

K-Means Clustering

- Initialization: Given K categories, N points in feature space. Pick K points randomly; these are initial cluster centers (means) m_1, \dots, m_K . Repeat the following:
 - Assign each of the N points, x_j , to clusters by nearest m_i (make sure no cluster is empty)
 - Re-compute mean m_i of each cluster from its member points
 - If no mean has changed, stop
- Effectively carries out gradient descent to minimize:

$$\sum_{i \in \text{clusters}} \left\{ \sum_{j \in \text{elements of } i^{\text{th}} \text{ cluster}} \|x_j - \mu_i\|^2 \right\}$$

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Slide credit: Christopher Rasmussen

Example: 3-means Clustering

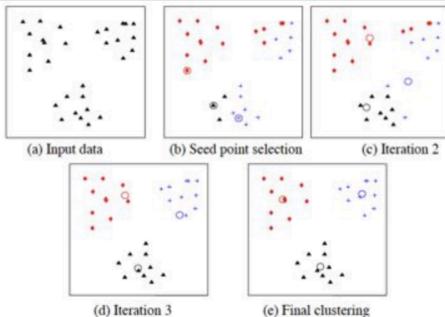


Figure 4 Illustration of K-means algorithm. (a) Two-dimensional input data with three clusters; (b) three seed points selected as cluster centers and initial assignment of the data points to clusters; (c) & (d) intermediate iterations updating cluster labels and their centers; (e) final clustering obtained by K-means algorithm at convergence.

Anil Jain, 2008

Example: 4-means Clustering: What should be the desired clustering?

Selection of Initial Cluster Centers:
... Does it matter?

How to judge quality of
clustering at any stage,
especially at the final
termination stage?

Visualization from Prof. Andrey Shabalin

K-Means

Minimizing squared distances to the center implies that the center is at the mean:

$$e(m_i) = \sum_{i=1}^{n_c} \sum_{j:c_j=i} \|x_j - m_i\|^2$$

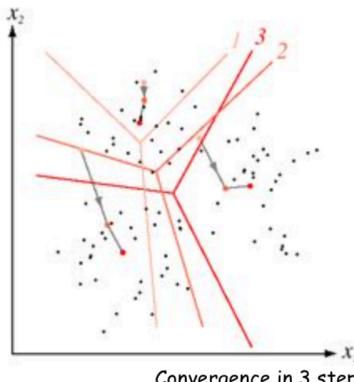
$$\frac{\partial e}{\partial m_k} = \sum_{j:c_j=k} -2(x_j - m_k) = 0$$

Derivative of error is zero at the minimum

$$m_k = \frac{\sum_{j:c_j=k} x_j}{\sum_{j:c_j=k} 1} = \frac{1}{n_k} \sum_{j:c_j=k} x_j$$

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Example: 3-means Clustering



from
Duda et al. 2005

Convergence in 3 steps

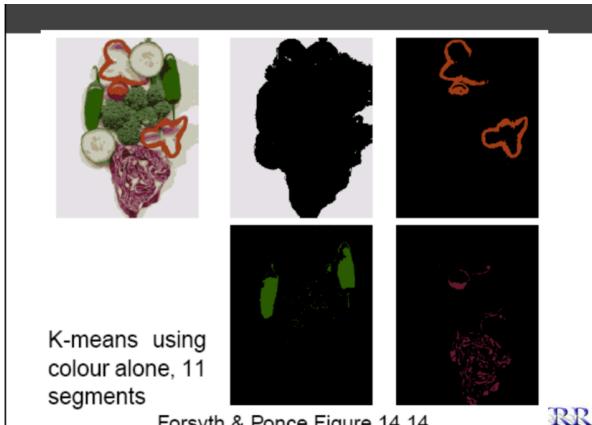
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Input Color Image Clusters on intensity Clusters on color



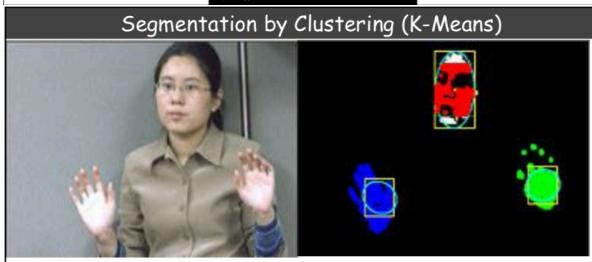
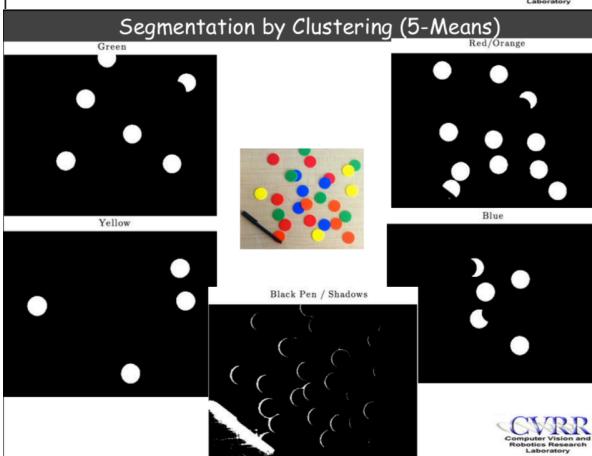
K-means clustering using intensity alone and color alone

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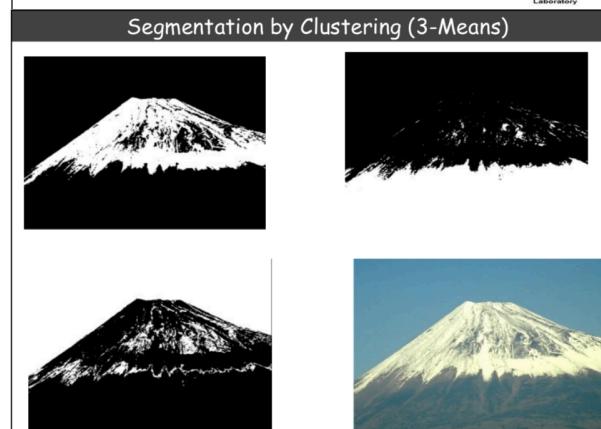


Forsyth & Ponce Figure 14.14

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