Stevens Institute of Technology

Robo-risk and Copula CoVaR of SPY

Authors: Xinhang Wang

Qi Yuan

Shihao Zhang

Advisor: Khaldoun Khashanah

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Abstract

In our study, a new method and forecasting model based on generalized autoregressive conditional heteroskedasticity (GARCH) model as well as multidimensional distributions are constructed for the risk measurement of the whole financial system. Historical returns from 2008 to 2015 are choosen and fitted by various marginal distribution models like ARMA-EGARCH. To reflect the impact of different sectors under stress, R-vine copula is raised to build a multivariate probability distribution. Besides, we applied CoVaR, the conditional Value-at-Risk, ΔCoVaR and VaR ratio to capture the dynamic systemic risk through a more accurate and reasonable way, creating a useful early warning indicator for the financial system which prevents risks in advance.

Keywords: ARMA-GARCH, Copula, CoVaR, Systemic Risk, Time Series Analysis

Contents

Chapter 1 Introduction

1.1 Background and Motivation

Over the past few decades, the global financial system has become more complicated. Although evolving changes in financial innovation and connectivity has improved the economy as a whole, they also brought about the negative impact of systemic risk.

During recent financial crisis of 2007-2009, losses spread across financial institutions, threatening the financial system as a whole. One lesson that we can learn from it is that large risk spillovers from one bank to another could exacerbate systemic risk. The discussion of the complexity and fragility of financial system and systemic risk was ignited after the outbreaks of subprime crisis and European debt crisis. Researchers began to realize that Value-at-Risk (VaR), which is the most widely used risk measure by financial institutions, is far from enough for risk management. VaR fails to capture the nature of systemic risk because it only focuses on individual institution in isolation, and tail co-movement and spill-over effect have been ignored. Therefore, finding a practical method for systemic risk measure to supervise the stability of financial system is essential and indispensable.

After referring to other definitions of great reference value, we could summarize some common points. First, systemic risk has a great influence to the instability of the entire financial system. Second, in addition to financial firms, systemic risk will also create large losses among non-financial firms because it can spillover to the whole sectors of the real economy. Last but not least, systemic risk has something to do with the interconnectedness of the financial system.

The motivation of this proposal was to find a new and effective method to measure systemic risk that can serve as a useful analytical tool for financial stability monitoring, and may provide some guidance for macro-prudential policy instead of only qualitatively revising regulations to avoid systemic risk. The main purpose was to investigate the relationship between interdependence among components and systemic risk. This project wanted to solve the problem by answering two questions. One was how to analyze the risk contribution of US equity sectors, another was how to measure the systemic risk.

In this proposal, we viewed the S&P 500 index as a system and the sectors in the S&P 500 as its components. The sectors are Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Telecommunication Services and Utilities. We considered systemic risk resulting not only from the financial sector but also from other sectors. The daily data was downloaded from Yahoo Finance.

1.2 Proposal Structure

This proposal was organized as follows: Chapter 2 presented the literature review in terms of CoVaR and copula applications. In Chapter 3, we presented the empirical dataset, systemic risk indicator application and dynamic time warping application. Chapter 4 talked about the methodology used in this proposal. We introduced ARMA-GARCH model and copulas, finally develop vine copula-based ARMA-EGARCH and ARMA-GJRGARCH model. Chapter 5 conducted empirical analysis of copula-based ARMA-GARCH model based on S&P 500 index and the sectors in the S&P 500. Chapter 6 summarized the key findings and presented future work.

Chapter 2 Literature Review

In this proposal, we presented the recent research about CoVaR application in risk management, and described the idea how copula modeling could be used into financial time series application, how we could evaluate systemic risk by constructing R-Vine Copula tree and select the parameter based on AIC.

2.1 CoVaR and Copula Application in Risk Management

Throughout research paper of Kuan-Heng Chen,[4](Chen, Kuan-Heng, and Khaldoun Khashanah 2015) we explored the idea that if we can design a specific indicator to measure the risk spillover intensity between financial markets, we could use quantitative methods to deeply study the risk spillover effect, which has great theoretical and practical significance in risk management. Based on this background, we introduce the CoVaR method proposed by Adian and Brunnermeier [2](Adrian, Tobias, and Markus K. Brunnermeier.2010), the Conditional-Value-at-Risk method, which attempts to measure the risk of portfolio potential loss in when other financial markets (or financial institutions) are under stress. Later, in 2009, the two scholars published several working papers in detail to elaborate on the CoVaR method. Compared with the VaR method, the CoVaR method incorporates risk spillover effects into the risk value framework as well as express the risk spillover intensity with a specific numerical value. It can be a more comprehensive and effective risk management technique in practice. For financial institutions, considering VaR with risk spillover effects (CoVaR) can more accurately measure actual risks and avoid underestimation (or overestimation) risk, which will improve the accuracy of risk management decisions. And for the regulatory authorities or investment company that focus on the risks of the entire financial system, because the CoVaR method accurately and effectively reflects the impact of a single financial institution (or financial market) on systemic risk, it can be known the extent to which financial institutions (or financial markets) contribute more to systemic risk. Then we can take specific action to avoid underside risk on higher risk level of financial institutions (or financial markets) and predict the systemic risk on market. CoVaR does provide a new perspective to manage system risks.

Based on our research, a large number of scholars have used CoVaR methods to measure the risk spillover effects of financial markets. Shen Yue empirically studied the systematic risk spillover effect of excessive housing price fluctuations by constructing the GARCH-copula-CoVaR model. The research shows that the systemic risk spillover effect of excessive housing price fluctuations is obvious, but there are differences in risk spillover effects at different economic levels. Although the literature has made good research results on the measurement of cross-market risk spillover effects using CoVaR, it is doubtful that most studies based on linear quantile regression to calculate CoVaR fail to escape the "linear infection relationship". Some scholars have jumped out of the traditional linear statistical correlation research framework, they have begun to use copula, hybrid copula, and time-varying copula functions to study financial contagion effects by discussing the structure of financial market dependence. But some of the relevant research ignores the spillover effect of market crush down. The risk spillover measure is mainly based on the binary copula, and when it has to face the "dimensional curse", it seems to be powerless applying into multi-dimensions. Brechmann & Czado[18] (Brechmann, Eike Christain, and Claudia Czado 2013) proved that C- and D-vines can be constructed by simple recursive conditioning, which is used in time series very often. And more scholars applied vine-copula to risk-contagion research, however, the existence of risk spillover was judged based on significant changes in conditional correlation, and it was still limited to C-vine and D-vine with fixed structure, which could not truly reflect the complex interdependent structure of high-dimensional market. Few scholars research the risk based on R-vine's non-linear quantitative measure of risk spillover effects between different markets. There is also no effective algorithm for the selection of structures.

Based on the above information and analysis, the R-vine copula function is used to analyze the complex interdependence between financial markets, and the pair-copula decomposition technique is used to simplify the dimensionality reduction. At the same time, we use McLeod-Li test to test the reliability of the non-linear dependence relationship of R vines. Finally, based on R-Vine Copula tree, we select the properly parameter and construct the CoVaR model to empirically analyze the risk spillover effects between markets, and test the validity of the model through posterior testing and try to accurately predict the risk spillover effect between different markets.

Chapter 3 Data Processing and Representation, the Risk Measure, Dynamic Time Warping

3.1 Data Representation Introduction

The emerging risk of U.S. financial market sector triggered by the subprime mortgage crisis in 2007 has quickly spread to other countries and regions, which eventually led to the major economic crisis on the global market. This fact fully demonstrates that the lack of consideration of the risk spillover effect under extreme market conditions may lead to a serious underestimation of the risk level of different market sectors. History has proved that because of the increasing globalization of economy and finance, the risk loss events of a single financial institution or financial market often spread rapidly to the entire financial system and bring systemic risk on market. And the volatility transmission mechanism between such markets is called risk spillover effect or volatility spillover effect. And the risk spillover effect treats different market sectors as an entire financial system.

It can be very challenging to estimate systemic risk on the market for the reason that the market structure is becoming more interconnected, especially when we consider the risk spillover effect. Furthermore, the numerous financing vehicles on the market make companies possible to making financing through the shadow banking system, typically from 2007 to 2009 on the US market. One big security default will immediately bring a financial institution into stress, and one financial institution in stress will immediately cause its counterparties and its investors into stress, on and on. Same reason, the instability in one industry sector can immediately bring damage to the stability of the entire financial system. So as a result, we considered all the sectors on the market into the systemic risk measuring.

When measuring the sector contributing risk to the entire market, the industry sector indices can be easily constructed. So the data was divided into two parts in the project: the market performance and different industry sectors performance. The first data part, SPDR S&P 500 index was selected as the market performance benchmark. According to Investopedia.com, Spider(SPDR) is a short form name for a Standard & Poor's depositary receipt, an exchange-traded fund(ETF) managed by State Street Global Advisors that tracks the Standard & Poor's 500 index(S&P 500). And the second data part, according to the Global Industrial Classification Standard (GICS) in 2015, the industry sectors on the market is classified into the following areas: Consumer Discretionary, Consumer Staples, Energy, Financial, Health Care, Industrial, Materials, Technology, Utilities, Telecommunications, as shown in table 3.1. The daily time-series data was selected from yahoo finance from 2007-01-01 to 2016-01-01, comprising 11 ETFs index times 2266 trading days observations, which included SPY ETF and 10 industry sector ETFs.

Table 3.1 The 10 S&P 500 sectors based on the Global Industrial Classification Standard (GICS)

	Sector Number	Sector Symbol	Sector Name
0	Market	SPY	S&P 500 Index
1	Sector1	XLY	Consumer Discretionary
2	Sector2	XLP	Consumer Staples
3	Sector3	XLE	Energy
4	Sector4	XLF	Financial
5	Sector5	XLV	Health Care
6	Sector6	XLI	Industrial
7	Sector7	XLB	Materials
8	Sector8	XLK	Technology
9	Sector9	XLU	Utilities
10	Sector10	IYZ	Telecommunications(iShares US ETF)

Besides the daily time-series data, the S&P Sector Weightings has to be determined, so that the synthetic market return(the sum of weighted sector return) can be calculated using 10 sector ETFs by the formula,

$$R_t = \sum_{i=1}^{10} w_{i,t} \times X_{i,t} \tag{3.1}$$

where $X_{i,t}$ is the sector i return at the time t. By the article 's&p 500 sector weightings report June 2018' posted on seekingalpha.com, the weightings $w_{i,t}$ can be listed in table 3.2. In our study, the data was only needed from 2007-01-01 to 2016-01-01.

