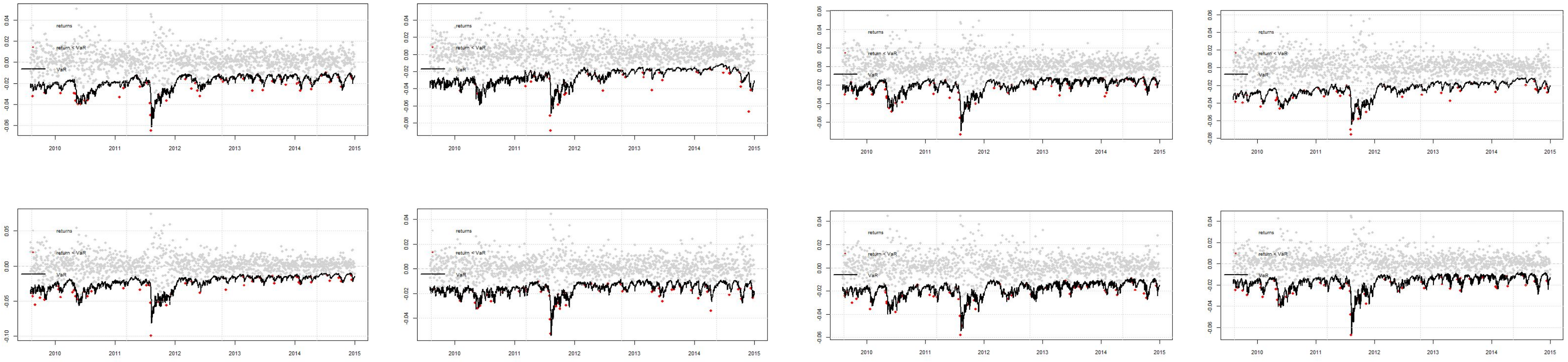
BASED ON ARMA(1,1)-EGARCH(1,1) MODEL

Contribution to systemic risk by sectors: XLF>XLE>XLK >XLV
(from 2008 to 2009)



30DAY ROLLING FORECAST OF COVAR (2009-2015)

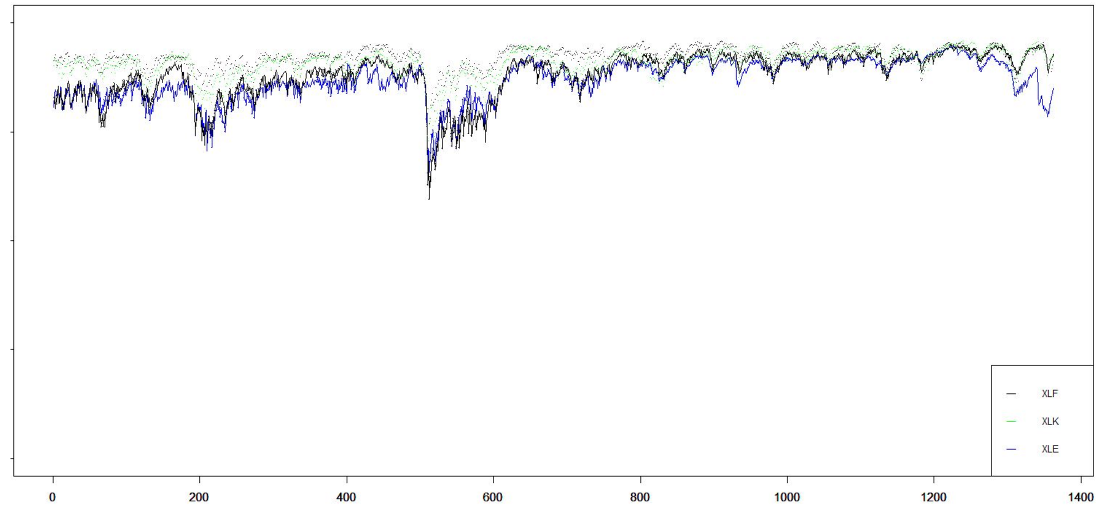
BASED ON ARMA-GJRGARCH MODEL



1. XLY 2. XLE
3. XLF 4. XLV

5. XLI 6. XLB
7. XLK 8. SPY

Contribution: XLF≈XLK>XLE
(from 2009 to 2015)



Summary

- Developed R-Vine Arma-Garch Model
  1. Could be applied to a high-dimensional distribution
  2. Utilize copula modeling in risk measurement
  3. Present a dependency structure

- Forecast one-day ahead VAR; compare risk contributions by sectors
  Financials sector is more sensitive to the systemic risk.

THANK YOU