Voting Paradoxes in the Real World

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Project Overview

The mathematical theory of voting is filled with instances of paradoxes and counterintuitive outcomes (specifically in elections containing 3 or more candidates). In this project, we examine the occurrence of such paradoxes in the real world, focusing specifically on "elections" in sports contexts such as the AP Top 25 Polls for college football and MVP (Most Valuable Player) voting in baseball.

Description of Data

- **AP Poll Data:** 11 seasons (2014-Present), 15-18 weeks per season, approx. **200 total polls ("elections")**, scraped from collegepolltracker.com.
- **MLB MVP Voting Data:** 12 seasons (2012-2023), 2 leagues per season (AL/NL), **24 total votes ("elections")**, scraped from bbwaa.com.

How the AP Top 25 College Football Poll Works

- Approximately 60 sports journalists are selected to vote on the Top 25 Poll by the Associated Press (AP).
- Every voter submits a ranked list of 25 teams.
- Each team is assigned points according to where they were ranked by pollsters: 25 points for first place, 24 points for second place, 1 point for 25th place, 0 points for Not Ranked.
- The points earned by each team are aggregated across voters. The official AP top 25 ranking is determined by the total number of points earned by each team.

How the MLB Baseball MVP Voting Works

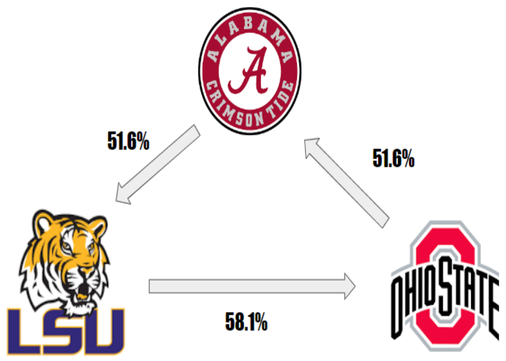
- MVP (Most Valuable Player) voting is conducted by the Baseball Writers' Association of America (BBWAA), which selects approximately 30 voters for both the National League (NL) and American League (AL).
- Every voter submits a ballot ranking their top 10 players.
- Each player is assigned points according to where they were ranked by pollsters: 14 points for first place, 9 points for second place, and 1 point for tenth place.
- The player with the most points is the MVP.

Notable Voting Paradoxes

- **Non-Transitive Cycle Paradox:** There exists a cycle of candidates A, B, and C such that a majority of voters rank A over B, B over C, and C over A.
- **Independence of Irrelevant Alternatives Paradox (IIA)**: The addition or removal of a candidate (team, player) changes the relative ranking of the remaining candidates.
- **Borda Condorcet Paradox:** The Borda count winner is different from the Condorcet winner and the Plurality winner.

Real World Examples of Voting Paradoxes I

Non-transitive Cycle Paradox: 2019 Week 10 College Football Poll



Example of non-transitive cycle: A majority of pollsters ranked Alabama over LSU (Louisiana State), LSU over Ohio State, and Ohio State over Alabama.

Independence of Irrelevant Alternatives Paradox: 2017 MLB National League MVP Voting

RANK	PLAYER
1	Stanton
2	Votto
3	Goldschmidt
4	Arenado
5	~~Blackmon~~

RANK	PLAYER
1	Votto
2	Stanton
3	Goldschmidt
4	Arenado

Suppose the 5th ranked player (Blackmon, "irrelevant alternative") is removed from all ballots. When recalculating the Borda Counts, the winner changes from Stanton to Votto.

Real World Examples of Voting Paradoxes II

Borda/Condorcet Paradoxes: 2014 Week 15 College Football Poll

1	Alabama (25)	1426 Points
2	Florida State (29)	1423 Points
3	Oregon (5)	1391 Points

- Alabama is the **Borda Winner** since it received the most Borda points using the official AP poll ranking method.
- Florida State is the **Plurality Winner** as it received the most first place votes (29 vs 25 vs 5).
- Florida State is also the **Condorcet Winner**, since it is the pairwise winner against all other teams.

Some Common Voting Methods: Borda Count, Plurality, Condorcet

Assume there are n candidates. Each voter ranks the candidates in the order of preference.

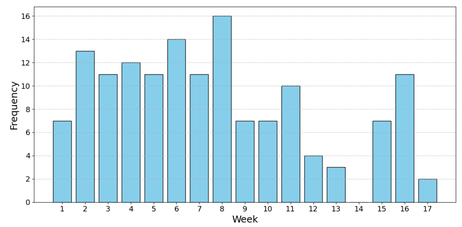
- **Borda Count Method**
  - Points are assigned based on the rank: Standard Borda Count: n points for 1st place, $n-1$ for 2nd place, etc, and 1 point for the last place.
  - The points for each candidate are added up and the candidate with the highest total points wins.
- **Plurality Method**
  - The candidate(s) with the most first place votes wins.
- **Condorcet Method**
  - For each pair of candidates A and B, the number of voters that rank A over B is compared to the number of voters that rank B over A.
  - A Condorcet winner is a candidate who can beat every other candidate in a head-to-head comparison.

Frequencies of Non-transitive Cycles

Non-transitive Cycles

- **College Football Poll Data (167 "elections")**
  - **Numbers of cycles found:** 137 cycles of length 3, 8 cycles of length 4, 1 cycle of length 5
  - **5-cycle example:** Coastal Carolina > Ole Miss > Utah > Virginia Tech > Wisconsin > Coastal Carolina
  - **Most extreme example:** 2019 Week 10, LSU (#1) > Ohio State (#3) > Alabama (#2) > LSU (#1)
- **Baseball MVP Voting Data (24 "elections")**
  - **Numbers of cycles found:** 9 cycles of length 3, 3 cycles of length 4, 1 cycle of length 5
  - **Most extreme example:** 2013 NL, Freeman (#5) > Kershaw (#7) > Votto (#6) > Freeman (#5)

Non-transitive Cycles Per Week (AP Poll)



Frequencies of IIA Paradoxes

- **College Football Poll Data**
  - **Number of Cases:** 32 cases if 1 team is removed, 146 cases if 2 teams are removed
  - **Most extreme example:** 2022 week 10, removing Michigan (#4) causes Ohio State (#2) and Tennessee (#3) to be reversed.
- **Baseball MVP Voting Data**
  - **Number of Cases:** 1 case if 1 player is removed, 9 cases if 2 players are removed
  - **Most extreme example:** 2017 NL, removing Blackmon (#5) causes Stanton (#1) and Votto (#2) to be reversed.

References

- Benoit, J. P. (1992). Scoring reversals: a major league dilemma. Social Choice and Welfare, 9(2), 89-97.
- Pacuit, E. (2011). Voting methods (Stanford Encyclopedia of Philosophy). http://plato.stanford.edu/entries/voting-methods/