Table 3.2 The S&P 500 Sector Weightings(%): 1995-2016

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
Technology	9.390	12.310	12.240	17.780	29.180	21.230	17.720	14.630	17.730	16.060	15.100
Financials	13.140	15.070	16.910	15.670	13.020	17.340	17.750	20.460	20.630	20.640	21.290
Health Care	10.820	10.380	11.350	12.010	9.310	14.360	14.360	14.770	13.310	12.680	13.340
Cons.Discret.	12.970	11.700	11.950	12.430	12.700	10.280	13.140	13.260	11.290	11.900	10.810
Industrials	12.630	12.750	11.650	9.970	9.910	10.570	11.290	11.500	10.960	11.790	11.350
Cons.Staples	12.800	12.620	12.360	11.280	7.170	8.100	8.190	9.480	11.000	10.480	9.450
Energy	9.140	9.180	8.510	6.410	5.550	6.570	6.340	5.990	5.790	7.160	9.310
Materials	6.050	5.680	4.530	3.110	3.000	2.300	2.610	2.820	3.050	3.090	2.990
Utilities	4.530	3.690	3.390	3.040	2.210	3.790	3.090	2.840	2.850	2.940	3.360
Real Estate	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Telecom	8.530	6.620	7.120	8.290	7.940	5.460	5.500	4.250	3.380	3.270	3.010

Table 3.2 The S&P 500 Sector Weightings(%): 1995-2016

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Technology	15.140	16.820	15.400	19.760	18.650	19.020	18.950	18.580	19.830	20.690	20.770
Financials	22.270	17.390	12.970	14.380	16.060	13.430	15.630	16.170	16.640	16.470	14.810
Health Care	12.030	12.000	14.920	12.640	10.910	11.850	12.050	13.010	14.110	15.160	13.630
Cons.Discret.	10.620	8.410	8.390	9.600	10.630	10.670	11.400	12.540	11.940	12.890	12.030
Industrials	10.840	11.520	11.060	10.300	10.950	10.690	10.130	10.930	10.450	10.050	10.270
Cons.Staples	9.250	10.270	13.060	11.360	10.630	11.540	10.640	9.790	9.870	10.060	9.370
Energy	9.820	12.940	13.140	11.510	12.030	12.270	11.040	10.240	8.450	6.500	7.560
Materials	2.960	3.360	2.970	3.600	3.740	3.500	3.620	3.500	3.170	2.760	2.840
Utilities	3.550	3.620	4.200	3.720	3.300	3.850	3.470	2.930	3.250	2.990	3.170
Real Estate	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.850
Telecom	3.510	3.660	3.880	3.150	3.110	3.170	3.070	2.310	2.290	2.430	2.660

Since the sum of the weighted sector return is a linear combination of sector weightings and sector indices return, the synthetic market daily returns and S&P 500 daily returns can be characterized in Figure 3.1. In this case, both series share the correlation up to 99 percent, and the daily returns on both sides are almost identical. The weighted data structure is fitted into the market daily returns well and we would discuss and compare the similarity of both time series later by using Dynamic Time Warping.

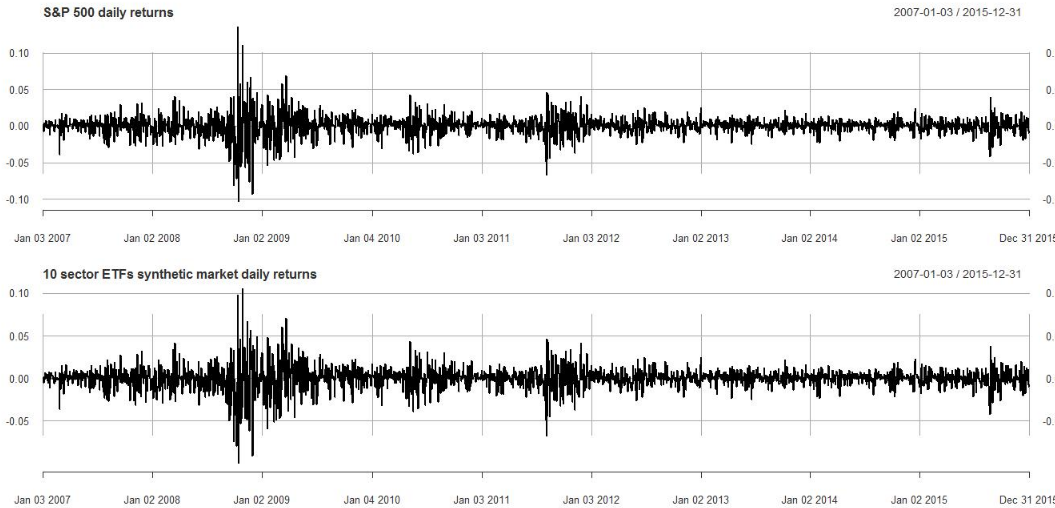


Figure 3.1 The market return and the sum of weighted sector return

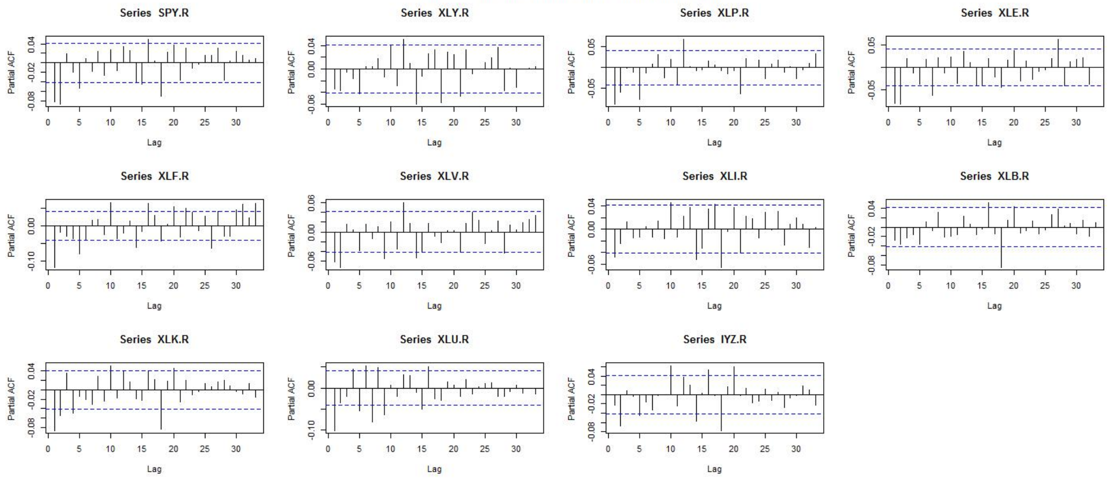
The statistics for the returns of 10 S&P 500 sector indices and the S&P 500 Index are shown in table 3.3. By observing the table result, every data series is proved to be non-normality by showing non-symmetrical distributions and leptokurtic distributions. Same statistics are shown by using the Jarque-Bera test that each series returns have very large χ^2, which reject the null hypothesis of normality at the $\alpha = 0.05$ significance level and prove the non-normality of the unconditional distribution of each time series returns. As much other financial time series returns, assuming the data fitted into the multivariate normal distribution may not be appropriate because all ten S&P 500 sector ETFs and the SPY ETF has rejected normality and shown with fat tail and high peak.

Table 3.3 Summary statistics for the 10 S&P 500 sector indices and S&P 500 Index

	Sector Name	Symbol	mean	sigma	p-value(mean)	skew	skew test statistics	p-value(skew)	kurt	kurt test statistics	p-value(kurt)	JBTest X squared
1	S&P 500	SPY	0.000244	0.01356	0.391653	-0.08537	-1.659053	1.902895	12.434405	120.82292	0	14632.8683
2	Consumer Discretionary	XLY	0.000372	0.014895	0.234616	-0.379541	-7.375864	2	6.540497	63.552859	0	4103.8291
3	Consumer Staples	XLP	0.000394	0.009079	0.038946	-0.336752	-6.544311	2	5.426404	52.727414	0	2830.7236
4	Energy	XLE	0.000101	0.019586	0.805977	-0.514467	-9.99797	2	10.636736	103.35529	0	10806.6421
5	Financial	XLF	-0.000106	0.023562	1.17017	-0.057516	-1.117746	1.736325	11.42092	110.975072	0	12344.1894
6	Health Care	XLV	0.00041	0.011223	0.082358	-0.280403	-5.449244	2	10.987858	106.767077	0	11454.6019
7	Industrial	XLI	0.000264	0.014704	0.393016	-0.239574	-4.655789	1.999997	5.550527	53.933489	0	2938.4713
8	Materials	XLB	0.000193	0.016975	0.589169	-0.290787	-5.651056	2	6.571712	63.856163	0	4120.045
9	Technology	XLK	0.000329	0.013772	0.255626	0.051851	1.007648	0.313623	8.933812	86.80828	0	7554.4705
10	Utilities	XLU	0.000222	0.012049	0.381324	0.30738	5.973506	0	11.116264	108.014777	0	11729.1328
11	Telecommunications	IYZ	0.0001	0.014843	0.749611	-0.137231	-2.666908	1.992345	9.011367	87.561871	0	7692.3074

Another features that generally existing in the financial time series data is the Autoregressive Conditional Heteroscedastic (ARCH) effect. The partial autocorrelation function checks whether the series is self-correlated in lags at the 0.05 significance level. Figure 3.2 clearly shows that ARCH effect exists in all sector indices and the series returns is serially correlated. For this reason, GARCH-type model will be used in the multivariate time series analysis, including the DCC-GARCH model (Dynamic Conditional Correlation GARCH model).

Figure 3.2 Series returns partial autocorrelation function



3.2 Risk measure using Value-at-Risk

When risk comes into the equity market, one useful method in risk management is the idea of Value-at-Risk (VaR), which can tell the manager how much loss the market or a financial institution will possibly suffer in the 5 percent significant level. And a copula is a very useful method when measuring multivariate VaR especially when the series are highly correlated in high dimensions under stress. By using copula structure, it is possible to determine beta risk when different industrial sectors under stress together in our case.

