William Yang
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Final Project Writeup

The spectrum auction is a complex mechanism that invites many different strategies. In this writeup, we will start by explaining the theoretical model that we constructed, including assumptions and simplifications, in order to analyze this auction. Then, we will derive the theoretical optimal response in this auction, creating a profit-maximizing agent. After that, we will demonstrate how we actually designed our agent using our theoretical results, and finally we will discuss what practical changes we had to make to our agent in order to optimize its real-life performance.

In order to make a model that we could construct an optimal response to, we had to start by making certain assumptions about agent behavior in the auction. First, we reasoned that agents will only submit a bid if a good's minimum bid is less than the agent's valuation for that good. Second, we assumed that bidders would be more likely to bid on goods that they had higher valuations for, since bidding on these goods made them more likely to make higher profits. Using these assumptions, we simplified the game in the following way: we assumed that the national bidder will bid on all goods whose minimum bids are less than its valuation for the good, and the regional bidders will bid on the four goods that they have the highest valuation for, until the minimum bid exceeds that valuation. The auction itself remained the same in this model, and there were still some unknowns, such as what prices the other bidders would use, but these assumptions simplify the game enough so that we could start thinking about how to best respond in this auction.

We sought to create an optimal response to this simplified version of the game. In order to do so, we needed to characterize two parts of our strategy: the amount that our agent would bid, and the selection of which goods our agents would bid on. Since a bid bundle consists of these 2 items, once we were able to determine the optimal response for both of these, we would be ready to construct our best-response agent.

First, we looked at what price would be optimal to have as an upper bound on our bids. Since this is a second-price auction, an agent would never want to have an upper bound on a bid equal to a value less than its valuation. It is optimal to set your upper bound on bids to a value at least as large as your valuation. This is because if you win the auction with a bid at or below your valuation, then the price of the good will surely be less than your valuation, since the price is the bid of the second-price bidder, and thus you earn a positive profit. However, we had to think about whether we wanted our agent to put an upper bound on its bids exactly at its valuation for the good, or above its valuation for the good. Without the global complement rule, we would have likely decided to bid up to exactly the valuation for a good, since winning a good for a price above one's evaluation would have no advantage. However, taking into account global complements, there is an advantage to winning a good for a very small loss. If you win multiple goods, then winning an additional good would cause your valuation for that good would increase by at least 20 percent, and it would cause your valuation for your other goods to increase by 20 percent. So, you could end up earning a significant amount more profit by slightly overbidding. It is also not unreasonable to assume that you may win multiple goods, since there are 18 goods in total and only 7 agents, which means the average agent will win 2.57 goods. Finally, since this is a competition, your goal is not only for you to earn a high profit, but for your opponents to earn a small profit. Thus, overbidding and winning a good that an opponent would otherwise win will not only cause them not to earn the profits they would have earned from that good, but it also prevents them from earning the additional 20 percent valuation on all of the items they won. This is why we decided that bidding slightly above our own valuation was the optimal strategy in this situation.

Second, we wanted to decide which items our agent would bid on. Since the national bidder has no restriction on the number of goods they can bid on, it is optimal to bid on all of them that have a minimum bid that is less than their valuation, plus goods with a minimum bid that is slightly higher than their valuation, as discussed in the previous paragraph. The regional bidders have a much more interesting situation. In order to maximize profit, you want to have high valuations for the goods that you bid on, but you also want these goods to have low prices. This means that the optimal bid bundle is not always the one for which your agent has the highest valuations. You also must account for the competition for goods; if one good is likely to have no other bidders, then it is extremely advantageous to bid on those goods, since you may win them at a very low price, earning a very high profit. This auction not only has some regional goods and some national goods, but also has some low-valuation goods and some high-valuation goods, with lots of asymmetry between which bidders can bid on which goods and against which other bidders, leading to certain more-demanded goods and certain less-demanded goods.

In order to determine which goods were more demanded than others, and how to respond to that, we had to consider each Agent 1 through 6 individually. First, we considered bidder 3, who was only eligible to bid on the high-value national goods (the ones with valuations in [0, 40] for them and [0, 20] for the national bidder) and their regional goods. For the regional goods, bidder 3 will be bidding against one opponent with a valuation in [0, 20], and for the national goods, bidder 3 will be bidding against one opponent with a valuation in [0, 20] and one opponent with a valuation in [0, 40]. The national goods are more likely to have high bids on them, since there are more bidders who are eligible to bid on them, and since their valuations come from higher distributions on average. This means that if bidder 3 had similar valuations for a regional and a national good, they would want to bid on the regional goods, since it is less likely to have competition. So, when our agent is bidder 3, instead of comparing the valuations of their goods and choosing to bid on those with the highest valuations, it is optimal to instead compare “weighted valuations”, which are equal to the valuation of the goods multiplied by some weight, depending on which good they are bidding on. For bidder 3, the weights of the regional goods would be larger than those of the national goods. For each one of the bidders one through six, we performed this analysis to determine which goods we should weigh higher, and which ones to weigh lower. These weights, along with our bid upper bounds, became our agent’s ideal response to the simplified auction. Together with deciding to slightly overbid, this created our best response, which is to slightly overbid, and to choose which goods to bid on using weighted valuations.

Now that we have analyzed our theoretical model, we will explain how we implemented our agent. We designed our agent to largely follow the theoretical principles and conclusions, including bidding above the valuation and weighting valuations to determine the optimal bundle. However, there are a few key decisions and changes in our implementation. These choices include determining the overbidding amount and the weights, bidding our maximum immediately, and maximizing value in later rounds. We will discuss each of these key decisions.

