Constructing Term Structure -- Nelson-Siegel

Equal Weighted

In[403]:=
$$\alpha = 1;$$

In[404]:=
$$R[t\_, \beta0\_, \beta1\_, \beta2\_] := \beta0 + \beta1 * \frac{1 - e^{-\alpha * t}}{\alpha * t} + \beta2 * \frac{1 - (1 + \alpha * t) * e^{-\alpha * t}}{\alpha * t};$$

In[405]:=
$$B[t\_, \beta0\_, \beta1\_, \beta2\_] := e^{-t * R[t, \beta0, \beta1, \beta2]};$$

In[406]:=
$$Cp[t\_, c\_, \beta0\_, \beta1\_, \beta2\_] := 0.5 * \sum_{i=1}^{2*t} c * e^{\frac{i}{2} R[i/2, \beta0, \beta1, \beta2]} + e^{-t * R[t, \beta0, \beta1, \beta2]};$$

In[407]:=
$$\{b0, b1, b2\} = \{\beta0, \beta1, \beta2\} /. \text{Minimize}\left[(1 - Cp[1, 0.78/100, \beta0, \beta1, \beta2])^2 + (1 - Cp[2, 0.91/100, \beta0, \beta1, \beta2])^2 + (1 - Cp[5, 1.26/100, \beta0, \beta1, \beta2])^2 + (1 - Cp[10, 1.70/100, \beta0, \beta1, \beta2])^2, \{\beta0, \beta1, \beta2\}\right][[2]]$$

Out[407]=
$$\{0.0274762, 0.00492522, -0.075346\}$$

Duration weighted

In[408]:=
$$Dur[t\_, c\_, \beta0\_, \beta1\_, \beta2\_] := \sum_{i=1}^{2*t} \frac{0.5 * c * (i/2) * e^{-(i/2) * R[i/2, \beta0, \beta1, \beta2]}}{Cp[t, c, \beta0, \beta1, \beta2]} + \frac{1 * t * e^{-t * R[t, \beta0, \beta1, \beta2]}}{Cp[t, c, \beta0, \beta1, \beta2]};$$

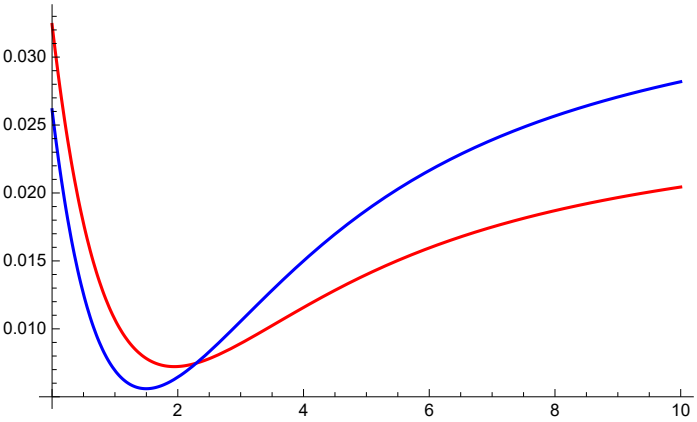
In[409]:=
$$\{B0, B1, B2\} = \{\beta0, \beta1, \beta2\} /. \text{Minimize}\left[(1/Dur[1, 0.54/100, \beta0, \beta1, \beta2]) * (1 - Cp[1, 0.54/100, \beta0, \beta1, \beta2])^2 + (1/Dur[2, 0.85/100, \beta0, \beta1, \beta2]) * (1 - Cp[2, 0.85/100, \beta0, \beta1, \beta2])^2 + (1/Dur[5, 1.59/100, \beta0, \beta1, \beta2]) * (1 - Cp[5, 1.59/100, \beta0, \beta1, \beta2])^2 + (1/Dur[10, 2.22/100, \beta0, \beta1, \beta2]) * (1 - Cp[10, 2.22/100, \beta0, \beta1, \beta2])^2, \{\beta0, \beta1, \beta2\}\right][[2]]$$

Out[409]=
$$\{0.0383766, -0.0122485, -0.0896862\}$$

Plot

In[410]:=
```
Plot[{R[t, b0, b1, b2], R[t, B0, B1, B2]}, {t, 0, 10}, PlotStyle → {Red, Blue}]
```

Out[410]=



Coupon Price

In[411]:=
```
Cp[5, 5 / 100, b0, b1, b2]
```

Out[411]=
```
1.19014
```

In[412]:=
```
y /.
  Solve[5 * (e^(-0.5 y) + e^-y + e^(-1.5 y) + e^(-2 y) + e^(-2.5 y) + e^(-3 y) + e^(-3.5 y) + e^(-4 y) + e^(-4.5 y) + e^(-5 y)) + 100 * e^(-5 y) ==
    118.012, y, Reals];
```

3. (d)

In[413]:=
```
rr = r /. Solve[∑_(i=11)^20 0.5 * 0.05 * e^(-r (0.5*i-5)) + 1 * e^(-r*5) == 1, r, Reals];
```