Value-at-Risk (VaR) gives us a numerical value on the quantile loss that a financial asset or portfolio investment may suffer in a given time horizon. VaR formula can be derived through the probability mass distribution *f(x)* of a given financial asset or portfolio investment. For a given significant level α, the possible risk exposure realization x^* is determined so that the probability of risk exposure lower than x^*, in this case, $P(x \leq x^*)$ is $1 - \alpha$.

$$P(x \leq x^*) = \int_{-\infty}^{x^*} f(x)dx = \alpha \qquad (3.2)$$

where the numerical value x^* is the quantile of the probability distribution.

3.3 Introduction about relative risk measure using CoVaR

According to Adrian & Brunnermeier (2011), Conditional Value at Risk (CoVaR) is a useful tool to describe dependence structure. By definition, $CoVaR_t^{i,j}$ is expressed when the financial asset y_j (generally expressed using asset returns) is at a risky level, the risk exposure of financial asset y_i is facing at the same time. Therefore, $CoVaR_t^{i,j}$ is the Conditional Value-at-Risk of asset y_i regarding the asset y_j, and the formula expression is:

$$Pr(y_{i,t} \leq CoVaR_t^{i,j} | y_{j,t-p} = VaR_{t-p}^{j}) = a \tag{3.3}$$

where $1 - \alpha$ is the confidence level. In our case, the confidence level is applied in 95 percent.

$CoVaR_t^{i,j}$ is also the Value-at-Risk for the asset y_i at a time t, which can be considered as the sum of the "exposure CoVaR" (called $\Delta CoVaR$) of a particular asset i conditional on j, $CoVaR^{i,j}$, and the unconditional VaR of the market system j, VaR^j. The $\Delta CoVaR$ that asset i's risk exposure contribution to j is:

$$\Delta CoVaR_t^{i,j} = CoVaR_t^{i,j} - VaR_t^{i} \tag{3.4}$$

where $\Delta CoVaR_t^{i,j}$ represents the Conditional Value-at-Risk y_i regarding the asset y_j. And we have to standardize $\Delta CoVaR_t^{i,j}$ in order to fully represents the risk exposure strength:

$$\%CoVaR_t^{i,j} = (\Delta CoVaR_t^{i,j} / VaR_t^{i}) * \ 100\% \tag{3.5}$$

Here if y_i represent the whole financial system(S&P 500), $\Delta CoVaR_t^{i,j}$ can better catch the change of general systemic risk when a single financial market y_j facing extremely risk level. By combing CoVaR and VaR, it can better represent the true risk level on the market.

3.4 Systemic Risk Indicator Application

According to Kuan-Heng Chen [3], the dynamic systemic risk indicator (SRI) is the linear combination of the weighted sum of the absolute value of the residual time series, and the formula is given by

$$SRI_t = \sum_{i=1}^{10} w_{i,t} \times |X_{i,t} - \widehat{X}_{i,t}| \tag{3.6}$$

where $w_{i,t}$ is the Sector Weightings Table mentioned before, $X_{i,t}$ is the sector i return at time t, $\widehat{X}_{i,t}$ is the reconstructed signals. The Systemic Risk Indicator (SRI) is constructed based on observing the residual term of ten variables including ten S&P 500 sector indices. And by using independent component analysis (ICA), we are transforming multidimensional observed signals into independent components for computational reduction, regardless of the potential dependence structure that may exist.

In other words, using ICA omit the dependency structure up to higher-order statistics. The market movement, SRI and VIX is shown in figure 3.3 after calculation. The true effect of SRI catching market movement would be discussed in the next paragraph.

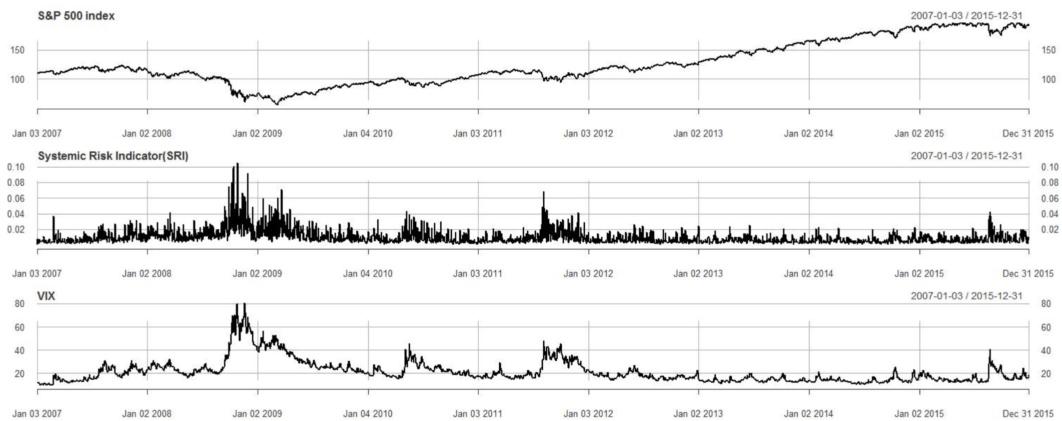


Figure 3. 3S&P 500 index, SRI and VIX

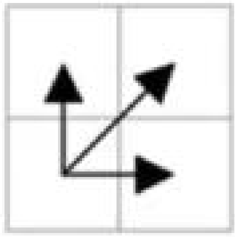
3.5 Dynamic Time Warping Application

Now we have two pairs of time series remains to compare and check similarity in figure 3.1 and figure 3.3. A very natural approach is to introduce the Dynamic Time Warping (DTW). DTW is a time series alignment algorithm developed originally for speech recognition. DTW algorithm is based on the idea of dynamic programming, which can measure the similarity of two lengths inconsistent time series, proposed by Japanese scholar Itakura. In our case, we could use it as a technical tool to comparing the similarity of a pair of time series. While the two time-series length is the same, the algorithms will still works.

The definition of DTW algorithms is given in this paragraph. Assuming we have two time series sequences $X = \{x_1,x_2,\cdots,x_n\}$ and $Y = \{y_1,y_2,\cdots,y_m\}$, whose length is n and m seperately, then the process of DTW computing the similarity of X and Y is shown:

Step 1: Construct a $n \times m$ matrix D, and the elements $d_{ij}$of the matrix, $d_{ij} = dist(x_i,y_j)$, where *dist* is the distance computation function, usually using Euclidean distance.

Step 2: Search matrix D for the shortest path from d_{11} to d_{nm} (usually using dynamic programming search). At the position of d_{ij}, the path search direction is shown in the below figure:



Step 3: Select the shortest path in distance from d_{11} to d_{nm} in the matrix as the similarity between time series sequence X and Y. As the algorithm formula is shown:

$$d_{ij}=dist(x_i,y_j)\\ D(i,j)=d_{i,j}+min\{D(i-1,j),D(i,j-1),D(i-1,j-1)\} \quad (3.7)$$

By applying Dynamic Time Warping algorithms into our data, we could understand the shortest distance path between two time-series. In other words, it could be possible to understand the similarity between a pair of time-series sequence as shown below.

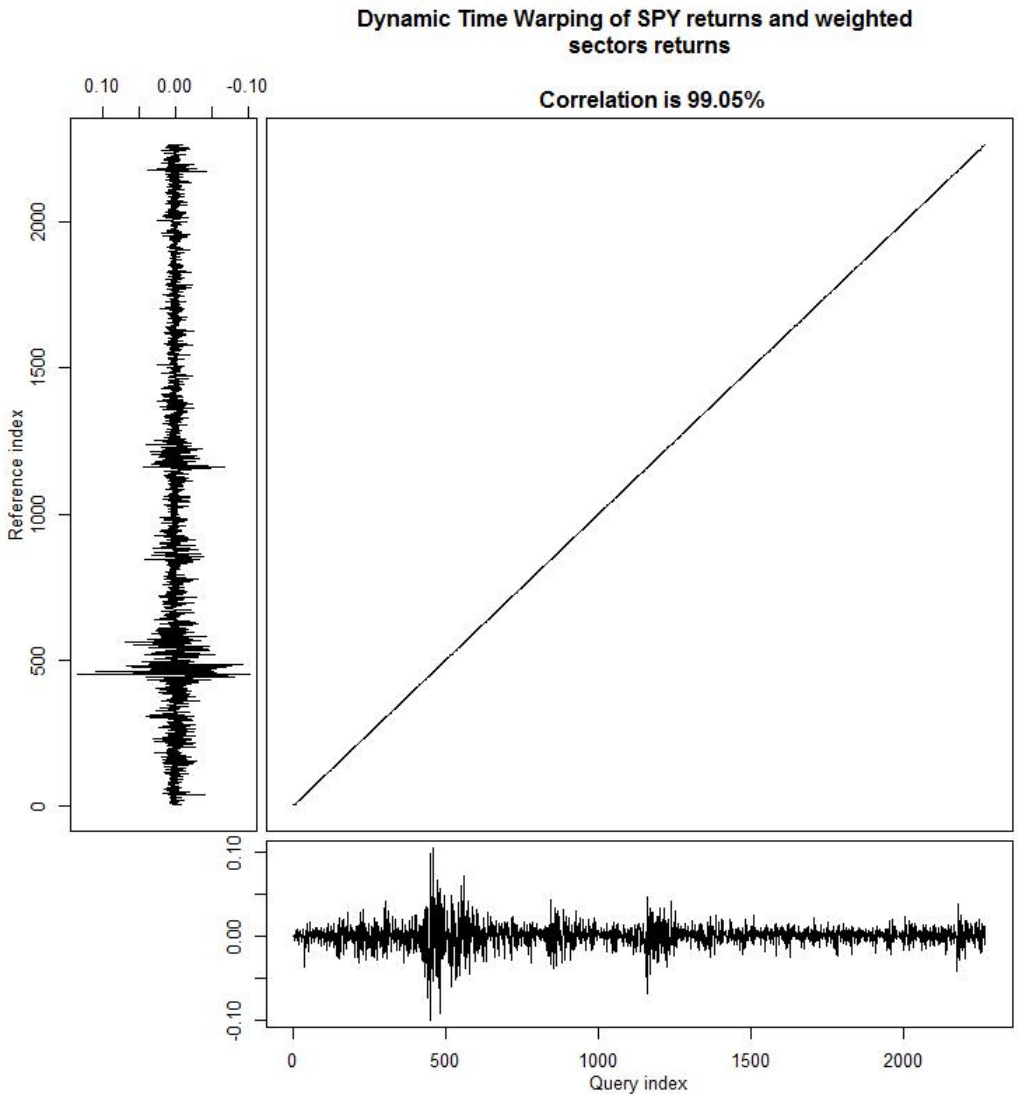


Figure 3.4 Dynamic Time Warping between SPY returns and weighted sectors sum returns

