Lists

Principles of Functional Programming

Lists

The list is a fundamental data structure in functional programming.

A list having $x_1, \ldots, x_n$ as elements is written `List(`$x_1, \ldots, x_n$`)`

Example

```
val fruit  = List(”apples”, ”oranges”, ”pears”)
val nums   = List(1, 2, 3, 4)
val diag3  = List(List(1, 0, 0), List(0, 1, 0), List(0, 0, 1))
val empty  = List()
```

There are two important differences between lists and arrays.

- Lists are immutable — the elements of a list cannot be changed.
- Lists are recursive, while arrays are flat.

Lists

```
val fruit  = List(”apples”, ”oranges”, ”pears”)
val diag3  = List(List(1, 0, 0), List(0, 1, 0), List(0, 0, 1))
```

The List Type

Like arrays, lists are homogeneous: the elements of a list must all have the same type.

The type of a list with elements of type `T` is written `scala.List[T]` or shorter just `List[T]`

Example

```
val fruit: List[String]    = List(”apples”, ”oranges”, ”pears”)
val nums : List[Int]       = List(1, 2, 3, 4)
val diag3: List[List[Int]] = List(List(1, 0, 0), List(0, 1, 0), List(0, 0, 1))
val empty: List[Nothing]   = List()
```

Constructors of Lists

All lists are constructed from:

- the empty list `Nil`, and
- the construction operation `::` (pronounced *cons*):
  `x :: xs` gives a new list with the first element `x`, followed by the elements of `xs`.

For example:

```
fruit = ”apples” :: (”oranges” :: (”pears” :: Nil))
nums  = 1 :: (2 :: (3 :: (4 :: Nil)))
empty = Nil
```

Right Associativity

Convention: Operators ending in ":" associate to the right.

`A :: B :: C` is interpreted as `A :: (B :: C)`.

We can thus omit the parentheses in the definition above.

Example

```
val nums = 1 :: 2 :: 3 :: 4 :: Nil
```

Operations on Lists

All operations on lists can be expressed in terms of the following three:

`head`	the first element of the list
`tail`	the list composed of all the elements except the first.
`isEmpty`	‘true‘ if the list is empty, ‘false‘ otherwise.

These operations are defined as methods of objects of type `List`. For example:

```
fruit.head       == ”apples”
fruit.tail.head == ”oranges”
diag3.head       == List(1, 0, 0)
empty.head       == throw NoSuchElementException(”head of empty list”)
```

List Patterns

It is also possible to decompose lists with pattern matching.

`Nil`	The `Nil` constant
`p :: ps`	A pattern that matches a list with a `head` matching `p` and a `tail` matching `ps`.
`List(p1, ..., pn)`	same as `p1 :: ... :: pn :: Nil`

Example

`1 :: 2 :: xs`	Lists of that start with `1` and then `2`
`x :: Nil`	Lists of length 1
`List(x)`	Same as `x :: Nil`
`List()`	The empty list, same as `Nil`
`List(2 :: xs)`	A list that contains as only element another list that starts with `2`.

Exercise

Consider the pattern `x :: y :: List(xs, ys) :: zs`.

What is the condition that describes most accurately the length `L` of the lists it matches?

○ `L == 3`

○ `L == 4`

○ `L == 5`

○ `L >= 3`

○ `L >= 4`

○ `L >= 5`

Exercise

Consider the pattern `x :: y :: List(xs, ys) :: zs`.

What is the condition that describes most accurately the length `L` of the lists it matches?

O `L == 3`

O `L == 4`

O `L == 5`

X `L >= 3`

O `L >= 4`

O `L >= 5`

Sorting Lists

Suppose we want to sort a list of numbers in ascending order:

- One way to sort the list `List(7, 3, 9, 2)` is to sort the tail `List(3, 9, 2)` to obtain `List(2, 3, 9)`.
- The next step is to insert the head `7` in the right place to obtain the result `List(2, 3, 7, 9)`.

This idea describes *Insertion Sort* :

```
def isort(xs: List[Int]): List[Int] = xs match
  case List()  => List()
  case y :: ys => insert(y, isort(ys))
```

Exercise

Complete the definition insertion sort by filling in the ???s in the definition below:

```
def insert(x: Int, xs: List[Int]): List[Int] = xs match
  case List() => ???
  case y :: ys => ???
```

What is the worst-case complexity of insertion sort relative to the length of the input list N?

```
O      the sort takes constant time
O      proportional to N
O      proportional to N log(N)
O      proportional to N * N
```

Exercise

Complete the definition insertion sort by filling in the ???s in the definition below:

```
def insert(x: Int, xs: List[Int]): List[Int] = xs match
  case List() => List(x)
  case y :: ys =>
    if x < y then x :: xs else y :: insert(x, ys)
```

What is the worst-case complexity of insertion sort relative to the length of the input list N?

```
O       the sort takes constant time
X       proportional to N
O       proportional to N * log(N)
O       proportional to N * N
```