In[414]:=
```
Array[k, 10];
```

In[415]:=
```
Do[k[i - 10] = e^(-rr (i/2 - 5)), {i, 11, 20, 1}];
```

In[416]:=
```
Array[v, 10];
```

In[417]:=
```
κ = 0.9;
σ = 0.1;
```

In[419]:=
$$\text{Do}\left[v[i-10] = \frac{1}{2*\kappa} * \frac{\sigma}{\kappa} * e^{-\kappa\left(\frac{i}{2}-5\right)} * \left(1 - e^{-2\kappa-5}\right), \{i, 11, 20, 1\}\right]$$

In[420]:=
```
Array[d1, 10];
Array[d2, 10];
```

In[422]:=
$$\text{Do}\left[\text{d1}[\text{i}-10]=\frac{1}{\text{v}[\text{i}-10]}*\text{Log}\left[\frac{\text{B}\left[\text{i}/2,\text{b0},\text{b1},\text{b2}\right]}{\text{k}[\text{i}-10]*\text{B}[5,\text{b0},\text{b1},\text{b2}]}\right]+\frac{\text{v}[\text{i}-10]}{2},\{\text{i},11,20,1\}\right]$$

In[423]:=
$$\text{Do}\left[\text{d2}[\text{i}-10]=\frac{1}{\text{v}[\text{i}-10]}*\text{Log}\left[\frac{\text{B}\left[\text{i}/2,\text{b0},\text{b1},\text{b2}\right]}{\text{k}[\text{i}-10]*\text{B}[5,\text{b0},\text{b1},\text{b2}]}\right]-\frac{\text{v}[\text{i}-10]}{2},\{\text{i},11,20,1\}\right]$$

In[424]:=
```
t = 5;
```

In[425]:=
$$\text{Poption}=\sum_{\text{i}=1}^{2\,\text{t}}\text{Boole}\left[5<\frac{\text{i}}{2}\right]*0.5*0.05*\left(\text{B}\left[\frac{\text{i}}{2},\text{b0},\text{b1},\text{b2}\right]*\text{CDF}[\text{NormalDistribution}[0,1],\text{d1}[\text{i}-10]]-\text{k}[\text{i}-10]*\text{B}[5,\text{b0},\text{b1},\text{b2}]*\text{CDF}[\text{NormalDistribution}[0,1],\text{d2}[\text{i}-10]]\right)+\left(\text{B}[10,\text{b0},\text{b1},\text{b2}]*\text{CDF}[\text{NormalDistribution}[0,1],\text{d1}[10]]-\text{k}[10]*\text{B}[5,\text{b0},\text{b1},\text{b2}]*\text{CDF}[\text{NormalDistribution}[0,1],\text{d2}[10]]\right)$$

Out[425]= {0.0867493}

In[426]:=
$$\text{Pcall}=\left(\text{Cp}\left[5,\ 5\ /\ 100,\ \text{b0},\ \text{b1},\ \text{b2}\right]-\text{Poption}\right)$$

Out[426]= {1.10339}

4. (g)

In[427]:=
```
ω = 0.38;
```

In[428]:=
```
γ = -1.5;
```

In[429]:=
```
t = 0;
```

In[430]:=
$$\text{Vt}=\Phi=\left(\int_{\text{t}}^{5}\left(\omega\ (5-\text{s})\ e^{\gamma\ (5-\text{s})}\right)^2 d\text{s}\right)^{\frac{1}{2}};$$

In[431]:=
$$\text{Ft}=\frac{\text{B}[5,\text{b0},\text{b1},\text{b2}]-\text{B}[10,\text{b0},\text{b1},\text{b2}]}{\sum_{\text{i}=1}^{10}\left(0.5\,\text{B}[5+0.5\,\text{i},\text{b0},\text{b1},\text{b2}]\right)};$$

In[432]:=
```
kk = 5 / 100;
```

In[433]:=
$$\text{dt1}=\frac{1}{\text{Vt}}\text{Log}\left[\frac{\text{Ft}}{\text{kk}}\right]+\frac{1}{2}\text{Vt};$$

In[434]:=
$$\text{dt2}=\frac{1}{\text{Vt}}\text{Log}\left[\frac{\text{Ft}}{\text{kk}}\right]-\frac{1}{2}\text{Vt};$$

In[435]:=
```
Bput[Ft_, kk_, Vt_, dt1_, dt2_] :=
  kk * CDF[NormalDistribution[0, 1], -dt2] - Ft * CDF[NormalDistribution[0, 1], -dt1];
```

In[436]:=
$$\text{A0}=\sum_{\text{i}=1}^{10}0.5\,\text{B}[5+0.5\,\text{i},\text{b0},\text{b1},\text{b2}];$$

In[437]:=
```
Pswaption = A0 * Bput[Ft, kk, Vt, dt1, dt2]
```

Out[437]=
```
0.0995704
```

In[438]:=
```
Cp[5, 5 / 100, b0, b1, b2] - Pswaption
```

Out[438]=
```
1.09056
```

3. (h)

In[439]:=
```
5 000 000 / (Cp[5, 5 / 100, b0, b1, b2] - Pswaption)
```

Out[439]=
```
4.58478 × 10^6
```