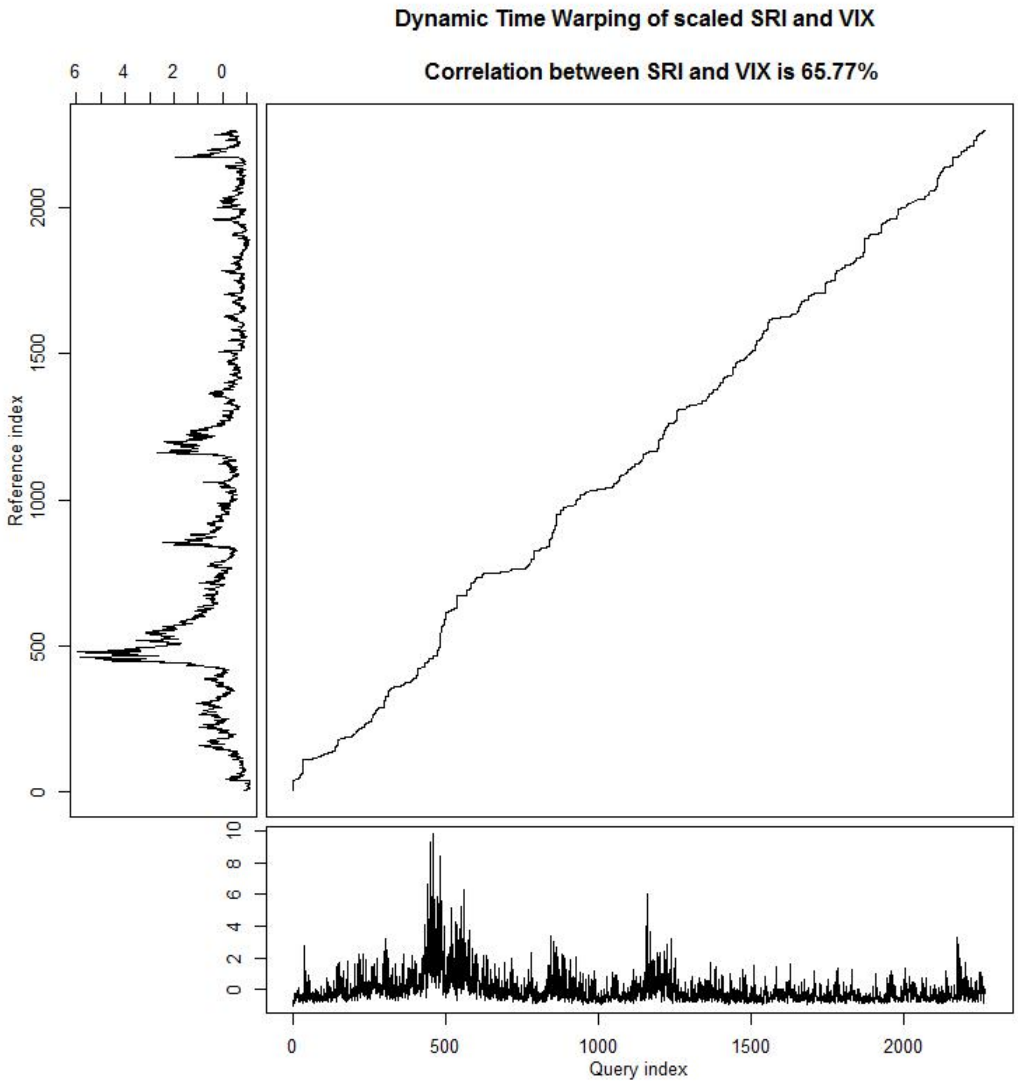


Figure 3.5 Dynamic Time Warping between scaled SRI and scaled VIX

In conclusion, by observing figure 3.4, the correlation between SPY returns and weighted sectors sum returns is up to 99 percent, and the shortest distance path is straight forward, hence it can be deduced that our selected sectors from S&P 500 almost perfectly matched the market movement of SPY. While in figure 3.5, the correlation between SRI using independent component analysis (ICA) and VIX is only 65.77 percent, and the shortest distance path is fluctuating up and right. Such an empirical result proves that the quick computation algorithms using ICA may not catch the market volatility very well since ICA omits the second order term movement and dependence structure between different industry sectors. While VIX is calculated by observing the market volatility through options and if we assume VIX does show the market risk level. And it could be better if we construct GARCH-Copula type model to fully represent different time series sequence's variance autocorrelation and dependence structure.

Chapter 4 Methodology

4.1 Overview

In this chapter, we presented vine Copula-based ARMA-GJRGARCH (1, 1) model to analyze each U.S. Equity sector's risk contribution which is referred as the ratio of the Value-at-Risk of a sector to the Value-at-Risk of the system. In order to investigate systemic risk in 10 S&P 500 sector indices in the U.S. stock market, We forecast one-day ahead VaR and one-day ahead VaR ratio from 2008 to 2009. Our results show that vine Copula-based ARMA-GJRGARCH(1, 1) is the appropriate model to forecast and analyze systemic risk.

4.2 Systemic Risk Measure

What we learned about risk management before is Value-at-Risk (VaR), which is the most widely used to measure and control the level of risk exposure. The definition of Value-at-Risk (VaR) is the biggest loss by which a position or portfolio could lose at a given probability $(1-\alpha)$ over a specific time frame [25] (Longerstaey and Spencer 1996). The probability is usually determined to be (1-95%), (1-99%), (1-99.9%). We calculate the VaR value of each sector through Copula-based ARMA-GJRGARCH (1, 1) model. Each sector i's risk contribution to the system j (S&P 500 index) at the confidence level α is denoted as $VaR_{1-\alpha}^{i\to j}$ ratio. The formula is expressed as the following.

$$VaR_{1-\alpha}^{i\to j} ratio = \frac{VaR_{1-\alpha}^{i}}{VaR_{1-\alpha}^{j}} \tag{4.1}$$

The sector with the higher ratio is the risk provider of the system.

4.3 ARMA-GARCH Model

ARMA stands for Autoregressive-moving-average. The AR part involves regressing the variable on its past lagged values to estimate future predictions. This model explains the current value of a return series which can be expressed as a function of its previous values. Different from AR model, the moving average (MA) model is a model that regresses on its previous values contains the lagged error terms and current error term. Since the MA model is a linear combination of a white noise sequence, the model is always weakly stationary. The ARMA model was introduced by Box, Jenkins and Reinsel [40](Box, George EP, Gwilym M. Jenkins, Gregory C. Reinsel

2015) which combines the AR(p) with MA(q), to overcome the difficulty of a great number of required parameters when modeling with the AR(p) and MA(q) models separately.

The autoregressive conditional heteroskedastic model (ARCH) was first introduced by Engle, which deals with modeling conditional volatility. In this model, conditional variance is allowed to be described by a quadratic function of its lagged error terms. Then Bollerslev [41] (Bollerslev, Tim 1986) presented an extension of the ARCH model, namely, the Generalized ARCH model. It is widely used in financial institutions for estimating conditional volatility as well as being a guidance in financial decisions makings involving risk analysis and portfolio selection. Chen and Khashanah[4] (K.-H. Chen and Khashanah 2015) implemented ARMA (p, q) -EGARCH (1, 1) with the Student's t distributed innovations for the marginal to account for the time-varying volatility. EGARCH model can effectively deal with asymmetric effects between positive and negative asset returns proposed by Nelson (Society 2017).

To explain the skewness, we replaced Student's t distributed innovations with skewed Student's t distributed innovation. ARMA(p, q)-EGARCH (1, 1) with the skewed Student's t distributed innovation can be written as[33] (Ghalanos 2014)

$$\begin{aligned} r_t &= \mu_t + \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \\ \ln(\sigma_t^2) &= \kappa_t + \alpha_t z_{t-1} + \zeta_t(|z_{t-1}| - E\{[z_{t-1}]\}) + \beta_t \ln(\sigma_{t-1}^2) \end{aligned} \tag{4.2}$$

In the equation, r_t is the log return, μ_t is the drift term, ε_t is the error term, z_t is the skewed Student's t distribution. The skewed student's t density function [42](Ferreira, José TA S., and Mark F. J. Steel 2017) can be written as

$$p(z|\zeta, f) = \frac{2}{\xi + \frac{1}{xi}} \left\{ f\left(\frac{z}{\xi}\right) I_{[0,\infty)}(z) + f(\xi z) I_{[0,\infty)}(z) \right\} \tag{4.3}$$

Here, ξ is the asymmetric parameter, f is a univariate pdf that is symmetric around 0, I_S is the indicator function on S.

In this paper, we tried another asymmetric model, GJRGARCH (GJR model) which was developed by Glosten, Jagannathan and Runkle [43](Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle 1993). It is based on the assumption that bad news has a higher impact than good news. This model is also designed to capture the leverage effect between asset return and volatility. Leverage coefficients of the GJR model can connect to the model through an indicator variable. In this case the leverage coefficients should be negative for the EGARCH model and positive for the GJR model when the asymmetric effect occurs. ARMA(p, q)-GJRGARCH (1, 1) model can be expressed as

$$
\begin{aligned}
r_t &= \mu_t + \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \\
\sigma_t^2 &= \omega + \alpha_t \sigma_{t-1}^2 + \gamma_t I_{t-1} \varepsilon_{t-1}^2 + \beta_t \varepsilon_{t-1}^2, \varepsilon_t = z_t \sigma_t
\end{aligned}
\tag{4.4}
$$

$$
\begin{cases} I_{t-j} = 1, & \varepsilon_{t-j} < 0 \\ I_{t-j} = 0, otherwise \end{cases}
$$

4.4 Sklar’s theory

Sklar's theorem states that any multivariate joint distribution can be written in terms of univariate marginal-distribution functions and a copula which describes the dependence structure between the variables. Let $H = F(x_1, x_2, ..., x_d)$ be a multivariate joint distribution function with margins $F_1, F_2, ..., F_d$, there exists a unique copula C such that for all $x = (x_1, x_2, ..., x_n) \in R^n$

$$
F(x_1, ..., x_d) = C\{F_1(x_1), ..., F_d(x_d)\} = C(u_1, ..., u_d) \tag{4.5}
$$

In this equation, $X_1, X_2, ..., X_d$ are random variables with continuous distribution functions $F_1, F_2, ..., F_d$.

