The Superposition Principle

You can build dynamic models with different components, for example a trend component plus a regression component, by using the principle of superposition. The idea is to think about the general form of the forecast function you want to have for prediction. You then write that forecast function as a sum of different components where each component corresponds to a class of DLM with its own state-space representation. The final DLM can then be written by combining the pieces of the different components.

For example, suppose you are interested in a model with a forecast function that includes a linear polynomial trend and a single covariate x_t, i.e.,

$$f_t(h) = k_{t,0} + k_{t,1}h + k_{t,3}x_{t+h}.$$

This forecast function can be written as $f_t(h) = f_{1,t}(h) + f_{2,t}(h)$, with

$$f_{1,t}(h) = (k_{t,0} + k_{t,1}h), \quad f_{2,t}(h) = k_{t,3}x_{t+h}.$$

The first component in the forecast function corresponds to a model with a 2-dimensional state vector, $\boldsymbol{F}_{1,t} = \boldsymbol{F}_1 = (1, 0)'$,

$$\boldsymbol{G}_{1,t} = \boldsymbol{G}_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

The second component corresponds to a model with a 1-dimensional state vector, $F_{2,t} = x_t$, $G_{2,t} = G_2 = 1$.

The model with forecast function $f_t(h)$ above is a model with a 3-dimensional state vector with $\boldsymbol{F}_t = (\boldsymbol{F}_1', F_{2,t})' = (1, 0, x_t)'$ and

$$\boldsymbol{G}_t = \text{blockdiag}[\boldsymbol{G}_1, G_2] = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

General Case. Assume that you have a time series process y_t with a forecast function $f_t(h) = \sum_{i=1}^{m} f_{i,t}(h)$, where each $f_{i,t}(h)$ is the forecast function of a DLM with representation

$$\{\boldsymbol{F}_{i,t}, \boldsymbol{G}_{i,t}, v_{i,t}, \boldsymbol{W}_{i,t}\}.$$

Then, $f_t(h)$ has a DLM representation $\{\boldsymbol{F}_t, \boldsymbol{G}_t, v_t, \boldsymbol{W}_t\}$ with $\boldsymbol{F}_t = (\boldsymbol{F}'_{1,t}, \boldsymbol{F}'_{2,t}, \ldots, \boldsymbol{F}'_{m,t})'$, $\boldsymbol{G}_t = \text{blockdiag}[\boldsymbol{G}_{1,t}, \ldots, \boldsymbol{G}_{m,t}]$, $v_t = \sum_{i=1}^{m} v_{i,t}$ and $\boldsymbol{W}_t = \text{blockdiag}[\boldsymbol{W}_{1,t}, \ldots, \boldsymbol{W}_{m,t}]$.