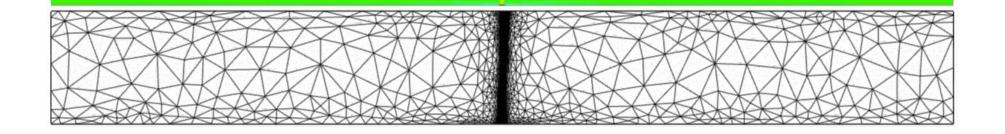


Modelling and

Numerical Methods





Saskia Goes



Stephen Neethling



Matthew Piggott

Aims

• Part 1: introduce mathematical essentials and physical equations for modelling a range of dynamic processes

• Part 2: provides complementary background on numerical methods that can be used to solve the equations of these physical systems

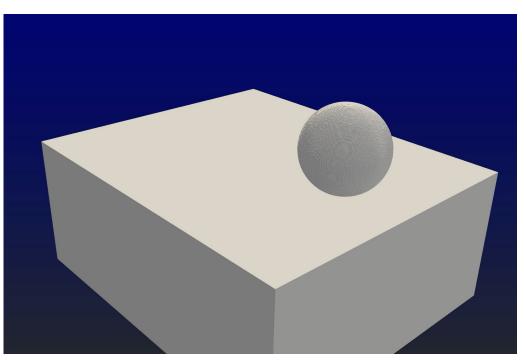
Example: compressible flow of deformable solids

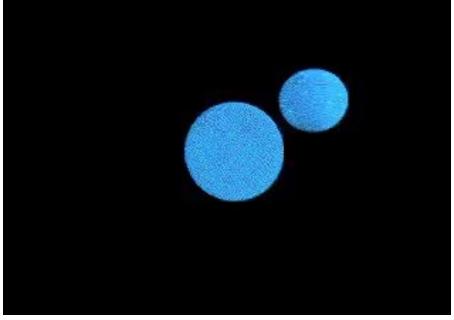
$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}$$

$$\rho \frac{D\mathbf{u}}{Dt} = \mathbf{f} - \nabla p + \nabla \cdot \bar{s}$$

$$\rho \frac{DE}{Dt} = -p \nabla \cdot \mathbf{u} + \operatorname{tr}(\bar{s} \cdot \nabla \mathbf{u})$$

Example: compressible flow of deformable solids





Halim, S.H., Crawford, I.A., Collins, G.S., Joy, K.H., Davison, T.M., 2020. Assessing the survivability of biomarkers within terrestrial material impacting the lunar surface. Icarus 114026. https://doi.org/10.1016/j.icarus.2020.114026

Canup, R.M., 2004. Simulations of a late lunar-forming impact. Icarus 168, 433–456. https://doi.org/10.1016/j.icarus.2003.09.028

Structure of course

- ➤ Week 1 Mathematical background: vectors, tensors, conservation principles in continuum mechanics and some analytical solutions
- ➤ Week 2-3 Numerical methods: Examples and numerical methods to solve fluid mechanical problems including common types of ODEs, PDEs.
- ➤ Assessment In week 3: CW1 (67% mark)- numerical assessment, CW2 (33% mark) analytical assessment

Some of you may already be familiar with the basic background that will be covered in part of the lectures but may have been taught to you in different way, review will be useful, and everyone will have same background for other course(s).

Outline of course

- ➤ Part 1: Analytical background
 - **1.** Intro vector/tensor calculus (SG)
 - **2.** Stress tensor (SG)
 - **3.** Kinematics and strain (*SG*)
 - **4.** Conservation equations (SG)
 - **5.** Dimensional Analysis (*SN*)
- ➤ Part 2: Numerical techniques (advanced)
 - **6.** Interpolation and quadrature (MP)
 - 7. Ordinary differential equations (MP)
 - **8.** Partial differential equations and finite difference (*MP*)

- ➤ Part 3: Numerical solutions
 - **9.** Potential flow (*SN*)
 - **10.** Navier-Stokes (SN)
 - **11.** Nonlinear rheology and turbulence (*SN*)
 - **12.** Finite Element Method (*MP*)

Schedule & Assessment

Lectures

- Morning (09:00—12:00)
- In person and streamed/recorded
- Using ppt and/or Jupyter notebooks

Workshops

- Afternoon (14:00—17:00)
- In person with GTA assistance

• 2 assessments –

- Coursework 1 (numerical): worth 67%, released Wed. 24 January 2024 @ 9:00 due Thu 25 January 2024 @ 17:00 (i.e. 2-day coursework submit as Jupyter notebook)
- Coursework 2 (analytical): worth 33%, released Fri. 26 January 2024 @ 9:30 due Friday 26 January 2023 @ 11:00 (i.e. 1.5-hour coursework, submit combined pdf of plots + scanned/photographed handwritten answers)

Course Schedule

Monday	Tuesday	Wednesday	Thursday	Friday
08-Jan	09-Jan	10-Jan	11-Jan	12-Jan
lecture workshop	lecture workshop	lecture	lecture workshop	lecture workshop
15-Jan	16-Jan	17-Jan	18-Jan	19-Jan
lecture workshop	lecture workshop	lecture	lecture workshop	lecture workshop
22-Jan	23-Jan	24-Jan	25-Jan	26-Jan
lecture workshop	lecture workshop	cw1 released 9:00	cw1 due 17:00	cw2 9:30- 11:00

lectures will be live and recorded. workshops will be GTA assisted. there are two pieces of coursework, cw1: numerical/notebook, cw2: analytical, handwritten