Sklar’s theorem is valuable because it connects each multivariate distribution with a copula and allows us to model dependence structure independently. Also, we could derive the joint density from Sklar’s Theorem. The joint density $f_{12}(x_1, x_2)$ is

$$
\begin{aligned}
f_{12}(x_1, x_2) &= \frac{\partial^2 F_{12}(x_1, x_2)}{\partial x_1 \partial x_2} \\
&= \frac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2} \frac{\partial F_1(x_1)}{\partial x_1} \frac{\partial F_2(x_2)}{\partial x_2} \\
&= c(F_1(x_1), F_2(x_2)) f_1(x_1) f_2(x_2)
\end{aligned}
\tag{4.6}
$$

$c(u_1, u_2) = \dfrac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2}$ is the copula density. For independent copula, $c(u_1, u_2) = 1$, and a important property of copula is invariance under increasing and continuous transformation. Patten [44]defined the conditional version of Sklar’s theorem. Let $F_{1,t}$ and $F_{2,t}$ be the continuous conditional distributions of $X_1 | F_{t-1}$ and $X_2 | F_{t-1}$, given the conditioning set F_{t-1}, let H_t be the joint conditional bivariate distribution of $(X_1, X_2 | F_{t-1})$. Then, there exists a unique conditional copula C_t such that

$$
H_t(x_1, x_2 | F_{t-1}) = C_t(F_{1,t}(x_1 | F_{t-1}), F_{2,t}(x_2 | F_{t-1}) | F_{t-1}) \tag{4.7}
$$

4.5 Copula Class

4.5.1 Elliptical Copula

(1) d-dimension Gaussian Copula with $u = (u_1,...,u_d)^T \in [0,1]^d$ can be described as

$$C_{Ga}(u_1,...,u_d;\sum\ ) = \Phi_{d,\Sigma}\ (\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)) \qquad (4.8)$$

Where $\sum$ is $d \times d$ correlation matrix, $\Phi()$ is standard normal distribution function. $\Phi^{-1}()$ is the inverse function of $\Phi()$. Gaussian copula has no tail dependence unless linear correlation coefficient $\rho = 1$.

(2) d-dimension Student-t Copula with $u = (u_1,...,u_d)^T \in [0,1]^d$ can be described as

$$C_t(u_1,...,u_d;\nu,\sum\ ) = t_{\nu,d,\Sigma}\ (t_\nu^{-1}(u_1),...,t_\nu^{-1}(u_d)) \qquad (4.9)$$

Where $t_\nu()$ is the student-t distribution function with degree of freedom ν, and σ is the correlation matrix, $t_\nu^{-1}()$ is the inverse function of $t_\nu()$. Student-t copula has tail dependence for all $\nu > 0$.

4.5.2 Archimedean Copula

d-dimensional Archimedean copula can be described as

$$C(u_1,...,u_n;\theta) = \phi\{\phi^{-1}(u_1) + ... + \phi^{-1}(u_d)\}, u_1,...u_d \in [0,1] \qquad (4.10)$$

where $\phi : [0,1] \to [0,+\infty]$ is the function which satisfies that $\phi(1) = 0, \phi(+\infty) = 1$, and ϕ is a decreasing function.

The following table gives a summary of some Archimedean copula, including the generator, domain of dependence parameter, range of attainable tau and tail dependence.

Copula	Generator	θ	τ range	Tail Dependence
Gumbel	$(-\log(t))^{\theta}$	$[1,+\infty)$	[0,1)	weak λ_l and strong λ_u
Clayton	$1/\theta(t^{-\theta}-1)$	$\theta \in [-1,+\infty)\setminus\{0\}$	[-1,1)\{0}	weak λ_u and strong λ_l
Joe	$-\log(1-(1-t)^{\theta})$	$\theta \in [-1,+\infty)$	[0,1)	weak λ_l and strong λ_u
Ali-Mikhail-Haq	$\log\left(\frac{1-\theta(1-t)}{t}\right)$	$\theta \in [-1,1]$	(0,1/3)	exhibit λ_l only when $\theta = 1$
Frank	$-\log\left(\frac{\exp(-\theta t-1)}{\exp(-\theta)-1}\right)$	$\theta \in R\setminus\{0\}$	(-1,1)	symmetric,weak λ_l and λ_u

Table 4.1 summary of some Archimedean copula

4.6 Vine Copulas

Even though the systemic risk could be computed by CoVaR, it cannot describe the interconnectedness among various sectors as well as the whole system. Our project tried to utilize R-vine copula to fully reflect the non-linear connecting structure, and derive a risk measurement to calculate CoVaR.

The first important factor is to construct proper marginal distributions to better describe characteristics of distributions of returns of ETFs. Then the vine copulas method helps to create a joint distribution arranging bivariate copulas together. n(n-1)/2 copulas could be constructed for n dimensional models and n-1 copulas are unconditional.

A regular vine is used to reflect the complex interdependency structure of portfolios, including a series of trees and each edge concludes a corresponding pair copula. A d-dimensional R-vine is built by d-1 trees. The function of vine copula could be written as

$$f(x_1,\cdots,x_n) = \Pi_{k-1}^{n} f(x_k) \times \Pi_{j=1}^{n-1}\Pi_{i=1}^{n-j} c_{j,j+1|1,\ldots,j-1} \qquad (4.11)$$

But at first, we should know the r-vine structure before we try to get the joint density function. In order to efficiently create a r-vine, various algorithms could be applied to get the optimal R-vine structure. For example, we could choose the optimal r-vine structure by maximizing the sum of the absolute value of Kendall-τ.

After determining the r-vine structure, each edge also needs an optimal pair copula to describe its dependency structure. In this time, AIC and BIC criteria can be used to find the best pair copula among bivariate copulas to show the connecting relationship between assets.

Chapter 5 Empirical Results

5.1 Marginal Distribution Estimation

By changing m, n, q, p in ARMA(m, n)-GARCH(q, p) model, we selected models according to AIC criteria and made sure there is no ARCH effect as well as auto-correlation in residuals.

Table 5.1 shows estimation results of ARMA(1,1)-EGARCH(1,1) model for the time period of 2008-2009.

	XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	IYZ	SPY
MU	-0.00101	-0.00046	-0.00123	-0.00303	-0.00104	-0.00163	-0.00111	-0.00089	-0.00074	-0.00101	-0.00119
AR1	0.498906	0.357572	0.215467	0.657196	0.268614	0.284799	0.582173	0.267347	0.555567	0.498906	0.446063
MA1	-0.59099	-0.53814	-0.36843	-0.78463	-0.40727	-0.42176	-0.66213	-0.41954	-0.62179	-0.59099	-0.58166
OMEGA	-0.13555	-0.25223	-0.19462	-0.09248	-0.43262	-0.09094	-0.15839	-0.15321	-0.2505	-0.13555	-0.17643
ALPHA1	-0.09925	-0.11054	-0.13611	-0.12408	-0.16652	-0.10626	-0.09815	-0.09515	-0.13363	-0.09925	-0.1387
BETA1	0.981851	0.971075	0.972697	0.985464	0.948977	0.988105	0.978529	0.980588	0.970523	0.981851	0.977794
GAMA1	0.106642	0.131765	0.095371	0.206146	0.174414	0.084513	0.135466	0.138162	0.189847	0.106642	0.141993
SKEW	1.045862	0.825422	0.792329	1.048364	0.895444	0.866102	0.88136	0.946577	0.879085	1.045862	0.876072
SHAPE	53.53903	59.69026	25.49902	12.4258	9.709441	25.05922	16.1263	10.85758	21.08718	53.53903	11.24047

TABLE 5.1 ESTIMATIONS OF PARAMETERS FROM 2008-1-1 TO 2009-5-1

Figure 5.1 shows results of ARCH test for six sectors: XLY(Consumer Discretionary), XLP(Consumer Staples), XLE(Energy), XLF(Financials), XLV(Healthcare), XLI(Industrials). It gives the p-value with different lags for various sectors. With p-value greater than 0.05, it is proved that residuals are with no ARCH effect after being fitted by the GARCH model we have chosen.

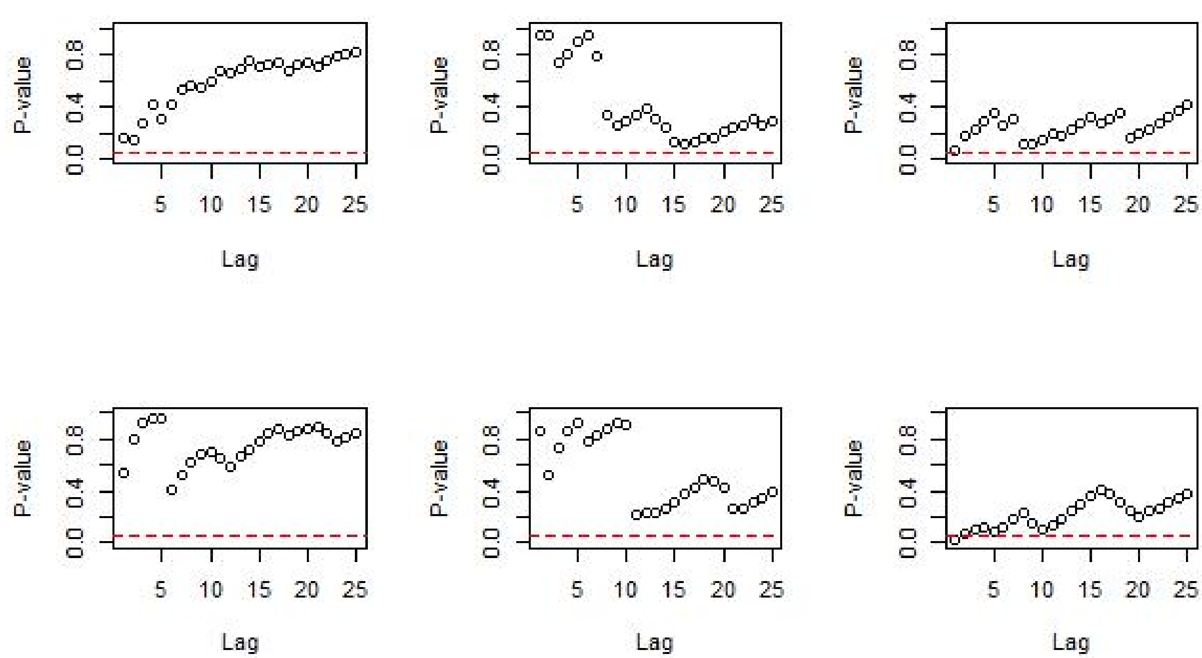


Figure 5.1 Results of ARCH test for residuals in 2008-2009

Table 5.2 shows estimations results for ARMA(1,1)-GJRGARCH(1,1) model for the time period of 2008-2015. It gives results of ARCH test and J-B test that after being fitted by GJRGARCH model, the original standardized residuals of eleven tickers are transformed into numbers with no Autoregressive Conditional Heteroscedastic (ARCH) effect.

	XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	IYZ	SPY
MU	0.00048	0.00042	0.00028	0.00031	0.00051	0.00032	0.00018	0.00048	0.00037	0.00018	0.0004
AR1	0.67421	0.61564	-0.50339	0.57847	0.56443	0.60213	0.71414	0.59233	0.73866	0.73349	0.59798
MA1	-0.72344	-0.68839	0.45211	-0.65215	-0.62272	-0.64036	-0.75645	-0.6419	-0.77753	-0.74976	-0.66827
OMEGA	0.00000226	0.00000188	0.00000298	0.00000281	0.00000395	0.00000211	0.00000159	0.00000318	0.00000209	2.76E-06	2.51E-06
ALPHA1	0.01623	0.00688	0.0166	0.04292	0	0	0.00384	0	0.03744	0.00145	0
BETA1	0.89851	0.88362	0.9151	0.88377	0.8693	0.90886	0.92843	0.88614	0.90535	0.92335	0.87201
GAMMA1	0.14397	0.16422	0.11087	0.13622	0.17551	0.1569	0.12096	0.18234	0.07328	0.1098	0.22121
SKEW	0.83854	0.86231	0.85673	0.90147	0.85758	0.83781	0.81058	0.84012	0.87569	0.86452	0.81998
DF	9.41401	10.35482	13.88961	6.72865	8.93365	11.42184	10.10042	7.44099	9.86119	9.25129	6.5213
ARCH	0	0	0	0	0	0	0	0	0	0	0
J-B TEST	1	1	1	1	1	1	1	1	1	1	1

TABLE 5.2 ESTIMATIONS OF PARAMETERS FROM 2008-1-1 TO 2015-1-1

For residuals during 2008-2015, since results of ARCH test is not ideal, we changed the model to ARMA(1,1)-GJRGARCH(1,1). And Figure 5.2 shows results of ARCH test for ARMA-GJRGARCH model for residuals during 2008-2015.

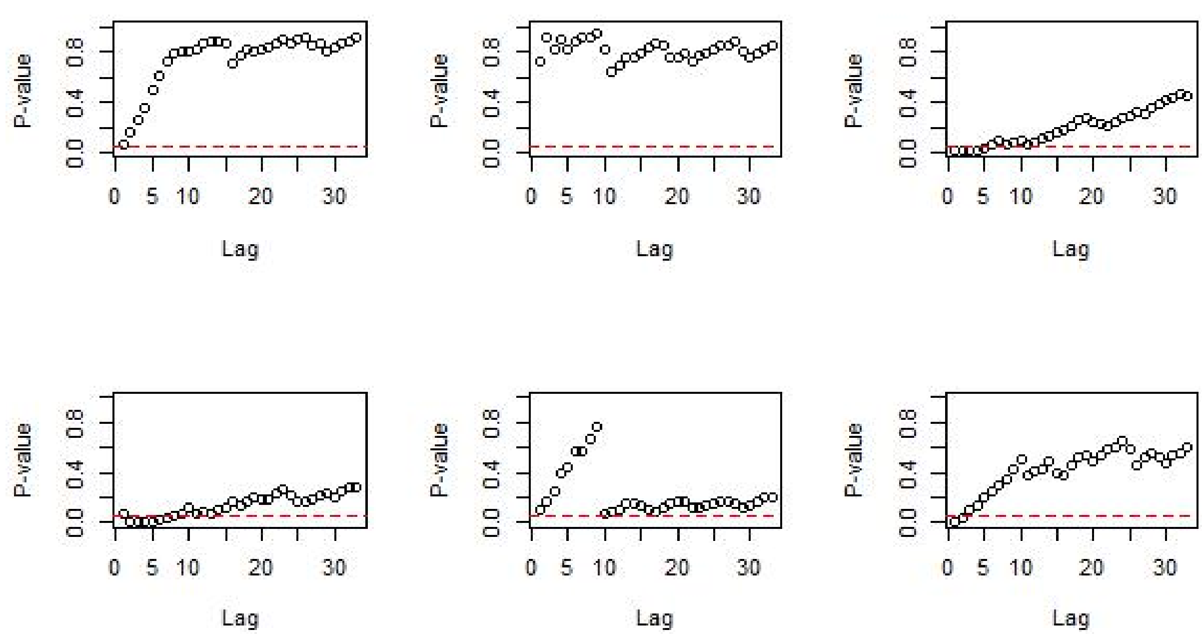


Figure 5.2 Results of ARCH test for residuals in 2008-2015

Figure 5.3 below presents the correlation of residuals of 11 sectors and SPY.

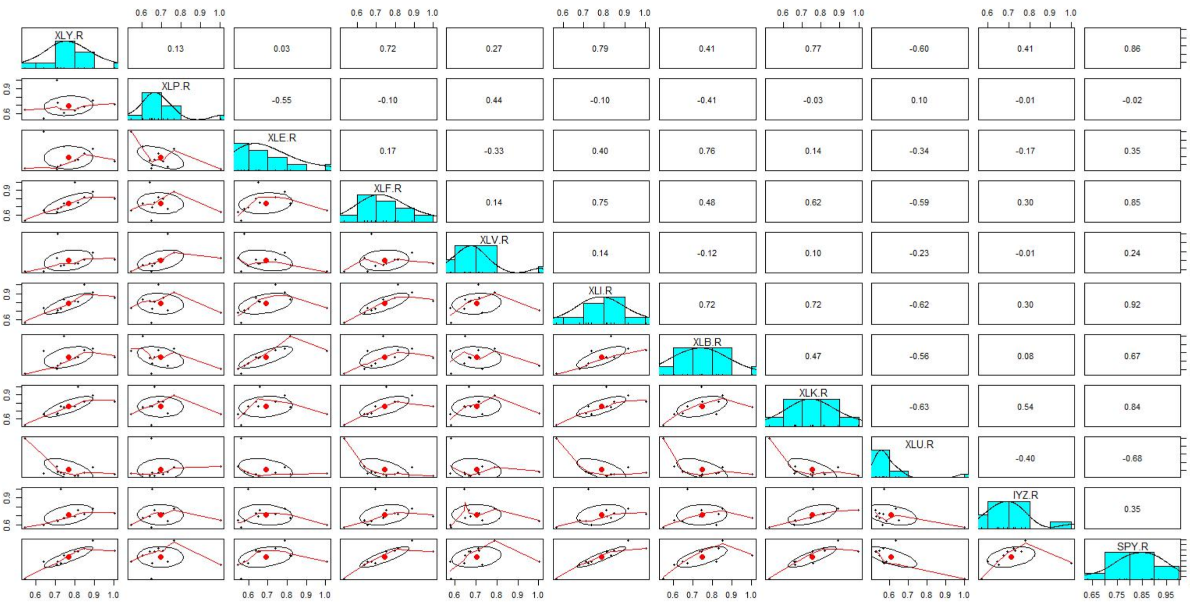


Figure 5.3 Correlation of Residuals from 2008 to 2009

5.2 Results of Copula Models

After choosing the marginal distribution, we select an optimal R-vine structure to simulate the interdependency of sectors in a financial system.

Figure 5.4 R-vine structure for residuals of 11 sectors and SPY from 2008 to 2009;
1: XLY, 2: XLP, 3: XLE, 4: XLF, 5: XLV, 6: XLI, 7: XLB, 8: XLK, 9: XLU, 10: IYZ, 11: SPY

A R-vine structure could describe the interconnectedness among sectors accurately. From figure 5.4, it can be indicated that SPY is in the center of the tree structure; XLE and XLB are connected to each other. So do XLY and IYZ.

After the R-vine structure has been decided, we could choose an optimal pair copula for each edge in the tree structure according to AIC and BIC criteria. Table 5.3 presents results of chosen pair copulas and corresponding parameters.