In the theoretical model, we concluded that it is best to bid above our valuations, but we did not decide how much to overbid. For our agent, we concluded that it is best to bid above our valuation by $\varepsilon/2$, and we will briefly explain why we decided to do so. Once again, we determined that overbidding is the optimal solution due to the rules of the multiple auction — the second-price auction, global complements rule, and the large value of epsilon — which also incentivize not only honest bidding, but also bidding above our valuation to win multiple goods. The choice of bidding ‘valuation + $\varepsilon/2$’ was determined using experimentation with the test agents, and also the intuition that we are looking to bid above our valuation, but not too much higher.

Our theoretical solution also indicates that we should determine our optimal bundle by comparing our weighted valuations, but we did not decide what values these weights should be. As the regional bidder, our agent must decide to construct a bundle by bidding on four of the six goods. This table below indicates how our agent decides which goods to bid on as the regional bidder:

Bidder	**National Goods**	**Regional Goods**
Bidder 1	A, B, 1.1C, 1.1D	1.2M, 1.3N

Bidder 2	1.2C, 1.2D, E, F	1.3N, 1.5O
Bidder 3	E, F, G, H	1.5O, 1.5P
Bidder 4	G, H, 1.2I, 1.2J	1.5P, 1.3Q
Bidder 5	1.1I, 1.1J, K, L	1.3Q, 1.2R
Bidder 6	K, L, A, B	1.2R, 1.2M

In the table, each letter represents our agent's valuation for that good. For example, if we were bidder 2 and our valuations were 10, 17, 28, 15, 13, 11, for goods C, D, E, F, N, O, respectively, then we would calculate the four largest valuations after applying the respective weights. In this case, we are comparing {1.2C, 1.2D, E, F, 1.3N, 1.5O} which gives {1.2*10, 1.2*17, 28, 15, 1.3*13, 1.5*11} = {12, 20.4, 28, 15, 16.9, 16.5}, so our agent would bid on goods D, E, N, and O. Once again, note that the strategies for bidder 2 and 4 are identical, and the strategies for bidder 1 and 5 are the same as well, since these strategies depend on whether the agent and its opponents are bidding on high value national goods. These specific weights are constructed using both experimentation and the intuition discussed in the theoretical model that we seek to bid on the goods that will be less demanded.

Another decision that must be made is whether our agent should bid low initially to gauge opponents' strategies, or bid high and secure a tentative allocation. We saw merits to initially bidding low, especially the fact that doing so does not strongly reveal our agent's preferences and valuations, and also that we can observe and adjust to opponents' strategies. However, we decided to bid high at our optimal maximum bid right away, which we have concluded to be valuation + $\varepsilon/2$. The factors that led to this decision to bid our maximum immediately are similar to those that incentivized us to overbid: the second-price auction and the global complements rule. Moreover, there is also another auction rule not previously mentioned that affected our decision-making process: the value of ε. In this auction, the value of ε is 2.5, which is an incredibly big value given that most valuations are U[0, 20]. This large value of epsilon ($\varepsilon = 2.5$) punishes agents who bid low. For instance, consider two agents who both have a valuation of 10 for a good. If the first agent bids high and submits a bid greater than 7.5, and the second agent bids low (<7.5), then in future rounds there is no incentive for the second agent to win the good, since the agent needs to submit a minimum bid of $7.5 + \varepsilon = 10$, in which case the second agent does not receive any utility. The first agent then wins the good with a comfortable margin, since the auction is second-price. Thus, in this simple example, we see that bidding high initially is better than bidding low. Therefore, due to the combination of all of these factors — the second-price auction, global complement rule, and high value of epsilon — we decided that our agent should bid above its valuation from the very first round. This ambitious first bid both increases the chances that our agent wins multiple goods and disincentives other agents from bidding against us.

However, this bidding strategy alone is not sufficient, since our agent needs to be able to learn, adapt, and respond to other bidders and the dynamics of the auction. Currently, our agent does not consider the minimum bids of each good, but only its valuations, when creating the bundles. This is a flaw because our agent needs to be able to assess which goods are more or less demanded by considering the minimum bids of the goods. And so, we adapted our strategy so that it maximizes utility after the first round; that is, if our agent discovers that switching to another bundle results in more utility, then the agent will switch to another bundle in this case. Our agent does this by calculating the difference between its valuation and the minimum bid for each good, and then bids on the four goods that give our agent the most utility as the regional bidder. This is a quick way for our agent to consider all $\binom{6}{4} = 15$ possible bundles and choose which one maximizes profit out of all of the ones that they are allowed to take. Similar to the first round, it is best to bid high initially, instead of bidding low and then incrementing our bids in future rounds. Thus, our agent also bids above its valuation by $\varepsilon/2$ in this case as well.

In conclusion, we have developed a simplified model of the complex Spectrum Auction and solved for optimal behavior for an agent in that game. More specifically, we solved using realistic assumptions about opponents' behavior so that the auction still largely resembles the original. With these simplifications, we determined that the optimal strategy is that our agent decides on the bundle by comparing the weighted valuations, and then bids above its valuation by $\varepsilon/2$. This analysis guided our agent's design in the first round of the auction; however, the later rounds of the auction are largely dependent on the actions of the other agents. Thus, after the first round, our agent maximizes utility by comparing the profits from the 15 possible bundles of goods. Therefore, the design and strategy of our agent effectively combines the theoretical and practical analysis discussed.