TREE	EDGE	FAMILY	COP	PAR	PAR2	TAU	UTD	LTD
1	11,5	17	SBB1	0.21	1.92	0.53	0.19	0.57
	11,2	2	t	0.78	8	0.57	0.32	0.32
	11,6	2	t	0.91	12.06	0.72	0.44	0.44
	7,3	7	BB1	0.65	1.82	0.58	0.54	0.55
	1,10	14	SG	65.1	0	0.98	-	0.99
	11,1	2	t	0.9	10.54	0.71	0.45	0.45
	11,7	7	BB1	0.74	1.74	0.58	0.51	0.58
	11,8	2	t	0.91	6.53	0.73	0.57	0.57
	11,4	17	SBB1	0.33	2.75	0.69	0.47	0.71
	11,9	7	BB1	0.69	1.41	0.47	0.37	0.49
2	2,5;11	1	N	0.31	0	0.2	-	-
	7,2;11	2	t	-0.2	13.04	-0.13	0	0
	7,6;11	14	SG	1.15	0	0.13	-	0.17
	11,3;7	5	F	1.4	0	0.15	-	-
	11,10;1	5	F	-1.09	0	-0.12	-	-
	4,1;11	19	SBB7	1.08	0.25	0.15	0.06	0.1
	4,7;11	1	N	-0.33	0	-0.21	-	-
	4,8;11	34	G270	-1.15	0	-0.13	-	-
	9,4;11	39	BB7_270	-1.14	-0.24	-0.17	-	-
3	7,5;2,11	5	F	-0.44	0	-0.05	-	-
	4,2;7,11	5	F	-1.14	0	-0.12	-	-
	3,6;7,11	24	G90	-1.07	0	-0.07	-	-
	4,3;11,7	24	G90	-1.22	0	-0.18	-	-
	4,10;11,1	1	N	0.15	0	0.09	-	-
	8,1;4,11	1	N	0.24	0	0.15	-	-
	8,7;4,11	2	t	-0.1	5.97	-0.06	0.02	0.02
	9,8;4,11	34	G270	-1.09	0	-0.08	-	-
4	4,5;7,2,11	23	C90	-0.1	0	-0.05	-	-
	3,2;4,7,11	5	F	-1.25	0	-0.14	-	-
	4,6;3,7,11	33	C270	-0.06	0	-0.03	-	-
	8,3;4,11,7	39	BB7_270	-1.06	-0.18	-0.11	-	-
	8,10;4,11,1	3	C	0.03	0	0.02	-	0
	9,1;8,4,11	33	C270	-0.09	0	-0.04	-	-
	9,7;8,4,11	5	F	-0.33	0	-0.04	-	-

5	3,5;4,7,2,11	5	F	-0.87	0	-0.1	-	-
	6,2;3,4,7,11	5	F	0.39	0	0.04	-	-
	8,6;4,3,7,11	13	SC	0.12	0	0.06	0	-
	9,3;8,4,11,7	1	N	0.2	0	0.13	-	-
	9,10;8,4,11,1	5	F	-0.61	0	-0.07	-	-
	7,1;9,8,4,11	33	C270	-0.1	0	-0.05	-	-
6	6,5;3,4,7,2,11	33	C270	-0.09	0	-0.04	-	-
	8,2;6,3,4,7,11	23	C90	-0.06	0	-0.03	-	-
	9,6;8,4,3,7,11	5	F	-0.86	0	-0.1	-	-
	1,3;9,8,4,11,7	1	N	-0.28	0	-0.18	-	-
	7,10;9,8,4,11,1	5	F	-0.67	0	-0.07	-	-
7	8,5;6,3,4,7,2,11	29	BB7_90	-1.07	-0.19	-0.12	-	-
	9,2;8,6,3,4,7,11	4	G	1.14	0	0.12	0.16	-
	1,6;9,8,4,3,7,11	2	t	0.17	12.2	0.11	0.01	0.01
	10,3;1,9,8,4,11,7	1	N	0.07	0	0.04	-	-
8	9,5;8,6,3,4,7,2,11	4	G	1.1	0	0.09	0.13	-
	1,2;9,8,6,3,4,7,11	14	SG	1.09	0	0.08	-	0.11
	10,6;1,9,8,4,3,7,11	5	F	0.38	0	0.04	-	-
9	1,5;9,8,6,3,4,7,2,11	3	C	0.1	0	0.05	-	0
	10,2;1,9,8,6,3,4,7,11	1	N	0.07	0	0.05	-	-
10	10,5;1,9,8,6,3,4,7,2,11	6	J	1.03	0	0.01	0.03	-

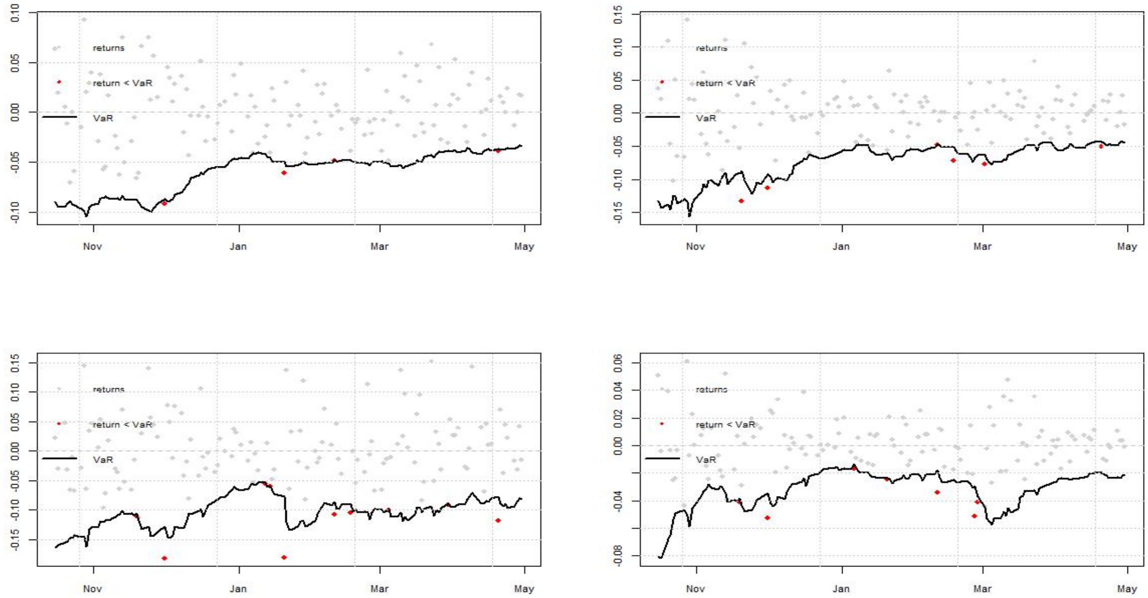
TABLE5.3 PAIR COPULAS AND PARAMETERS OF R-VINE TREE STRUCTURE

From column 'Cop', it is given that different sectors are connected by different types of copulas, like student-t copula. When tau is greater than 0, the connection between two sectors are positive and the spillover risk is more likely to be higher; when tau is less than 1, the connection of two sectors are negative, and the spillover risk is likely to be small.

5.3 Estimation of Systemic Risk

With given marginal distributions and the interdependency structure of 11 ETF sectors and SPY, the systemic risk could be calculated by CoVaR, ΔCoVaR and %CoVaR.

Figure 5.5 shows prediction results of the one-day ahead worst 5% loss for following sectors: XLY, XLE, XLF, XLV, XLI, XLB, XLK, SPY. It indicated that the systemic risk at the beginning of the chosen time period is relatively high because of the financial analyst happening in 2008. What's more, the volatility of financials sector is even larger than other sectors.



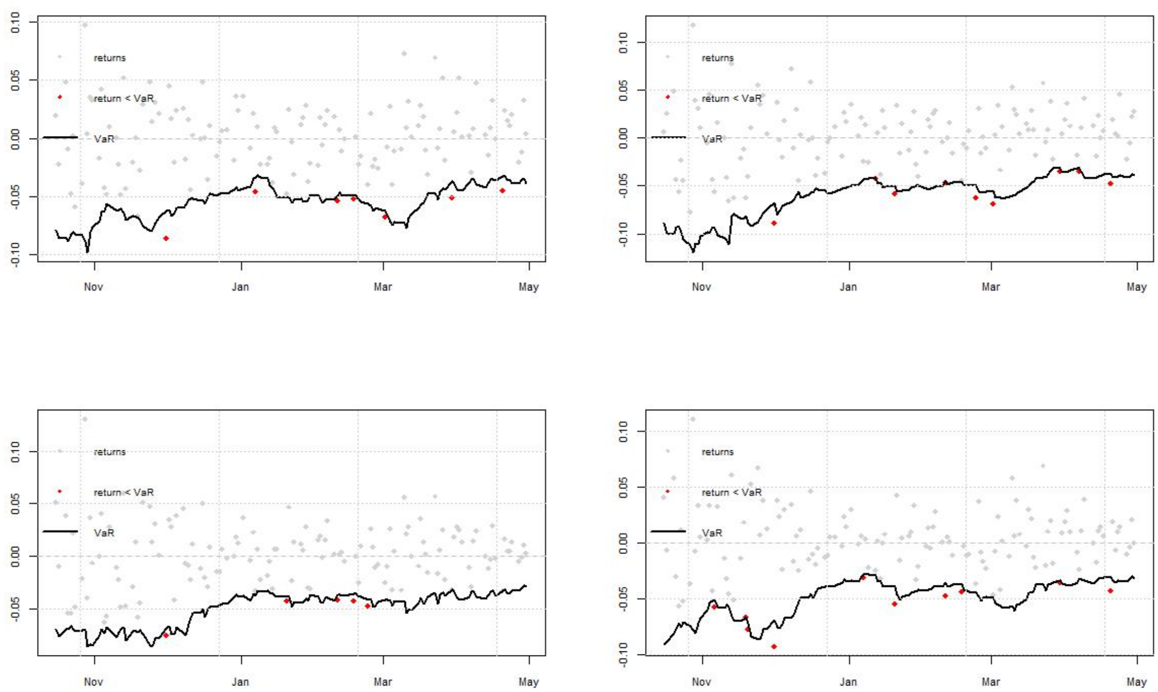


Figure 5.5 10Day rolling forecast CoVaR of ETF and SPY from 2008 to 2009

Figure 5.6 shows the result of rolling CoVaR for various sectors and SPY during 2008 and 2009. It is clear to see that financials sector during the chosen time period occupies a larger proportion of the whole financial systemic risk (CoVaR[SPY]).

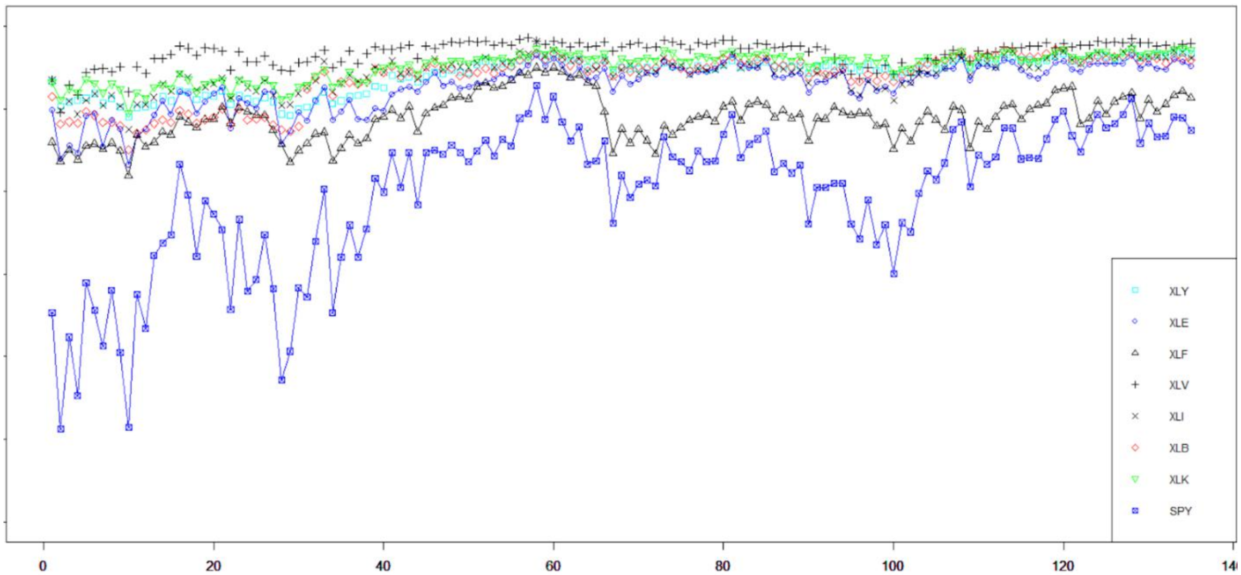
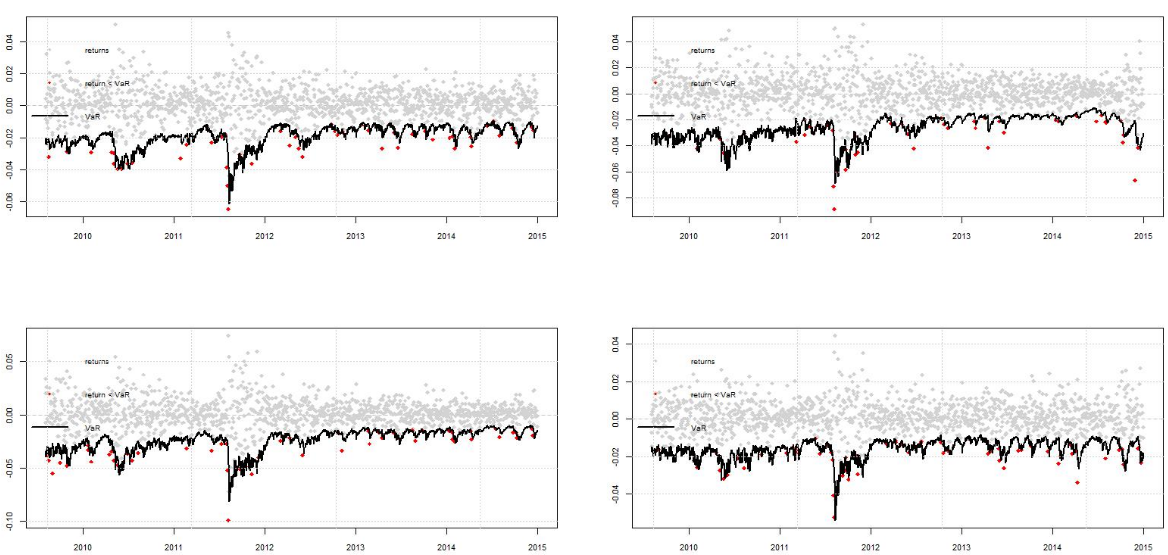


Figure 5.6 10Day rolling forecast of CoVaR for ETF sectors and SPY for 2008-2009

Figure 5.7 presents 30 day rolling CoVaR while applying the R-vine arma-garch model from 2008 to 2015.



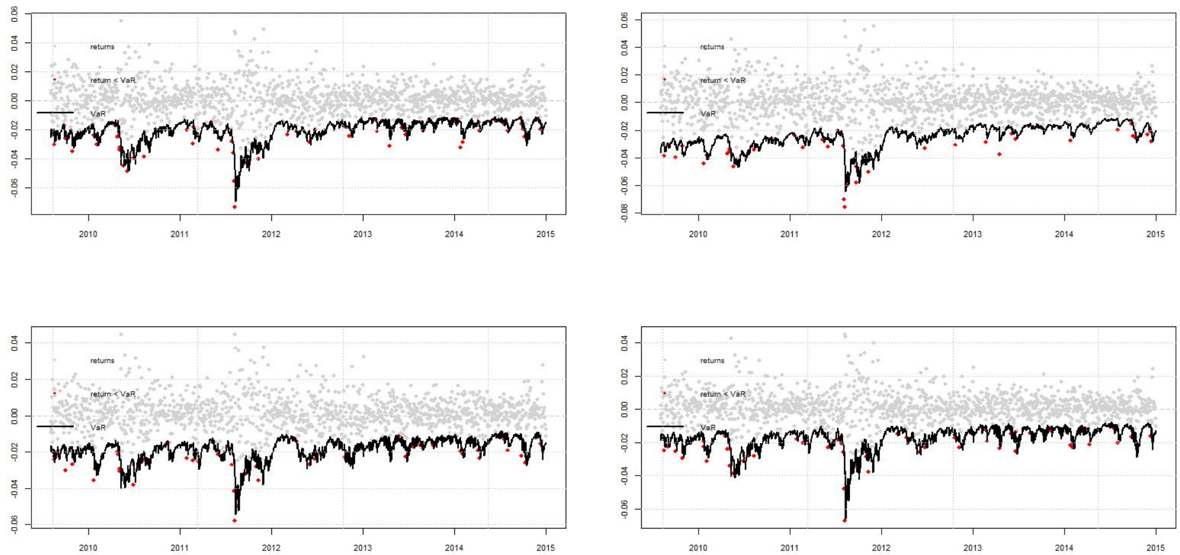


Figure 5.7 30Day rolling forecast of CoVaR for ETF sectors and SPY from 2008 to 2015

And we also draw rolling CoVaR into one single chart:

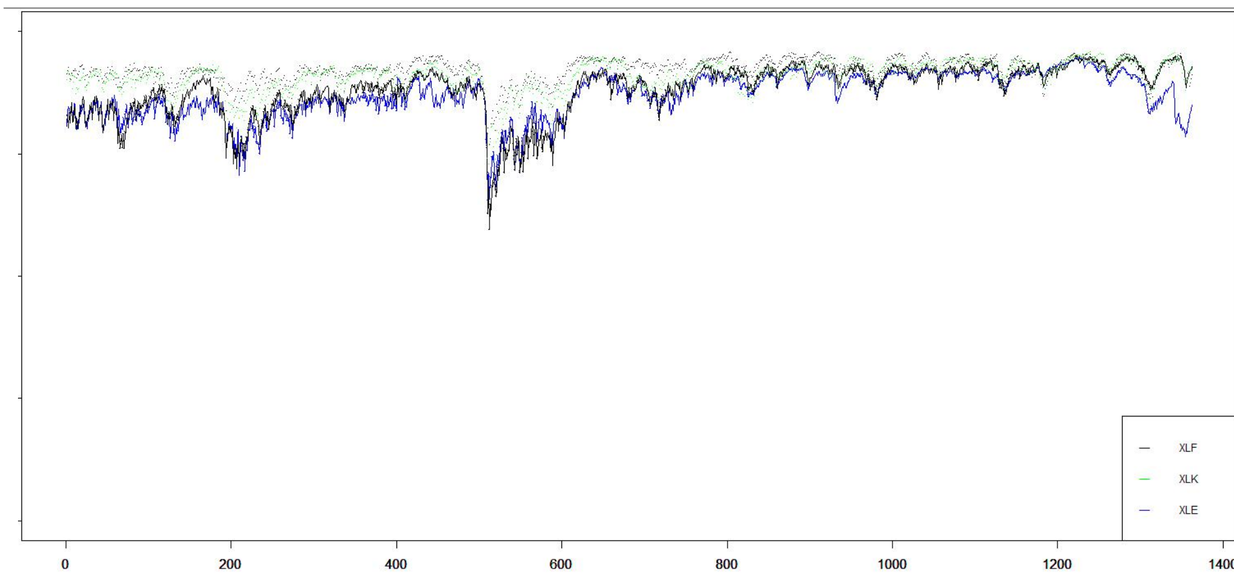


Figure 5.8 30Day rolling forecast of COVAR for ETF sectors from 2008 to 2015

From the figure above, it is shown that in 2011, the systemic risk is relatively high and the rolling CoVaR is much more volatile. Considering the timeline of the financial crisis, during 2011, Portugal requested activation of aid mechanism; at the end of 2011, Span and Cyprus requested financial support, which partially explain the downward peak of rolling CoVaR in 2011.

During such period, both financials and energy sectors have higher CoVaR, indicating that they could be more sensitive to the systemic risk.

Chapter 6 Conclusion

One of basic problems of a financial system is how to manage and predict the systemic risk, especially under a financial crisis. Our project tried to create and apply a R-vine ARMA-GARCH model for risk calculation, combining Conditional Value-at-Risk (CoVaR) and copula functions together.

A dynamic forecasting method is used for calculating the systemic risk, eliminating the potential biases from marginal or joint distributions. We focused our project on historical returns of 11 ETF sectors and SPY, fitting them to different GARCH models to find the best one for specific time periods as well as combining copula functions together. Besides, in order to make results to be more reasonable, we computed covar based on dynamic copulas and used %CoVaR to better present the marginal distribution of systemic risks and to see how the systemic risk changed over time.

Results show that ETF sectors we have chosen are highly connected to some extent. Financials, Energy, Technology sectors are more sensitive to the volatility of the whole financial system and the systemic risk. R-vine arma-garch model presents that sectors all show high co-movement before the financial crisis. For time periods with less risk, the value of ΔCoVaR and CoVaR is much more stable.

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Group Member Contribution

Xinhang Wang: Construct R-vine tree structure, and simulate new residuals for all sectors. Utilize ARMA-GARCH vine copula model to compute dynamic copula CoVaR; build data visualizations for rolling VaR for various time periods.

Qi Yuan: Fit log returns to ARMA-EGARCH model and ARMA-GJRGARCH model and build multivariate probability distribution by R-Vine copula

Shihao Zhang: Data processing and representation, the risk measure, dynamic time warping